Section 1.2. Subgraphs

Note. We complete our very brief survey of graph theory terms.

Definition. A *subgraph* of graph X is a graph Y such that $V(Y) \subseteq V(X)$ and $E(Y) \subseteq E(X)$. If $V(Y) = V(X)$ then Y is a *spanning subgraph* of X. A subgraph Y of X is an *induced subgraph* if two vertices of $V(Y)$ are adjacent in Y if and only if they are adjacent in X. See Figure 1.3.

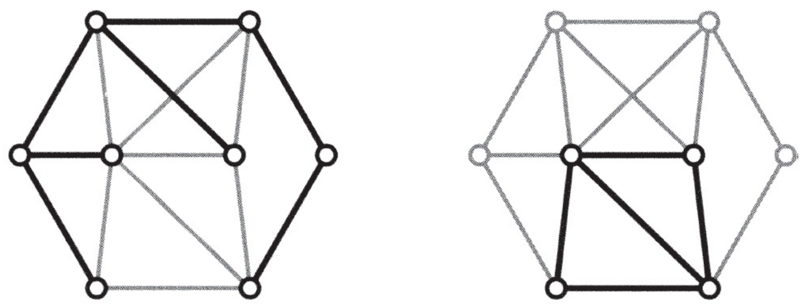

Figure 1.3. A spanning subgraph and an induced subgraph of a graph

Definition. A subgraph of a given graph that is complete is called a *clique*. A set of vertices that induces an empty subgraph is an *independent set*. The size of the largest clique in a graph X is denoted by $\omega(X)$, and the size of the largest independent set is denoted by $\alpha(X)$.

Definition. A *path* of length r from x to y in a graph is a sequence of $r + 1$ distinct vertices starting with x and ending with y such that consecutive vertices are adjacent. If for any two distinct vertices in graph X there is a path from one vertex to the other then graph X is *connected*, otherwise X is *disconnected*.

Note. A graph X is disconnected if and only if we can partition its vertices into two nonempty sets, R and S, such that no vertex in R is adjacent to a vertex in S. This is proved in Exercise 3.1.4 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics #244 (2008, Springer).

Definition. If graph X is disconnected such that nonempty sets R and S partition the $V(X)$ where no vertex of R is adjacent to a vertex in S, then X is the (edge) *disjoint union* of the two subgraphs induced be R and S. A maximal connected induced subgraph of graph X is a *connected component* of X.

Definition. A *cycle* is a connected graph where every vertex has exactly two neighbors. An *acyclic graph* is a graph which contains no cycles. A connected acyclic graph is a *tree* (an acyclic not necessarily connected graph is called a *forest*). A spanning subgraph containing no cycles is a *spanning tree*. A *maximal spanning forest* in graph X is a spanning subgraph consisting of a spanning tree from each connected component.

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