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Downward Continuation of the Free-Air Gravity Anomalies to the Ellipsoid Using the Gradient Solution and Terrain Correction - An Attempt of Global Numerical Computations

Y. M. Wang
Department of Geodetic Science and Surveying
The Ohio State University

Abstract

The formulas for the determination of the coefficients of the spherical harmonic expansion of the disturbing potential of the earth are defined for data given on a sphere. In order to determine the spherical harmonic coefficients, the gravity anomalies have to be analytically downward continued from the earth's surface to a sphere - at least to the ellipsoid. The goal of this paper is to continue the gravity anomalies from the earth's surface downward to the ellipsoid using recent elevation models. The basic method for the downward continuation is the gradient solution (the g_1 term). The terrain correction has also been computed because of the role it can play as a correction term when calculating harmonic coefficients from surface gravity data.

The fast Fourier transformation has been applied to the computations.

1. Introduction

The formulas for the determination of the coefficients of the spherical harmonics of the earth's gravitational potential require the free-air anomalies to be given on a sphere, at least at a simple surface, e.g., ellipsoid or sea level. Thus we have to continue the free-air gravity anomalies downward to a sphere or an ellipsoid.

The validity of the analytically downward continuation of the free-air gravity anomalies inside the earth is guaranteed by Runge's theorem (Moritz, 1980, p. 67). Of course these gravity anomalies are not the original gravity anomalies inside the earth. The downward continuation gives a fictitious gravity anomaly on the ellipsoid that generates a disturbing potential T on and outside the earth.

Moritz (1980) suggested that the free-air anomalies be continued to the point level. We can take also the ellipsoid as the reference surface and use this method to continue the gravity anomalies down to the ellipsoid.

Pellinen (1966) studied the methods for the determination of the coefficients of the spherical harmonics of the earth's gravitational potential and he added a term, which can be easily transformed into terrain correction, to the free-air anomalies.

In this paper we carried out some numerical investigation with the above mentioned methods. The terrain correction and the gradient solution were computed on a global basis.

2. Mathematical formulation

We continue the free-air anomalies downward to the ellipsoid by using the "gradient solution (Moritz, 1966, p. 68):

$$\Delta g^{*} = \Delta g + g_1 = \Delta g - h\frac{\partial \Delta g}{\partial h} \tag{1}$$

where Δg^*, Δg are the gravity anomalies on the ellipsoid and on the earth's surface, respectively.

The gradient of the gravity anomaly Δg is given by

$$\frac{\partial \Delta g}{\partial h} = \frac{R^2}{2\pi}\iint_\sigma \frac{\Delta g - \Delta g_P}{\ell_0^3}\, d\sigma \tag{2}$$

where $\ell_0 = 2R \sin \psi/2$, ψ is the angular distance between the current point and the computation point p, σ is the unit sphere, and R is the mean radius of the earth.

Pellinen (1966, p. 70) suggested that

$$\Delta g^* = \Delta g + G' \tag{3}$$

where

$$G' = \frac{R^2}{4\pi}\iint_\sigma \frac{(h-h_p)(\Delta g - \Delta g_p)}{\ell_0^3}\, d\sigma \tag{4}$$

The relationship between the gravity anomaly Δg and the elevation h is assumed as

$$\Delta g = a + bh, \qquad b = 2\pi k\rho \approx 0.11 \text{ mgal/meter} \tag{5}$$

a is a constant.

Taking the plane approximation and using the assumption (5) we get

$$g_1 = -h_p k\rho \iint_\tau \frac{h-h_p}{\ell_0^3}\, dxdy \tag{6}$$

$$G' = C = \frac{1}{2} k\rho \iint_\tau \frac{(h-h_p)^2}{\ell_0^3}\, dxdy \tag{7}$$

with $\ell_0 = [(x-x_p)^2 + (y-y_p)^2]^{1/2}$, where τ is the two-dimensional plane.

3. Computations of the Terrain Correction and the Gradient Solution on a global Basis

The elevation data in 5′ x 5′ mean block values are available from the National Geophysical Data Center, Boulder, Colorado as ETOPO5. This elevation data was used for the computation of the g_1 term and the terrain correction on a global basis.

The integration region is taken as 15° in latitude extent and 30° in longitude. The boundary (or overlap) of the integration is taken 50 km. This satisfies the accuracy for most situations (Noë, 1980). But the boundary is not large enough for the Himalaya Mountains. In the computations we took a 250 km boundary for the high mountain areas and a 50 km boundary for the flat areas.

The g_1 term and the terrain correction are computed in 5′ mean block values by using the FFT (fast Fourier transformation) technique. All computations are completed at the Instruction and Research Computer Center at Ohio State University. The CPU time required was about 10^{-3} second for each point on the IBM 3081 at OSU.

The statistics of the g_1 term and the terrain correction are exhibited in Tables 1 and 2.

Table 1. Statistics of the g_1 Term in 5′ and 30′ Mean Block Values
Unit: mgal

Block Size	Mean Value	Standard Dev.	Max. Value	Min. Value
5′	0.27	±2.56	442.14	-78.88
30′	0.27	±1.54	45.08	-10.47
1°	0.27	±1.24	25.52	- 5.16

Table 2. Statistics of the Terrain Correction in 5′ and 30′ Mean Block Values
Unit: mgal

Block Size	Mean Value	Standard Dev.	Max. Value	Min. Value
5′	0.23	±1.01	183.57	0.0
30′	0.23	±0.82	25.24	0.0
1°	0.23	±0.74	17.77	0.0

It shows that the mean values of the g_1 term and of the terrain correction are almost the same. Of course, the g_1 term is larger and rougher (larger standard deviation, larger maximum values).

After the computations of the g_1 term and terrain correction in 5′ mean block values on a global basis, g_1 and the terrain correction were expanded in the spherical harmonics up to 180^{th} order. The RMS (root mean square) values of the degree variances of the g_1 term and the terrain correction are about 2 percent of the RMS values of the degree variance of the OSU86E gravity model.

The total contributions of the g_1 term and the terrain correction on the geoid and the deflections of the vertical are shown in Table 4.

Table 4. RMS values of the corrections of the geoid undulation and the deflections of the vertical due to the g_1 and TC

correction	RMS of δN	RMS of $\delta\theta$
g_1	0.7071 m	0.1129 secs
TC	0.7065 m	0.0880 secs

4. Conclusion

The g_1 term and the terrain correction were computed in 5′ mean block values on a global basis. The maximum of the g_1 term is 442 mgal located in the Himalaya Mountains. The maximum of the terrain correction is 184 mgal. The influence of the g_1 term and the terrain correction on the geoid undulation has been considered. It takes the order of 1 meter (RMS of correction of the geoid undulation). For the deflections of the vertical the RMS of the corrections of the g_1 term and the terrain correction is on the order 0.1".

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