

Event Horizon Telescope Memo Series

EHT Memo 2017-CE-02

Calibration & Error Analysis WG

A conceptual overview of single-dish absolute amplitude calibration

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Sept 15, 2017 - Version 1.0

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Abstract

This document presents an outline of common single-dish calibration techniques and key differences between centimeter-wave and millimeter-wave observatories in naming schemes and measured quantities. It serves as a conceptual overview of the complete single-dish amplitude calibration procedure for the Event Horizon Telescope, using the Submillimeter Telescope (SMT) as the model station.

Note: This document is not meant to be used as a general telescope guide or manual from an engineering perspective. It contains a number of common approximations used at observatories as an attempt to reason through the methods used and the specific calibration information needed to calibrate VLBI amplitudes from Event Horizon Telescope observing runs. This document can be used in conjunction with similar calibration outlines from other stations for procedural comparisons.

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Relevant terminology

Relevant variables introduced in this document (brightness temperatures approximated with the Rayleigh-Jeans approximation):

- *C*_{hot}: Counts measured when looking at the hot load (vane)
- C_{cold}: Counts measured when looking at the cold load (liquid nitrogen)
- *C*_{on}: Counts measured observing a target
- C_{sky} : Counts measured when looking at blank sky
- T_{cold} : temperature of the cold load
- $T_{\rm rx}$: receiver noise temperature
- *T*_{amb}: ambient temperature around the observatory, as measured by a weather station (physical temperature)
- *T*_{sky}: temperature of the atmospheric emission (the brightness temperature of the sky)
- *T*_{cab}: physical temperature of the receiver cabin (this is assumed to be the same as the ambient temperature)
- T_{cal} : derived temperature to give a correct temperature scale for the signal band
- *T*_{inject}: injected known temperature (of calibrator or noise diode) in the signal chain
- T_{sys} : system noise temperature of the system
- *T*^{*}_{sys}: effective system noise temperature (corrected for atmospheric attenuation)

Efficiency and correction terms:

- r_{sb} : sideband ratio since the SMT has a sidebandseparating receiver, $r_{sb} = \frac{g_i}{g_s} \ll 1$ since no signal comes from the image band but some leakage can still be present
- AM: amount of airmass in the line of sight of the receiver (elevation-dependent)
- au_0 : atmospheric opacity at the zenith
- $e^{-\tau}$: atmospheric attenuation factor, which damps the signal based on atmospheric opacity in the line of sight $\tau = \tau_0 \times AM$
- el: elevation of the antenna dish for a particular observation (in degrees)
- g(el): elevation-dependent gain curve correcting for changing illumination of the main reflector and ground contributions as the dish moves and tilts to different elevations
- η₁: forward efficiency representing the fraction of power received through the forward atmosphere (accounting for rearward losses)
- η_{taper} : efficiency loss due to non-uniform illumination of the aperture plane by the tapered radiation pattern
- η_{block} : aperture blockage efficiency due to blocking of the feed by the sub-reflector (including its support legs)
- $\eta_{\text{spillover}}$: feed spillover efficiency past the main reflector it is the ratio of the power intercepted by the reflective elements to the total power
- η_{Ruze} : surface error efficiency (or Ruze loss) calculated from Ruze's formula (Ruze 1952)
- η_A: aperture efficiency approximated for the SMT, a combination of various efficiencies (= η_{taper} × η_{block} × η_{spillover} × η_{Ruze})
- Ageom: geometric area of the SMT dish
- A_{eff} : effective area of the SMT dish (= $\eta_A A_{geom}$)

1 Introduction to standard singledish T_{svs} calibration techniques

The following is an outline of the different calibration procedures for cm-wave and mm-wave observatories and the different quantities they output. The equations provided here contain various approximations commonly used but are not exact from an engineering perspective. They are only meant to serve as guidelines for a quick understanding of the outputs of the two different techniques.

1.1 The antenna-based system-equivalent flux densities (SEFDs)

A telescope's system-equivalent flux density (SEFD) is simply the noise contribution of the system, given by the system noise temperature, and all losses and gains, converted to a flux density scale. The SEFDs can be calculated using system noise temperature T_{sys} measurements and all efficiencies and contributions to source attenuation and noise, and one can determine the sensitivity of the telescope when compared to other telescopes in the array. The higher a telescope's SEFD, the lower its sensitivity. Ultimately, the flux density of a source is simply the telescope's SEFD, which contains all system and telescope parameters and efficiencies, multiplied by the ratio of signal to noise power (defined as r_{SN}) of the source detection. The equation for the SEFD can be subdivided into three main components, each with station-based variations for how they are determined and measured. The three components to the SEFD are:

- 1. T_{sys} : the total noise characterization of the system, given by the system noise temperature
- 2. $\frac{e^{r}}{\eta_{1}}$: the correction terms for attenuation of the source signal by the atmosphere and rearward losses (ohmic losses, rearward spillover and scattering) of the telescope
- 3. *G*: The antenna gain, including all the loss terms from the telescope and the conversion from a temperature scale (K) to a flux density scale (Jansky), given by the "degrees per flux density unit" factor (DPFU) in K/Jy and the normalized elevation-dependent gain curve g(el): $G = DPFU \times g(el)$

This gives the following general equation for a telescope's

SEFD:

$$\text{SEFD} = \frac{T_{\text{sys}}e^{\tau}}{\eta_{\text{I}}G} \tag{1}$$

The flux density of a source detected with a given ratio of signal to noise power r_{SN} is then:

$$S_{\text{source}} = \text{SEFD} \times r_{\text{s/N}} = \frac{r_{\text{s/N}} \times T_{\text{sys}}e^{\tau}}{\eta_{1}G}$$
 (2)

For mm-observatories, which measure the effective system noise temperature $T_{\text{sys}}^* = T_{\text{sys}} \frac{e^{\tau}}{\eta_1}$ directly using the chopper technique (explained in the next section), the SEFD equation can be rewritten in only two components, the effective system noise temperature and the antenna gain:

$$SEFD = \frac{T_{sys}^*}{G}$$
(3)

For the SMT, the SEFD at zenith is of order 13 000 Jansky.

1.2 The receiver noise temperature

1.2.1 Two-load (hot and cold) calibration

During a two-load calibration (also called cold calibration), the Y-factor and the receiver noise temperature are measured using voltage or counts measurements with a hot and a cold load. In principle the receiver noise temperature can be estimated from T_{sys} measurements at very low opacities ($\tau \ll 1$) by extrapolating a linear fit of airmass versus T_{sys} to zero airmass. However, it is highly recommended to measure a receiver noise temperature at least once an observing night, as this yields more accurate T_{sys} measurements rather than backtracking in post-processing. The Y-factor is calculated with the following:

$$Y = \frac{C_{\text{hot}}}{C_{\text{cold}}},\tag{4}$$

where the numerator is C_{hot} , the counts obtained from the hot load and the denominator is C_{cold} , the counts obtained from the cold load. The Y-factor also enables an easy diagnostic of the sensitivity of the receiver. A high Y-factor means little receiver noise, and thus sensitive observations (of course what constitutes "high" depends on the type of receiver and the observing frequency).

Then the receiver noise temperature is determined as follows:

$$T_{\rm rx} = \frac{T_{\rm hot} - YT_{\rm cold}}{Y - 1} \tag{5}$$

Here temperatures are used, where $T_{\rm hot}$ is the temperature of the hot load (for the SMT this is near room temperature ~ 290K) and $T_{\rm cold}$ is the cold load temperature (for the SMT this is the temperature of liquid nitrogen ~ 77K).

1.3 The effective system noise temperature

A lot of confusion comes from mixtures of complicated calibration documents for different types of calibrations. This section is an attempt to approximately explain two of the common techniques for T_{sys} measurements (chopper wheel common for mm-telescopes, direct for cm-telescopes), what they output and what they mean for data processing. The following outlines are modelled for an SMT-like telescope, thus with a sideband-separating (SSB) receiver.

1.3.1 Clarification of system temperature jargon

We define the system noise temperature as the contributions by the receiver and the sky to a source measurement (assuming T_{CMB} is negligible), where η_{l} is the forward efficiency, accounting for rearward efficiency loss due to ohmic losses, rear spillover and scattering:

$$T_{\rm sys} = T_{\rm rx} + T_{\rm sky} = T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l}e^{-\tau})$$
(6)

Before entering the atmosphere, the source signal is defined as Sig = T_{source} . After attenuation by the atmosphere, the signal becomes Sig= $\eta_1 e^{-\tau} T_{\text{source}}$, where the exponential is the atmospheric attenuation factor (τ is the opacity in the line of sight) and η_1 embodies rearward efficiency losses. Therefore, the ratio of signal to noise power of a telescope must depend on this received signal (not taking into account ground and ambient contributions):

$$r_{\rm s/N} = \frac{\eta_{\rm l} e^{-\tau} T_{\rm source}}{T_{\rm sys}} = \frac{\eta_{\rm l} T_{\rm source}}{e^{\tau} T_{\rm sys}}$$
(7)

We thus define the **effective system noise temperature** T^*_{sys} :

$$T_{\rm sys}^* = \frac{e^{\tau}}{\eta_{\rm l}} T_{\rm sys} = T_{\rm rx} \frac{e^{\tau}}{\eta_{\rm l}} + T_{\rm atm} (\frac{e^{\tau}}{\eta_{\rm l}} - 1)$$
(8)

The effective system temperature is the best description of the sensitivity of a telescope: the system sensitivity drops rapidly (exponentially) as opacity increases.

1.3.2 Direct (switched noise diode) method

This method is commonly used at cm-observatories, such as the VLBA. The system noise temperature is obtained using a known source or a switched noise diode with a known temperature placed in the signal chain. The equation is the following, where C_{sky} represents the counts on blank sky, so only receiver noise and sky contribute, and $C_{on,cal}$ represents counts on the calibrator (or diode), such that the signal contains the source, the receiver and the sky. T_{inject} is the temperature of the diode or the brightness temperature of the source (known for common calibrators), which turns the counts scale to a temperature scale. When the telescope is pointed at blank sky in the calibration procedure, without the source signal, the temperature contribution is entirely noise from the receiver and the atmospheric emission, and thus is the system noise temperature T_{sys} :

$$T_{\rm off,cal} = T_{\rm rx} + T_{\rm sky} = T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l}e^{-\tau})$$
 (9)

When the telescope is pointed at the calibrator (or diode) of a known brightness (or physical) temperature, the source signal is **added** to the temperature contribution:

$$T_{\rm on,cal} = T_{\rm rx} + T_{\rm sky} + T_{\rm inject}$$
(10)

The system temperature is then determined in the following way:

$$T_{\text{sys}} = \frac{C_{\text{sky}}}{C_{\text{on,cal}} - C_{\text{sky}}} T_{\text{inject}}$$
(11)
$$= \frac{T_{\text{off,cal}}}{T_{\text{on,cal}} - T_{\text{off,cal}}} T_{\text{inject}}$$

$$= \frac{(T_{\text{rx}} + T_{\text{sky}})}{(T_{\text{rx}} + T_{\text{sky}} + T_{\text{inject}}) - (T_{\text{rx}} + T_{\text{sky}})} T_{\text{inject}}$$

$$= \frac{(T_{\text{rx}} + T_{\text{sky}})}{T_{\text{inject}}} T_{\text{inject}} = T_{\text{rx}} + T_{\text{sky}}$$
(12)

Since the brightness temperature of the source observed (or the diode temperature) is determined outside the atmosphere, the system noise temperature calculated with this method does not include effects on sensitivity due to atmospheric attenuation (e^{τ} term). This is because the contribution of the source or diode is added to the signal chain (as opposed to the chopper technique that blocks everything but the receiver noise, explained and derived in the next section). This method does not provide an effective system temperature directly, only the receiver and sky contributions to the noise (which cannot be disentangled from each other). In order to obtain the effective system temperature, opacity measurements during observations must be obtained. This is done either by using water vapor radiometers (or tipping radiometers) or by using the telescope as a tipper using sky tips. Tipping radiometers are notoriously unreliable (although water vapor radiometers perform very well), and sky tips must be done very often (every 10 min) and take up valuable observing time. This makes this method highly cumbersome for frequencies at which the atmosphere cannot be neglected.

The effective system temperature is thus:

$$T_{\rm sys}^* = \frac{e^{\tau}}{\eta_{\rm l}} T_{\rm sys},\tag{13}$$

such that the effective (opacity-corrected) antenna temperature of a source (where C_{on} is the telescope signal on target) can be given using ON-OFF measurements as:

$$T_{\rm A}^{*} = \frac{C_{\rm on} - C_{\rm sky}}{C_{\rm sky}} T_{\rm sys}^{*} = \frac{T_{\rm on} - T_{\rm off}}{T_{\rm off}} T_{\rm sys}^{*} = \frac{e^{\tau}}{\eta_{\rm l}} T_{\rm A} \quad (14)$$

1.3.3 Chopper (or single load) calibration

The chopper (or single load) calibration technique is commonly used by (sub)mm observatories. The system noise temperature is obtained by placing an ambient temperature load T_{hot} that has properties similar to a blackbody in front of the receiver, blocking everything but the receiver noise. As long as $T_{atm} \sim T_{hot}$, this method automatically compensates for rapid changes in mean atmospheric absorption.

For calibration of source measurements, we want to obtain the effective sensitivity of the system, not a comparison between the receiver and sky contributions to noise. Therefore, we want to obtain the effective system noise temperature T_{sys}^* to calibrate source measurements.

To first order, the chopper method directly measures T_{sys}^* . This is obtained via the following equation:

$$T_{\rm sys}^* = T_{\rm hot} \frac{C_{\rm sky}}{C_{\rm hot} - C_{\rm sky}} = T_{\rm rx} \frac{e^{\tau}}{\eta_{\rm l}} + T_{\rm atm} (\frac{e^{\tau}}{\eta_{\rm l}} - 1),$$
 (15)

where C_{sky} is the voltage/count signal on blank sky and τ is the opacity in the line of sight.

How does the chopper technique directly provide T_{sys}^* ? This is shown simply by investigating the exact output by the chopper technique. The chopper system temperature equation is given in telescope counts, where C_{hot} are the counts measured when the blocker/chopper/vane is in place, and C_{sky} is our usual blank sky counts. In terms of temperatures, the temperature contribution when the blocker is in place T_{block} is defined as:

$$T_{\text{block}} = T_{\text{rx}} + T_{\text{hot}},\tag{16}$$

where T_{hot} is the temperature of the hot load itself. The load completely blocks the sky emission, which changes the calibration equations from the direct (or diode) calibration method. As seen in the direct method, the blank sky contribution is simply the system noise temperature:

$$T_{\rm off} = T_{\rm rx} + T_{\rm sky} = T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l}e^{-\tau})$$
(17)

We can thus write the chopper equation (eq. 15) in terms of temperatures:

$$T_{\rm sys}^* = T_{\rm hot} \frac{C_{\rm sky}}{C_{\rm hot} - C_{\rm sky}} = T_{\rm hot} \frac{T_{\rm off}}{T_{\rm block} - T_{\rm off}}$$
(18)

$$= T_{\rm hot} \frac{T_{\rm rx} + T_{\rm sky}}{(T_{\rm rx} + T_{\rm hot}) - (T_{\rm rx} + T_{\rm sky})}$$
(19)

We assume the hot load is at ambient temperature, and so $T_{\text{hot}} = T_{\text{amb}} = T_{\text{atm}}$. This gives:

$$T_{\rm sys}^* = T_{\rm hot} \frac{T_{\rm rx} + T_{\rm sky}}{(T_{\rm rx} + T_{\rm hot}) - (T_{\rm rx} + T_{\rm sky})}$$
(20)

$$= T_{\rm atm} \frac{T_{\rm rx} + T_{\rm sky}}{(T_{\rm rx} + T_{\rm atm}) - (T_{\rm rx} + T_{\rm sky})}$$
(21)

As we have defined $T_{sky} = T_{atm}(1 - \eta_l e^{-\tau})$, we can simplify:

$$T_{\rm sys}^* = T_{\rm atm} \frac{T_{\rm rx} + T_{\rm sky}}{(T_{\rm rx} + T_{\rm atm}) - (T_{\rm rx} + T_{\rm sky})}$$
(22)

$$= T_{\text{atm}} \frac{T_{\text{rx}} + T_{\text{atm}}(1 - \eta_{\text{l}}e^{-\tau})}{(T_{\text{rx}} + T_{\text{atm}}) - (T_{\text{rx}} + T_{\text{atm}}(1 - \eta_{\text{l}}e^{-\tau}))}$$
(23)

$$= T_{\rm atm} \frac{T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l}e^{-\tau})}{T_{\rm atm} - T_{\rm atm} + \eta_{\rm l}e^{-\tau}T_{\rm atm}}$$
(24)

$$= T_{\rm atm} \frac{T_{\rm rx} + T_{\rm atm} (1 - \eta_l e^{-\tau})}{\eta_l e^{-\tau} T_{\rm atm}}$$
(25)

$$=\frac{T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l}e^{-\tau})}{\eta_{\rm l}e^{-\tau}}$$
(26)

Finally we obtain:

$$T_{\rm sys}^* = T_{\rm hot} \frac{C_{\rm sky}}{C_{\rm hot} - C_{\rm sky}} = \frac{T_{\rm rx} + T_{\rm atm}(1 - \eta_{\rm l} e^{-\tau})}{e^{-\tau}}$$
(27)

$$= T_{\rm rx} \frac{e^{\tau}}{\eta_{\rm l}} + T_{\rm atm} (\frac{e^{\tau}}{\eta_{\rm l}} - 1) = \frac{e^{\tau}}{\eta_{\rm l}} (T_{\rm rx} + T_{\rm sky})$$
(28)

If we compare the chopper effective system noise temperature to the system temperature from the direct method:

$$T_{\rm sys}^* = \frac{T_{\rm sys}}{\eta_{\rm l} e^{-\tau}} \tag{29}$$

To first order, the chopper calibration (or alternatively named the single-load calibration) corrects for atmospheric attenuation of an observed source and rearward losses of the telescope by directly measuring T_{sys}^* . It is also worth noting that during VLBI observing, the quarter wave-plate is added to the signal chain to convert linear to circular polarization: any losses associated with the addition of the wave-plate will be automatically calibrated and included in the T_{sys}^* measurement from the chopper technique in the same way as the atmospheric and rearward losses.

1.4 Getting a flux density

We have defined the antenna temperature (modified for measured quantities at a telescope) as:

$$T_{\rm A}^* = \frac{C_{\rm on} - C_{\rm sky}}{C_{\rm sky}} T_{\rm sys}^*$$
(30)

To get a flux density, we must correct for the aperture efficiency η_A (determined through different loss terms or planet flux measurements) and gain curve g(el) as a function of elevation of the telescope and convert from a temperature scale to a flux density scale (where *k* is the Boltzmann constant), dependent on the geometric area of the dish A_{geom}:

$$S = \frac{T_{\rm A}^*}{\eta_{\rm A}g(\rm el)} \frac{2k}{A_{\rm geom}}$$
(31)

The equation above is then the final expression to obtain a flux density for a given source. If we expand using all the terms we've discussed, we get the following:

$$S = \frac{T_{\rm A}^*}{\eta_{\rm A}g(\rm el)} \frac{2k}{A_{\rm geom}} = \frac{1}{\eta_{\rm A}g(\rm el)} \frac{C_{\rm on} - C_{\rm sky}}{C_{\rm sky}} T_{\rm sys}^* \frac{2k}{A_{\rm geom}}$$
(32)

$$= \frac{1}{\eta_{A}g(\text{el})} \frac{C_{\text{on}} - C_{\text{sky}}}{C_{\text{sky}}} \frac{T_{\text{sys}}}{\eta_{1}e^{-\tau}} \frac{2k}{A_{\text{geom}}}$$
(33)

$$= \frac{T_{\rm on} - T_{\rm off}}{T_{\rm off}} \frac{T_{\rm sys}}{\eta_{\rm A}g({\rm el})\eta_{\rm l}e^{-\tau}} \frac{2k}{A_{\rm geom}}$$
(34)

Now the flux density is rewritten also in terms of a system noise temperature determined with the direct method.

We can subdivide the flux density equation into three major parts:

1. The ratio of signal to noise power of the observed source as measured by the telescope (thus attenuated by the atmosphere):

$$r_{\rm s/N} = \frac{T_{\rm on} - T_{\rm off}}{T_{\rm off}}$$

- 2. The total noise characterization of the system, including the correction term for atmospheric absorption, given by the effective system noise temperature: $T_{sys}^* = \frac{e^{\tau}}{m} T_{sys}$
- 3. The antenna gain *G*, including all the loss terms from the telescope and the conversion from a temperature scale (K) to a flux density scale (Jansky), given by the "degrees per flux density unit" factor (DPFU) and the gain curve:

$$DPFU = \frac{\eta_A A_{geom}}{2k} \text{ giving } G = DPFU \times g(el)$$

We can thus simplify the flux density equation using the three main terms actually measured by the SMT:

$$S = \frac{r_{\rm s/N} \times T_{\rm sys} e^{\tau}}{\eta_{\rm l} G} = \frac{r_{\rm s/N} \times T_{\rm sys}^*}{G}$$
(35)

1.4.1 Determining a gain curve

As previously mentioned, the characterization of the antenna gain G is subdivided into two quantities that must be separately provided for the calculation of the SEFDs: the gain curve g(e) and the DPFU (explained in the next section). The characterization of the telescope's geometric (opacity-free) gain curve is an important part of the flux density calibration, and is particularly crucial for the EHT a priori amplitude calibration due to the low-elevation observations of some science targets (including Sgr A*) for the northern hemisphere stations.

Telescopes do not have perfect surfaces, and must thus suffer some losses of signal due to distorted illumination of the main reflector as they slowly move to different elevations. This large-scale surface deformation affects the received signal and is not taken into account in the measurements leading to the efficiency and DPFU characterization. These losses can be determined by tracking sources through a wide range of elevations, and thus measure an elevationdependent gain curve for the telescope, where the maximum (g = 1) is set where the telescope is expected to be most efficient. The source measurements used to obtain a gain curve must of course be calibrated for all other effects, including telescope efficiency (through the DPFU) and the atmospheric attenuation of the signal (through T_{sys}^{s}). At the SMT, this is done by observing two sources (usually K3-50 and W75N, a planetary nebula and a star-forming region, due to their similar up-time plots and wide range of elevation) contiguously, tracked as they increase and decrease in elevation from the tree-line to transit and vice-versa.

The gain curve is estimated by fitting a polynomial (usually second-order for standard radio-dishes). If more than one source is used, this is done once the flux density measurements are normalized around a plateau (to a relative gain scale). This normalized gain curve must be written in the form of a second order polynomial (in the standard VLBA format for ANTAB), where 'el' is the elevation in degrees:

$$g(el) = a_2(el)^2 + a_1(el) + a_0$$
 (36)

Each parameter must not be rounded to the uncertainties of the fit but instead many significant figures should be provided. Uncertainties for each parameter as outputted by the polynomial fit must also be provided, along with the full covariance matrix of the fit parameters. This will help determine an error estimate for the gain curve and propagate to the error estimation of the final SEFDs. Additionally, a plot of the relative gains (normalized fluxes) versus elevation and the fitted polynomial should be provided if possible, as shown in Fig. 1.



Figure 1: Example of a normalized geometric gain curve plot.

1.4.2 Determining the DPFU

The degrees per flux density unit (or DPFU) is the characterization of the temperature to flux density scale of a telescope. The DPFU is used to calibrate the telescope measurements to a flux density scale and is obtained using known flux calibrators, particularly planets, or by bootstrapping near-in-time observations of non-planet sources from telescopes with well-defined and accurate flux density measurements. This enables to check the flux density scale obtained by the telescope by directly measuring an aperture efficiency.

The DPFU is estimated with the following equation, where $k = 1.38 \times 10^{-23}$ J/K = 1.38×10^3 Jy/K:

$$DPFU = \frac{\eta_A A_{geom}}{2k} [K/Jy]$$
(37)

The geometric area A_{geom} is simply the area of the dish, where *D* is the dish diameter:

$$A_{\text{geom}} = \frac{\pi D^2}{4} \tag{38}$$

The aperture efficiency is the most difficult part of the estimation of the DPFU. It represents the efficiency of the telescope compared to a telescope with a perfect collecting area (uniform illumination, no blockage or surface errors) and it is determined using observations of known calibrator sources, usually planets. The observed planet fluxes are then compared to expected planet brightness temperatures from a planet simulation software for a perfect telescope at the given frequency and beam width.

The aperture efficiency η_A is found using the following equation, where T_A^* is the observed effective antenna temperature, g(el) is the telescope gain curve, k is the Boltzmann constant, A_{geom} is the geometric area of the telescope and $S_{beam,sim}$ is the expected flux density of the planet in the telescope beam from the simulation program used:

$$\eta_{\rm A} = \frac{2k}{A_{\rm geom}} \frac{T_{\rm A}^*}{g({\rm el}) \, S_{\rm beam,sim}} \tag{39}$$

Or similarly the DPFU is directly given by:

$$DPFU = \frac{T_A^*}{g(el) S_{beam,sim}}$$
(40)

For extended sources, it is important to calibrate the flux density observed in the beam because some emission might not be picked up by the telescope. The aperture efficiency is only concerned by the main beam flux density, and so the following equation is used to calibrate the simulated flux density in the beam for an extended source, where S_{sim} is the expected total flux density of the source:

$$S_{\text{beam,sim}} = S_{\text{sim}} \times K \tag{41}$$

Here *K* is the following, where θ_{mb} is the half-power beamwidth in arcseconds of the primary lobe of the telescope beam pattern (telescope beam diameter) and θ_s is the diameter in arcseconds of the observed extended source, usually given by the simulation program:

$$x = \frac{\theta_{\rm s}}{\theta_{\rm mb}} \sqrt{\ln(2)} \tag{42}$$

$$K = \frac{1 - e^{-x^2}}{x^2} \tag{43}$$

This *K* factor is the ratio of the beam-weighted source solid angle and the solid angle of the source on the sky. It is in fact the integral of the antenna pattern of the telescope (approximated as a normalized gaussian) $P(\theta, \phi) = e^{-\ln 2(2\theta/\theta_{mb})^2}$ and a disklike source with a uniform brightness distribution $\Psi(\theta, \phi) = 1$ over the size of the extended source. This serves very well for our a priori calibration purposes¹.

$$K = \frac{\Omega_{\text{sum}}}{\Omega_{\text{s}}} = \frac{1}{\Omega_{\text{s}}} \int_{\text{source}} P(\theta - \theta', \phi - \phi') \Psi(\theta', \phi') d\Omega' \quad (44)$$

$$K = \frac{1}{\Omega_{\rm s}} \int_{\rm source} P(\theta - \theta', \phi - \phi') d\Omega'$$
(45)

To minimize the number of approximations used by different planet simulation softwares, the expected total flux density can be estimated by:

$$S_{\rm sim} = \frac{2h}{c^2} \frac{v^3 \Omega_{\rm s}}{e^{\frac{hv}{kT_{\rm B}}} - 1},$$
 (46)

where v is the observing frequency in Hz, h is the Planck constant, c is the speed of light (in m/s), $T_{\rm B}$ is the mean brightness temperature for the planet (assuming a disk of uniform temperature) from the simulation program, and $\Omega_{\rm s}$ is the solid angle of the source on the sky in steradians. Since we are dealing with very small objects, the latter can be approximated using the small angle approximation, where θ_s is the apparent diameter in radians of the planet observed:

$$\Omega_{\rm s} \simeq \frac{\pi \theta_{\rm s}^2}{4} \tag{47}$$

Of course this process heavily depends on assumptions made in the planet calibration, such as accurate predicted planet brightness temperatures from available software, telescope beam width used, stable weather conditions and a well-calibrated instrument in terms of pointing and focus.

An average value for the aperture efficiency can be estimated from the individual measurements during a particular observing run, but it is preferable to keep the timedependence of the variable if a telescope's efficiency is expected to vary with temperature and sunlight, causing systematic differences in the telescope performance between day-time and night-time observing.

Even more preferable, a plot of long-term trends of the aperture efficiency, using additional measurements outside EHT observing or even from previous years, would greatly help understand the time-dependent nature of the aperture efficiency of a particular telescope. As the scatter between individual measurements can be caused by various factors, such as unstable weather or changing pointing/focus accuracy, it is not always representative of the true aperture efficiency change in the observations. A trend exhibited in the long-term as a function of time would be more reliable to estimate an aperture efficiency for a particular scan. Such a plot is shown in Fig. 2, as an example from the JCMT.



Figure 2: Example of long-term trend for the time-dependent aperture efficiency η_A .

¹More detail on this method in Calibration of spectral line data at the IRAM 30m radio telescope by C. Kramer.

If a UT time-dependence is found for a particular station, a fit for this dependence must be provided, as well as the covariance matrix for the fit parameters, for error analysis of the a priori deliverables. A fit to the UT time-dependence would be the most robust against various observing effects from day to day and session to session and should be very stable over the years, provided no major work has been done on the telescope. For telescopes with no visible timedependence, a mean aperture efficiency (or DPFU) will suffice, with the appropriate error estimate.

We can also write the aperture efficiency as the combination of various individual forward efficiencies, each closely approximated for the telescope via various measurements:

$$\eta_{\rm A} = \eta_{\rm taper} \times \eta_{\rm block} \times \eta_{\rm spillover} \times \eta_{\rm Ruze} \tag{48}$$

Each efficiency term corresponds to an aspect of the telescope feed ²:

- η_{taper} is the efficiency loss due to non-uniform illumination of the aperture plane by the tapered radiation pattern/feed function (also formally known as the illumination efficiency). It is the most important contributor to the aperture efficiency.
- η_{block} is the aperture blockage efficiency due to blocking of the feed by the sub-reflector (including its support legs)
- $\eta_{\text{spillover}}$ is the feed spillover efficiency past the main reflector it is the ratio of the power intercepted by the reflective elements beyond the edge of the sub-reflector and primary to the total power. It is due partly to cold sky and partly to a warm background, and is elevation-dependent.
- η_{Ruze} is the surface error efficiency (also called "Ruze loss" or scattering efficiency) calculated from Ruze's formula (Ruze 1952). It is due to small scale, randomly distributed deviations of the reflector from the perfect paraboloidal shape. Ruze's formula is presented below, where σ is the surface rms (accounting for small-scale deviations from a perfect surface through dish holography) and λ is the observing wavelength:

$$\eta_{\text{Ruze}}(\lambda) = e^{\frac{-16\pi^2 \sigma^2}{\lambda^2}} \tag{4}$$

In summary, the aperture efficiency accounts for all forward losses of the telescope, which come from different contributions. As previously mentioned in section 1.3.3, the chopper technique itself account for the rearward losses of the telescope automatically. These losses are also outlined in the following section.

1.4.3 Other efficiencies

The main beam efficiency of a telescope is the fraction of observed power in the main lobe of the telescope beam pattern. Let the beam solid angle (the full antenna pattern) be Ω_A and the main beam solid angle (the main lobe) be Ω_{mb} . The main beam efficiency is written as the ratio between the total beam and main beam solid angles:

$$\eta_{\rm mb} = \frac{\Omega_{\rm mb}}{\Omega_{\rm A}} \tag{50}$$

It is estimated with the following, where $T_{\rm mb}$ is the main beam temperature of a source that fills the main beam, as estimated from the simulation program:

$$\eta_{\rm mb} = \frac{S_{\rm beam,sim}}{T_{\rm mb}} \frac{\eta_{\rm A} A_{\rm geom}}{2k}$$
(51)

It should be noted that the main beam efficiency is not the same as the aperture efficiency and should not be used to determine telescope DPFUs and SEFDs.

The forward efficiency η_1 represents the fraction of power received through the forward atmosphere (in other terms it is the coupling of the receiver to the cold sky) and is written as the ratio between the solid angle over the forward hemisphere of the telescope and the beam solid angle and it is typically close to unity (but drops with frequency due to loss of receiver sensitivity):

$$\eta_{\rm l} = \frac{\Omega_{2\pi}}{\Omega_{\rm A}} \tag{52}$$

The only way to estimate it is via sky-dips, by measuring the atmospheric emission with elevation:³

$$T_{\rm A}({\rm el}) = \eta_{\rm l} T_{\rm atm} (1 - e^{-\tau/\sin({\rm El})}) + (1 - \eta_{\rm l}) T_{\rm amb}$$
 (53)

It is important to note that sky-dips measure both the atmospheric opacity and the forward efficiency so they need to be disentangled. Fortunately, this is not an issue for the EHT because the chopper technique implicitly corrects for the 49) forward efficiency η_1 (see Section 1.3.3).

²More detail on the measurement of the different losses, see Baars, J., *The paraboloidal reflector antenna in radio astronomy*, Springer, 2007. ³Overview in: Kramer, C., *Millimeter Calibration*, presentation at IRAM Summer School 2013, IRAM, Granada, Spain.

2 Miscellaneous explanations

2.1 VLBI and the Event Horizon Telescope array

2.1.1 Determining the antenna-based SEFD for VLBI

The SEFD needed for calibration of single-dish on-off observations and that for VLBI are identical. The equation for the antenna-based SEFD for VLBI observations is thus:

$$SEFD = \frac{T_{sys}^*}{G} = \frac{T_{sys}^*}{DPFU \times g(el)}$$
(54)

It is important to note that the SEFD contains corrections for system noise, atmospheric absorption, antenna gain terms and temperature-to-Jansky conversion.

2.1.2 A brief overview of a priori amplitude calibration

For VLBI observations, there are very few suitable calibrators that do not become resolved on some baselines, thus we cannot use the primary calibrator scaling to calibrate VLBI amplitudes. An alternative approach is to calibrate the VLBI amplitudes using the system temperatures and collecting areas of the individual antennas. The visibility amplitudes can be calibrated in units of flux density by multiplying the normalized visibility amplitudes by the geometric mean of the SEFDs of the two antennas concerned (TMS Section 10.1.). On a baseline between two telescopes, for example the SMT and the LMT, which both use the chopper method, the amplitude calibration for the correlated source signal $r_{\rm corr,SMT-LMT}$ (compensated for digitization and sampling losses) on that baseline is given by:

$$S_{\text{SMT-LMT}} = \sqrt{\text{SEFD}_{\text{SMT}} \times \text{SEFD}_{\text{LMT}} r_{\text{corr},\text{SMT-LMT}}},$$
 (55)

where SEFD_{SMT} and SEFD_{LMT} are determined as shown above and $S_{\text{SMT-LMT}}$ is then the source signal in Jansky on that baseline.

Since the SEFDs for the telescopes are expected to include the effective system noise temperature, which corresponds to a signal plane above the atmosphere, then the resulting visibility amplitudes will be corrected for atmospheric losses.

2.1.3 Double-sideband (DSB) receivers

It is worth noting that the equations presented in the previous sections for amplitude calibration are modeled after the SMT, which has a sideband-separating receiver. However, a few stations in the Event Horizon Telescope array have double-sideband (DSB) receivers, which lead to some modifications of the equations for amplitude calibration. The most relevant difference between SSB and DSB receiver is the handling of measured signals. For an SSB receiver, all the measured signal comes from only one sideband, but for a DSB receiver it comes from two sidebands folded together into one single larger band, usually used for spectral-line observing. However, for continuum VLBI with the EHT, only one sideband of the DSB receiver systems is used as the signal sideband and gets correlated, but the rest of the telescope continues to operate as a DSB system. Therefore, the sensitivity of the measurements during EHT observing (through one sideband) is about a factor of two lower than the normal operation of the telescope as a perfect DSB system. This rescaling of the telescope sensitivity from two sidebands to one is done by correcting T_{sys}^* .

For a measured effective system temperature from a perfect DSB system $T^*_{\text{sys,DSB}}$, the actual effective system temperature for VLBI observing with only one sideband is:

$$T_{\rm sys}^* = 2T_{\rm sys,DSB}^* \tag{56}$$

For EHT observing we use half the number of sidebands, thus the telescope sensitivity must drop by a factor of two, leading to the effective system temperature increasing by the same factor. However, if the telescope does not have a perfect DSB system but one sideband has more gain than the other, then the equation becomes, more generally:

$$T_{\rm sys}^* = (1 + r_{\rm sb}) T_{\rm sys,DSB}^*,$$
 (57)

where the sideband ratio (r_{sb}) is the ratio of source signal power in the remaining sideband to the signal power in the sideband of interest (the sideband to be correlated). For a perfect DSB system, the gains of each sideband are equal, giving $r_{sb} = 1$, which gives back Eq. 56. For a perfect SSB system, where all signal is in one sideband, $r_{sb} = 0$ and this gives back simply T_{sys}^* needed for the EHT. Once this correction is applied to T_{sys}^* , the rest of the amplitude calibration process remains the same. For planet scans to determine the telescope's DPFU, the signal is collected by both sidebands in a DSB system, thus the effective antenna temperature is usually measured in DSB mode. This is sufficient to reflect the conversion from Kelvin to Jansky within the aperture efficiency and DPFU estimation. It should be noted that the correction from a DSB system to an SSB system for VLBI should only be done on T_{sys}^* , otherwise the resulting SEFDs would be double-corrected for a DSB system.

2.2 Telescopes not using the chopper technique

As explained above, the result for telescopes like the SMT and the LMT, which both use the chopper (or single-load) technique, is very clean and simple. Now what happens when there is a telescope in the array that does not use the chopper technique but instead uses the direct (or noise diode) method?⁴

In that case, on baselines with telescopes with the chopper method, there will be inconsistencies in the amplitude calibration if the same corrections are applied in post-processing to both stations on that particular baseline. This is precisely because the chopper technique gives T_{sys}^* and the direct method only gives T_{sys} .

Fortunately, as explained in the previous section, the relationship between the two is well-understood and T_{sys}^* can easily be determined from the direct method using opacity measurements. If the telescope has a tipping radiometer or water vapor radiometer nearby measuring opacities, this can give a fairly good estimate for $T_{sys}^* = e^{\tau}T_{sys}$.

However, some aspects of radiometers hinder this approach:

- The radiometer does not always point in the same direction as the telescope, thus under a varying or partly cloudy sky the opacities from the radiometer are not entirely accurate to the observations.
- The radiometer can have something blocking and corrupting the measurements (as on Mt Graham due to the LBT)
- The radiometer does not always measure an opacity

at the observing frequency but instead is converted (sometimes not so accurately) from a different frequency

Another possible solution is to use the telescope itself as a tipper: using the dish to observe blank sky through a big elevation range in the direction of observing to determine the relationship between elevation and sky temperature and get an estimate of the zenith opacity.

This tipping method solves the radiometer issues of getting an opacity in the direction of observing and at the right observing frequency. However, these tipping scans are required very frequently, every 10 minutes or so, and take up valuable observing time just to get accurate opacities.

An alternative is then to obtain opacities using approximations in post-processing. The system noise temperature, as measured in the direct method, is defined as seen previously. For $\tau_0 \ll 1$, we can approximate:

$$T_{\rm sys} \approx T_{\rm rx} + T_{\rm atm}(1 - e^{-\tau}) \approx T_{\rm rx} + T_{\rm atm} \times \tau_0 \times AM$$
 (58)

By fitting a least-squares (or as it is done for the GMVA, a linear fit to the lower envelope) of T_{sys} as a function of airmass, the extrapolation of the fit will give an approximation for the receiver noise temperature T_{rx} . If the telescope frequently does a dual-load (cold cal) calibration to refresh values for the receiver temperature, these values are usually more accurate to use.

With this linear relationship (or measured T_{rx}) and every variable but the sky opacity known, measured or approximated by the telescope, we can get the sky opacity at the zenith and thus correct the system noise temperature for the atmospheric attenuation:

$$\tau_0 = -\frac{1}{\mathrm{AM}} \ln \left(1 - \frac{T_{\mathrm{sys}} - T_{\mathrm{rx}}}{T_{\mathrm{atm}}} \right) \tag{59}$$

2.3 T_{sys} or T_{sys}^* ?

A crucial part of the amplitude calibration process is to determine which variables are actually provided by each telescope in the context of the entire EHT array. Are all

⁴As far as the EHT is concerned, there are no stations in the array at this time without the chopper technique. However, this information could be potentially useful for the a priori calibration of GMVA or HSA observations related to EHT, which are a mixture of mm- and cm-observatories.

telescopes providing T_{sys} like cm-observatories do? Or are some telescopes providing in fact T_{sys}^* but labeling it as T_{sys} (as is commonly done by mm-observatories)? Discrepancies in notation and a heavy background knowledge in the context of cm-observatories can cause misunderstandings of the calibration information provided by the telescopes. However, there is a nice way to do a quick check in postprocessing⁵.

In order to visually understand the difference between the two variables, simulated measurements of system noise temperatures for the SMT are presented. Using the standard chopper equation, the calibration temperature was approximated to $T_{cal} = T_{amb} = 280$ K, the receiver noise temperature was set to $T_{rx} = 60$ K, and we have used a constant zenith opacity of $\tau_0 = 0.2$, common for the SMT, for consistency. Figure 3 shows the effective system noise temperature T_{sys}^* using the chopper technique equation and the direct method system noise temperature $T_{sys} \approx e^{-\tau}T_{sys}^*$ as a function of airmass. It is clear that both temperatures indeed do vary with airmass, but T_{sys}^* is a lot more sensitive because in addition it corrects for the increasing attenuation of a signal from outside the atmosphere, automatically determined with the chopper technique.

It is a misconception to assume that because T_{sys} does not contain that term, it does not vary with airmass. T_{sys} is inherently dependent on airmass because the sky brightness temperature T_{sky} , representing atmospheric noise, increases with airmass as the telescope looks through a larger layer of atmosphere. This effect is also present in T_{sys}^* , which, in addition, corrects for the increasing signal attenuation that is also elevation (airmass) dependent.

In the previous section, we introduced a useful tool to tell the two system noise temperatures apart. For the case of T_{sys} , as shown by eq. 59, it is possible to untangle a zenith opacity from T_{sys} and T_{rx} measurements. In the case of T_{sys}^* , because the opacity relationship is much more complicated, it would not be valid. If we were to apply eq. 59 using T_{sys}^* instead of T_{sys} , the opacities at zenith obtained would be highly inaccurate when compared to, for example, radiometer measurements during the same observing window.

This reasoning was thus applied to the SMT to see if what was previously called " T_{sys} " was really T_{sys} . For example, we can take the system noise temperatures and receiver temperature measured during the gain curve measurements for 2017. These were measured in the lapse of a few hours, thus minimizing opacity fluctuations due to changing weather.



Figure 3: Simulated system temperatures from the chopper and direct method calibration techniques show divergence as a function of airmass.



Figure 4: Zenith opacities obtained by assuming the telescope provides T_{sys} are much larger than what is actually measured by the SMT tipper.

We use the opacity equation:

$$\tau_0 = -\frac{1}{\mathrm{AM}} \ln \left(1 - \frac{T_{\mathrm{sys}} - T_{\mathrm{rx}}}{T_{\mathrm{atm}}} \right) \tag{60}$$

Recall that at this point, what is plugged in as T_{sys} is what is measured by the chopper technique (in fact it is T_{sys}^* but this was still unknown).

The zenith opacities obtained from that equation were then compared to the measured zenith opacities by the tipping radiometer on the telescope scan-by-scan. These results are presented in Fig.4. It is clear that the zenith opacities obtained from " T_{sys} " are completely different, incredibly high and inconsistent with the tipper measurements. This is of course because " T_{sys} " is in fact T_{sys}^* , which diverges and is increasingly larger than T_{sys} as a function of airmass. Thus the zenith opacity equation does not work for what is outputted by the chopper technique at the SMT, and this output is definitely not simply T_{sys} but something much

⁵This check only works if the telescopes provide an elevation for each " $T_{\rm sys}$ ", or alternatively these can be extracted from the VLBI Monitor database

more sensitive to opacity: T_{sys}^* . For a telescope that is genuinely providing T_{sys} , the opacity equation would give results much more in-line with the measured opacities from its tipper/radiometer.

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