Improving Drag Correlations for Modeling of Real Particle Fluidization

James M. Parker

CPFD Software, LLC

Outline

- Drag fundamentals
- Ideal drag
- Agglomerative drag

Dimensionless Particle Drag

$$F_{drag} = F_{Stokes} \underbrace{F\left(\varepsilon, \mathrm{Re}, \phi\right)}_{\text{Dimensionless}} \qquad F_{Stokes} = 3\pi\mu d_p U \qquad \mathrm{Re} = \rho_f d_p U / \mu_f$$

Close pack or minimum fluidization

Ergun correlation with sphericity:

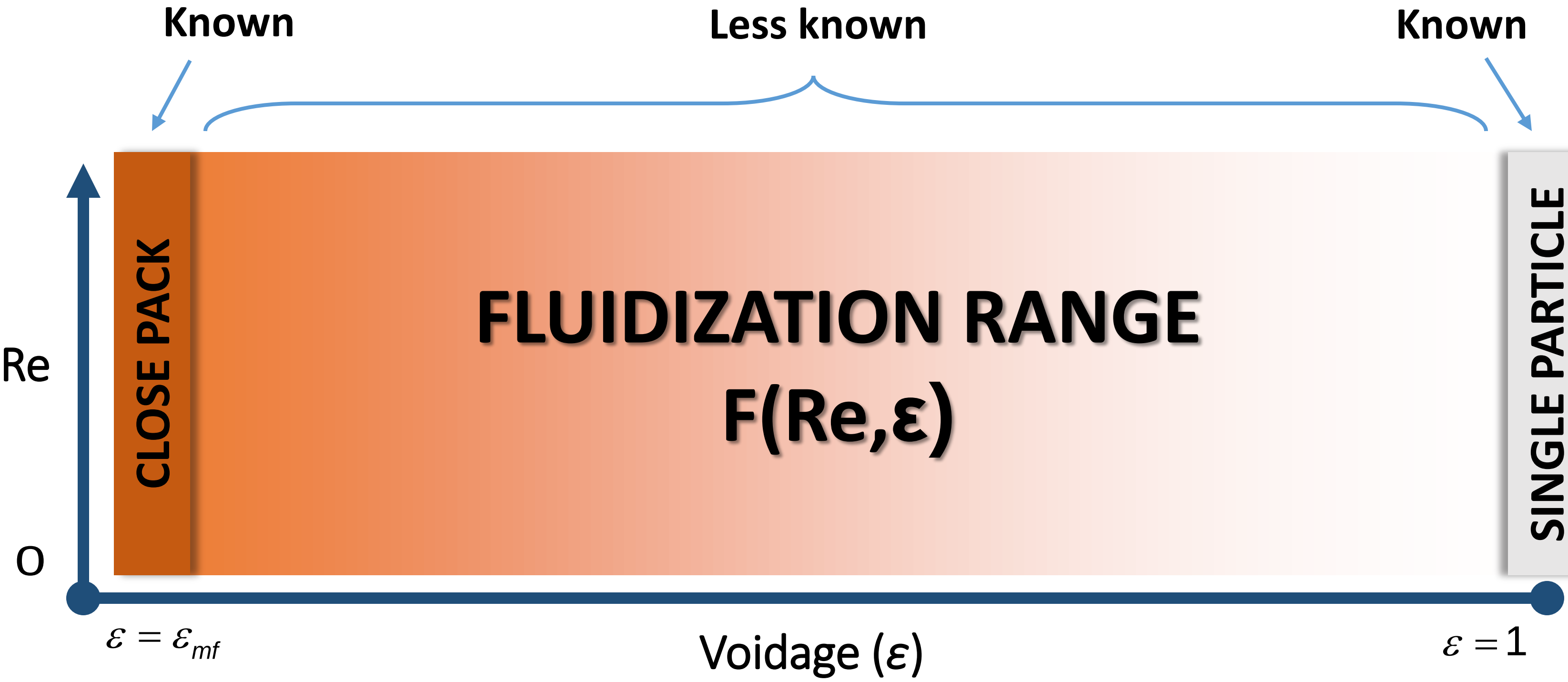
$$F_p(\varepsilon_{mf}, \mathrm{Re}, \phi) = F_{mf}(\varepsilon_{mf}, \mathrm{Re}, \phi) = \frac{a}{18} \frac{1-\varepsilon_{mf}}{\phi^2 \varepsilon_{mf}{}^2} + \frac{b}{18} \frac{\mathrm{Re}}{\phi \varepsilon_{mf}{}^2}$$

Single particle

Schiller-Nauman correlation:

$$F_p(1, \mathrm{Re}) = F_{sp}(\mathrm{Re}) = \begin{cases} 1 + 0.15\,\mathrm{Re}^{0.687} & \mathrm{Re} < 1000 \\ \dfrac{0.44}{24}\mathrm{Re} & \mathrm{Re} \geq 1000 \end{cases}$$

Particle drag map



Known

Less known

Known

Re

O

CLOSE PACK

FLUIDIZATION RANGE

F(Re,ε)

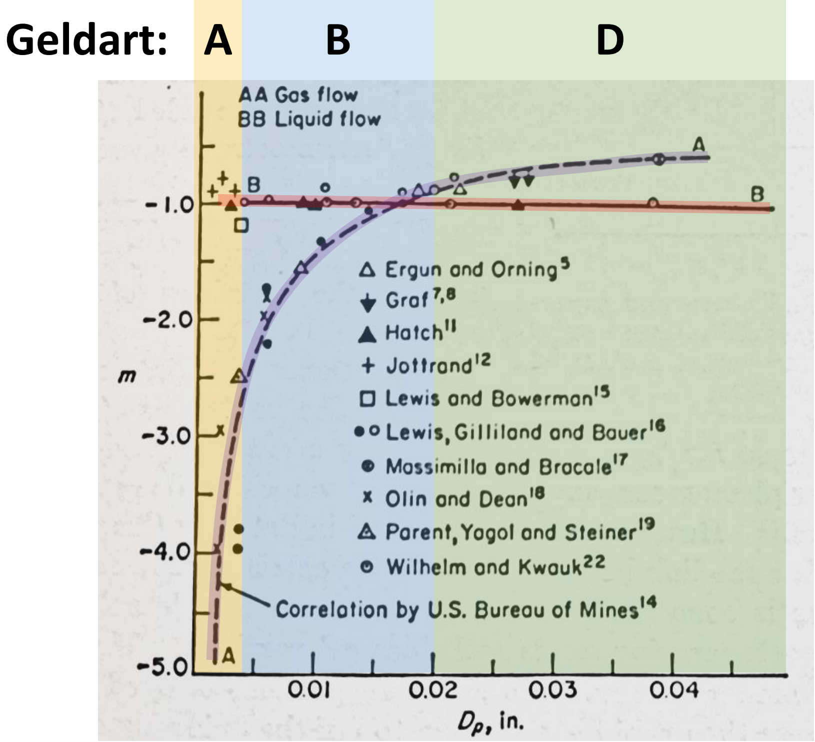
SINGLE PARTICLE

$\varepsilon = \varepsilon_{mf}$

Voidage (ε)

$\varepsilon = 1$

Fluidization close to minimum fluidization



Fluidization by M. Leva (1959) examined fluidization right above minimum

$$\Delta P = \frac{200 G \mu L_e (1-\varepsilon)^2}{d_p{}^2 \phi^2 \rho_f \varepsilon^3} \qquad \ln G = m \ln\left(\frac{1-\varepsilon}{\varepsilon^3}\right) + \text{Const}$$

- *m* = -1 for packed beds
- Above fluidization:
  - m ≈ -1 for liquid-solid systems
  - m ≠ -1 for gas-solid systems, as function of particle size

Particulate and Agglomerative systems

Particulate systems:

  - Spacing between particles remains roughly uniform as bed expands
  - Ergun's equation remains valid up to 80% voidage (Leva, 1959)
  - **Wen and Yu model is for *particulate* fluidization**

- Agglomerative systems:
  - Particles cluster into larger groups causing **deviation** from ideal, particulate drag
  - Can occur immediately upon fluidization

Drag for real systems

Look for deviations from particulate drag

- Could have multiple sources such as Van der Waals forces, static electricity, liquid bridging, etc.
- Particle size dependence
- Zero deviation at minimum fluidization or for single particle

$$F_p(\varepsilon,\mathrm{Re},\phi) = \underbrace{F_{\mathrm{ideal}}(\varepsilon,\mathrm{Re},\phi)}_{\text{Ideal drag}} \times \underbrace{f(\varepsilon,\mathrm{Re},\phi)}_{\text{Deviation}}$$

Correlation for Ideal Particulate Drag

- Requirements:
    - Satisfy both close pack and single particle extremes
    - Continuous function of voidage
    - Match experimental data

- Popular models do not satisfy these requirements
    - Wen and Yu does not satisfy close pack
    - Ergun does not satisfy single particle drag
    - Wen-Yu/Ergun Blend (Gidaspow) is discontinuous at $\varepsilon = 0.8$

New drag forms for particulate fluidization

Form A: A Wen-Yu form (power of voidage) is used with an exponent that is adjusted to match Ergun's drag for all Re at minimum fluidization

$$F_p(\varepsilon,\mathrm{Re}) = F_{sp}\left(\mathrm{Re}\right)\varepsilon^{\beta} \qquad \beta = \frac{\ln\left(F_{mf}(\mathrm{Re}) / F_{sp}(\mathrm{Re})\right)}{\ln\left(\varepsilon_{mf}\right)}$$

The expression can be further simplified by rearrangement

Form A

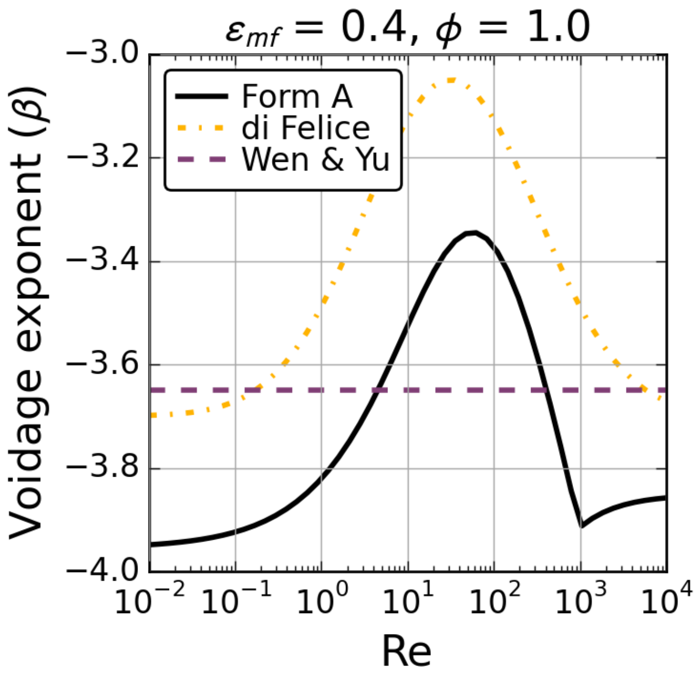
$$F_p(\varepsilon,\mathrm{Re}) = F_{sp}\left(\mathrm{Re}\right)^{\chi} F_{mf}^{\;1-\chi} \qquad \chi = 1 - \frac{\ln\varepsilon}{\ln\varepsilon_{mf}}$$

Comparison of Form A with expression of di Felice

- The analysis of di Felice (1994) found a Reynolds number dependence for the exponent and proposed the following expression:

$$\beta = 3.7 - 0.65\exp\left(-\frac{\left(1.5-\log(Re)\right)^2}{2}\right)$$

- The di Felice expression does not guarantee close-pack drag, but the shape is similar



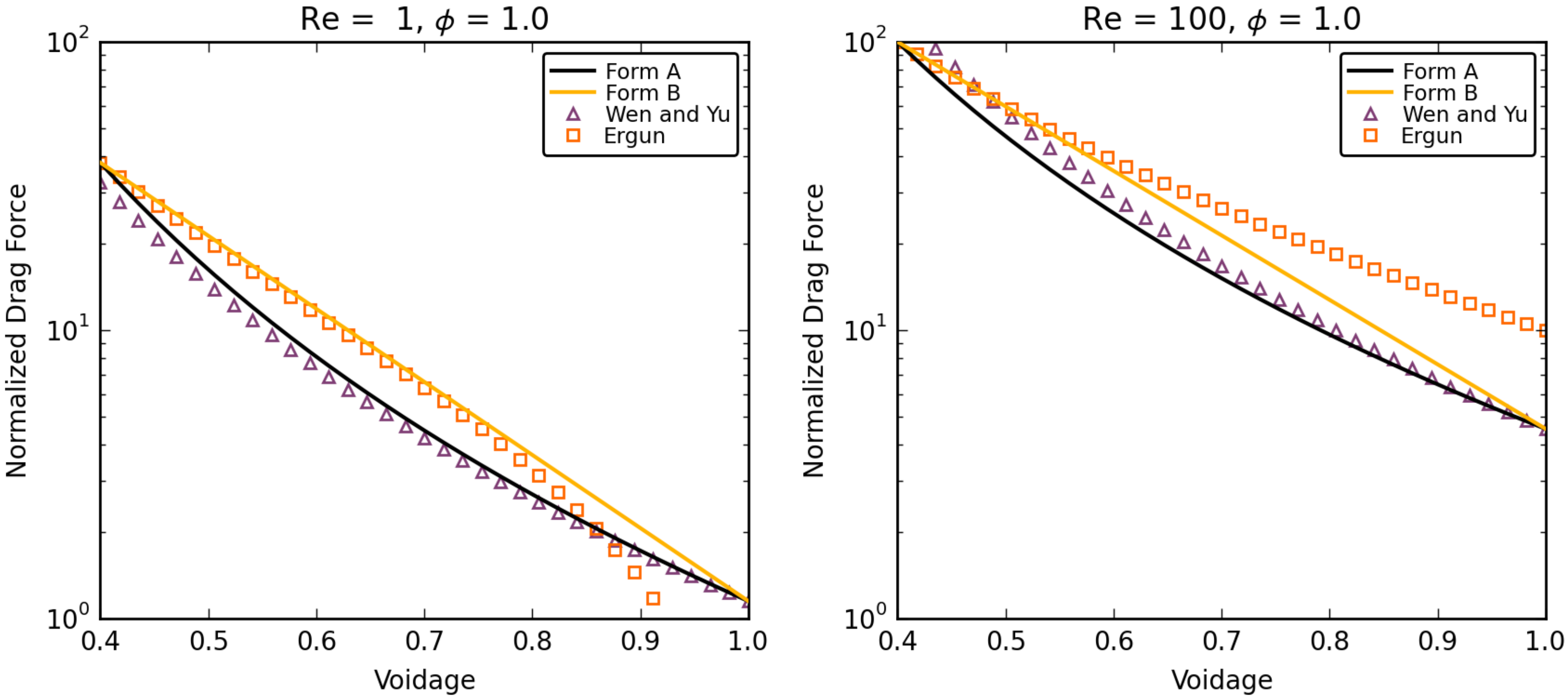
New drag forms for particulate fluidization

Form B: It was observed by Leva (1959) that the Ergun's model is applicable up to voidages of 0.8 and this same cutoff is commonly used in the Gidaspow Wen Yu / Ergun blend. Logarithmic interpolation between Ergun's drag and the Schiller-Nauman drag reproduces this observation well

Form B

$$F_p(\varepsilon,\mathrm{Re}) = F_{sp}\left(\mathrm{Re}\right)^{\zeta} F_{mf}(\varepsilon, Re)^{1-\zeta} \qquad \zeta = \frac{\varepsilon - \varepsilon_{mf}}{1-\varepsilon_{mf}}$$

Proposed models vs Existing models



Validation against experimental data

Data sets used for validation:

- Wen and Yu (1966): Bed expansion data for 191 & 500 micron glass balls in water
- Liu, Kwauk, and Li (1996): Bed expansion data for 54 micron FCC catalyst in supercritical CO_2 (8 and 9.4 MPa)
- Jottrand (1952): Bed expansion data for 20, 29, 43, 61, 86, and 113 micron sand in water
- Wilhelm and Kwauk (1948): Bed expansion data for 373, 556, and 1000 micron sea sand in water
- Lewis, Gilliland, and Bauer (1949): Settling of 100 and 150 micron glass in water

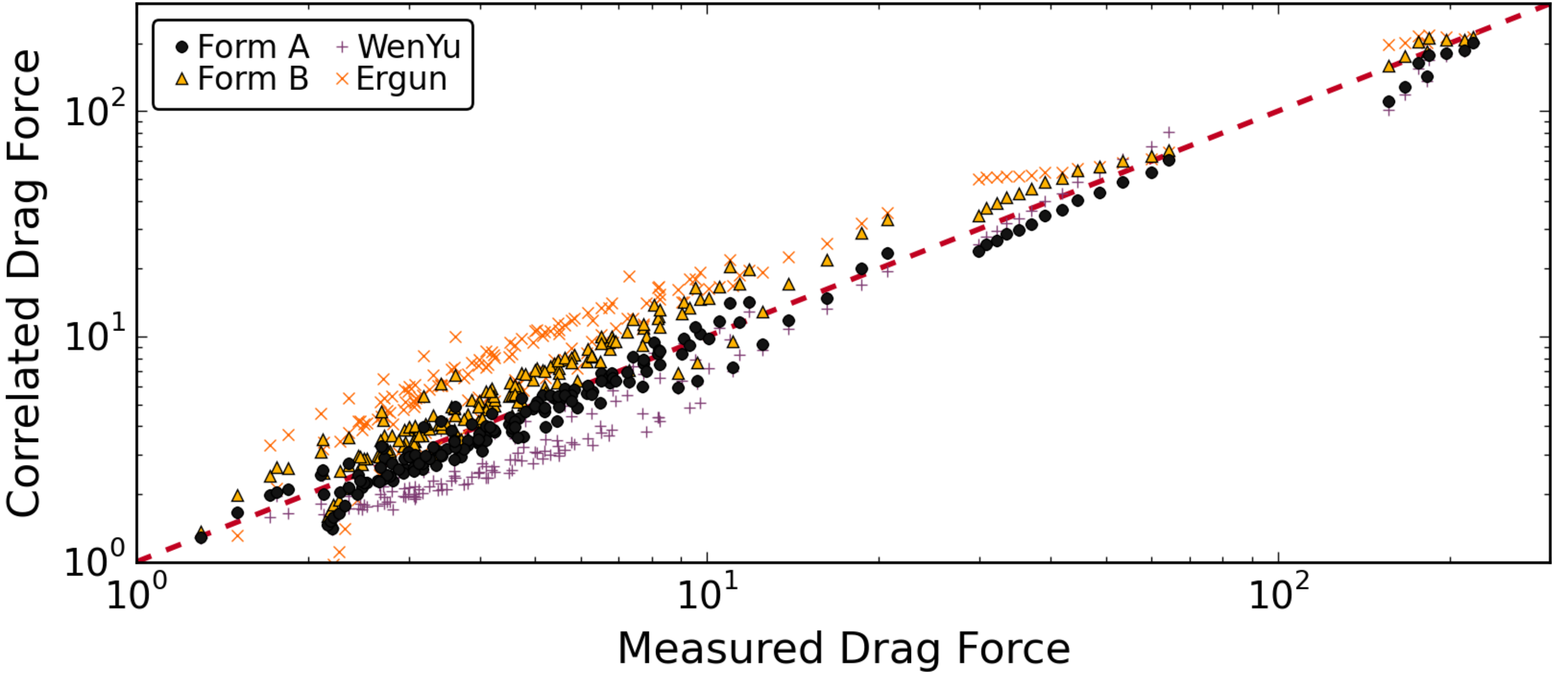
Analysis Approach

- Bed expansion is generally comprised of data showing
  - Superficial velocity
  - Bed height or voidage
  - Pressure drop
- Non-dimensional drag is related to the Archimedes number in a fluidized bed (di Felice, 1994)

$$F_{\mathrm{meas}}(\varepsilon,\mathrm{Re}) = \mathrm{Ar}\,\frac{\varepsilon}{18\,\mathrm{Re}} \qquad \mathrm{Ar} = \frac{d_p^{\,3}\rho_f\left(\rho_s - \rho_f\right)g}{\mu_f^{\,2}}$$

- Particle sphericity is estimated from close pack pressure drop data where possible

Correlations vs Measured Data



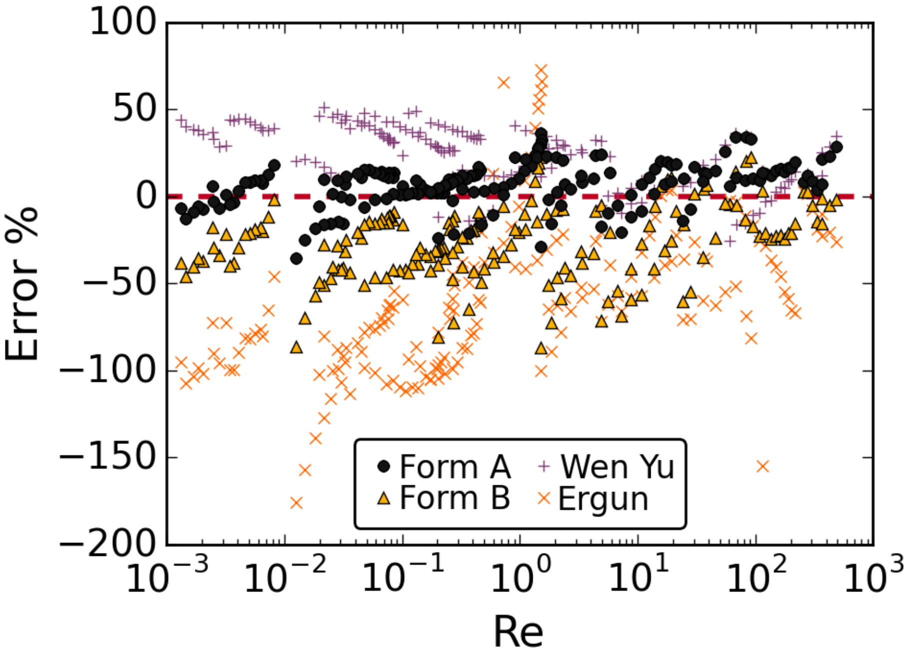
Error Analysis of different drag models

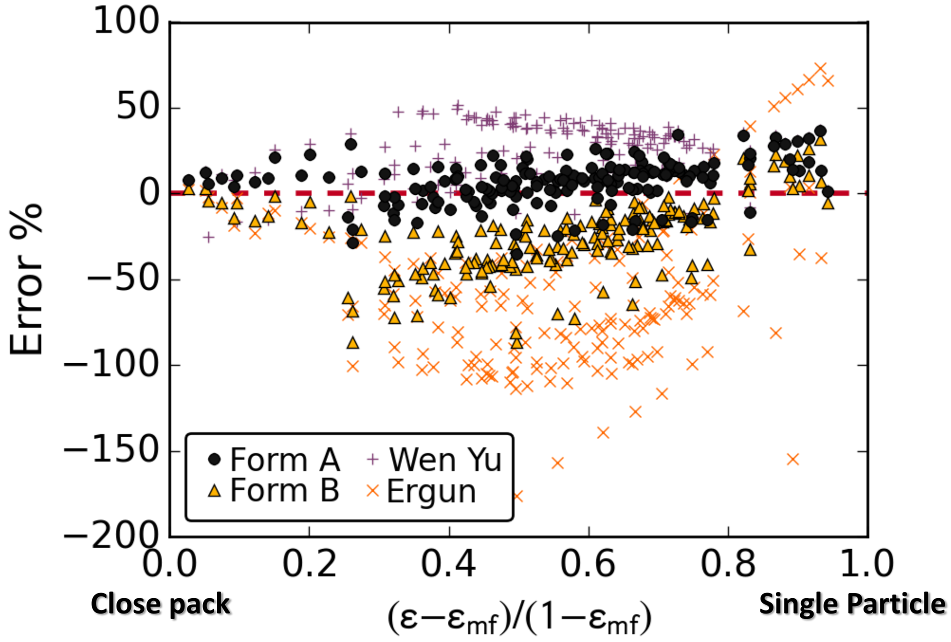
Drag Model	Error	Ergun Coeffs
Form A	**11.9%**	**a = 180, b = 1.8**
Ergun*	20.5%	a = 180, b = 1.8
Gidaspow*	23.1%	a = 150, b = 1.75
Wen and Yu	26.0%	
Ergun*	26.1%	a = 150, b = 1.75
Form B	28.0%	a = 180, b = 1.8
Ergun	64.2%	a = 180, b = 1.8

$$\text{Average error} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{F_{meas}-F_{corr}}{F_{meas}}\right|_i$$

* Sphericity is assumed = 1, as is common practice for these models

Particulate Drag Error dependence on Re and Voidage





Close pack

Single Particle

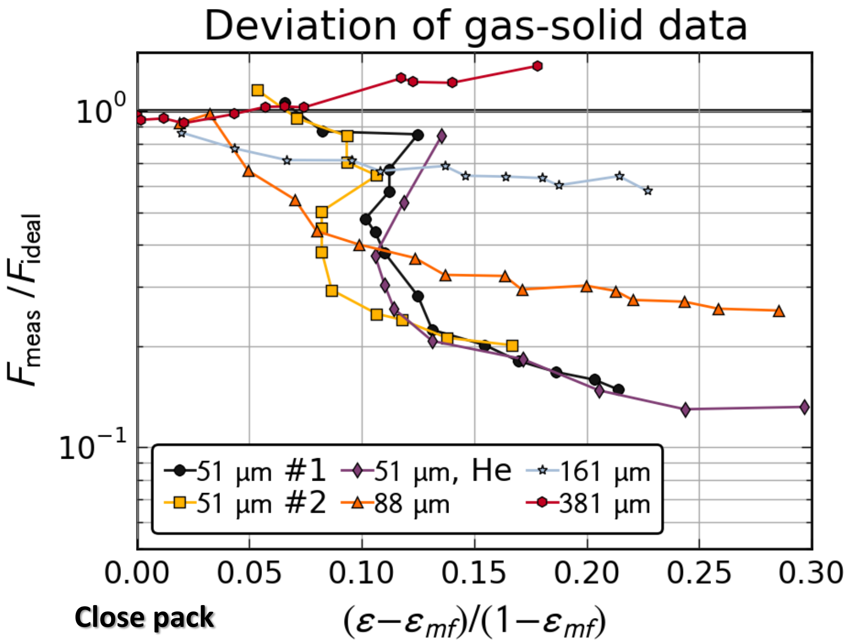
US Bureau of Mines data for agglomerative systems

- Bureau of Mines Bulletin 504 (Leva, 1951) contains a large set of bed expansion data for the fluidization of sand by different gases
- Select data was analyzed as part of preliminary work:
  - 51 micron round sand in air (X 2)
  - 51 micron round sand in helium
  - 88 micron round sand in air
  - 161 micron round sand in air
  - 381 micron round sand in air

Measured drag force vs Ideal prediction

A comparison of measured drag force vs the ideal prediction shows:

- Deviation between ideal and measured *increases* as particle sizes become smaller
- Deviation approaches zero at minimum fluidization



Close pack

Simple agglomeration model

- The following simple agglomeration model is proposed which consists of:
  - The ideal model for particulate fluidization (Form A)
  - Correction term for agglomeration which includes “***α***”, an agglomeration constant with units of length

$$F_{\text{drag}} = 3\pi\mu d_p U \times F_p(\varepsilon,\text{Re})$$

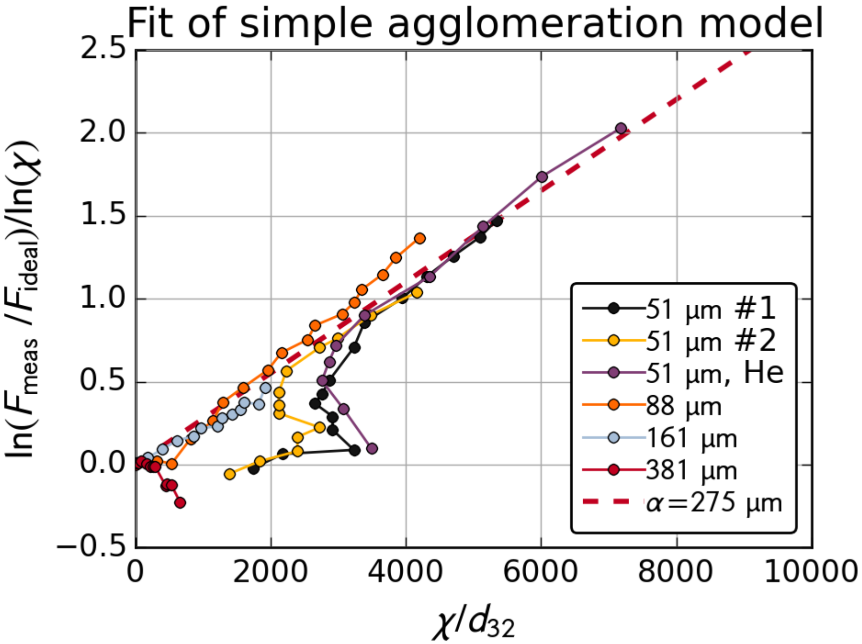
$$F_p(\varepsilon,\text{Re}) = \left(F_{sp}\right)^{\chi}\left(F_{mf}\right)^{1-\chi}\,\chi^{(a\chi/d_p)} \qquad \chi = 1 - \frac{\ln\varepsilon}{\ln\varepsilon_{mf}}$$

Estimating agglomeration constant

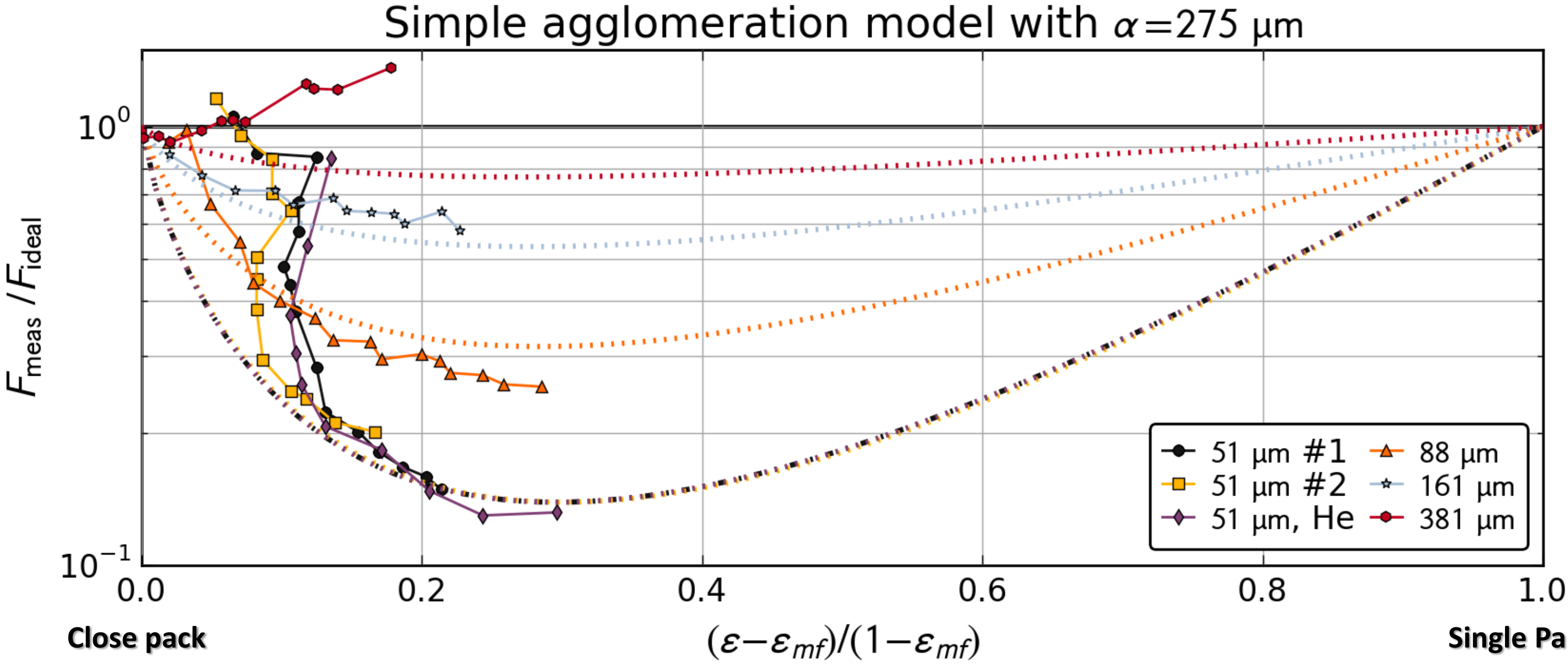
The proposed agglomerative drag model correction:

$$\ln(F_{\text{meas}} / F_{\text{ideal}}) / \ln(\chi) = a \frac{\chi}{d_p} \qquad \chi = 1 - \frac{\ln \varepsilon}{\ln \varepsilon_{mf}}$$

Constant α = 275 µm shows decent fit



Simple agglomeration model vs experimental data



Close pack

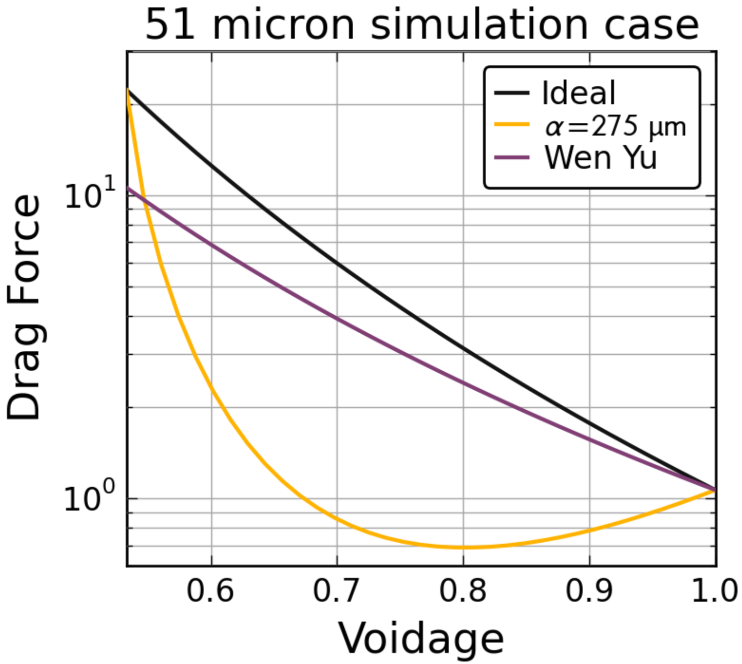
Single Particle

Symbols: experimental data; dashed lines: model fit

Using the model in Barracuda

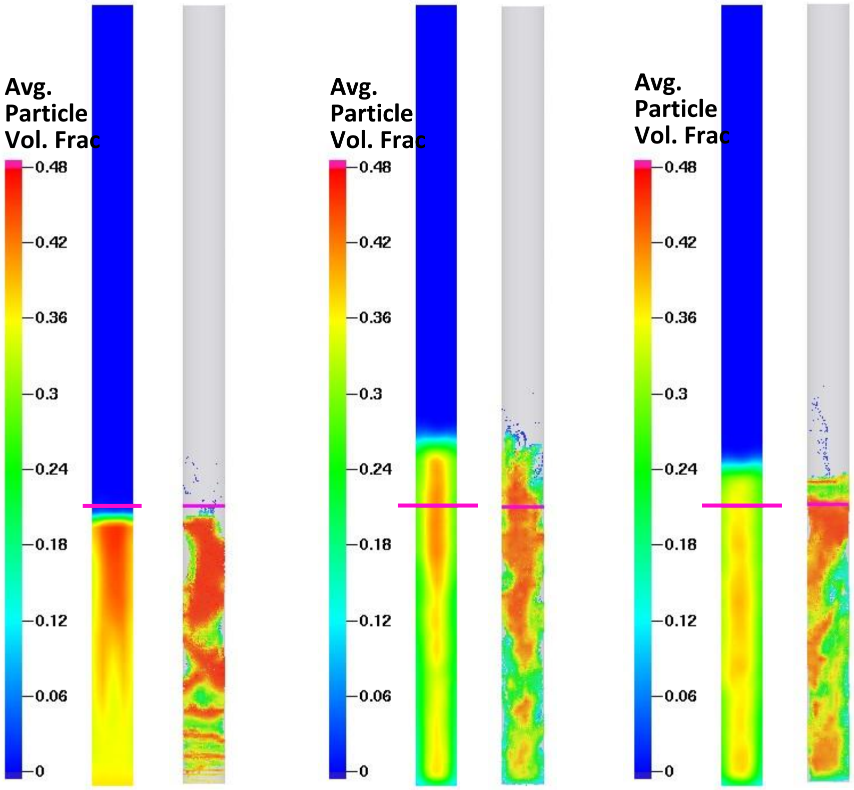
Case from Bureau of Mines dataset:

- Particle size: 0.00202” (51 µm)
- Column diameter: 2.5”
- Gas flow: 2.73 lb/hr air
- Initial column height 33.8 cm
- Fluidized height: 42.5 cm (25% expansion)



Using the model in Barracuda VR

- Simple agglomeration produces the following improvements in simulation
- Better agreement with bed expansion data (magenta line)
- Fluidization behavior affected by drag model



Simple Agglomeration
a = 275 um

Ideal (Particulate)

Wen - Yu

Conclusions

- Particle drag is a fundamental calculation for CFD simulations
- Agglomerative effects exist for gas-solid particles which cause the drag force to deviate from ideal.
- A model for particulate fluidization was proposed and validated against data
- **Work in progress** - A simple agglomeration model was proposed for gas-solid systems
- Improvements to bed expansion and changes to fluidization patterns are observed with the simple agglomeration model

Proposed drag model

Recommended form

$$F_{sp}(\mathrm{Re}) = \begin{cases} 1 + 0.15\,\mathrm{Re}^{0.687} & \mathrm{Re} < 1000 \\ 0.44 / 24\,\mathrm{Re} & \mathrm{Re} \geq 1000 \end{cases} \qquad F_{mf}(\varepsilon_{mf}, \mathrm{Re}) = \left(\frac{180(1 - \varepsilon_{mf})\phi^{-1} + 1.8\,\mathrm{Re}}{18\phi\varepsilon_{mf}{}^{2}} \right)$$

$$F_p(\varepsilon, \mathrm{Re}) = \left(F_{sp}\right)^{\chi} \left(F_{mf}\right)^{1-\chi} \chi^{(a\chi/d_p)} \qquad \chi = 1 - \frac{\ln \varepsilon}{\ln \varepsilon_{mf}}$$

$$F_{\mathrm{drag}} = 3\pi\mu d_p U \times F_p(\varepsilon, \mathrm{Re})$$

α = 275 microns from UBM sand data

References

- Di Felice, R (1994). The Voidage Function for Fluid-Particle Interaction Systems. *International Journal of Multiphase Flow*, 20(1):153.
- Leva, M. Weintraub, M., Grummer, M., Pollchik, M., and Storch, H. (1951). Fluid Flow Through Packed and Fluidized Systems. Bureau of Mines Bulletin 504.
- Leva, M (1959). *Fluidization*. McGraw-Hill Book Company, Inc., New York.
- Lewis, W., Gilliland, E., and Bauer, W. (1949). Characteristics of Fluidized Particles. *Industrial and Engineering Chemistry*, 41(6): 1104.
- Liu, D., Kwauk, M., and Li, H. (1996). Aggregative and Particulate Fluidization – The Two Extremes of a Continuous Spectrum. *Chemical Engineering Science*, 51(17): 4045.
- Jottrand, R. (1952). An Experimental Study of the Mechanism of Fluidization. *Journal of Applied Chemistry*, Supplementary Issue 1: S17.
- Wen, C. and Yu, Y. (1966). Mechanics of Fluidization. *Chemical Engineering Progress Symposium Series*, 62(62): 100
- Wilhelm, R. and Kwauk, M. (1948). Fluidization of Solid Particles. *Chemical Engineering Progress,* 44(3): 201.