

Computer Aided Analysis of Underdrawings in Infrared Reflectograms †

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Abstract

Recent developments in computer vision are providing powerful tools for the evaluation of data gathered by art historians and archaeologists. New camera hardware allows new insights into cultural heritage, especially if infrared cameras are concerned, since they allow the study of structures that are visually hidden. In this paper preliminary results of developing a system for automatic analysis of infrared reflectograms are presented. We concentrate on an algorithm for the automatic segmentation of strokes in underdrawings - the basic concept of the artist - in ancient panel paintings and the removal of cracks in infrared images. The purpose of the stroke analysis is the determination of the drawing tool used to draft the painting. This information allows significant support for a systematic stylistic approach in the analysis of paintings. Stroke segmentation in paintings is related to the extraction and recognition of handwriting, therefore similar techniques to segment the strokes from the background incorporating boundary information are used. Results of the algorithms developed are presented for both test panels and real reflectograms.

1. Introduction

Europe's rich cultural heritage is one of its important assets. Recovering more of this heritage and making it accessible to the public must be a concern of research work also in the future. For example panel paintings from 1400 to 1520 have been of great influence to European culture during this period and beyond. To learn more about the unknown drawing technique below the colored surface can give new insights into the working of famous artists or painting schools and so increase significantly cultural knowledge and awareness. Interdisciplinary projects between the field of art history and computer based image analysis have brought new aspects in both of the fields to save, protect and extend cultural heritage. While art historians benefit from new objective analysis methods and improved efficiency due to computer based solutions, for technicians a new field of applica-

tion was opened, which requires the adaptation and development of algorithms to the specific needs of art history.

A current project develops a computer based analysis system of underdrawings in medieval paintings. Underdrawings are the basic concept of an artist when he starts the creation of his work of art. Therefore the art historians and restorers are interested in investigations of these underdrawings. Moreover, a systematic analysis, starting with medieval paintings, over a longer period will bring insights into the practice in painting schools which is still rarely examined up to now.

Normally the underdrawing is hidden by covering paint layers and is invisible to the observer in the visible light spectrum. Using sensors that are sensible in the near infrared, especially in the spectral range from 1000 nm to 2400 nm, underdrawings of paintings can be visualized, even below the hardly penetrable colors blue and green in the paint layer. Figure 1 (a) shows an image of a panel painting and the visualization of the underdrawing by an IR-reflectogram (b) taken from a detail of the painting as outlined in (a).

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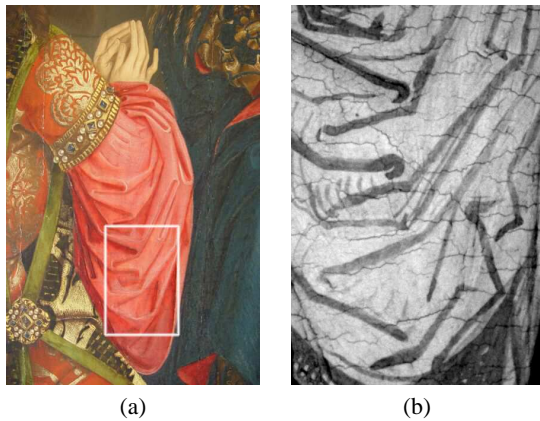


Figure 1: "Adoration of the Kings", master of the Schottenstift (1470): (a) color image (b) image taken in the near IR range (700nm–900 nm)

To be able to investigate image processing methods for the analysis of underdrawings a specific acquisition system with a high resolution infrared camera is necessary. In contrast to other infrared projects^{1,2,3} our goal is not only to digitize, visualize, and improve images of underdrawings, but to analyze the structure of the underdrawings with methods of image processing and pattern recognition to obtain insights from the unknown working procedure in medieval painting schools of famous artists.

In this paper we present a system that will use the IR-reflectography technique to obtain digital images of underdrawings and that will apply image analysis methods to extract objective and reproducible information to support the experts in studying underdrawings. Although parts of the system are well-known algorithms and have been published elsewhere^{4,5}, this paper will introduce a new challenging field for image analysis on works of art. The paper is organized as follows. First a motivation will show the need for a computational support for the analysis of paintings. Section 3 gives art historic facts, necessary for the development of algorithms. The components of the system are described in Section 4, with emphasis on the image preprocessing for crack elimination, stroke segmentation and stroke feature extraction. Section 5 presents and discusses the results of applying the algorithms developed to IR-reflectograms as well as test images. Section 6 will conclude with a brief overview of work in progress and future work.

2. Motivation

Since the late 1960s examination of paintings with IR photography and IR-reflectography opened a new window for the art historian, restorer, and conservator into the working process of artists^{6,7,8,9,10,11}. It helped to visualize the underdrawing on the ground of a painting and offered so far totally

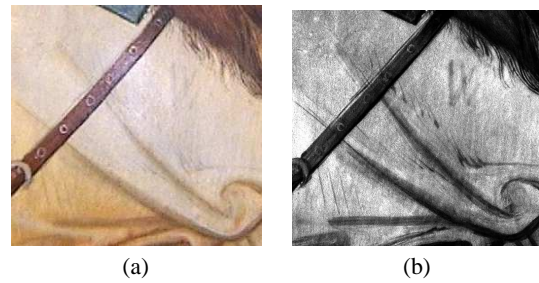


Figure 2: Color instructions: (a) the original painting (b) color instruction "W" in the right-top part of the underdrawing

unknown types and forms of composition design from the 14th to the 16th century. During the last twenty years techniques of IR-reflectography have developed further worldwide and have been more and more adapted to the needs of:

- Examination of paintings^{12,13}
- Restoration and conservation¹⁴
- Fake detection¹

Meanwhile, many leading museums and centers for art research and conservation (i.e. Victoria and Albert Museum, Harvard Museum) have installed their own IR equipment for regular use and specialized research projects are on the way in several countries. But due to the lack of state of the art digital IR cameras, the potential of digital image analysis for this field of research is not used. Therefore a significant support for a required systematic stylistic approach in the analysis of medieval and Renaissance paintings using underdrawings is still missing¹⁵.

From the conservator's point of view answers on individual parts of the working process will be given concerning:

- Execution between the first concept and the final result
- Visualization of paint instructions in terms of written color names (Figure 2b shows an example of a paint instruction visible in the infrared image "W = white")
- Differentiation between freehand drawings and drawings applied with different kinds of stencils¹⁴.

Aside from the large number of images acquired and the improvement of these images – this is already state of the art¹⁶ – the innovation of this project is the computer-based analysis of the structure of the underdrawing. Analysis of IR-reflectograms is performed primarily by visual inspection only. It is visual, that the analysis of a large number of images has been made by naked eye examination only. The restricted human optical retentiveness complicates the comparison of different underdrawings concerning drawing tools, drawing materials, and stroke characteristics.

3. Art Historic Background

In conservation and art history three prominent questions are of particular interest. The first question deals with the development of underdrawings and their relations to other drawings and between underdrawings and the covering painting. Secondly, art historians and restorers are interested in the style of the underdrawing, and whether the underdrawing is sketchy, freehand or a copy from a template. Finally an important question is, what kind of materials and drawing tools are used in an underdrawing³.

The system presented in this paper will contribute to answering the last question, while providing answers concerning the style or developments of underdrawings will be part of future research. In order to analyze the strokes with respect to the drawing tools used, the visual appearance is investigated. The following section gives a characterization.

3.1. Characterizing Drawing Tools / Materials

Drawing tools used in medieval panel paintings can be categorized into two different types, into those that are fluid and into a group consisting of dry drawing material³. In Figure 3 six examples of a stroke for both of the groups are depicted. Three strokes represent the class of drawing tools using fluid materials (a,c,e) and three strokes represent dry materials (b,d,f). These examples have been taken from a panel prepared for our experiments by a restorer.

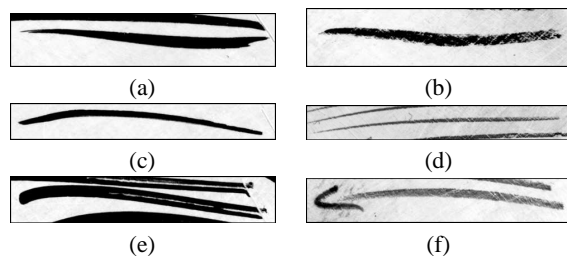


Figure 3: Stroke details showing tools using fluid materials on the left, brush (a), quill (c), reed pen (e) and dry material tools on the right, black chalk (b), silver point (d) and graphite (f).

Our analysis approach is based on the observation that prominent characteristics of drawn strokes are variations of shape and variations of the intensity in the drawing direction. Table 1 gives an overview of the characteristics of the two groups of drawing tools. The first characteristic we analyzed is the boundary of a stroke. It can be observed that there are variations in smoothness depending on the drawing tool used. While strokes applied with a pen or brush using a fluid medium show a smoother boundary, the boundary of strokes applied with a dry material, e.g. black chalk or graphite is less smooth.

Table 1: Characteristics of different drawing tools and materials

| Tools/Materials | Characteristics |
|--|--|
| fluid materials - paint or ink applied by pen or brush | fluid lines - continuous and smooth - vary in width and density - pooling of paint at the edges - droplet at the end - different endings (brush/pen) |
| dry materials - charcoal - chalks - metal points - graphite | dry lines - less variation in width - less continuous - more granular |

4. System Overview

The analysis system, according to the standard process of digital image analysis¹⁷, consists of an acquisition step, a preprocessing step, an image processing step and finally a classification step. A schematic overview is given in Figure 4. Since the system is still under development, we can only present the image processing part in more detail. In the following sections the processing steps and the subtasks will be discussed.

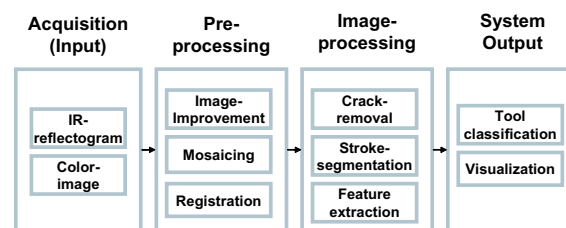


Figure 4: System overview

4.1. Acquisition

The quality of an image acquired and therefore the quality of the information of the images has a great influence on the success of the image processing and analysis phase¹⁸. Van Asperen De Boer¹⁹ showed that the range of optimal transmittance for many visually opaque paint layers is located in the region around 2 μm , which is only accessible by special electronic imaging devices. For our purpose we use a Focal Plane Array Camera (FPA) with a PtSi sensor, which has a sensitivity range from 1.0 μm to 5.7 μm that can be adapted by using a band pass filter. FPA cameras have higher

thermal stability, higher resolution and less geometric distortions than Vidicon cameras. The digital images captured fulfill the requirements with respect to radiometric resolution, geometric distortions, pixel-resolution and sensitivity constancy over time for the application of further processing steps¹⁸.

4.2. Preprocessing

When acquiring an image with an infrared array, noise from the detector as well as from the illumination source is to be expected¹⁸. Equally distributed detector noise is reduced by calculating the average image from a series of images, taken with the same camera setup²⁰.

Due to non-uniform illumination or inhomogeneities in the sensitivity of the sensor array, the intensity values of the digital image are vary (radiometric distortions). The deviations of the intensity values are measured in images taken from a uniform colored test-plane. This allows corrections of sensor response in already acquired images²¹.

An important preprocessing step for building IR-reflectograms of larger paintings is mosaicing. Panel paintings can have sizes of $2m$ by $2m$ or even larger, but the resolution and the pixel number of the camera is limited. In order to get a complete IR-reflectogram of a painting, smaller sub images are stitched together into one larger image. The alignment of images depends on the geometry of the acquisition setup, i.e. how the camera is moved with respect to the object. In the simplest case these are pure image-plane translations. This can be obtained if the camera is mounted on an XY-shift unit. This acquisition setup further allows the combination of the two overlapping parts by simple averaging. Finally changes of the brightness in different images, which is usually a result of automatic gain control have to be corrected. Using a positioning unit, which shifts the camera and thus the image plane within a virtual plane, planar image mosaicing methods²² can be applied.

4.3. Image Processing

One major goal of the project is to identify the drawing tools used by the painter to create the underdrawing from the appearance of the strokes in the IR-reflectogram. A step towards the identification is the segmentation of the individual strokes. From the segmentation point of view, cracks are treated as structural noise and will produce artifacts in the segmentation step. To overcome this problem, our intention is to eliminate the cracks while keeping the boundaries of the strokes as accurately as possible for further analysis. The following sections will present (1) a mathematical morphology based method for detection and elimination of cracks, (2) an edge-based method for segmentation of the strokes, and (3) finally the detection of features to differentiate between drawing tools.

4.3.1. Crack Removal

During the aging of the paintings, climactic fluctuations cause changes in the dimensionality of the panels. While younger pigment layers are elastic enough to follow contractions, a network of fine cracks (craquelé) may cover the whole painting during the aging process. The example of an IR-reflectogram depicted in Figure 1(b) shows several dark thin horizontally aligned lines representing the cracks in the ground layer. The pattern of the cracks is determined by the background used. In the case of wooden panels, the cracks are primarily oriented perpendicular to the grain²³.

Willingen et al.²⁴ have studied the appearance of cracks and determined features to classify different types of cracks. They differentiate between features of individual cracks (smooth, jagged, depth, thickness etc.) and features of crack patterns (distance between cracks, type of junctions). A similar problem has been treated by Giakoumis and Pitas²⁵. They use a three step process which first detects the cracks using a top-hat operator, separates them from brush strokes using color information, and then fills them in. In contrast to this work, we are working on greyscale images (recorded in the infra-red region). We therefore have no color information for separating the cracks from the brush strokes. The information we start with is that cracks are usually thinner than the brush strokes, and that they have a favored orientation. To take this information into account, we make use of a morphological opening²⁶ with a viscous reconstruction step, which detects the cracks and fills them in in one step. Abas and Martinez²⁷, on the other hand, are interested in the structure of the crack network. They use a top-hat operator to extract the cracks, and then extract descriptive information about the crack network so as to classify it.

In the context of this paper, we are interested in the ability of the viscous morphological reconstruction to reconstruct small details while preventing certain elements from being reconstructed. Viscosity is added to standard morphological reconstruction by including an opening after each geodesic dilation step. The challenges faced in the art history application are illustrated schematically in the simple binary example shown in Figure 5a. In this image, we wish to preserve the thick line and all its details as accurately as possible, while removing the thin lines which intersect it. The thin lines are known to have a diameter of less than 10 pixels, but intersections can sometimes result in thicker regions. An opening of Figure 5a with a disc-shaped structuring element of radius 5 is shown in Figure 5b. As expected, the details on the thick line have been smoothed, however, not all the thin lines have been successfully removed. Using a larger structuring element would smooth the thick line even more, and reconstruction cannot be used as the thin lines intersect the thick line, and would therefore be reconstructed too.

The use of viscous reconstruction is a good solution to this problem. We begin by creating the marker image shown in Figure 5c by eroding Figure 5a by a disc of radius 5 pix-

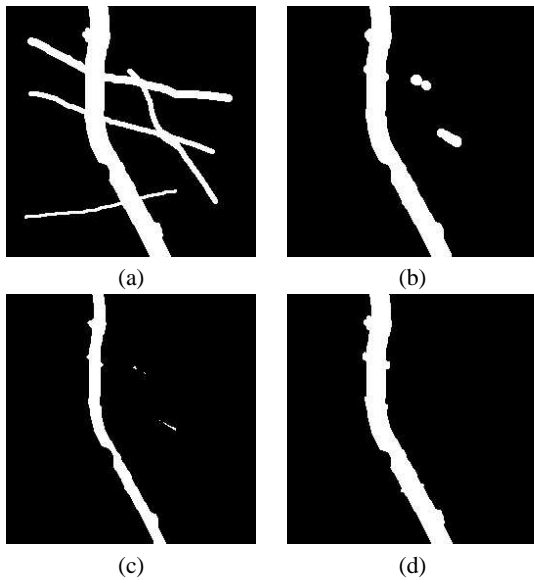


Figure 5: (a) Initial image. (b) Opening of image (a) with a disc of radius 5 pixels. (c) Marker image obtained by eroding image (a) with a disc of radius 5 pixels. (d) Viscous reconstruction for mask (a) from marker (c). Images are of size 256 by 256 pixels.

els. To reconstruct the initial image, we use a 3×3 pixel square structuring element for the geodesic erosion and a disc of radius 5 pixels as structuring element for the associated opening. In order to reconstruct the small details, we append an extra geodesic dilation onto the reconstruction algorithm. The result of this reconstruction is shown in Figure 5d.

4.3.2. Stroke Segmentation and Feature Extraction

Stroke segmentation in paintings is related to the extraction and recognition of handwriting²⁸. Letters and words in Western languages and symbols or signs in Chinese or Japanese languages are built of manually drawn strokes or lines. Many approaches start with thresholding and thinning methods. While these methods are fast and save resources, valuable information for a more detailed analysis of strokes requires an approach that also incorporates boundary information²⁹. We used Doermann's segmentation algorithm in the segmentation part of our approach, since it provides both the boundary of a stroke and its intensity profiles, which will be used to characterize strokes. Figure 6 gives an overview of our approach consisting of three basic steps, segmentation, boundary refinement and feature extraction.

Segmentation In the Step I, first edgels $E_i(x, y)$ located at the stroke contour are detected by a Canny edge detector. Second, based on the hypothesis that the gradient vectors of the edgels point in opposite directions, the set of edgels

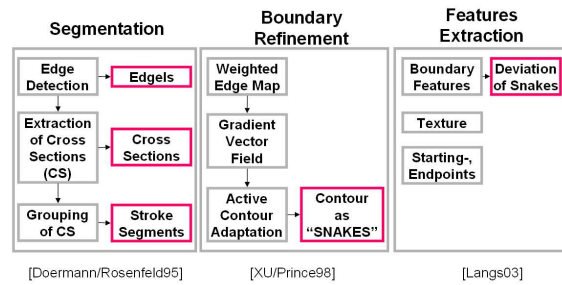


Figure 6: Schematic diagram of our approach

are grouped into distinctive pairs (cross sections). Finally, neighboring cross sections are grouped into sets and represent a stroke segment. Figure 7(a) shows the cross sections grouped into one stroke segment and the polygonal boundary. For further algorithmic details of we refer to⁵.

Boundary refinement In Step II the approximation of the stroke boundary by a closed polygon is refined by "snakes", a method based on active contours³⁰. After determining the principal component of the edgel distribution, the contour is split into two sides ("top" and "bottom" boundary) that are treated separately. A set of gray value profiles, perpendicular to the axis, represent the domain for the snake algorithm. Figure 7(b) shows the equidistant profiles in the original image, and arranged to form an image (c). The snake moves through this domain to minimize an energy functional determined by inner parameters controlling rigidity and tension of the snake and an external energy influenced by a gradient vector flow in order to provide accurate and fast convergence to boundary concavities.

Feature extraction Contour estimates with different levels of elasticity provide descriptive information by means of deviation against each other. We used two succeeding snakes. The first rigid snake was initialized on the coarse contour estimate. The second, more elastic snake proceeds from this position. Figure 7(d) shows the converged rigid and non-rigid snakes. MEAN of the deviation and standard deviation (SDV) of the deviation between the two snakes are used as descriptive features. For more details refer to³¹.

4.4. System Output

The system will provide objective support for the interpretation of underdrawings with high quality visualizations of IR-reflectograms combined with color images on the one hand, on the other hand a description and classification of details of the underdrawing with respect to drawing tools and materials.

When the objects of interest are detected and described by features (e.g. boundary, shape, orientation, color, and the

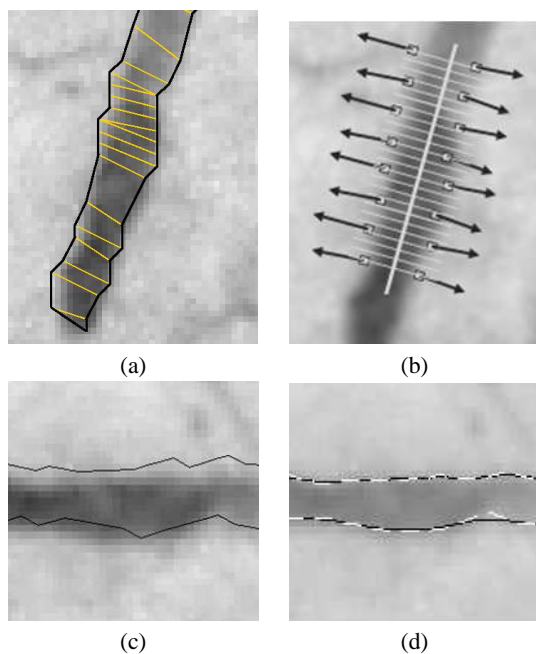


Figure 7: Segmentation and Refinement (a) cross sections and polygonal boundary (b) edgels with axis and gray value profiles (c) initial "top" and "bottom" boundary (d) converged "rigid" (black) and "non-rigid" snakes (white-dashed)

like) these features may be the input to the classification stage. Classification basically consists of two tasks ¹⁷:

- investigation of the relation between the image features and the object classes
- the actual classification task, that selects an optimal set of features which allows the different object classes to be distinguished with minimum effort and minimal errors

Results of the methods described above have to be presented in a visual form. The registration of images from different sources will allow the provision of a visual overlay of, e.g. the image of the paint layer and the segmented strokes of the underdrawing.

5. Experimental Results and Discussion

The methods developed will be applied to IR-reflectograms and test panels. Since the acquisition of original panels with an IR camera is ongoing work, IR-reflectograms with different drawing tools are not available at present. We therefore tested the segmentation and feature extraction algorithm on test panels and the crack removal algorithm on IR-reflectograms.

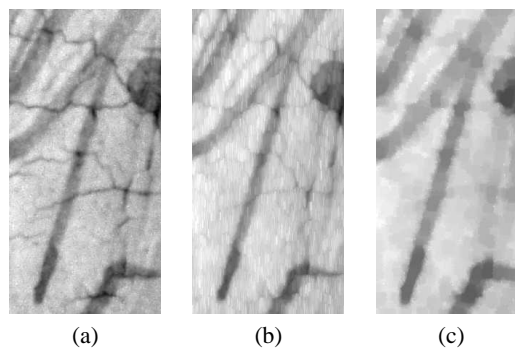


Figure 8: (a) Initial image. (b) Erosion of (a) by a vertical line of length 10 pixels. (c) Viscous reconstruction of (a) using (b) as a marker.

5.1. Crack Removal

To show the application of the crack elimination algorithm we use Figure 8a, which corresponds to the lower sub-region from the IR-reflectogram in Figure 1b. For the erosion step we took a priori information into account, namely that a large majority of the cracks have a preferred orientation, as discussed in the introduction. For the image under consideration, this preferred orientation is horizontal. We therefore take as our marker image an erosion of the initial image by a vertical line of length 10 pixels, shown in Figure 8b. The viscous reconstruction from this marker image, using a 3×3 pixel square for the geodesic reconstruction, and a disc-shaped structuring element of radius 6 for the opening step, is shown in Figure 8c. While the cracks are eliminated efficiently, the structure in the strokes remains. For more details we refer to Hanbury et al. ⁴.

5.2. Stroke Segmentation Results

In our experiments we studied the differences of three types of drawing tools - brush, chalk and graphite. Test panels (21cm x 30cm) containing sets of the mentioned strokes have been prepared by a restorer. The test panels were digitized using a flat-bed scanner with an optical resolution of 1200 dpi. Details from images, as depicted in Figure 9 have been cropped manually. Figure 9 (a) shows a series of brush strokes, (c) chalk strokes and (e) graphite strokes, all applied in bottom up direction.

The result of the segmentation step is illustrated in Figure 9 (b),(d) and (f) respectively. The boundary of the stroke segments, consisting of at least 20 cross sections are depicted. The segmentation algorithm works well for most of the brush strokes and graphite strokes. Problems arise e.g. at left stroke in Figure 9(a), which is not segmented completely, since the stroke width parameter was set too narrow. The segmentation algorithm still has problems with overlapping strokes like the "arrow top" in the left most stroke of

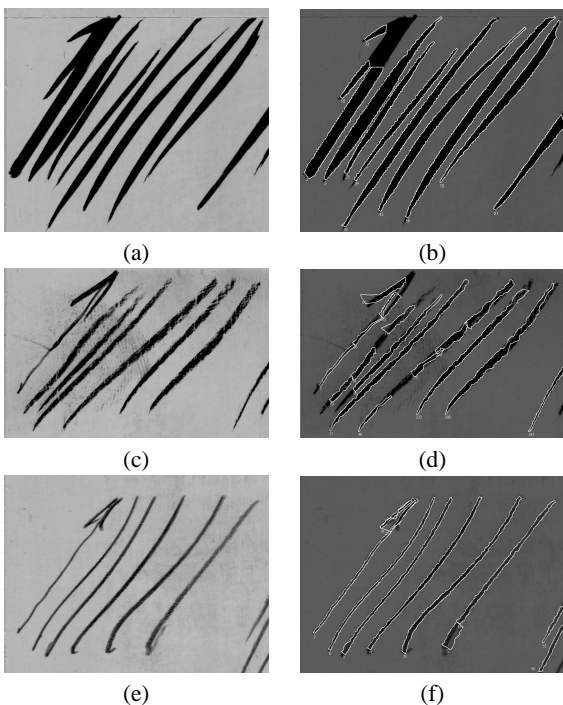


Figure 9: The left column shows details from the test panel with strokes used in our experiments: brush strokes (873x729 pixel) (a), chalk strokes (992x631 pixel)(c) and graphite strokes (989x729) pixel)(e). The right column shows an overlay of the detected boundaries of the segmentation step

Figure 9(f) and (d). Problems occur with the chalk strokes in Figure 9(d) which are segmented into many small segments due to the inhomogeneity of the strokes. This necessitates a further processing step, that will be handled together with the overlapping problem.

5.3. Feature Extraction Results

For the refinement and feature extraction step, the stroke segments shown are used. First, the refinement step is initialized by the boundary of the segmentation step. Figure 10(a,c,e) shows the detected boundary of the segmentation step for three example strokes. The refinement algorithm, i.e. the adaptation of the two snakes with different rigidity, is applied separately to the "top" and "bottom" boundary of a stroke. Figure 10(b,d,f) shows the example strokes together with an overlay of the more elastic (dotted bright line) and more rigid snake (underlying black line). It can be observed that the deviation of the rigid and elastic snake is smaller from the brush stroke than those from the black chalk and graphite strokes.

To show the differences calculated, the SDV- and MEAN-values of the deviations of the two snakes, i.e. two values,

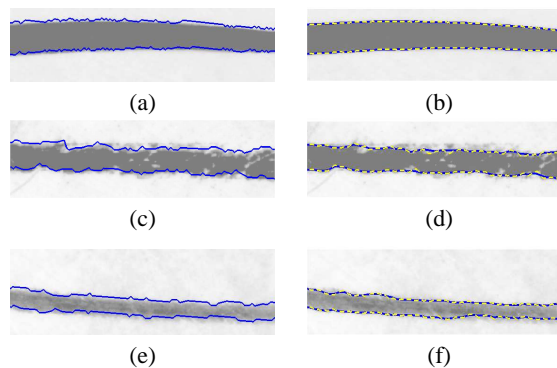


Figure 10: Details from the test panel showing stroke used in our experiments: brush strokes (a), chalk strokes (c) and graphite strokes (e). The right column shows an overlay of the snakes to corresponding stroke sample (b,d,f)

one for the "top" and one for the "bottom" boundary, are plotted in the diagram of Figure 11. The MEAN values of the brush strokes (denoted as circles) are concentrated near zero, while there is a higher variation of the MEAN graphite strokes (denoted as "x") and brush strokes (denoted as stars). Similarly, the standard deviation SDV of brush strokes is below 0.2 for all but two of the stroke borders. The SDV values for chalk and graphite is between 0.2 and 1.6 in our samples. So using the SDV feature will allow to distinguish between brush, i.e. a fluid drawing tool, and graphite and chalk respectively as dry drawing tools. Using a combination of SDV and MEAN the data of our samples can be used to differentiate between graphite and chalk, since most of the chalk values are positioned right and above the graphite values. Still, these results are preliminary and experiments with more samples are necessary. Furthermore the reliability of this differentiation can be improved if a set of strokes is considered. As can be observed in underdrawings, in certain regions of a drawing, a group of strokes are applied with the same drawing tool, e.g. as hatches or cross hatches.

6. Conclusion and Outlook

In this paper we presented a first step towards a system for automatic analysis of IR-reflectograms. We have demonstrated the application of viscous morphological reconstruction to eliminate thin lines (cracks), while retaining as much detail as possible in the thicker lines (the brush strokes). The suggested approach works well except in more complicated regions of a painting where the brush strokes have a similar width to the cracks. Further work on separating strokes and cracks based on their smoothness remains to be done. The results from the boundary analysis algorithm show that the contour feature extracted to initialize the snakes that model the contour are a promising way to obtain satisfactory results, although improvements in the segmentation are needed

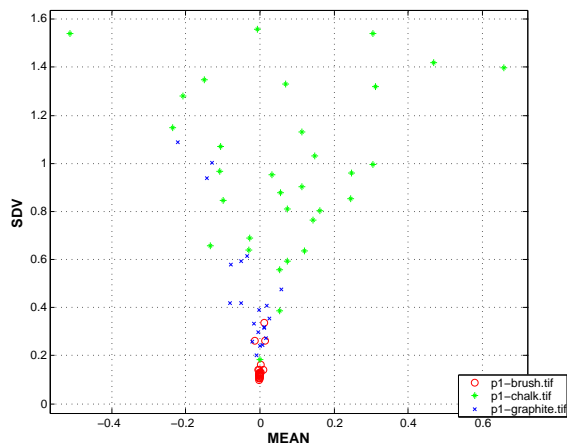


Figure 11: Standard deviation (SDV) and MEAN of the snake deviations. The deviations are measured on the "top" and "bottom" boundary of the individual brush, chalk and graphite strokes.

to cope with sudden variations of the contour, as shown in Figure 9. The first results show, that the visual appearance of the boundary of a stroke can be used for discrimination. Further experiments with more samples are necessary to validate our method.

We further plan to incorporate additional features, like the texture of the different types of strokes, to get a measure for granularity of a stroke. Furthermore we have noticed, that in some cases, there is a difference between the "top" and "bottom" boundary of a stroke in dry drawing tools. This observation has to be proved and evaluated. As reported, some problems occur in the segmentation step if the strokes are interrupted. One of our goals is therefore to improve the robustness of the segmentation step and to extend the approach to segment overlapping and crossing stroke formations as e.g. reported in ³².

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