CHAPTER X

CATEGORICAL SYLLOGISMS

Section 1. The type of logical, deductive, mediate, categorical Inference is a Syllogism directly conformable with the Dictum: as–

All carnivores (M) are excitable (P);

Cats (S) are carnivores (M):

.'. Cats (S) are excitable (P).

In this example P is predicated of M, a term distributed; in which term, M, S is given as included; so that P may be predicated of S.

Many arguments, however, are of a type superficially different from the above: as–

No wise man (P) fears death (M);

Balbus (S) fears death (M):

.'. Balbus (S) is not a wise man (P).

In this example, instead of P being predicated of M, M is predicated of P, and yet S is given as included not in P, but in M. The divergence of such a syllogism from the Dictum may, however, be easily shown to be superficial by writing, instead of No wise man fears death, the simple, converse, No man who fears death is wise.

Again:

Some dogs (M) are friendly to man (P);

All dogs (M) are carnivores (S):

.'. Some carnivores (S) are friendly to man (P).

Here P is predicated of M undistributed; and instead of S being included in M, M is included in S: so that the divergence from the type of syllogism to which the Dictum directly applies is still greater than in the former case. But if we transpose the premises, taking first All dogs (M) are carnivores (P),

then P is predicated of M distributed; and, simply converting the other premise, we get– Some things friendly to man (S) are dogs (M):

whence it follows that–

Some things friendly to man (S) are carnivores (P);

and this is the simple converse of the original conclusion.

Once more:

No pigs (P) are philosophers (M);

Some philosophers (M) are hedonists (S):

.'. Some hedonists (S) are not pigs (P).

In this case, instead of P being predicated of M distributed, M is predicated of P distributed; and instead of S (or part of it) being included in M, we are told that some M is included in S. Still there is no real difficulty. Simply convert both the premises, and we have: No philosophers (M) are pigs (P); Some hedonists (S) are philosophers (M).

Whence the same conclusion follows; and the whole syllogism plainly conforms directly to the Dictum.

Such departures as these from the normal syllogistic form are said to constitute differences of Figure (see Section 2); and the processes by which they are shown to be unessential differences are called Reduction (see Section 6).

Section 2. Figure is determined by the position of the Middle Term

in the premises; of which position there are four possible variations. The middle term may be subject of the major premise, and predicate of the minor, as in the first example above; and this position, being directly conformable to the requirements of the Dictum, is called the First Figure. Or the middle term may be predicate of both premises, as in the second of the above examples; and this is called the Second Figure. Or the middle term may be subject of both premises, as in the third of the above examples; and this is called the Third Figure. Or, finally, the middle term may be predicate of the major premise, and subject of the minor, as in the fourth example given above; and this is the Fourth Figure.

It may facilitate the recollection of this most important point if we schematise the figures thus:
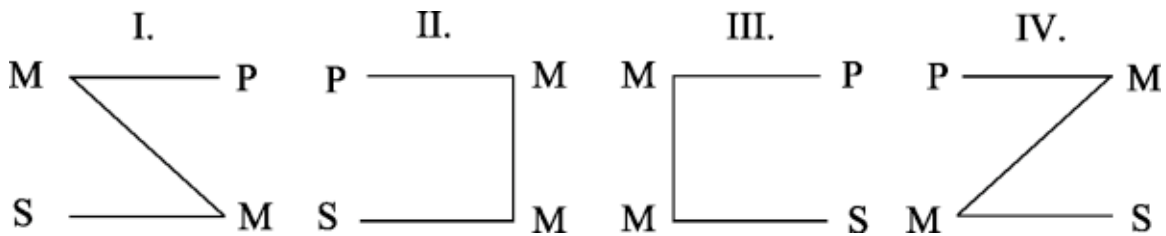
The horizontal lines represent the premises, and at the angles formed with them by the slanting or by the perpendicular lines the middle term occurs. The schema of Figure IV. resembles Z, the last letter of the alphabet: this helps one to remember it in contrast with Figure I., which is thereby also remembered. Figures II. and III. seem to stand back to back.

Section 3. The Moods of each Figure are the modifications of it which arise from different combinations of propositions according to quantity and quality. In Figure I., for example, four Moods are recognised: A.A.A., E.A.E., A.I.I., E.I.O.

A. All M is P; A. All S is M: A. .'. All S is P.

E. No M is P; A. All S is M: E. .'. No S is P.

A. All M is P; I. Some S is M: I. .'. Some S is P.

E. No M is P; I. Some S is M: O. .'. Some S is not P.

Now remembering that there are four Figures, and four kinds of propositions (A. I. E. O.), each of which propositions may be major premise, minor premise, or conclusion of a syllogism, it appears that in each Figure there may be 64 Moods, and therefore 256 in all. On examining these 256 Moods, however, we find that only 24 of them are valid (i.e., of such a character that the conclusion strictly follows from the premises), whilst 5 of these 24 are needless, because their conclusions are 'weaker' or less extensive than the premises warrant; that is to say, they are particular when they might be universal. Thus, in Figure I., besides the above 4 Moods, A.A.I. and E.A.O. are valid in the sense of being conclusive; but they are superfluous, because included in A.A.A. and E.A.E. Omitting, then, these 5 needless Moods, which are called 'Subalterns' because their conclusions are subaltern (chap. vii. Section 2) to those of other Moods, there remain 19 Moods that are valid and generally recognised.

Section 4. How these 19 Moods are determined must be our next inquiry. There are several ways more or less ingenious and interesting; but all depend on the application, directly or indirectly, of the Six Canons, which were shown in the last chapter to be the conditions of Mediate Inference.

(1) One way is to begin by finding what Moods of Figure I. conform to the Dictum. Now, the Dictum requires that, in the major premise, P be predicated of a term distributed, from which it follows that no Mood can be valid whose major premise is particular, as in I.A.I. or O.A.O. Again, the Dictum requires that the minor premise be affirmative ("with which term another is identified"); so that no Mood can be valid whose minor premise is negative, as in A.E.E. or A.O.O. By such considerations we find that in Figure I., out of 64 Moods possible, only six are valid, namely, those above-mentioned in Section 3, including the two subalterns. The second step of this method is [Pg 125]to test the Moods of the Second, Third, and Fourth Figures, by trying whether they can be reduced to one or other of the four Moods of the First (as briefly illustrated in Section 1, and to be further explained in Section 6).

(2) Another way is to take the above six General or Common

Canons, and to deduce from them Special Canons for testing each Figure: an interesting method, which, on account of its length, will be treated of separately in the next section.

(3) Direct application of the Common Canons is, perhaps, the simplest plan. First write out the 64 Moods that are possible without regard to Figure, and then cross out those which violate any of the Canons or Corollaries, thus:

AAA, ~~AAE~~ (6th Can. b). AAI, ~~AAO~~ (6th Can. b)._~~AEA~~ (6th Can. a) AEE, ~~AEI~~ (6th Can. a) AEO,_~~AIA~~ (Cor. ii.) ~~AIE~~ (6th Can. b) AII, ~~AIO~~ (6th Can. b)_~~AOA~~ (6th Can. a) ~~AOE~~ (Cor. ii.) ~~AOI~~ (6th Can. a) AOO

Whoever has the patience to go through the remaining 48 Moods will discover that of the whole 64 only 11 are valid, namely:

A.A.A., A.A.I., A.E.E., A.E.O., A.I.I., A.O.O., E.A.E., E.A.O., E.I.O., I.A.I., O.A.O.

These 11 Moods have next to be examined in each Figure, and if valid in every Figure there will still be 44 moods in all. We find, however, that in the First Figure, A.E.E., A.E.O., A.O.O. involve illicit process of the major term (3rd Can.); I.A.I., O.A.O. involve undistributed Middle (4th Can.); and A.A.I., E.A.O. are subalterns. In the Second Figure all the affirmative Moods, A.A.A., A.A.I., A.I.I., I.A.I., involve undistributed Middle; O.A.O. gives illicit process of the major term; and A.E.O., E.A.O. are subalterns. In the Third Figure, A.A.A., E.A.E., involve illicit process of the minor term (3rd Can.); A.E.E., A.E.O., A.O.O., illicit process of the major term. In the Fourth Figure, A.A.A. and E.A.E. involve illicit process of the minor term; A.I.I., A.O.O., undistributed Middle; O.A.O. involves illicit process of the major term; and A.E.O. is subaltern.

Those moods of each Figure which, when tried by these tests, are not rejected, are valid, namely:

Fig. I.–A.A.A., E.A.E., A.I.I., E.I.O. (A.A.I., E.A.O., Subaltern);

Fig. II.–E.A.E., A.E.E., E.I.O., A.O.O. (E.A.O., A.E.O., Subaltern);

Fig. III.–A.A.I., I.A.I., A.I.I., E.A.O., O.A.O., E.I.O.;

Fig. IV.–A.A.I., A.E.E., I.A.I., E.A.O., E.I.O. (A.E.O., Subaltern).

Thus, including subaltern Moods, there are six valid in each Figure. In Fig. III. alone there is no subaltern Mood, because in that Figure there can be no universal conclusion.

Section 5. Special Canons of the several Figures, deduced from the Common Canons, enable us to arrive at the same result by a somewhat different course. They are not, perhaps, necessary to the Science, but afford a very useful means of enabling one to thoroughly appreciate the character of formal syllogistic reasoning. Accordingly, the proof of each rule will be indicated, and its elaboration left to the reader. There is no difficulty, if one bears in mind that Figure is determined by the position of the middle term.

Fig. I., Rule (a): The minor premise must be affirmative.

For, if not, in negative Moods there will be illicit process of the major term. Applying this rule to the eleven possible Moods given in Section 4, as remaining after application of the Common Canons, it eliminates A.E.E., A.E.O., A.O.O.

(b) The major premise must be universal.

For, if not, the minor premise being affirmative, the middle term will be undistributed. This rule eliminates I.A.I., O.A.O.; leaving six Moods, including two subalterns.

Fig. II. (a) One premise must be negative.

For else neither premise will distribute the middle term. This rule eliminates A.A.A., A.A.I., A.I.I., I.A.I.

(b) The major premise must be universal.

For else, the conclusion being negative, there will be illicit process of the major term. This eliminates I.A.I., O.A.O.; leaving six Moods, including two subalterns.

Fig. III. (a) The minor premise must be affirmative.

For else, in negative moods there will be illicit process of the major term. This rule eliminates A.E.E., A.E.O., A.O.O.

(b) The conclusion must be particular.

For, if not, the minor premise being affirmative, there will be illicit process of the minor term. This eliminates A.A.A., A.E.E., E.A.E.; leaving six Moods.

Fig. IV. (a) When the major premise is affirmative, the minor must be universal.

For else the middle term is undistributed. This eliminates A.I.I., A.O.O.

(b) When the minor premise is affirmative the conclusion must be particular.

Otherwise there will be illicit process of the minor term. This eliminates A.A.A., E.A.E.

(c) When either premise is negative, the major must be universal.

For else, the conclusion being negative, there will be illicit process of the major term. This eliminates O.A.O.; leaving six Moods, including one subaltern.

Section 6. Reduction is either–(1) Ostensive or (2) Indirect. Ostensive Reduction consists in showing that an argument given in one Mood can also be stated in another; the process is especially used to show that the Moods of the second, third, and fourth Figures are equivalent to one or another Mood of the first Figure. It thus proves the validity of the former Moods by showing that they also essentially conform to the Dictum, and that all Categorical

Syllogisms are only superficial varieties of one type of proof.

To facilitate Reduction, the recognised Moods have all had names given them; which names, again, have been strung together into mnemonic verses of great force and pregnancy:

Barbara, Celarent, Darii, Ferioque prioris:_Cesare, Camestres, Festino, Baroco, secundæ:_Tertia, Darapti, Disamis, Datisi, Felapton,_Bocardo, Ferison, habet: Quarta insuper addit_Bramantip, Camenes, Dimaris, Fesapo, Fresison.

In the above verses the names of the Moods of Fig. I. begin with the first four consonants B, C, D, F, in alphabetical order; and the names of all other Moods likewise begin with these letters, thus signifying (except in Baroco and Bocardo) the mood of Fig. I., to which each is equivalent, and to which it is to be reduced: as Bramantip to Barbara, Camestres to Celarent, and so forth.

The vowels A, E, I, O, occurring in the several names, give the quantity and quality of major premise, minor premise, and conclusion in the usual order.

The consonants s and p, occurring after a vowel, show that the proposition which the vowel stands for is to be converted either (s) simply or (p) per accidens; except where s or p occurs after the third vowel of a name, the conclusion: then it refers not to the conclusion of the given Mood (say Disamis), but to the conclusion of that Mood of the first Figure to which the given Mood is reduced (Darii).

M (mutare, metathesis) means 'transpose the premises' (as of Camestres).

C means 'substitute the contradictory of the conclusion for the foregoing premise,' a process of the Indirect Reduction to be presently explained (see Baroco, Section 8).
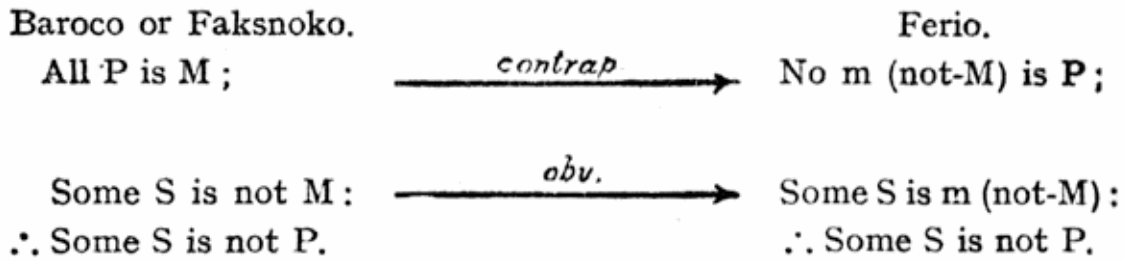
The other consonants, r, n, t (with b and d, when not initial), occurring here and there, have no mnemonic significance.

What now is the problem of Reduction? The difference of Figures depends upon the position of the Middle Term. To reduce a Mood of any other Figure to the form of the First, then, we must so manipulate its premises that the Middle Term shall be subject of the major premise and predicate of the minor premise.
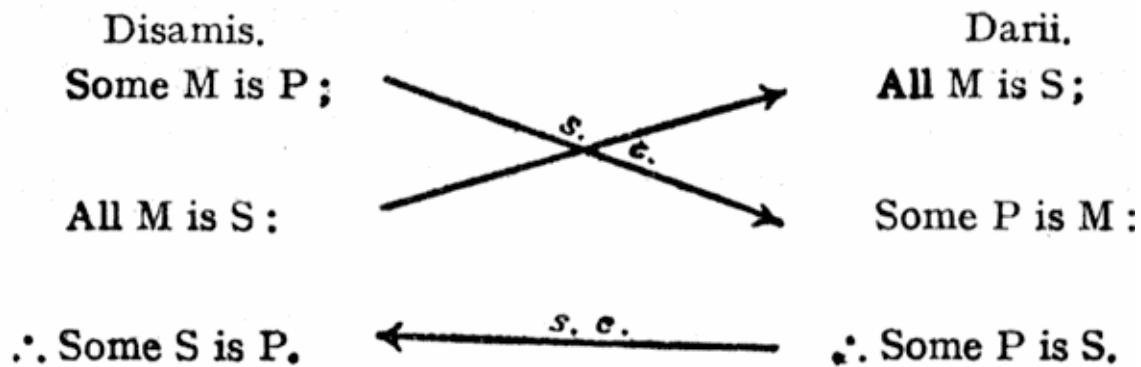
Now in Fig. II. the Middle Term is predicate of both premises; so that the minor premise may need no alteration, and to convert the major premise may suffice. This is the case with Cesare, which reduces to Celarent by simply converting the major premise; and with Festino, which by the same process becomes Ferio. In Camestres, however, the minor premise is negative; and, as this is impossible in Fig. I., the premises must be transposed, and the new major premise must be simply converted: then, since the transposition of the premises will have transposed the terms of the conclusion (according to the usual reading of syllogisms), the new conclusion must be simply converted in order to prove the validity of the original conclusion. The process may be thus represented (s.c. meaning 'simply convert')

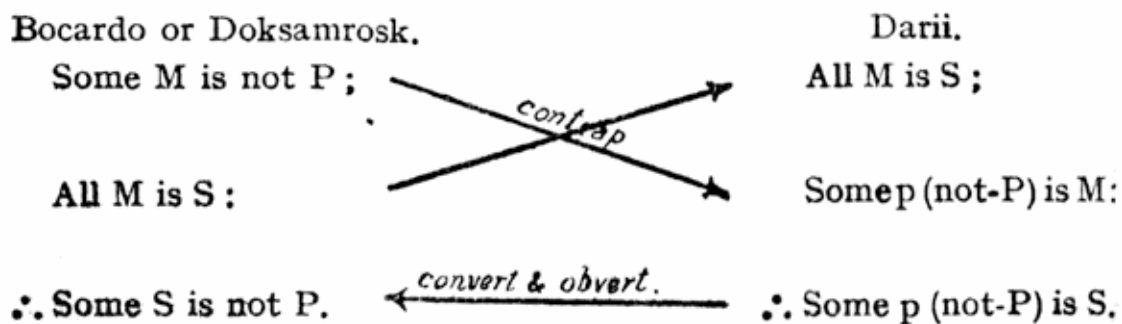| Camestres.		Celarent.
All P is M ;		No M is S ;
No S is M :		All P is M ;
∴ No S is P.	s. c.	No P is S.
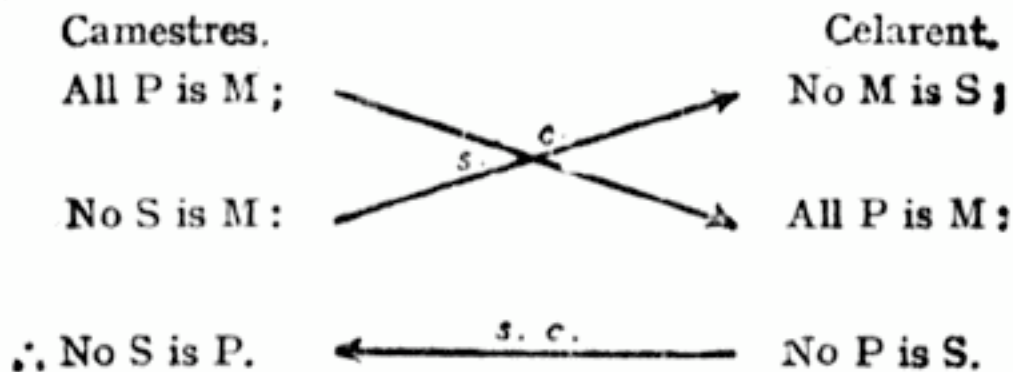
The Ostensive Reduction of Baroco also needs special explanation; for as it used to be reduced indirectly, its name gives no indication of the ostensive process. To reduce it ostensively let us call it Faksnoko, where k means 'obvert the foregoing premise.' By thus obverting (k) and simply converting (s) (in sum, contrapositing) the major premise, and obverting the minor premise, we get a syllogism in Ferio, thus:

Baroco or Faksnoko.
 All P is M ;   ———— *contrap* ————→ No m (not-M) is P ;

 Some S is not M : ———— *obv.* ————→ Some S is m (not-M) :
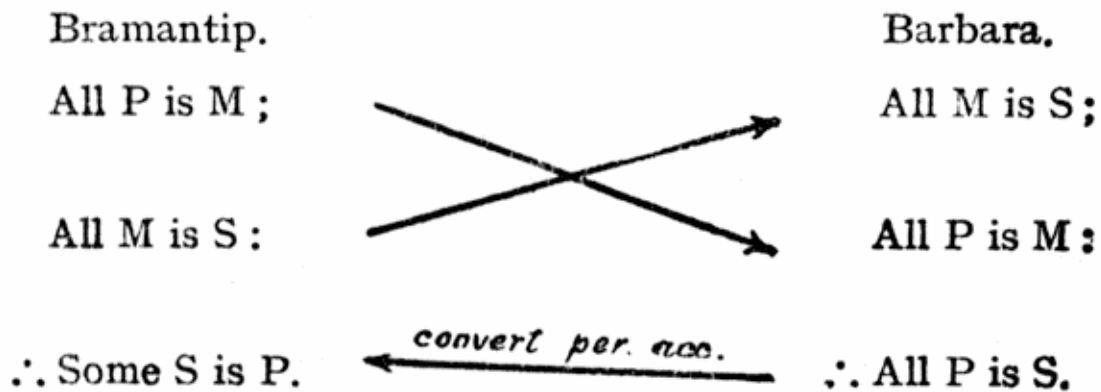∴ Some S is not P.   ∴ Some S is not P.

Ferio.

In Fig. III. the middle term is subject of both premises; so that, to reduce its Moods to the First Figure, it may be enough to convert the minor premise. This is the case with Darapti, Datisi, Felapton, and Ferison. But, with Disamis, since the major premise must in the First Figure be universal, we must transpose the premises, and then simply convert the new minor premise; and, lastly, since the major and minor terms have now changed places, we must simply convert the new conclusion in order to verify the old one. Thus:

Disamis.   Darii.
 Some M is P ;   All M is S ;

 All M is S :   Some P is M :

∴ Some S is P. ←———— *s. c.* ———— ∴ Some P is S.

Bocardo, like Baroco, indicates by its name the indirect process. To reduce it ostensively let its name be Doksamrosk, and proceed thus:

Bocardo or Doksamrosk.   Darii.
 Some M is not P ;   All M is S ;

 All M is S :   Some p (not-P) is M :

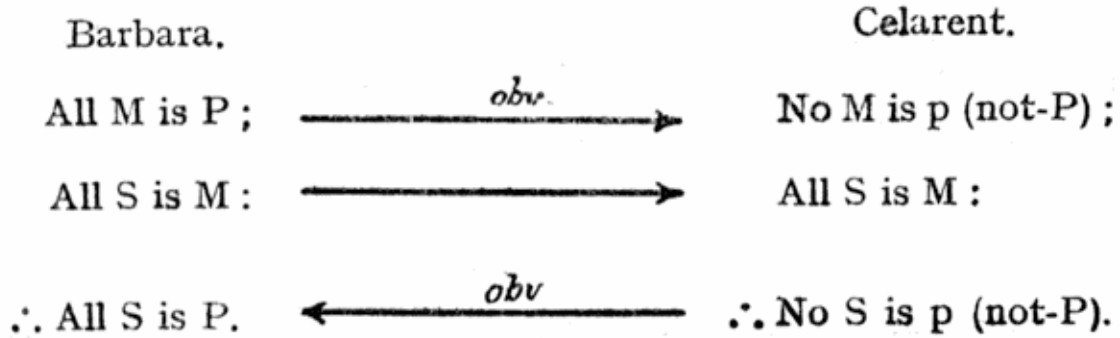∴ Some S is not P. ←—— *convert & obvert.* —— ∴ Some p (not-P) is S.

In Fig. IV. the position of the middle term is, in both premises, the reverse of what it is in the First Figure; we may therefore reduce its Moods either by transposing the premises, as with Bramantip, Camenes, and Dimaris; or by converting both premises, the course pursued with Fesapo and Fresison. It may suffice to illustrate by the case of Bramantip:

Bramantip.

All P is M ;

All M is S :

∴ Some S is P.

convert per. acc.

Barbara.

All M is S ;

All P is M :

∴ All P is S.

This case shows that a final significant consonant (s, p, or sk) in the name of any Mood refers to the conclusion of the new syllogism in the First Figure; since p in Bramantip cannot refer to that Mood's own conclusion in I.; which, being already particular, cannot be converted per accidens.

Finally, in Fig. I., Darii and Ferio differ respectively from Barbara and Celarent only in this, that their minor premises, and consequently their conclusions, are subaltern to the corresponding propositions of the universal Moods; a difference which seems insufficient to give them rank as distinct forms of demonstration. And as for Barbara and Celarent, they are easily reducible to one another by obverting their major premises and the new conclusions, thus:

Barbara.   **Celarent.**

All M is P ; ——————*obv.*——————▶ No M is p (not-P) ;

All S is M : ——————————————▶ All S is M :

∴ All S is P. ◀——————*obv*—————— ∴ No S is p (not-P).

There is, then, only one fundamental syllogism.

Section 7. A new version of the mnemonic lines was suggested in Mind No. 27, with the object of (1) freeing them from all meaningless letters, (2) showing by the name of each Mood the Figure to which it belongs, (3) giving names to indicate the ostensive reduction of Baroco and Bocardo. To obtain the first two objects, l is used as the mark of Fig. I., n of Fig II., r of Fig. III., t of Fig. IV. The verses (to be scanned discreetly) are as follows:

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    Balala,  Celalel, Dalii,   Felioque prioris:
                {Faksnoko}
    Cesane,  Camenes, Fesinon, {Banoco,} secundÃ_:
    Tertia,  Darapri, Drisamis,Darisi, Ferapro,
    Doksamrosk    }, Ferisor habet:  Quarta insuper
addit.
    Bocaro   }
    Bamatip, Cametes, Dimatis, Fesapto, Fesistot.
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De Morgan praised the old verses as "more full of meaning than any others that ever were made"; and in defence of the above alteration it may be said that they now deserve that praise still more.

Section 8. Indirect reduction is the process of proving a Mood to be valid by showing that the supposition of its invalidity involves a contradiction. Take Baroco, and (since the doubt as to its validity is concerned not with the truth of the premises, but with their relation to the conclusion) assume the premises to be true. Then, if the conclusion be false, its contradictory is true. The conclusion being in O., its contradictory will be in A. Substituting this A. for the minor premise of Baroco, we have the premises of a syllogism in Barbara, which will be found to give a conclusion in A.,
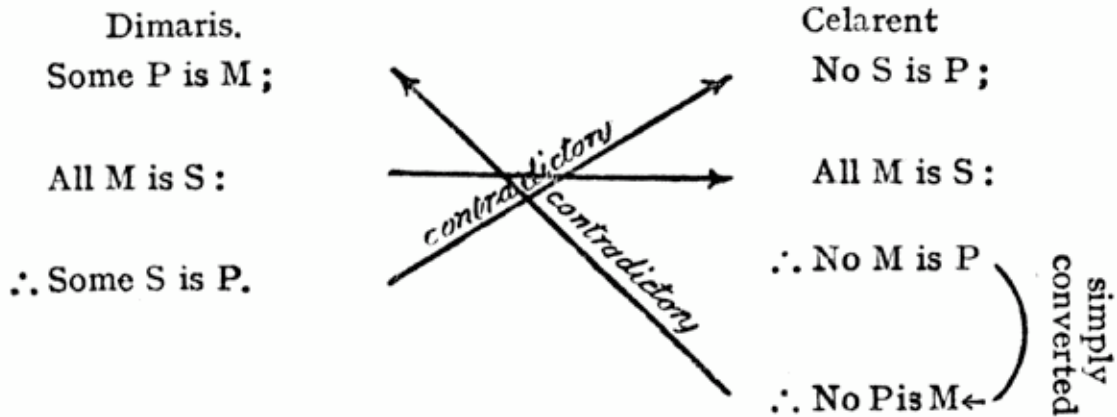
contradictory of the original minor premise; thus:

Baroco.		Barbara.
All P is M ;		All P is M ;
Some S is not M :		All S is P :
∴ Some S is not P		All S is M.
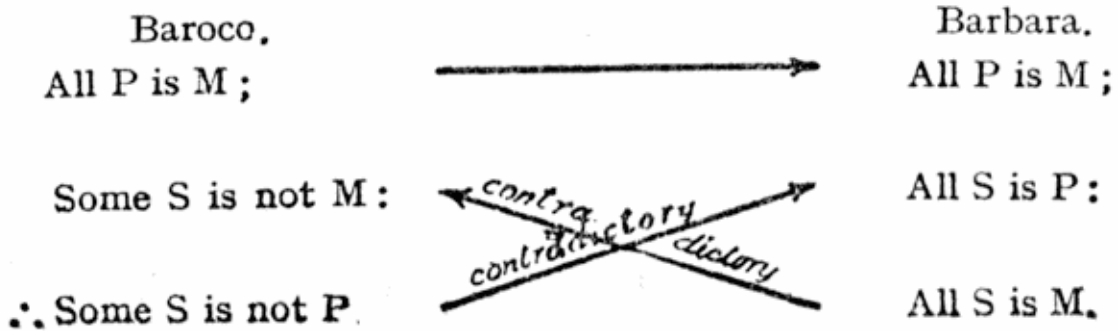
But the original minor premise, Some S is not M, is true by hypothesis; and therefore the conclusion of Barbara, All S is M, is false. This falsity cannot, however, be due to the form of Barbara, which we know to be valid; nor to the major premise, which, being taken from Baroco, is true by hypothesis: it must, therefore, lie in the minor premise of Barbara, All S is P; and since this is contradictory of the conclusion of Baroco Some S is not P, that conclusion was true.

Similarly, with Bocardo, the Indirect Reduction proceeds by substituting for the major premise the contradictory of the conclusion; thus again obtaining the premises of a syllogism in Barbara, whose conclusion is contradictory of the original major premise. Hence the initial B in Baroco and Bocardo: it points to a syllogism in Barbara as the means of Indirect Reduction (Reductio ad impossibile).

Any other Mood may be reduced indirectly: as, for example, Dimaris. If this is supposed to be invalid and the conclusion false, substitute the contradictory of the conclusion for the major premise, thus obtaining the premises of Celarent:

The conclusion of Celarent, simply converted, contradicts the original major premise of Dimaris, and is therefore false. Therefore the major premise of Celarent is false, and the conclusion of Dimaris is true. We might, of course, construct mnemonic names for the Indirect Reduction of all the Moods: the name of Dimaris would then be Cicari.

Section 9. The need or use of any Figure but the First has been much discussed by Logicians. Since, in actual debate, arguments are rarely stated in syllogistic form, and, therefore, if reduced to that form for closer scrutiny, generally have to be treated with some freedom; why not always throw them at once into the First Figure? That Figure has manifest advantages: it agrees directly with the Dictum; it gives conclusions in all four propositional forms, and therefore serves every purpose of full affirmation or denial, of showing agreement or difference (total or partial), of establishing the contradictories of universal statements; and it is the only Figure in which the subject and predicate of the conclusion occupy the same positions in the premises, so that the course of argument has in its mere expression an easy and natural flow.

Still, the Second Figure also has a very natural air in some kinds of negative arguments. The parallelism of the two premises, with the middle term as predicate in both, brings out very forcibly the necessary difference between the major and minor terms that is involved in their opposite relations to the middle term. P is not, whilst S is, M, says Cesare: that drives home the conviction that S is not P. Similarly in Camestres: Deer do, oxen do not, shed their

horns. What is the conclusion?

The Third Figure, again, furnishes in Darapti and Felapton, the most natural forms of stating arguments in which the middle term is singular: Socrates was truthful; Socrates was a Greek: .'. Some Greek was truthful.

Reducing this to Fig I., we should get for the minor premise, Some Greek was Socrates: which is certainly inelegant. Still, it might be urged that, in relation to proof, elegance is an extraneous consideration. And as for the other advantage claimed for Fig. III.–that, as it yields only particular conclusions, it is useful in establishing contradictories against universals–for that purpose none of its Moods can be better than Darii or Ferio.

As for Fig. IV., no particular advantage has been claimed for it. It is of comparatively late recognition (sometimes called the 'Galenian,' after Galen, its supposed discoverer); and its scientific claim to exist at all is disputed. It is said to be a mere inversion of Fig. I.; which is not true in any sense in which Figs. II. and III. may not be condemned as partial inversions of Fig. I., and as having therefore still less claim to recognition. It is also said to invert the order of thought; as if thought had only one order, or as if the order of thought had anything to do with Formal Logic. Surely, if distinction of Figure be recognised at all, the Fourth Figure is scientifically necessary, because it is inevitably generated by an analysis of the possible positions of the middle term.

Section 10. Is Reduction necessary, however; or have not all the Figures equal and independent validity? In one sense not only every Figure but each Mood has independent validity: for any one capable of abstract thinking sees its validity by direct inspection; and this is true not only of the abstract Moods, but very frequently of particular concrete arguments. But science aims at unifying knowledge; and after reducing all possible arguments that form categorical syllogisms to the nineteen Moods, it is another step in the same direction to reduce these Moods to one form. This is the very nature of science: and, accordingly, the efforts of some Logicians to expound separate principles of each Figure seem to be supererogatory. Grant that they succeed; and what can the next

step be, but either to reduce these principles to the Dictum, or the Dictum and the rest to one of these principles? Unless this can be done there is no science of Formal Logic. If it is done, what is gained by reducing the principles of the other Figures to the Dictum, instead of the Moods of the other Figures to those of the first Figure? It may, perhaps, be said that to show (1) that the Moods of the second, third, and fourth Figures flow from their own principles (though, in fact, these principles are laboriously adapted to the Moods); and (2) that these principles may be derived from the Dictum, is the more uncompromisingly gradual and regular method: but is not Formal Logic already sufficiently encumbered with formalities?

Section 11. Euler's diagrams are used to illustrate the syllogism, though not very satisfactorily, thus:
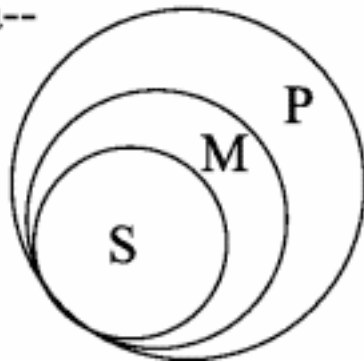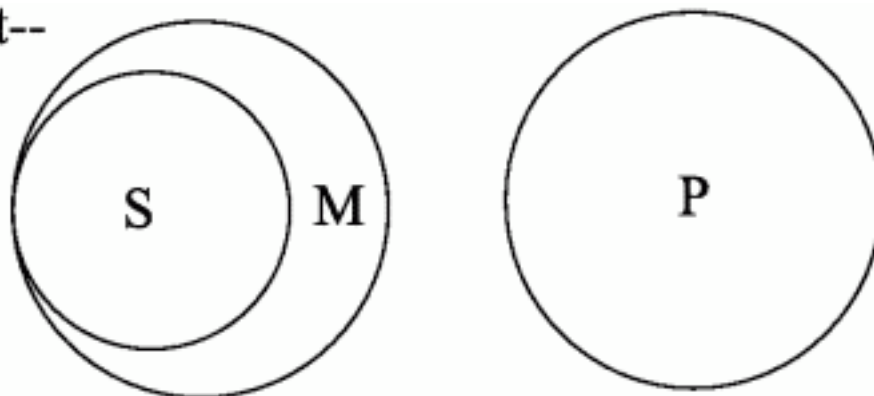
Barbara--

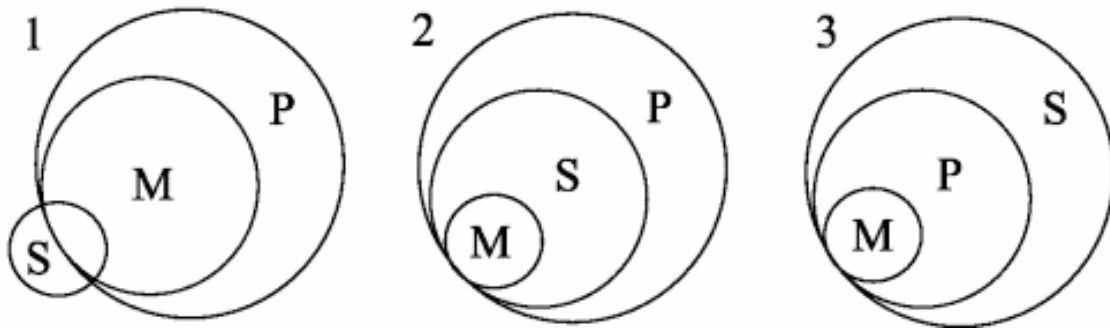Fig. 5.

Celarent--

Fig. 6.

Darii--

Fig. 7.

Remembering that 'Some' means 'It may be all,' it is plain that any one of these diagrams in Fig. 7, or the one given above for Barbara, may represent the denotative relations of P, M and S in Darii; though no doubt the diagram we generally think of as representing Darii is No. 1 in Fig. 7.

Remembering that A may be U, and that, therefore, wherever A occurs there may be only one circle for S and P, these syllogisms may be represented by only two circles, and Barbara by only one.
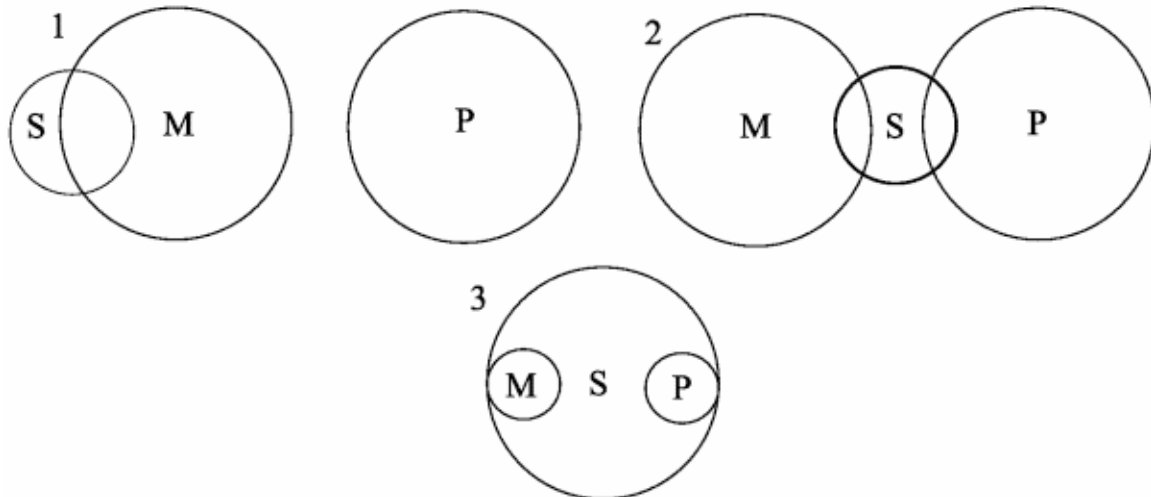
Ferio--

Fig. 8.

Here, again, probably, we generally think of No. 1 as the diagram

representing Ferio; but 2, or 3, or that given above for Celarent, is compatible with the premises.

If instead of dealing with M, P, and S, a concrete example be taken of Darii or Ferio, a knowledge of the facts of the case will show what diagram is suitable to it. But, then, surely it must be possible to do without the diagram. These diagrams, of course, can be used to illustrate Moods of the other Figures.