DOCUMENT RESUME

ED 090 309 TM 003 597

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TITLE	The Effect of Multiple Fallible Covariables in Analysis of Covariance and Two Correction Methods.
PUB DATE	74
NOTE	22p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, Illinois, April, 1974)
EDRS PRICE	MF-$0.75 HC-$1.50 PLUS POSTAGE
DESCRIPTORS	*Analysis of Covariance; *Mathematical Models; Research Problems; *Statistical Analysis; *Statistical Bias; *Tests of Significance; True Scores

ABSTRACT

When random assignment has been accomplished and an analysis of covariance (ANCOVA) is being used to correct for initial differences among treatment groups, use of unreliable covariables not only decreases the power of ANCOVA, but also causes ANCOVA to test biased treatment effects. Several correction procedures have been suggested for the single fallible covariable design. The purpose of this paper is to extend the earlier work by describing two alternative correction procedures for the multiple fallible covariable design, demonstrate their properties in terms of population parameters, and investigate empirically the sampling distributions of their test statistics, i.e., probability of Type I error and power. (Author)

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THE EFFECT OF MULTIPLE FALLIBLE COVARIABLES IN ANALYSIS OF COVARIANCE AND TWO CORRECTION METHODS

TM 003 592

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One of the assumptions underlying classical analysis of covariance (ANCOVA) with random covariables is that the covariables are observed free from errors of measurement. When random assignment of experimental units to levels of the treatment independent variable is an aspect of the experimental design, failure to meet the perfectly reliable covariables assumption decreases the statistical power of ANCOVA but does not cause it to test biased treatment effects. When random assignment is not an aspect of the design, however, and ANCOVA is being used to correct for initial differences among treatment groups on the covariables, use of less than perfectly reliable covariables not only decreases the power of ANCOVA, but also causes ANCOVA to test biased treatment effects. Several correction procedures have been suggested for the single fallible covariable design. The purpose of the proposed paper is to extend the earlier work by describing two alternative correction procedures for the multiple fallible covariable design, demonstrate their properties in terms of population parameters, and empirically investigate the sampling distributions of their test statistics, i.e., probability of Type I error and power.

Before we proceed, an important caveat is necessary. In our opinion there is no perfectly acceptable solution to the problem of estimating causal relationships from quasi or naturally occurring experiments. Perhaps ANCOVA is a useful procedure in some situations, but it is clearly limited by the case and ingenuity used in selecting the covariables. The problem we are addressing is not how to select a useful set of covariables or even whether that task can ever be accomplished. Rather, we are concerned with the effects of errors of measurement in situations where a useful set of covariables has been identified.

The paper first presents a brief review of past work on the single fallible covariable problem. Next, the effects of errors of measurement in multiple linear regression are incorporated into the multiple covariable model. Finally, two proposed solutions to the multiple fallible covariable problem are described and their properties investigated, first analytically and then via a Monte Carlo study.

Review of ANCOVA with Fallible Covariable

That ANCOVA tests biased treatment effects when there are initial differences on a random fallible covariable can be seen through inspection of the null hypothesis for a one-way ANCOVA. The null hypothesis can be stated

$$\sum_{i=1}^{I} [\alpha_{Y_i} - \beta_{Y.X}\alpha_{X_i}]^2 = 0,$$

where α_{Y_i} is the i^{th} treatment effect on the dependent variable Y, α_{X_i} is the i^{th} treatment effect on the random fallible covariable X, and $\beta_{Y.X}$ is the least squares pooled within i slope of the regression of Y on X. Although errors of measurement satisfying classical measurement assumptions do not cause bias in the estimation of α_{Y_i} and α_{X_i}, they do cause a bias in using the least squares regression coefficient as an estimate of the regression coefficient defined on the latent true variables, i.e., the structural relationship of Y on X. The bias of the least squares regression coefficient for estimating the structural relationship is a function of the reliability of the random covariate and can be stated as

$$\beta_{Y'.X'} = 1/\rho_{XX}\beta_{Y.X},$$

where primes denote latent true variables and ρ_{XX} denotes the reliability of X.

For a fallible covariable the effect of using the least squares regression coefficient in ANCOVA is a function of the values of α_{X_i}. If as in a random assignment design the α_{X_i}'s are all zero, ANCOVA will test the desired hypothesis. For quasi experiments, however, the α_{X_i}'s are typically not zero, and so ANCOVA tests biased estimates of the adjusted treatment effects. The bias can result in either a spurious rejection of the null hypothesis or a spurious retention of the null hypotheses, depending upon the values of the α_{X_i}'s (Porter, 1967).

Although the bias of the least squares regression coefficient for estimating the structural relation has been recognized since 1878 (Adcock, 1878), the information was not considered in work on ANCOVA until as late as 1960 (Lord, 1960). Lord was first to consider the problem, and proposed a large sample solution which is restricted to a two group design with two observations on a single fallible covariable. Building on Lord's work, Porter proposed an estimated true scores solution which at least in theory is not limited by the complexity of the design to be analyzed (Porter, 1969; Porter and Chibucos, 1974), but is limited to a single covariate and does require an estimate of the reliability of the covariable. Briefly, estimated true scores ANCOVA is computationly identical to traditional ANCOVA with the single exception that estimated true scores are used as a substitute covariable (Porter and Chibucos, 1974). A Monte Carlo investigation of the two procedures for the single covariate two group design indicated that both were equally satisfactory (Porter, 1967). In the same study, the utility of estimated true scores ANCOVA was also demonstrated for a one way layout with four treatment groups.

More recently DeGracie (1968) has proposed a solution quite similar to estimated true scores ANCOVA, and cited the above mentioned Monte Carlo investigation as support for the utility of his test statistic. Stroud (1972) has also proposed a solution for the two group case with a single fallible covariable and argued that it is readily extendable to more complex designs. As yet, however, no small sample distributional investigations have been done on the Stroud statistic. We conclude our review by noting the apparent lack of work on the multiple fallible covariable problem.

ANCOVA with Multiple Fallible Covariables

Unfortunately the problem of multiple fallible covariables in ANCOVA is more complex. Consider the null hypothesis for a one-way ANCOVA with two random fallible covariables

$$\sum_{i=1}^{I} [\alpha_{Y_i} - \beta_X \alpha_{X_i} + \beta_Z \alpha_{Z_i}]^2 = 0,$$

where again the α's denote treatment effects on the dependent variable Y and the covariables X and Z, and β_X and β_Z are the pooled within i least squares regression coefficients for predicting Y from X and Z respectively. As before, the estimates of the α's are left unbiased by the introduction of errors of measurement which follow classical assumptions, but the least squares regression coefficients are biased estimates of the corresponding regression coefficients for the latent true variables. The nature of the bias has been given by Cochran (1968) as

$$\beta_X = \frac{\beta'_X\rho_{XX}(1 - \rho'^2_{XZ}\rho_{ZZ}) + \beta'_Z\beta'_{Z.X}(1 - \rho_{ZZ})\rho_{XX}}{1 - \rho'^2_{XZ}\rho_{XX}\rho_{ZZ}},$$

$$\text{and} \quad \beta_Z = \frac{\beta'_X\beta'_{X.Z}\rho_{ZZ}(1 - \rho_{XX}) + \beta'_Z\rho_{ZZ}(1 - \rho'^2_{XZ}\rho_{XX})}{1 - \rho'^2_{XZ}\rho_{XX}\rho_{ZZ}},$$

where primes denote statistics for the latent true variables, ρ_{XX} and ρ_{ZZ} are the reliabilities of X and Z respectively, and ρ'_{XZ} is the correlation between the latent true X and Z. A useful restatement of the above two expressions in terms of bivariate statistics is

$$\beta_X = \frac{\rho_{XX}\beta'_{Y.X} - \rho_{XX}\rho_{ZZ}\beta'_{Y.Z}\beta'_{Z.X}}{1 - \rho'^2_{XZ}\rho_{XX}\rho_{ZZ}}$$

$$\text{and} \quad \beta_Z = \frac{\rho_{ZZ}\beta'_{Y.Z} - \rho_{XX}\rho_{ZZ}\beta'_{Y.X}\beta'_{X.Z}}{1 - \rho'^2_{XZ}\rho_{XX}\rho_{ZZ}}$$

A solution to the multiple fallible covariable problem requires a procedure that provides unbiased estimates of the regression coefficients defined on the latent true variables. The substitution of estimated true scores for the observed covariables does not adequately solve the general problem, but does provide a solution in the restricted case of uncorrelated latent true covariables.

For uncorrelated latent true covariables

$$\rho'_{XZ} = 0 \text{ and } \beta'_{X.Z} = \beta'_{Z.X} = 0$$

the two regression coefficients become $\beta_X = \rho_{XX}\beta'_{Y.X}$ and $\beta_Z = \rho_{ZZ}\beta'_{Y.Z}$. Applying estimated true scores gives $\beta_{\hat{X}} = \beta_X/\rho_{XX} = \beta'_{Y.X}$ and $\beta_{\hat{Z}} = \beta_Z/\rho_{ZZ} = \beta'_{Y.Z}$.

The point of breakdown for the estimated true scores solution to the general problem provided a suggestion for a new procedure, Method A. Use of estimated true scores corrected bivariate regression coefficients between the dependent variable and each covariable, but left uncorrected the bivariate regression coefficients among the covariables. Thus Method A consisted of 1) substituting estimated true scores for each observed covariable, and 2) correcting for attenuation the relationships between the estimated true scores covariables.

A second approach to the solution of the general problem, Method B, was motivated by the simplified situation which exists for uncorrelated covariables. Method B can be described for two covariables as follows:
1) one covariable is transformed to make it orthogonal to the other;
2) estimated true scores are substituted for the two orthogonal covariables and computations proceed as for regular ANCOVA.

<u>Method A</u>

First, consider the effects of Method A on the pooled within regression coefficients for a one-way ANCOVA having two fallible covariables. Using standard ANCOVA procedures the population regression coefficient for one of the covariables, X, is

$$\beta_X = \frac{SWZ \cdot SWYX - SWXZ \cdot SWYZ}{SWX \cdot SWZ - SWXZ^2} ,$$

where SW denotes a sum of squares within. Substituting estimated true scores for X and Z replaces

$$SWX \text{ with } \rho_{XX}^2 SWX$$

$$SWZ \text{ with } \rho_{ZZ}^2 SWZ$$

$$SWXZ \text{ with } \rho_{XX}\rho_{ZZ} SWXZ$$

$$SWYX \text{ with } \rho_{XX} SWYX$$

$$SWYZ \text{ with } \rho_{ZZ} SWYZ \ ,$$

where ρ_{XX} and ρ_{ZZ} are the sample reliabilities of X and Z respectively. It follows that

$$\beta_{\hat{X}} = \frac{\rho_{ZZ}^2 \rho_{XX} SWZ \cdot SWYX - \rho_{ZZ}^2 \rho_{XX} SWXZ \cdot SWYZ}{\rho_{ZZ}^2 \rho_{XX}^2 SWX \cdot SWZ - \rho_{ZZ}^2 \rho_{XX}^2 SWXZ} \ ,$$

where $\hat{X}$, $\overline{X}_{i.} - \rho_{XX}(Xij - \overline{X}_{i.})$, denotes estimated true scores for X. The expression can be simplified to

$$\beta_{\hat{X}} = \frac{\frac{1}{\rho_{XX}} \frac{SWYX}{SWX} - \frac{1}{\rho_{XX}} \frac{SWXZ}{SWX} \cdot \frac{SWYZ}{SWZ}}{1 - \frac{SWXZ}{SWX} \cdot \frac{SWXZ}{SWZ}} \ .$$

Further correction for attenuation of the relationship between the covariates by dividing SWXZ the square root of the product of the reliabilities of X and Z results in

$$\beta_{\hat{X}} = \frac{\frac{1}{\rho_{XX}} \frac{SWYX}{SWX} - \sqrt{\frac{\rho_{ZZ}}{\rho_{XX}}} \frac{1}{\rho_{XX}} \frac{SWXZ}{SWX} \cdot \frac{1}{\rho_{ZZ}} \frac{SWYZ}{SWZ}}{1 - \frac{1}{\rho_{XX}} \frac{1}{\rho_{ZZ}} \frac{SWXZ}{SWX} \cdot \frac{SWXZ}{SWZ}} \ .$$

By substitution

$$\beta_{\hat{X}} = \frac{\beta'_{Y.X} - \sqrt{\frac{\rho_{ZZ}}{\rho_{XX}}} \beta'_{Z.X}\beta'_{Y.Z}}{1 - \rho'^2_{XZ}} \ .$$

Thus when $\rho_{ZZ} = \rho_{XX}$, $\beta_{\hat{X}}$ is equal to the regression coefficient for the latent true X. Similar steps result in the parallel conclusion that the regression coefficient $\beta_{\hat{Z}}$, provided by Method A, is identical to the regression coefficient for the latent true Z when $\rho_{ZZ} = \rho_{XX}$. Since substitution of estimated true scores does not change the means of the covariables, it follows that Method A tests the desired hypothesis.

Method B

Method B starts with a transformation on the second covariable that results in a new variable which is orthogonal to the first covariable. The transformation used was

$$W_{ij} = Z_{ij} - \beta_{Z.X} X_{ij} \quad ,$$

where $\beta_{Z.X}$ denotes the pooled within regression coefficient for predicting Z from knowledge of X. It should be noted that for perfectly reliable X and Z, use of covariables X and W does not change the hypothesis tested.

The null hypothesis tested by Method B is

$$\sum_{i=1}^{I} [\alpha_{Y_i} - \frac{1}{\rho_{XX}} \beta_{Y.X} \alpha_{X_i} - \frac{1}{\rho_{WW}} \beta_{Y.W} \alpha_{W_i}]^2 = 0 \quad ,$$

where $\beta_{Y.X}$ and $\beta_{Y.W}$ are bivariate regression coefficients since X and W are uncorrelated. The question is whether this null hypothesis is identical to the desired null hypothesis stated in terms of the latent true variables Y, X, and Z?

Since W is a new variable, its reliability must first be estimated. By definition

$$\rho_{WW} = \frac{var(W')}{var(W)} \quad ,$$

where W' denotes the latent true W. But

$$var(W) = var(Z) + \beta^2_{Z.X} var(X) - 2\beta_{Z.X} cov(X,Z)$$

$$= var(Z)(1 - \rho^2_{XZ}) \quad ,$$

and

$$\text{var}(W') = \text{var}(Z') + \beta^2_{Z.X}\text{var}(X') - 2\beta_{Z.X}\rho_{XZ}\sqrt{\text{var}(Z)\text{var}(X)}$$

$$= \rho_{ZZ}\text{var}(Z) + \rho^2_{XZ}\frac{\text{var}(Z)}{\text{var}(X)}\rho_{XX}\text{var}(X) - 2\rho_{XZ}\sqrt{\frac{\text{var}(Z)}{\text{var}(X)}}\rho_{XZ}\sqrt{\text{var}(X)\text{var}(Z)}$$

$$= \text{var}(Z)(\rho_{ZZ} - 2\rho^2_{XZ} + \rho^2_{XZ}\rho_{XX}) \quad .$$

Thus

$$\rho_{WW} = \frac{\rho_{ZZ} - 2\rho^2_{XZ} + \rho^2_{XZ}\rho_{XX}}{1 - \rho^2_{XZ}} \quad .$$

Further

$$\beta_{Y.W} = \frac{\text{cov}(Y,W)}{\text{var}(W)}$$

$$= \frac{\text{cov}(Y,W) - \beta_{Z.X}\text{cov}(Y,X)}{\text{var}(Z)(1 - \rho^2_{XZ})}$$

$$= \frac{\beta_{Y.Z} - \beta_{X.Z}\beta_{Y.X}}{1 - \rho^2_{XZ}} \quad ,$$

and

$$\mu_W = \mu_Z - \beta_{Z.X}\mu_X \quad .$$

The last two terms in the squared quantity for stating the null hypothesis for Method B can now be restated as

$$-\frac{1}{\rho_{XX}}\beta_{Y.X}(\mu_{X_{i.}} - \mu_{X..}) - \frac{1}{\rho_{WW}}\beta_{Y.W}(\mu_{Z_{i.}} - \beta_{Z.X}\mu_{X_{i.}} - \mu_{Z..} + \beta_{Z.X}\mu_{X..})$$

$$= -(\beta_{Y.X}/\rho_{XX} - \beta_{Y.W}\beta_{Z.X}/\rho_{WW})(\mu_{X_{i.}} - \mu_{X..}) - \frac{\beta_{Y.W}}{\rho_{WW}}(\mu_{Z_{i.}} - \mu_{Z..}) \quad .$$

By further substitution

$$\beta_{Y.X}/\rho_{XX} - \beta_{Y.W}\beta_{Z.X}/\rho_{WW}$$

becomes

$$\frac{\beta'_{Y.X}\left(1 - \frac{2\rho^2_{XZ}}{\rho_{ZZ}} - 2\rho^2_{XZ}\frac{\rho_{XX}}{\rho_{ZZ}}\right) - \beta'_{Y.Z}\beta'_{Z.X}\rho_{XX}}{1 - 2\frac{\rho^2_{XZ}}{\rho_{ZZ}} + \rho^2_{XZ}\frac{\rho_{XX}}{\rho_{ZZ}}},$$

and $\beta_{Y.W}/\rho_{WW}$ becomes

$$\frac{\beta'_{Y.Z} - \beta'_{X.Z}\beta'_{Y.X}\rho_{XX}}{1 - 2\frac{\rho^2_{XZ}}{\rho_{ZZ}} + \rho^2_{XZ}\frac{\rho_{XX}}{\rho_{ZZ}}},$$

where again primes denote statistics for the latent true variables. Since the two expressions do not simplify to the regression coefficient for the latent true X and Z covariables, it follows that Method B does not test the desired hypothesis. In retrospect the error in logic was that the transformation forced the manifest variables to be orthogonal, but not their latent true counterparts.

Monte Carlo Study

Thus far we have considered whether various modifications of ANCOVA test the correct hypothesis when there are multiple fallible covariables in a quasi experiment. The conclusions were: 1) if the latent true covariables are uncorrelated, estimated true scores ANCOVA tests the desired hypothesis; 2) when the latent true covariables are correlated but have equal reliability Method A tests the correct hypothesis, and 3) Method B does not appear to test the hypothesis of interest under any circumstances. The remaining question to be answered was how do the small sample distributions of the various test statistics behave?

A computer program for the CDC 6500 computer system at the Michigan State University Computer Center was written to get empirical F distributions of the estimated true score ANCOVA when two fallible covariates were independent of each other, and of the two proposed correction methods when two fallible covariables were related. All distributions were based on 1000 samples and empirical α's and power were reported for nominal levels of .10, .05, and .01. The number of treatment groups was varied at 2 and

4 in a one-way layout. The sample size per treatment was fixed at 40. The bivariate correlation for each latent true covariate with the dependent variable was fixed at .7 as was the reliability of each covariate. The correlation between the two latent true covariates was varied at .0 and .2. Covariable treatment means were 6.0 and 0.0 for two treatments and 6.0, 3.0, 3.0, and 0.0 for four treatment groups. Finally the non central case was created by adding the value one half standard deviation of the marginal distribution of the dependent variable to each observation on the dependent variable in one treatment group.

A pseudo-random unit normal deviate generator was used to generate observations from a trivariate normal distribution with known parameters. The unit normal generator involved two stages. First, the multiplicative congruent method was used to generate sixteen pseudo-random numbers from a uniform distribution. Second, the sixteen numbers were summed and linearly rescaled to provide a pseudo-random unit normal deviate via the Central Limit Theorem.

Fallible covariables were created by adding random normal deviates, rescaled by the desired standard error of measurement, to the latent true covariate values. Reliabilities for calculating estimated true scores were estimated by first generating two fallible covariable values on each covariate for each unit of analysis, and then correlating the two sets of values for each covariate.

Estimated True Scores ANCOVA with Independent Covariables

The results of the Monte Carlo investigation of estimated true scores ANCOVA for two uncorrelated fallible covariables are provided in Tables 1 through 5. As stated previously the sample size per treatment group was 40, and the correlations of latent true covariables with the dependent variable were each .7 as were the reliabilities of each covariable.

Empirical Type I errors, statistical power, average mean squares, and average adjusted means for the two treatment design are given in Table 1. The Type I error rates for estimated true scores ANCOVA were slightly liberal but within two standard errors for all three nominal values. By contrast the results using latent true covariables were slightly conservative but also within two standard errors. The inappropriateness of using fallible covariables in ANCOVA for quasi experiments was clearly supported by the .999 empirical Type I errors for all three nominal values.

As was expected, use of latent true covariables resulted in substantially greater power than estimated true scores ANCOVA. The difference in power is explained by two factors. First, the multiple correlation of estimated true scores is identical to that for the fallible covariables, while the multiple correlation for the latent true covariables is consistent with a correction for attenuation. Second, the variance of the estimated true scores covariables are equal to the variance of the corresponding fallible covariables multiplied by the respective squared reliabilities. The variance of the latent true covariables, however, are equal to the variance of the corresponding fallible covariables multiplied by the respective reliabilities. The smaller variance of estimated true scores covariables operates to further dampen the power of the procedure.

The slight liberal tendency of estimated true scores ANCOVA reflected also in the average mean squares for the central case, which showed the average mean square between to be slightly larger than the average mean square within. Support for the earlier analytic demonstration that the procedure tests the correct null hypothesis was given by the average adjusted means. For the central case the average adjusted means were -.055 and -.0002 which were very close to the desired values of zero.

Average pooled within regression coefficients and cumulative distributions for the 1000 samples are provided in Table 2. The average coefficients were .703 and .707 for the two covariables which were very close to the .7 value of the population coefficient for the latent true covariables. As was expected the standard errors for the regression coefficients were substantially larger for estimated true scores than for latent true covariables. The reasons are the same as those given previously for the discussion of statistical power.

Tables 3 and 4 contain comparable data for the four treatment group design. Number of treatment groups did not noticeably alter the results for ANCOVA using latent true and fallible covariables. The estimated true scores ANCOVA empirical Type I errors, however, were markedly discrepant from the nominal values, i.e., .177, .109, and .037 for nominal values of .10, .05, and .01 respectively. This was true despite the average adjusted means being quite close to the desired value of zero, i.e., -.021, -.017, -.011, -.0020. Further, the average pooled within regression coefficients, as shown in Table 4, were .703 and .700.

It was hypothesized that the use of sample reliabilities for calculating estimated true scores was the cause of the liberal nature of the F test statistic. Therefore, the simulations were replicated using population reliabilities. The empirical Type I errors and statistical powers for the replications are reported in Table 5. For both the 2 and 4 treatment groups designs the empirical Type I errors for estimated true scores ANCOVA were slightly closer to the nominal values than they had been using estimated reliabilities. For the 4 treatment group design, however, the empirical Type I errors were still quite liberal, i.e., .163, .097, .029 for nominal values of .10, .05, and .01 respectively. The liberalness of the estimated true scores test statistic for the four treatment design is consistent with, but much more pronounced than, that found for a single fallible covariable (Porter, 1967). With a single fallible covariable, however, the empirical Type I error rates were still within the bounds of practical utility, i.e., .111, .058, and .013. These simple fallible covariable results were for the same parameters except sample size which was twenty rather than forty per treatment group.

Methods A and B

The results of the Monte Carlo investigation of Methods A and B for two correlated fallible covariables are presented in Tables 6 and 7. The earlier analytic demonstrations suggested that Method A should test the right hypothesis while Method B should not. Nevertheless, Method B was investigated on the chance that it might have some practical utility. The parameters of the Monte Carlo simulations were as before, with the exception that the latent true covariables had a .2 intercorrelation.

Empirical Type I error, statistical power, average mean squares, and average adjusted means for the two treatment designs are given in Table 6. The average adjusted means for Methods A and B supported our analytic findings. The averages for Method A were in close agreement with the desired zero values, i.e., -.070 and -.002, while the averages for Method B were not, i.e., =1.074 and =.003. Further support for the analytic work is provided in Table 7. The average values fo the regression coefficients for Method A were in close agreement with the desired .58 value, i.e., .586 and .591, while for Method B there was little agreement, i.e., .705 and .640.

Unfortunately the empirical Type I error rates were not within the range of practical utility for either method. The finding for Method B was not surprising, but we had greater hopes for Method A. The too liberal nature of the F test statistic for Method A stemmed from a far too large average adjusted mean square for treatments, i.e., 179.1.

Several modifications of Method A were proposed in an attempt to decrease the adjusted mean square for treatments, none of which resulted in empirical Type I error rates within the bounds of practical utility. One modification was motivated by the argument that the reliability of a covariate for the total sample should be greater than the pooled within treatment reliability. Thus the estimated true scores and correction to the within treatment cross products were calculated using pooled within reliabilities while the correction to the total cross products used the reliabilities for the total sample. The effect was only a minor reduction in the Type I error. Another modification left the total cross products uncorrected, and resulted in a far too conservative F test. Additional modifications involved correcting the relationships between the dependent variables and the covariables for attentuation.

Conclusion

Thus far we have been notably unsuccessful in discovering a solution to the problem of multiple fallible covariables in quasi experiments. For two uncorrelated covariables, estimated true scores ANCOVA appeared to satisfactorily solve the problem for a two group design but not a four group design. Method A tested the correct hypothesis for two correlated fallible covariables of equal reliability, but provided a far too liberal test of that hypothesis. Method B had no useful properties.

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Table 1

Empirical type I error, statistical power, average mean square and average adjusted means for estimated true scores ANCOVA with $t = 2$, $nb = 40$, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .00$, $\mu_{X1} = \mu_{Z1} = 6.0$, $\mu_{XZ} = \mu_{Z2} = \mu_{Z2} = 0$, $\mu_{Y1} = 8.4$, $\mu_{Y2} = 0.0$

	CENTRAL			NON-CENTRAL		
	.10	.05	.01	.10	.05	.01
TRUE COV.	.092	.044	.005	.973	.950	.814
EST. TRUE	.115	.063	.013	.200	.128	.046
FALLIBLES	.999	.999	.999	1.000	1.000	1.000

MEANS SQU.	BETWEEN	WITHIN	BETWEEN	WITHIN
TRUE COV.	.0178(.0006)	.0200(.00001)	.2694(.0203)	.0200(.00001)
EST. TRUE	.3411(.2253)	.3150(.0026)	.5292(.4906)	.3150(.0026)
FALLIBLES	9.3305(13.25)	.3150(.0026)	13.2614(20.38)	.3150(.0026)

ADJ. MEANS	T1	T2	T1	T2
TRUE COV.	-0.002(.018)	-0.0007(.0005)	.497(.018)	*
EST. TRUE	-0.055(.508)	-0.0002(.0115)	.444(.508)	*
FALLIBLES	2.523(.208)	-0.0008(.0086)	3.022(208)	*

Primes denote parameters on latent true variables.

*same as in central case.

Table 2

Empirical cumulative distribution of regression coefficients from estimated true scores ANCOVA with t = 2, nb = 40, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .00$, $\beta'_X = .70$, $\beta'_Z = .70$

	TRUE COV		EST. TRUE		FALLIBLES	
	X	Z	X	Z	X	Z
.50			4	1	572	569
.55			22	21	873	878
.60			92	83	981	975
.65	0	0	258	237	997	998
.70	490	506	515	494	1000	1000
.75	1000	999	746	743		
.80		1000	884	875		
.85			958	956		
.90			985	978		
.95			990	995		
MEAN	.701	.700	.703	.707	.489	.490
VAR	.0003	.0002	.0069	.0065	.0029	.0030

Primes denote parameters on latent true variables.

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Table 3

Empirical type I error, statistical power, average means squares and average adjusted mean for estimated true scores ANCOVA with $t = 4$, $nb = 40$, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .00$, $\mu_{X1} = \mu_{Z1} = 6.0$, $\mu_{X2} = \mu_{X3} = \mu_{Z2} = \mu_{Z3} = 3.0$, $\mu_{X4} = \mu_{Z4} = 0.0$, $\mu_{Y1} = 8.40$, $\mu_{Y2} = \mu_{Y3} = 4.20$, $\mu_{Y4} = 0.0$

	CENTRAL .10	CENTRAL .05	CENTRAL .01	NON-CENTRAL .10	NON-CENTRAL .05	NON-CENTRAL .01
TRUE COV.	.103	.052	.014	1.000	1.000	1.000
EST. TRUE	.177	.109	.037	.785	.706	.813
FALLIBLES	1.000	1.000	1.000	1.000	1.000	1.000

M.S.	CENTRAL BETWEEN	CENTRAL WITHIN	NON-CENTRAL BETWEEN	NON-CENTRAL WITHIN
TRUE COV.	.0205(.0003)	.0198(.000005)	1.0128(.0295)	*
EST. TRUE	.4094(.1132)	.3125(.0013)	1.4050(.7911)	*
FALLIBLES	6.0588(2.969)	.3125(.0012)	9.3968(5.216)	*

ADJ. MEANS	CENTRAL T1	CENTRAL T2	CENTRAL T3	CENTRAL T4	NON-CENTRAL T1	NON-CENTRAL T2	NON-CENTRAL T3	NON-CENTRAL T4
TRUE COV.	.005(.010)	.003(.003)	.003(.003)	-.0006(.0004)	.505(.010)	*	*	*
EST. TRUE	-.021(.237)	-.017(.065)	-.011(.066)	-.0020(.0115)	.479(.237)	*	*	*
FALLIBLES	2.525(.112)	1.259(.032)	1.262(.034)	-.0023(.0077)	3.025(.112)	*	*	*

Primes denote parameters on latent true variables.

*same as in central case.

Table 4

Empirical cumulative distribution of regression coefficient from estimated true scores ANCOVA with $t = 4$, $nb = 40$, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .00$,

$\beta'_X = .70$, $\beta'_Z = .70$

	TRUE COV		EST. TRUE		FALLIBLES	
	X	Z	X	Z	X	Z
.50			0	0	612	600
.55			1	2	953	944
.60			25	29	998	997
.65	0	0	178	196	1000	1000
.70	532	497	490	517		
.75	1000	1000	796	814		
.80			959	943		
.85			991	993		
.90			998	999		
.95			1000	1000		
MEAN	.699	.700	.703	.700	.490	.490
VAR	.00014	.00012	.0031	.0033	.0014	.0014

Primes denote parameters on latent true variables.

Table 5

Empirical type I error and statistical power for estimated true scores ANCOVA using population reliabilities with $nb = 40$, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .00$

	CENTRAL			NON-CENTRAL		
	$\alpha = .10$	$\alpha = .05$	$\alpha = .01$	$\alpha = .10$	$\alpha = .05$	$\alpha = .01$
T = 2						
TRUE COV	.099	.045	.008	.963	.943	.819
EST. TRUE	.100	.056	.012	.219	.127	.032
FALLIBLES	.999	.999	.992	.999	.999	.999
T = 4						
TRUE COV	.098	.045	.007	1.000	1.000	1.000
EST. TRUE	.163	.097	.029	.789	.703	.479
FALLIBLES	1.000	1.000	1.000	1.000	1.000	1.000

Primes denote parameters on latent true variables.

Table 6

Empirical type I error, statistical power, average mean squares and average adjusted means for Method A and Method B with t = 2, nb = 40, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$,

$\rho'_{YZ} = .70$, $\rho'_{XZ} = .20$, $\mu_{X1} = \mu_{Z1} = 6.0$, $\mu_{X2} = 0.0$, $\mu_{Y1} = 6.96$,

$\mu_{Y2} = 0.0$

	CENTRAL			NON-CENTRAL		
	.10	.05	.01	.10	.05	.01
TRUE COV.	.106	.055	.010	.377	.254	.100
METHOD A	1.000	1.000	1.000	1.000	1.000	1.000
METHOD B	.180	.096	.027	.120	.057	.013
FALLIBLES	.980	.956	.845	.998	.996	.979

MS	BETWEEN	WITHIN	BETWEEN	WITHIN
TRUE COV	.188(.066)	.184(.0009)	.507(.313)	*
METHOD A	179.1(945.6)	.429(.0050)	204.56(1205.7)	*
METHOD B	1.242(1.99)	.399(.0043)	.733(.958)	*
FALLIBLES	5.980(9.48)	.398(.0043)	9.369(16.05)	*

ADJ. MEAN	T1	T2	T1	T2
TRUE COV	.015(.155)	-.000(.005)	.515(.155)	*
METHOD A	-.070(.606)	-.002(.014)	.429(.606)	*
METHOD B	.514(.566)	-.003(.015)	.014(.566)	*
FALLIBLES	1.848(.236)	-.001(.011)	2.349(.236)	*

Primes denote parameters for latent true variables.

*same as in central case.

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Table 7

Empirical distribution of regression coefficients for Method A and Method B cumulative with t = 2, nb = 40, $\rho_{XX} = .70$, $\rho_{ZZ} = .70$, $\rho'_{YX} = .70$, $\rho'_{YZ} = .70$, $\rho'_{XZ} = .20$, $\beta'_X = .58$, β'_Z .58

	TRUE COV		METHOD A		METHOD B		FALLIBLES	
	X	Z	X	Z	X	Z	X	Z
.50	53	63	189	160	29	56	881	868
.55	261	248	366	331	76	153	982	975
.60	639	638	564	552	169	353	1000	996
.65	910	912	732	743	316	562		100
.70	993	994	886	876	496	762		
.75	997	1000	961	937	672	874		
.80	1000		986	979	805	943		
.85			995	994	901	981		
.90			997	999	958	995		
.95			999	999	976	997		
MEAN	.581	.582	.586	.591	.705	.640	.427	.431
VARIANCE	.0025	.0025	.0093	.0092	.0129	.0090	.0035	.0037

Primes denote parameters on latent true variables.

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