Jet vs. Jettiness – the event shape in p + A and e + A collisions

Jianwei Qiu

Brookhaven National Laboratory

Based on works by Z. Kang, X. Liu, S. Mantry, J. Qiu, …
D. Kang, C. Lee, I. Stewart, …
1204.5469, 1303.6952, 1303.6954,
1312.0301, 1404.6706, …

The 2nd International Conference on the Initial Stages in High-Energy Nuclear Collisions (IS2014)

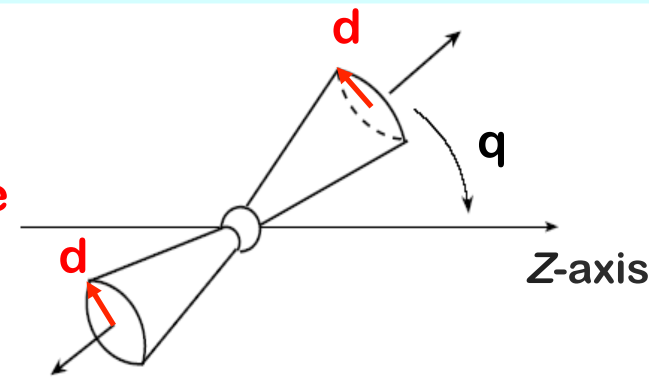
Embassy Suites Napa Valley, Napa, CA, December 3rd - 7th, 2014

Jets

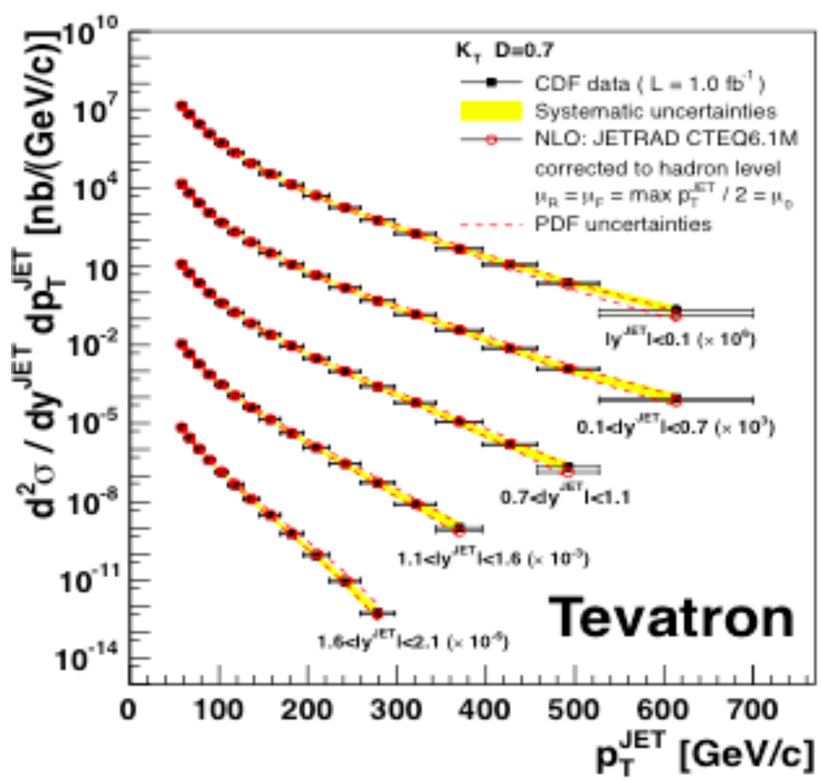
❑ Definition:

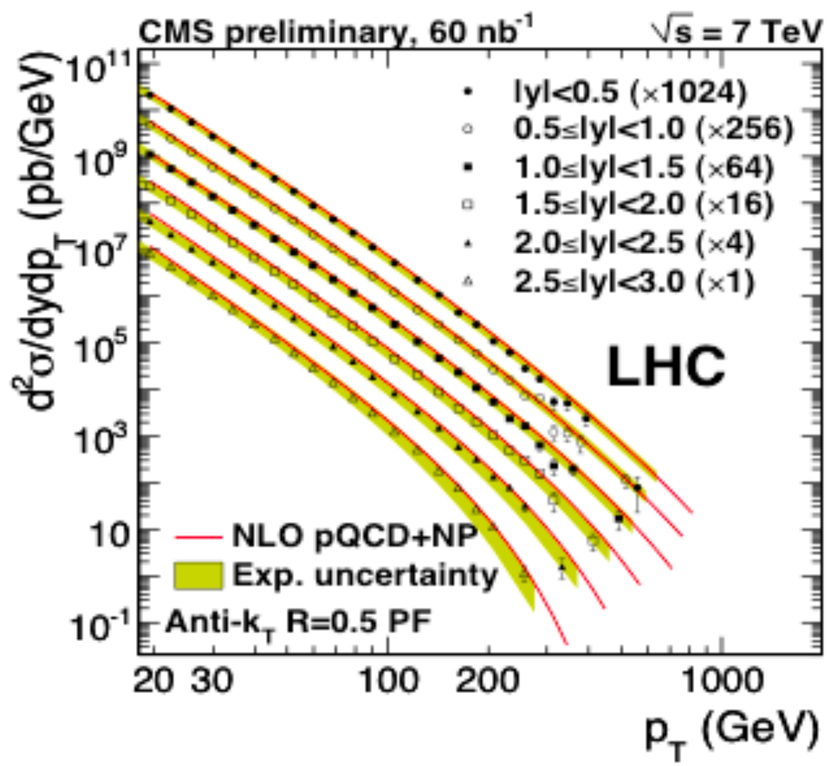
✧ Inclusive cross section with limited phase space

✧ “footprints” or “trace” of quarks and gluons



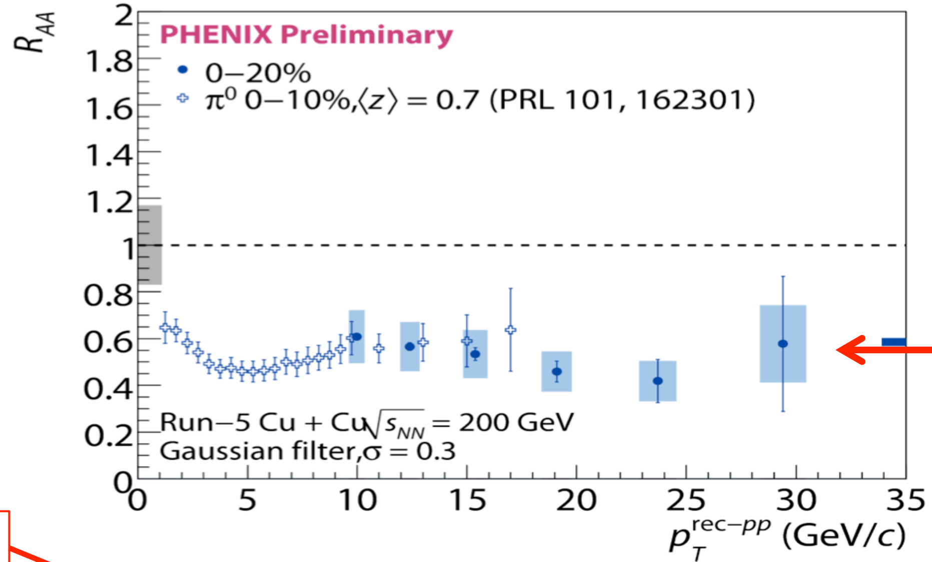
Sterman & Weinberg, PRL 1977





Suppression of jets – Jet quenching

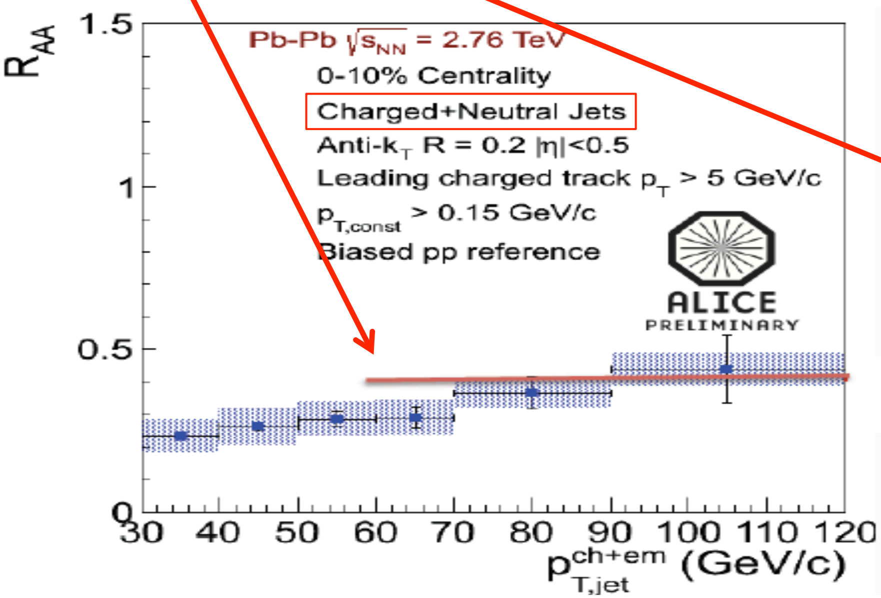
- Jets vs. leading hadron:

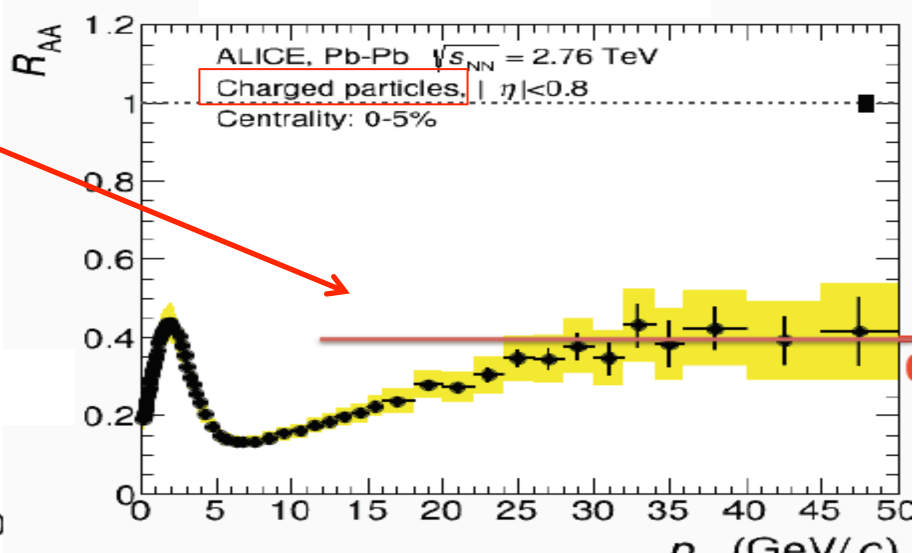


Narrow jet

Same suppression as leading hadron

Similar R_{AA}





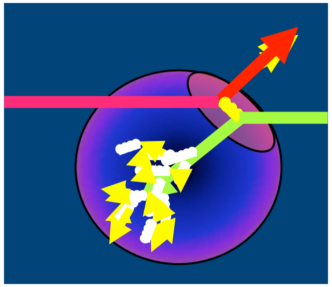
ALICE PLB 720 52-62 (2013)

Where does the lost energy go?

❑ Medium induced radiation:

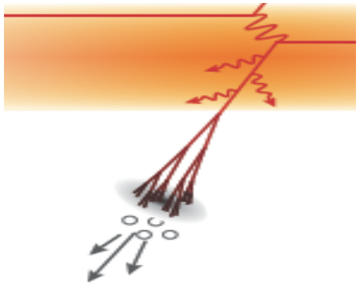
✧ Small angle in/near cone

No suppression if the cone is bigger enough!



✧ Thermalize with the medium:

Radiation is gone!



✧ Broaden the jet

Jet cone dependence!

❑ Where does the lost energy go?

We do not know, since we did not keep track of every particles

❑ What if we do keep track of every particles?

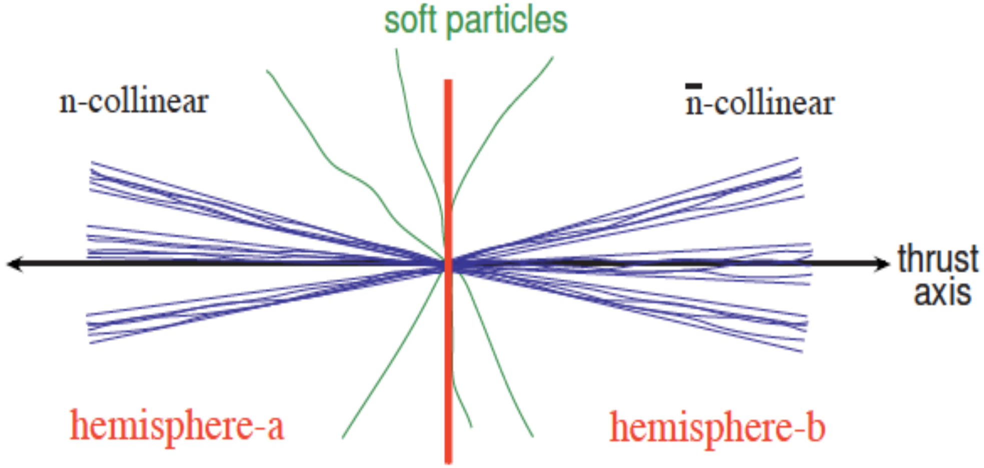
We should know the full event shape!

Event shapes

- ❑ **Event shapes are theoretically cleaner (more inclusive!):**

- ❑ **Thrust, as an example:**

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



- ✧ **Two jet configurations obtained in the limit:**

$$T \to 1$$

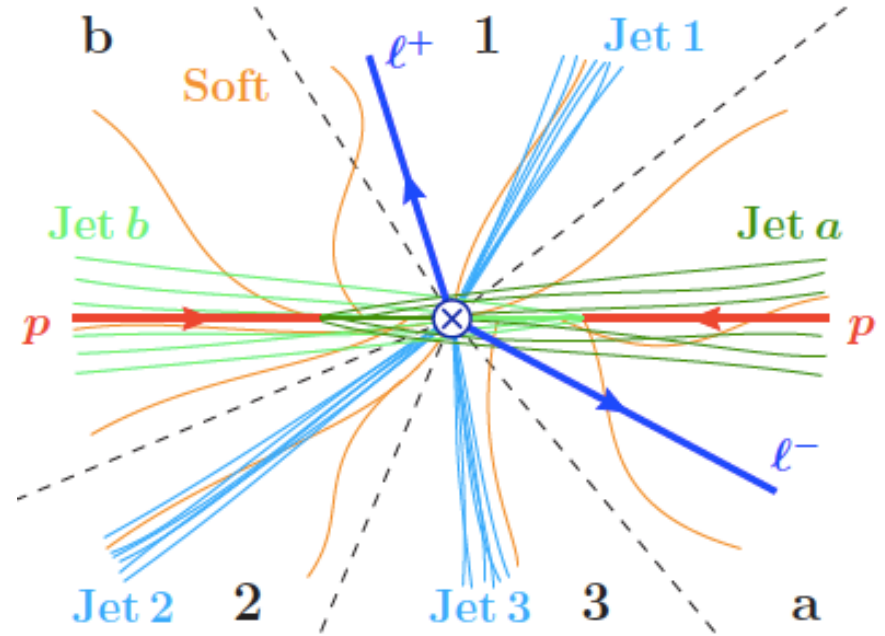
- ✧ **Resummation of logarithms of (1-T), corresponds to a resummation of the jet veto logs**

- ✧ **Structure of resummation is simpler, *no jet algorithm dependence* (jet algorithm dependence begins at NNLO with two emissions)**

N-Jettiness

- **Event structure:**

  $pp \to \text{leptons plus jets}$



- **N-Jettiness:**

  (Stewart, Tackmann, Waalewijin, 2010)

$$\tau_N^i = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

The sum include all final-state hadrons *excluding* more than N jets

Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust), and is complementary to jets

- **N-infinitely narrow jets – isolated single hadron(s) (jet veto):**

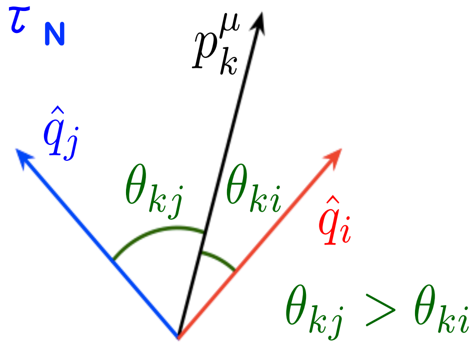
  As a limit of N-Jettiness: $\tau_N \to 0$

N-Jettiness – implementation

- Steps for implementation:
  - ✧ Use a standard jet algorithms to find N-jets
  - ✧ Initial reference vectors = momenta of the N-jets + hadron beam directions

    (reference vectors are the only information used from the jet algorithm)
  - ✧ Calculate value for the N-jettiness global event shape: τ_N

    (new reference directions from the minimization)
  - ✧ Select events with N narrow well-separated jets and impose veto on additional jets



- New "jet" momenta = sum of momenta in jet regions

$$P_i^\mu = \sum_k p_k^\mu \prod_{j \neq i} \theta\left(\hat{q}_j \cdot p_k - \hat{q}_i \cdot p_k\right)$$

- N-jettiness momentum = sum of jettiness from each region:

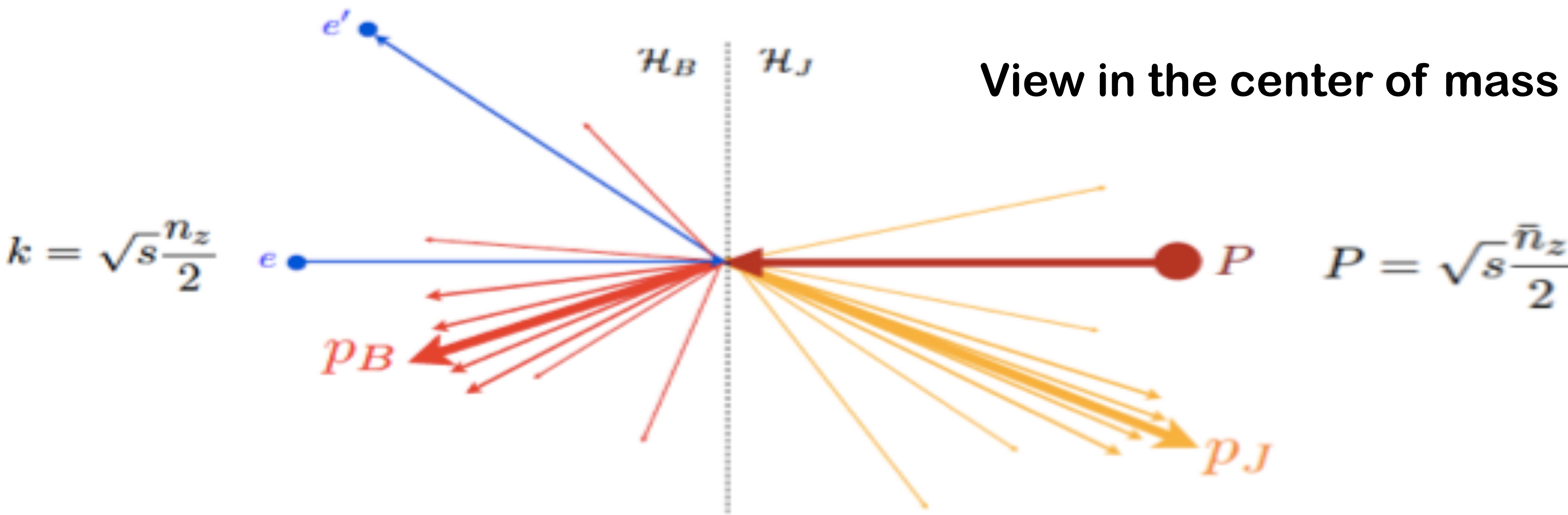
$$\mathcal{T}_N = \sum_i \mathcal{T}_N^i \equiv \sum_i 2\hat{q}_i \cdot P_i$$

- Dependence on Jet algorithms is power suppressed

1-Jettiness cross section in DIS

Kang, Mantry, Qiu, 2012

View in the center of mass frame



$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Very much “like” the calculation for the “Thrust”

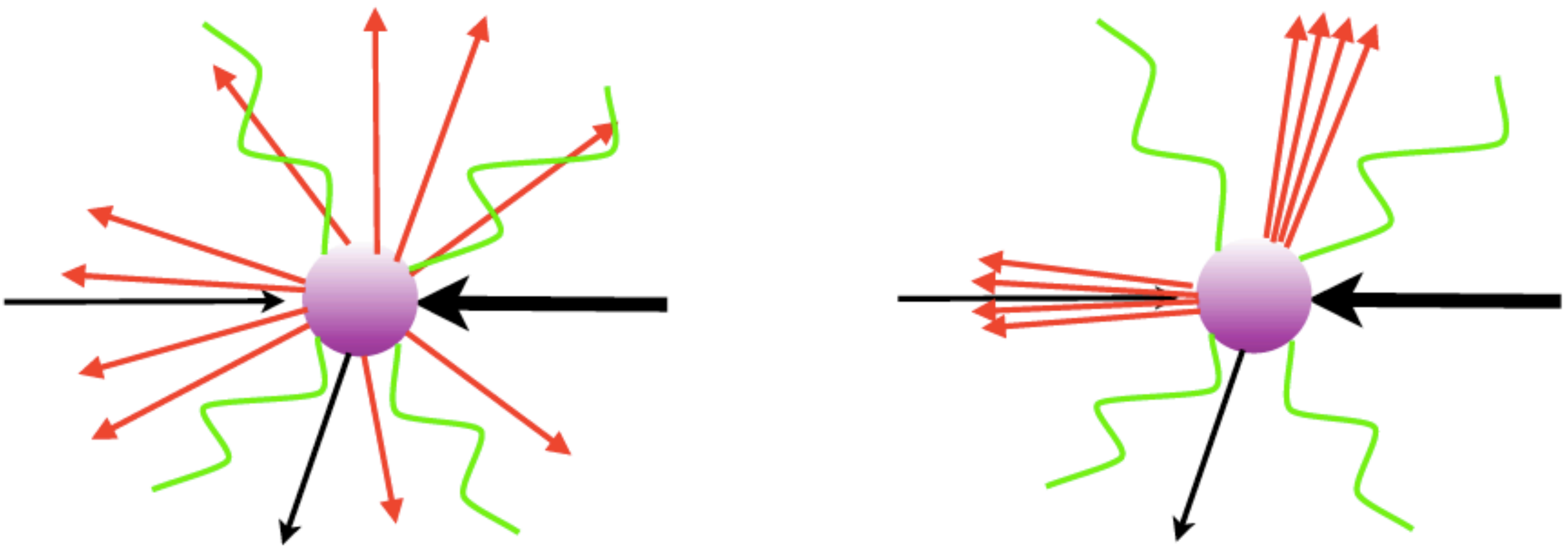
(Minimization vs maximization!)

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{du\, dP_{J_T}\, d\tau_1}$$

1-jettiness: global event shape

Event shape with 1-Jettiness

- Configurations of large and small 1-jettiness:



$$\tau_1 \sim P_{J_T} \qquad\qquad \tau_1 \ll P_{J_T}$$

- 1-jettiness distributions can be a probe of nuclear structure and dynamics.

Most importantly, the radiation pattern following the additional scattering

Event shapes for DIS

- Event shapes have been studied before in DIS:
  - Breit Frame
  - Thrust, NLL +NLO (Anyonelli, Dasgupta, Salam, 99)
  - Broadening, NLL +NLO (Dasgupta, Salam, 01)
  - Non-Global Event Shapes (Dasgupta, Salam, 01, 02)
- 1-jettiness global event shape for DIS was first introduced about a year ago: (Z.Kang, SM, Qiu, 12)
  - 1-jettiness factorization in SCET
  - Proposed as probe of nuclear physics
  - Proton target, NLL results
- More recently: (Z. Kang, Liu, SM, Qiu, 13)
  - NNLL resummation
  - Variety of nuclear targets: Proton, C, Ca, Fe, Au, Ur
- Most recently: (D. Kang, Lee, Stewart, 13)
  - Also, considered 1-jettiness with NNLL resummation
  - Introduced two new variations of 1-jettiness and their factorization
  - Analysis restricted to proton target
- **Matching from small τ to large τ** **(Z. Kang, S. Mantry, X. Liu, 13)**

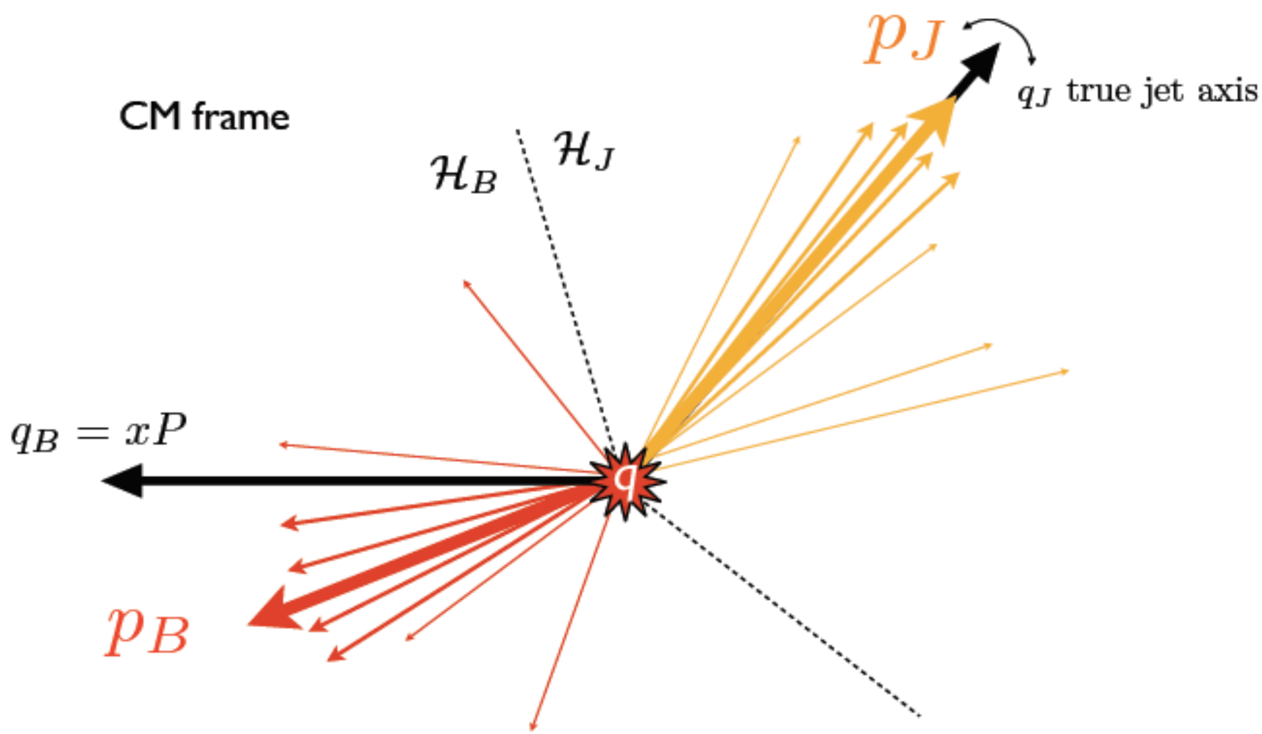
Three ways to define the 1-jettiness

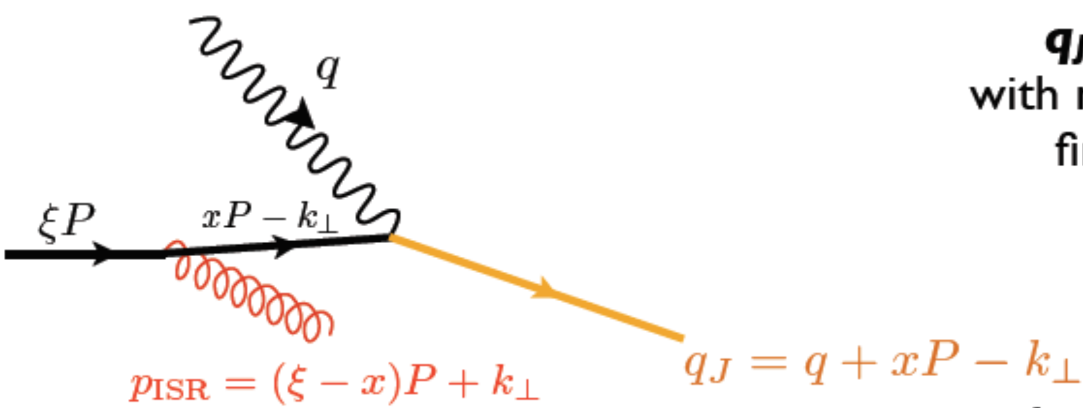
τ_1^a

$$q_B = xP$$

$$q_J = \text{true jet axis}$$

Kang, Mantry, Qiu (2012)



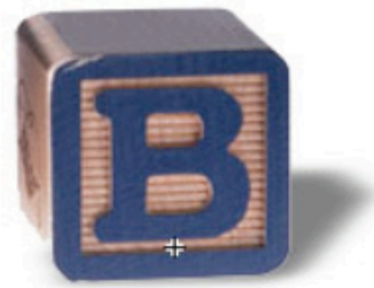


$k_\perp \sim Q\lambda$

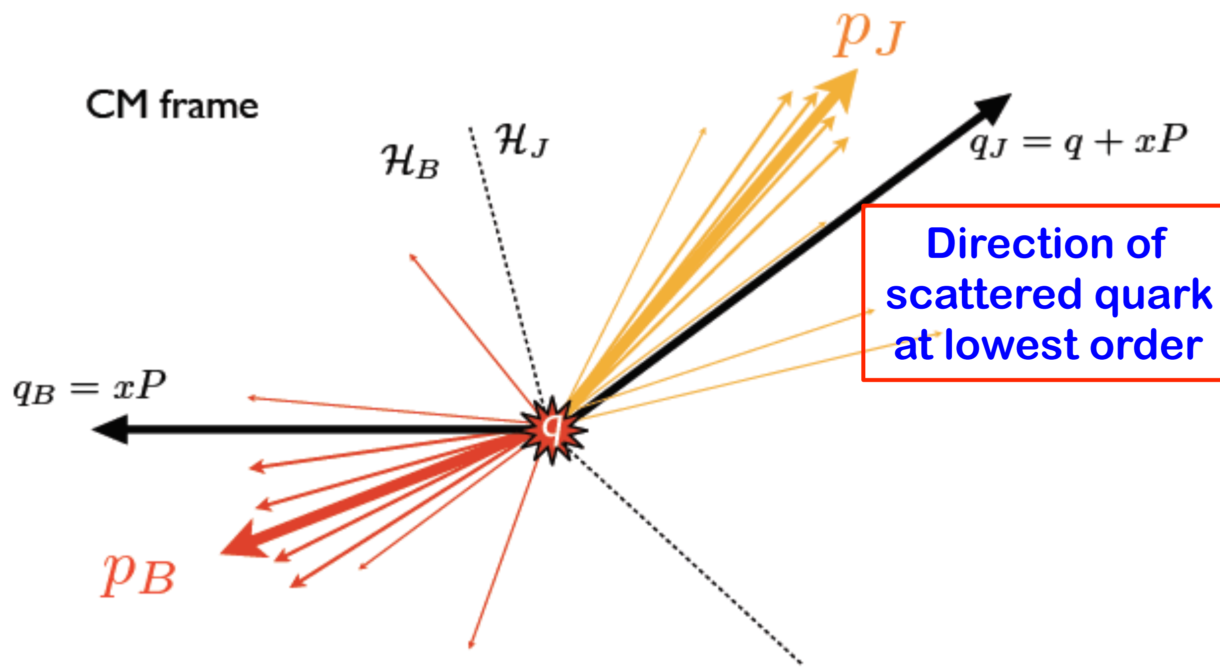
q_J is **A**ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization

depends on momenta of final-state hadrons

Three ways to define the 1-jettiness



$$\tau_1^b$$



$$q_B = xP$$

$$q_J = q + xP$$

same as DIS thrust
by Antonelli, Dasgupta, Salam (1999)

$\boldsymbol{q_J}$ no longer exactly aligned with jet, but simpler in that $q+xP$ is given only by lepton and initial-state proton momenta

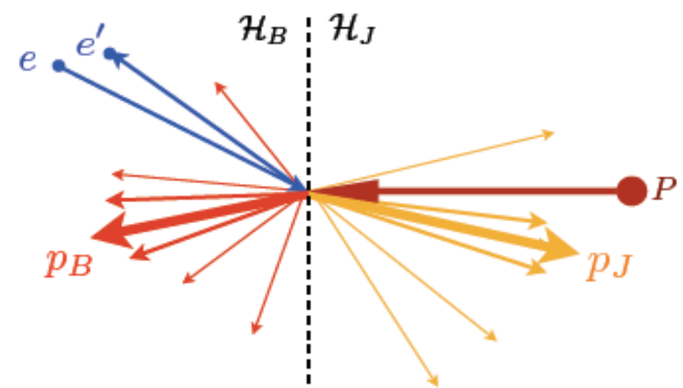
Boost

Breit frame:

$$q = (Q, 0, 0, Q)$$

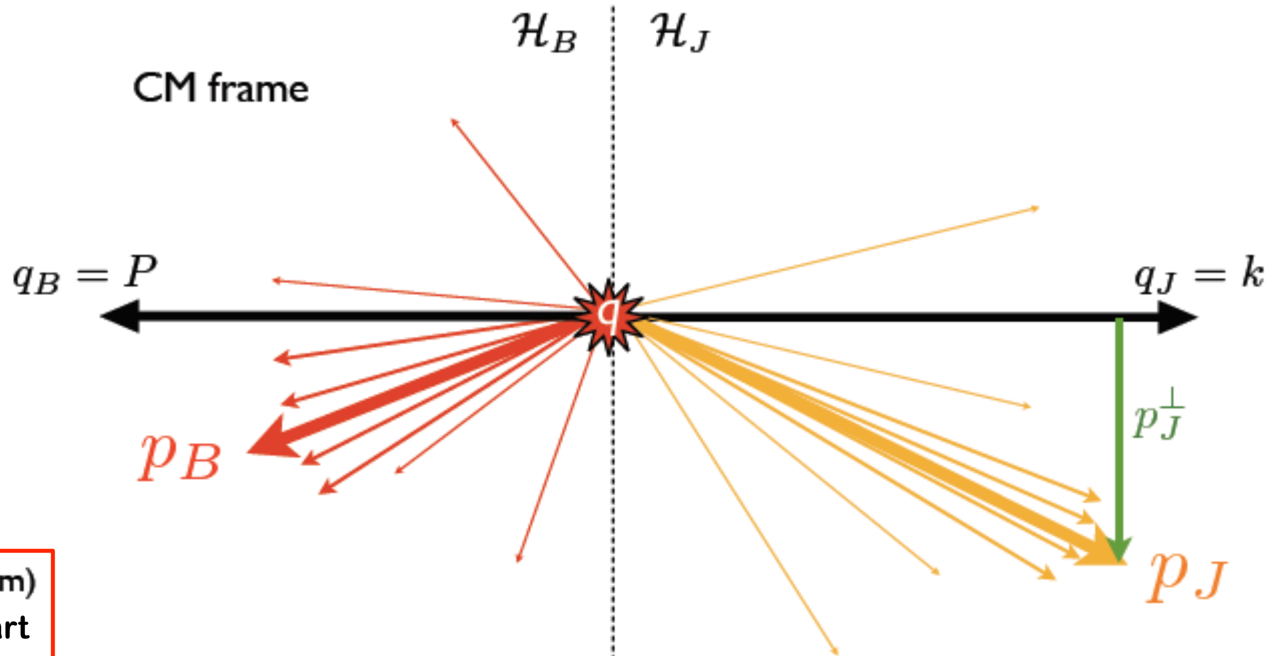
$$q_B = Q\bar{n}_z \quad q_J = Qn_z$$

1-jettiness regions are hemispheres in Breit frame



Three ways to define the 1-jettiness

τ_1^c



$q_B = P$

$q_J = k$

(electron momentum)
Kang, Lee, Stewart 2013

measures thrust in back-to-back hemispheres in **C**enter-of-momentum frame

momentum transfer ***q*** itself has a nonzero transverse component:

$$q = y\sqrt{s}\frac{n_z}{2} - xy\sqrt{s}\frac{\bar{n}_z}{2} + \sqrt{1-y}\,Q\hat{n}_\perp$$

seemingly simplest definition: *in practice* hardest to calculate!

Restriction: $p_J^\perp$ has to be small for 1-jettiness τ_1^c to be small $\Rightarrow 1 - y \sim \lambda^2$

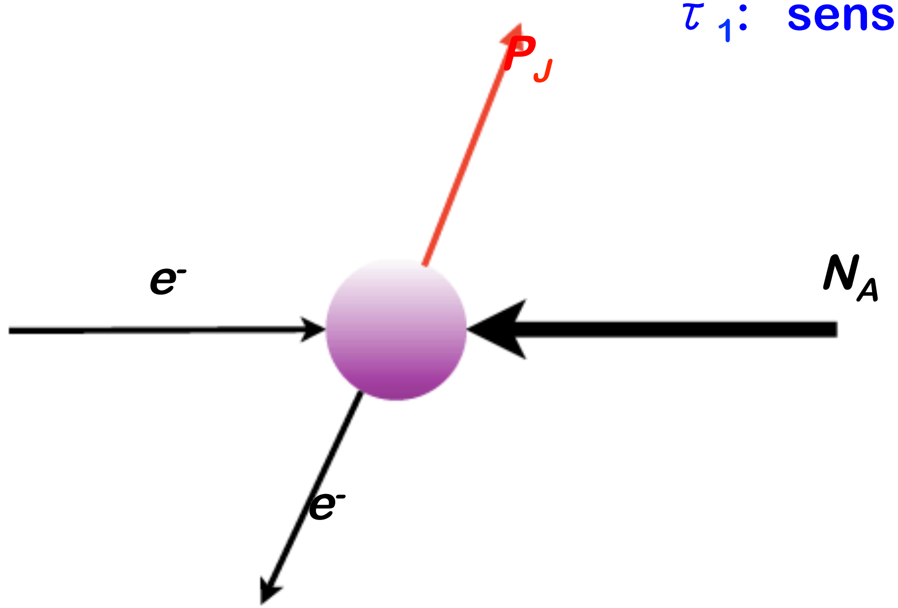
Tree-level 1-jettiness distribution

Kang, Mantry, Qiu, PRD (2012)

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

Two scales observables!

- P_T: localized probe
- τ_1: sensitive to event shape

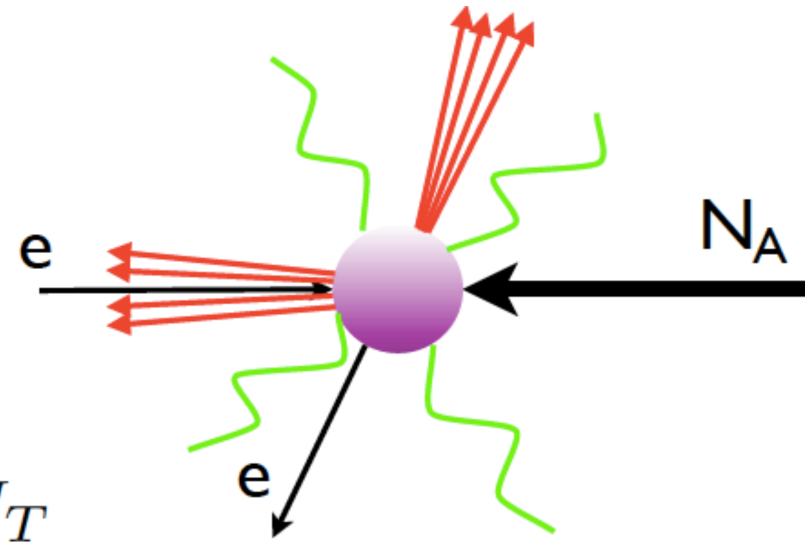


- Tree-level distribution in 1-jettiness:

$$\frac{d^3\sigma^{(0)}}{dy dP_{JT} d\tau_1} = \sigma_0 \delta(\tau_1) \sum_q e_q^2 \frac{1}{A} f_{q/A}(x_A, \mu)$$

Hierarchy of energy scales

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$



- Hierarchy of scales:

$$Q_s, \Lambda_{QCD} \ll \tau_1 \ll P_{J_T}$$

Q_s, Λ_{QCD} → Nuclear scales

τ_1 → 1-jettiness

P_{J_T} → Jet transverse momentum

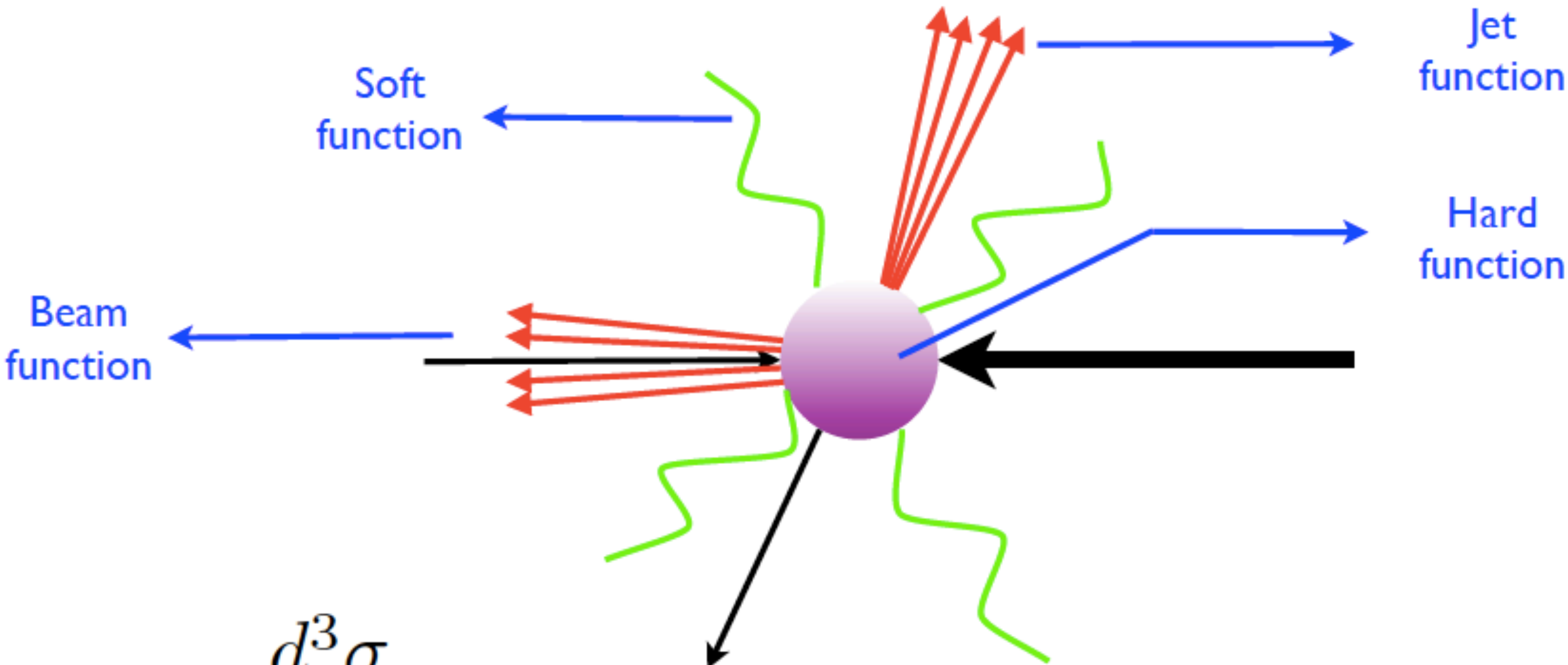
Hard	$\mu_H \sim P_{J_T}$
Beam, Jet	$\mu_B \sim \mu_J \sim \sqrt{\tau_1 P_{J_T}}$
Soft	$\mu_S \sim \tau_1$
Nuclear	$Q_s^2(A) \sim A^\alpha \Lambda_{QCD}^2$

- Jet-veto Sudakov logarithms:

$$\sim \alpha_s^n \ln^{2n}(\tau_1 / P_{J_T})$$

Factorization – SCET

- Schematic form of factorization:



$$\frac{d^3\sigma}{dy\,dP_{JT}\,d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S},$$

Hard function · Beam function · Jet function · Soft function

Factorized cross section

- Detailed form of factorization:

$$\frac{d^3\sigma}{dy dP_{JT} d\tau_1} = \frac{\sigma_0}{A} \sum_{q,i} e_q^2 \int_0^1 dx \int ds_J \int dt_a$$

Hard function ⟶

$$\times H(xAQ_e P_{J_T} e^{-y}, \mu; \mu_H) \delta\left[x - \frac{e^y P_{J_T}}{A(Q_e - e^{-y} P_{J_T})}\right]$$

Jet function ⟶

$$\times J^q(s_J, \mu; \mu_J) B^q(x, t_a, \mu; \mu_B)$$

⟵ Beam function

$$\times \mathcal{S}\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right),$$

⟵ Soft Function

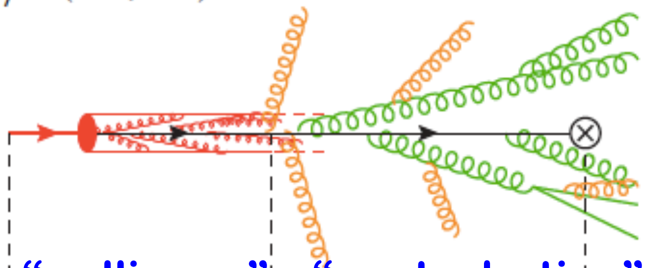
- Beam function matching onto the PDF:

(Fleming, Leibovich, Mehen; Jouttenus, Stewart, Tackmann, Waalewijin)

$$B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} \mathcal{I}^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B)$$

- Tree-level matching:

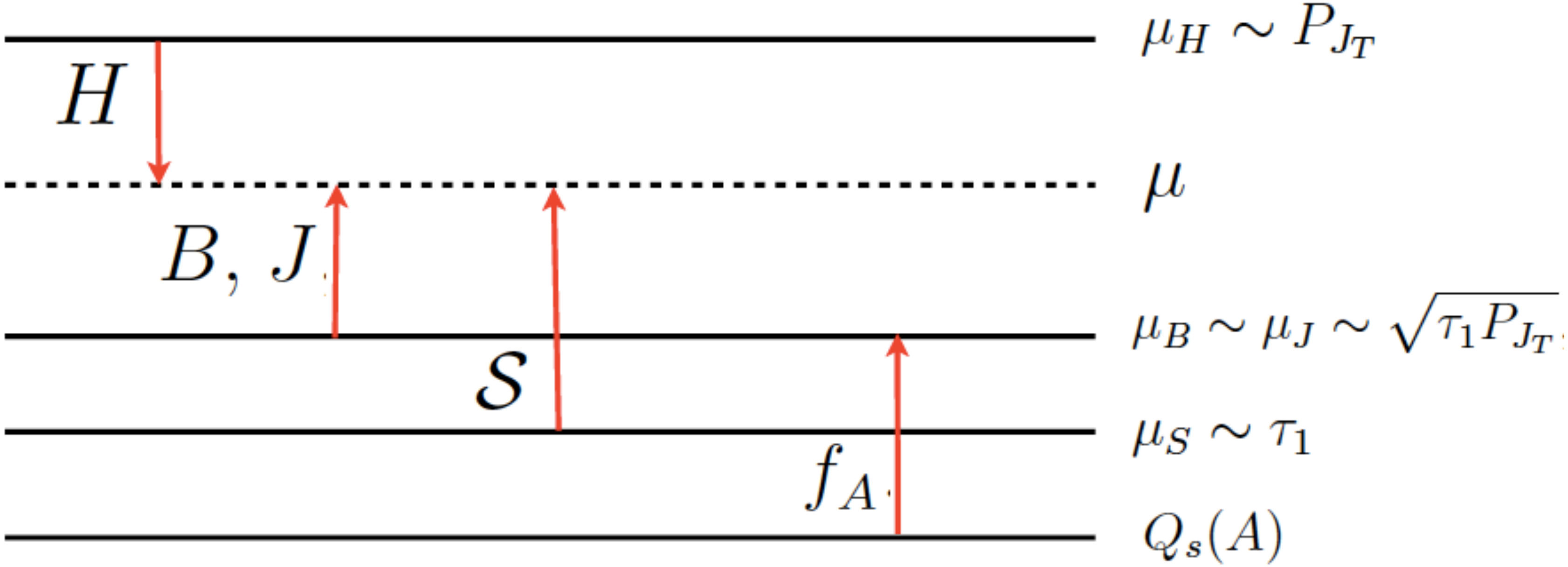
$$B^q(x, t_a, \mu_B) = \delta(t_a) f_{q/A}(x, \mu_B)$$



~ "collinear" "perturbative"

Resummation

- Resummation achieved through renormalization group equations:



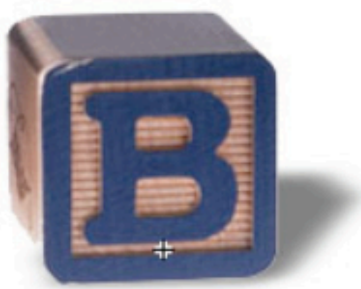
- All objects in factorization formula have well defined evolution equations:

$$
\begin{aligned}
\mu\frac{d}{d\mu}H(Q^2,\mu) &= \gamma_H\, H(Q^2,\mu),\\
\mu\frac{d}{d\mu}B_A^q(x,t,\mu) &= \int dt'\,\gamma_B(t-t',\mu)\,B_A^q(x,t',\mu),\\
\mu\frac{d}{d\mu}J(s,\mu) &= \int ds'\,\gamma_J(s-s',\mu)J(s',\mu),\\
\mu\frac{d}{d\mu}\mathcal{S}(k_a,k_J,\mu) &= \int dk_a' \int dk_J'\,\gamma_S(k_a-k_a',k_J-k_J',\mu)\mathcal{S}(k_a',k_J',\mu)
\end{aligned}
$$

Differences between the three definitions

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Z. Kang, Mantry, Qiu, 2012



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

D. Kang, Lee, Stewart, 2013

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\right) \times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

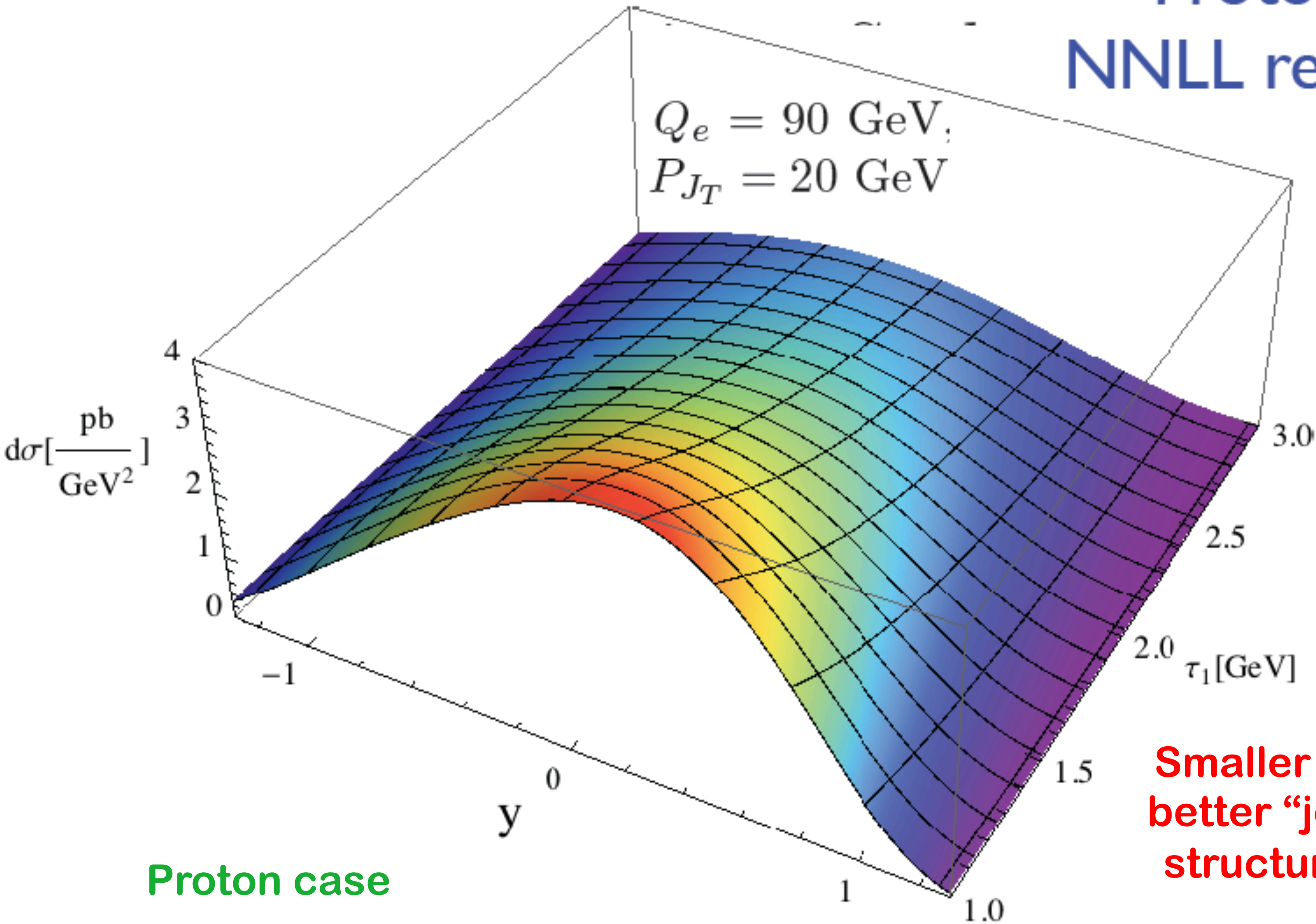
D. Kang, Lee, Stewart, 2013

1-jettiness and rapidity distribution

- One can study distributions in the space of :

$$\{A, Q_e, P_{J_T}, y, \tau_1\}$$

Proton target:
NNLL resummation



Larger τ
less "jet" structure

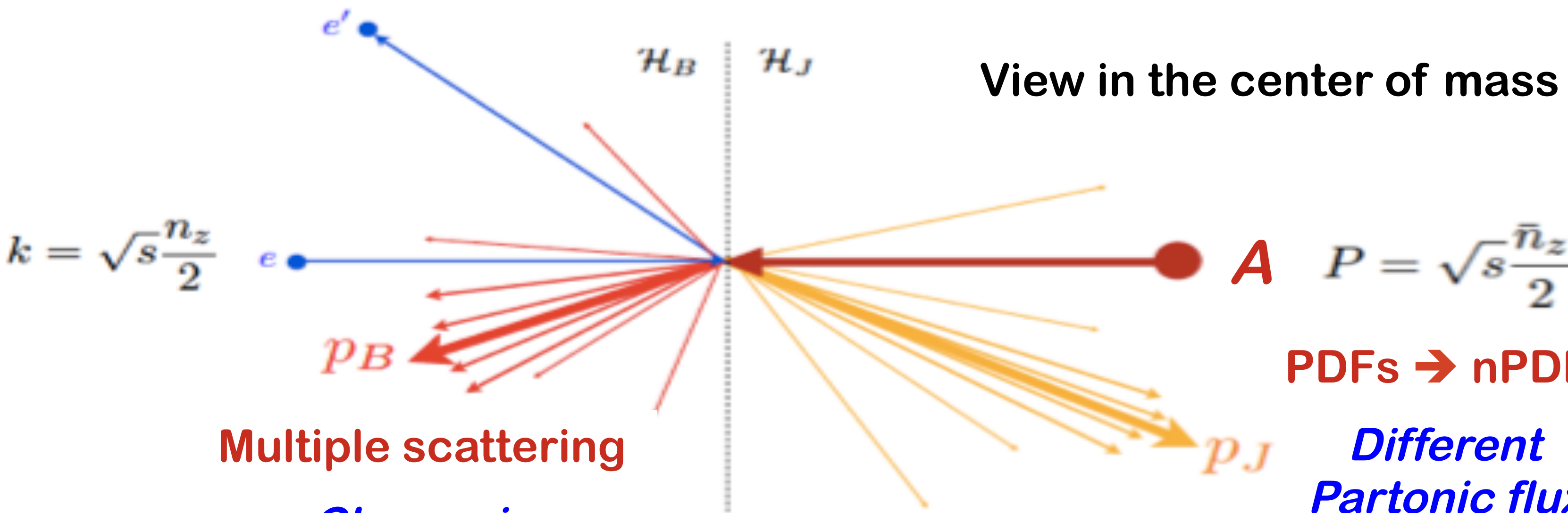
Smaller τ
better "jet" structure

Proton case

1-Jettiness cross section in e+A DIS

Kang, Mantry, Qiu, 2012, 2013

View in the center of mass frame



PDFs → nPDFs

Multiple scattering

Different Partonic flux

Change in radiation pattern

❑ Same definition:

$$\tau_1 = \frac{2}{Q^2} \sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

Additional variable: *A*

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{du\, dP_{J_T}\, d\tau_1}$$

$d\tau_1$ → 1-jettiness: global event shape

1-Jettiness cross section in e+A DIS

Leading power case

- Factorization formula:

$$\frac{d^3\sigma}{dy\,dP_{J_T}\,d\tau_1}\Big|_{EPS09} = \sigma_0 \sum_{q,i} e_q^2 \int_{x_*}^{1} \frac{dx}{x} \int ds_J \int dt_a \times H(\xi^2,\mu;\mu_H) J^q(s_J,\mu;\mu_J) \mathcal{I}^{qi}\left(\frac{x_*}{x}, t_a, \mu; \mu_B\right) \times \mathcal{S}\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right) \boxed{f_{i/A}^{EPS09}(x,\mu_B),}$$

- Lower limit of Bjorken-x integration:

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

→ Determines the Bjorken-x region

See appendices of arXiv: 1303.3063 for details on each functions and their evolutions

Similar leading power factorization formula for p+A collisions

Two beam functions, two soft functions

Nuclear PDFs

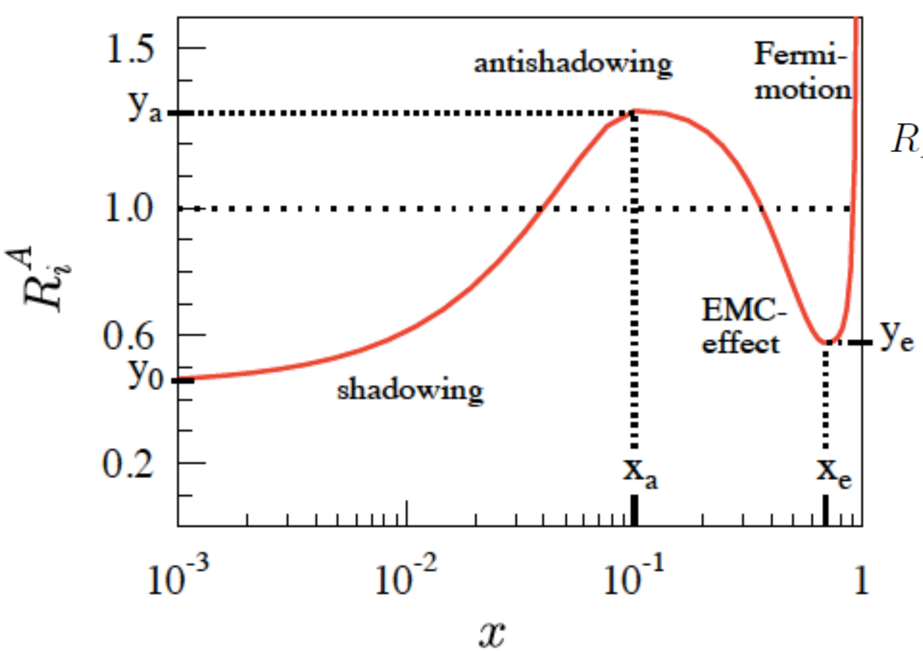
- At leading twist, we directly probe nuclear PDFs

(Eskola, Paukunnen, Salgado)

$$
\begin{aligned}
f_{u/A}^{EPS09}(x,\mu) &= \frac{Z}{A}\, R_u^A(x,\mu)\, f_{u/p}(x,\mu) + \frac{A-Z}{A}\, R_d^A(x,\mu)\, f_{d/p}(x,\mu), \\
f_{d/A}^{EPS09}(x,\mu) &= \frac{Z}{A}\, R_i^A(x,\mu)\, f_{d/p}(x,\mu) + \frac{A-Z}{A}\, R_u^A(x,\mu)\, f_{u/p}(x,\mu), \\
f_{s,c,b/A}^{EPS09}(x,\mu) &= R_{s,c,b}^A(x,\mu)\, f_{s,c,b/p}(x,\mu), \\
f_{g/A}^{EPS09}(x,\mu) &= R_g^A(x,\mu)\, f_{g/p}(x,\mu),
\end{aligned}
$$

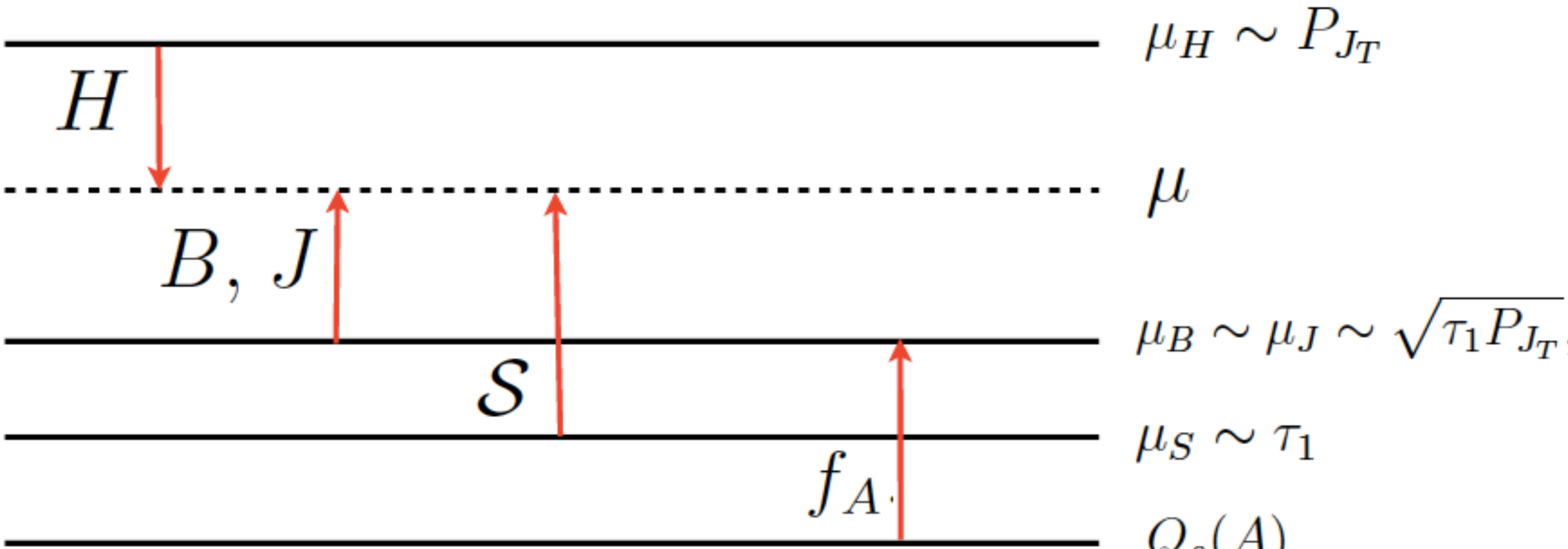
EPS09 PDF set

- Schematic behavior of nuclear modification factors



$$
R_L(x,\mu) = \frac{\sum_q e_q^2 f_{q/A}^{EPS09}(x,\mu)}{\sum_q e_q^2 f_{q/p}(x,\mu)}
$$

Scale variations – theory uncertainty

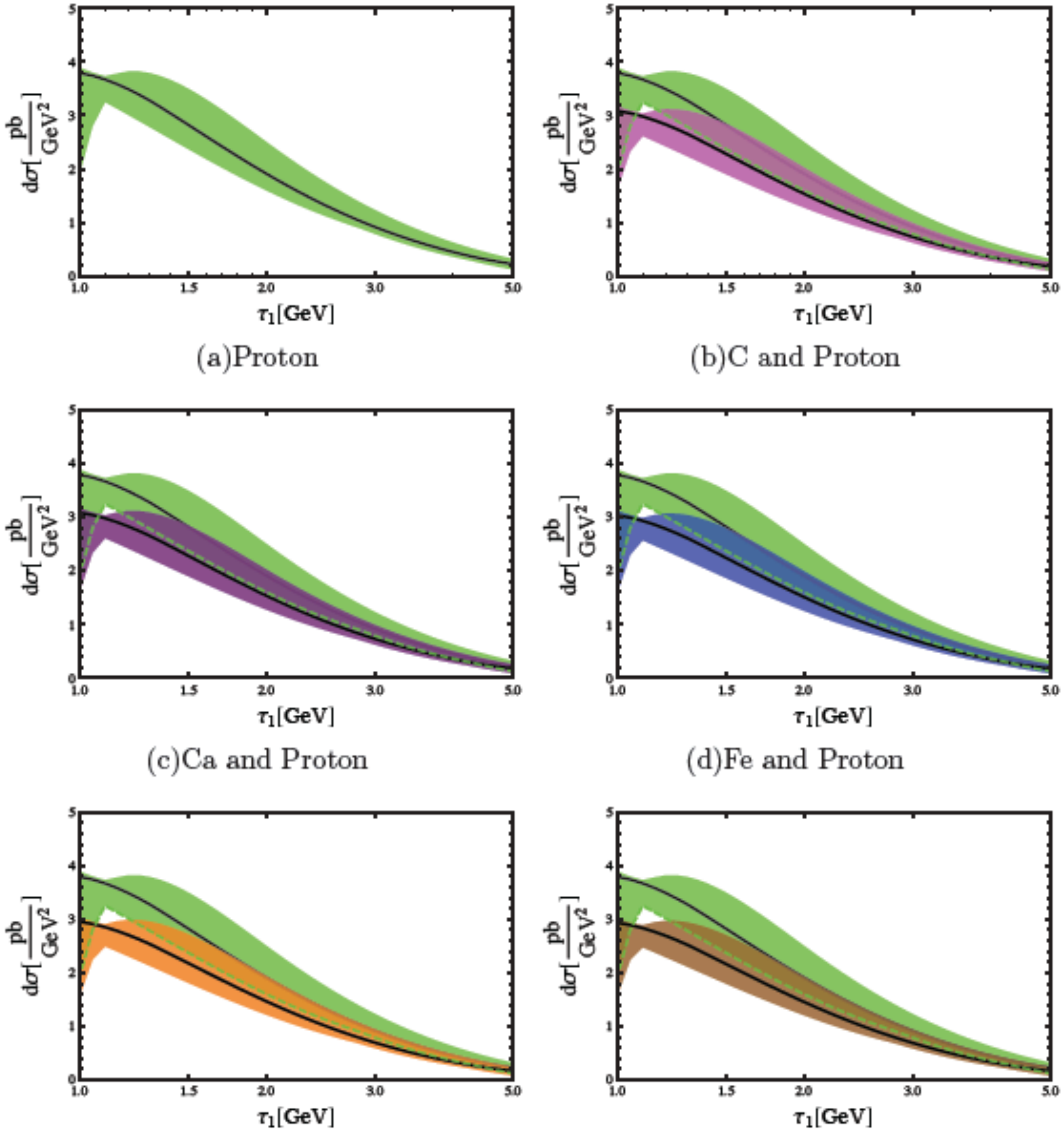


- Four independent scale variations employed to estimate perturbative uncertainties: (Stewart, Tackmann, Waalewijin)

$$(a)\ \mu = \mu_H = r\sqrt{\xi^2},\ \mu_B = r\sqrt{Q_a\tau_1},\ \mu_J = r\sqrt{Q_J\tau_1},\ \mu_S = r\,\tau_1,$$
$$(b)\ \mu = \mu_H = \sqrt{\xi^2},\ \mu_B = \sqrt{Q_a\tau_1},\ \mu_J = \sqrt{Q_J\tau_1},\ \mu_S = r^{-\frac{1}{4}\ln\frac{\tau_1}{\xi}}\,\tau_1,$$
$$(c)\ \mu = \mu_H = \sqrt{\xi^2},\ \mu_B = r^{-\frac{1}{4}\ln\frac{\tau_1}{\xi}}\sqrt{Q_a\tau_1},\ \mu_J = \sqrt{Q_J\tau_1},\ \mu_S = \tau_1,$$
$$(d)\ \mu = \mu_H = \sqrt{\xi^2},\ \mu_B = \sqrt{Q_a\tau_1},\ \mu_J = r^{-\frac{1}{4}\ln\frac{\tau_1}{\xi}}\sqrt{Q_J\tau_1},\ \mu_S = \tau_1,$$

$$\xi^2 \equiv \frac{P_{J_T}^2}{1 - e^{-y}P_{J_T}/Q_e} \qquad r = \{1/2, 2\}$$

1-jettiness distribution in e+A for various nuclei



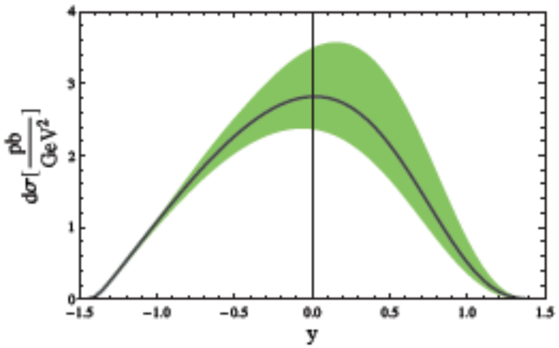
(a)Proton
(b)C and Proton
(c)Ca and Proton
(d)Fe and Proton
(e)Au and Proton
(f)Ur and Proton

NNLL resummation

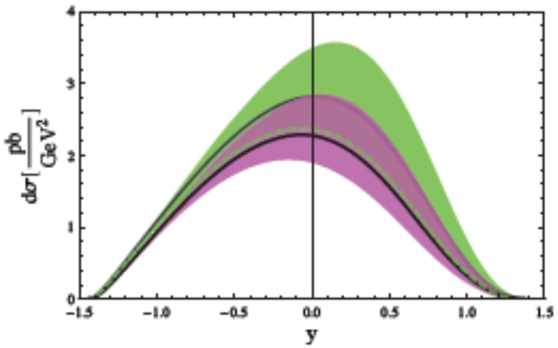
$$Q_e = 90 \text{ GeV},$$
$$P_{J_T} = 20 \text{ GeV}$$
$$y = 0$$

Effect of nPDFs and smearing

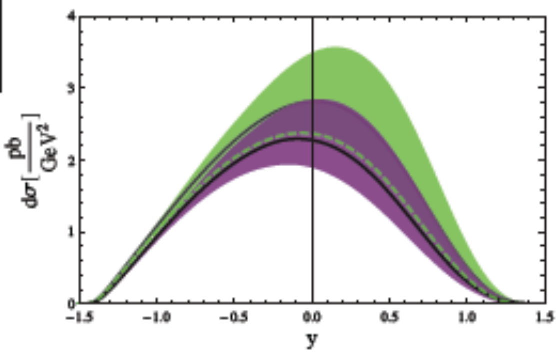
Jet rapidity distributions in e+A for various nuclei



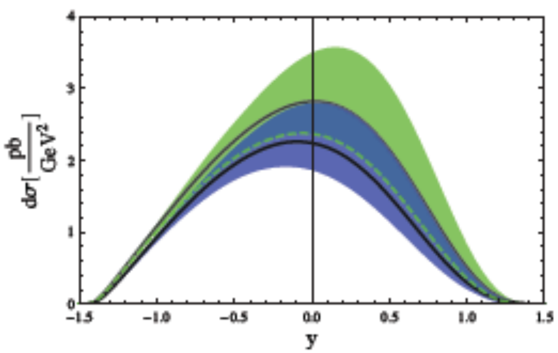
(a) Proton



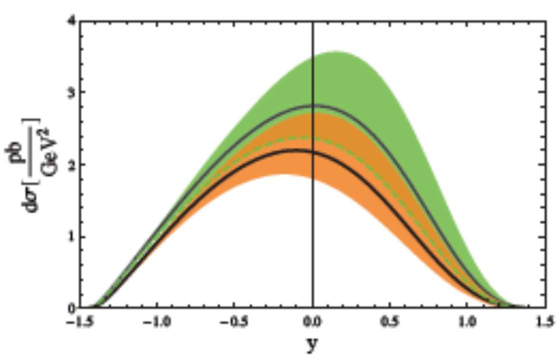
(b) Proton and C



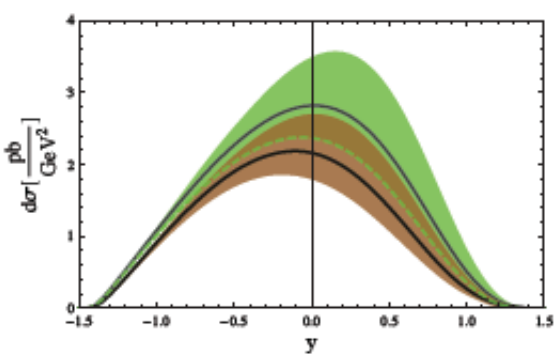
(c) Proton and Ca



(d) Proton and Fe



(e) Proton and Au



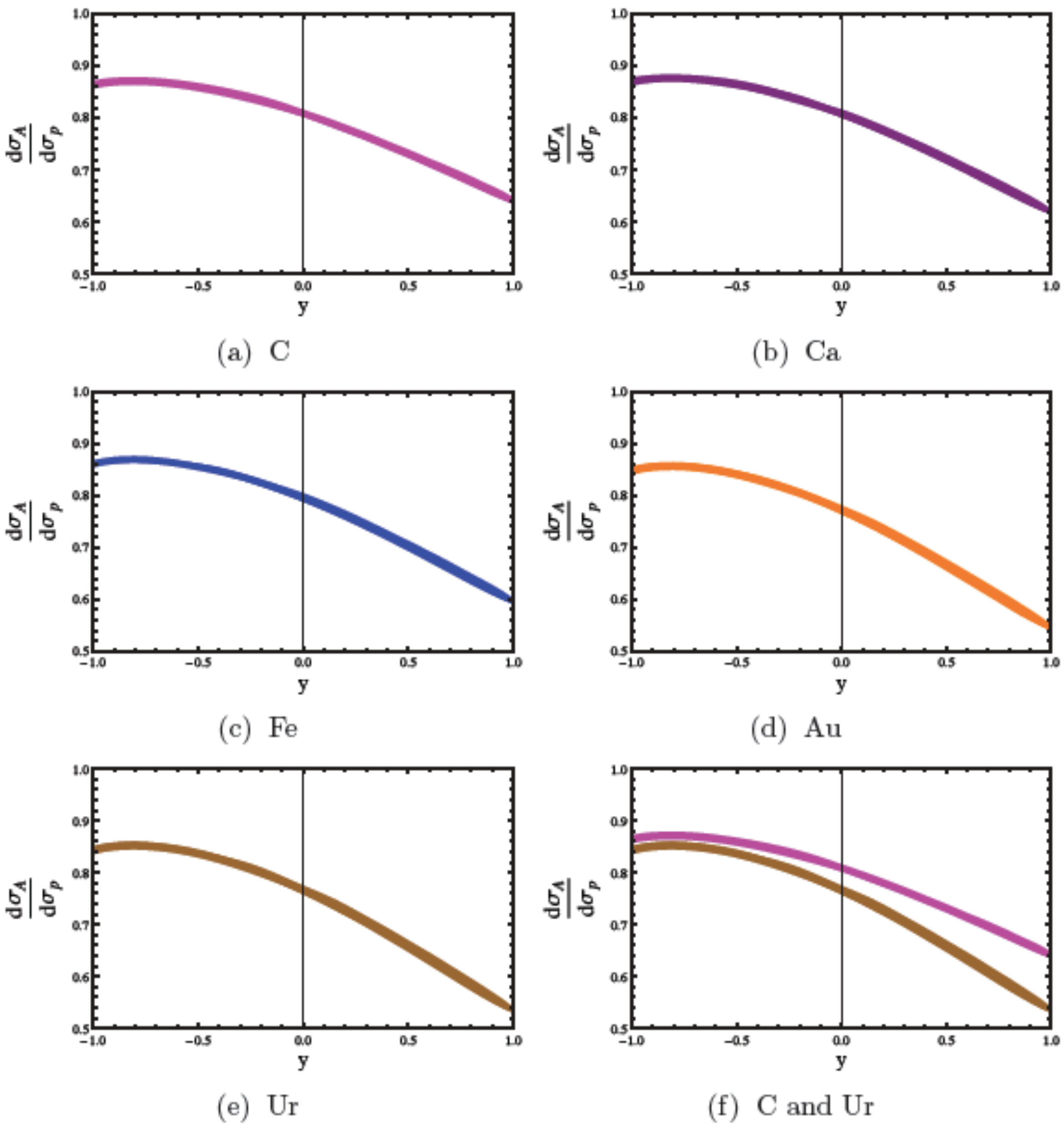
(f) Proton and Ur

NNLL resummation

$$Q_e = 90\ \text{GeV},$$
$$P_{J_T} = 20\ \text{GeV}$$
$$\tau_1 = 1.5\ \text{GeV}$$

Effect of nPDFs and smearing

Jet rapidity: Nuclei over Proton



$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

NNLL resummation

$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$\tau_1 = 1.5 \text{ GeV}$$

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

$$x_* \in [0.2, 0.7]$$

Effect of nPDFs and smearing

Matching from low τ to high τ

Kang, Liu, Mantry, 1312.0301

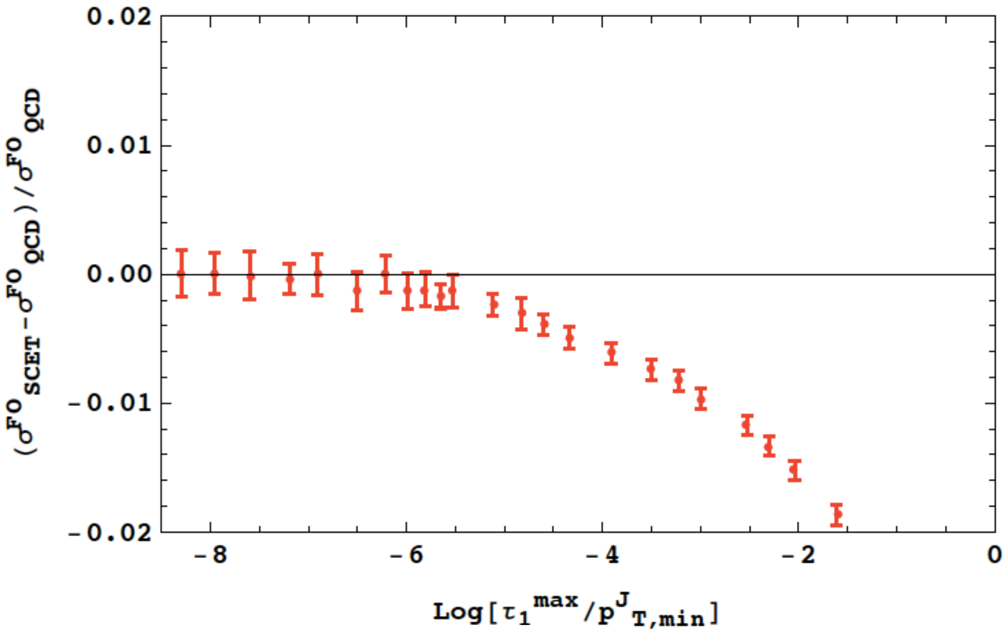
- **Low τ vs high τ :**

  Low τ : Resummation by using SCET

  High τ : Fix order perturbative calculation

- **Matching:**

$$d\sigma = \left[d\sigma_{\rm resum} - d\sigma^{FO}_{\rm resum}\right] + d\sigma^{FO}$$



Three regions:

$$\tau_1 \sim \Lambda_{QCD},$$
$$\Lambda_{QCD} \ll \tau_1 \ll P_{J_T},$$
$$\tau_1 \sim P_{J_T},$$

Beam funciton:

$$B \sim \mathcal{I} \otimes f$$

$$d\sigma_{\rm resum} \equiv \frac{d^3\sigma_{\rm resum}}{dy dP_{J_T} d\tau_1} \sim H \otimes B \otimes J \otimes \mathcal{S}$$

$$\mu_H \sim P_{J_T}, \qquad \mu_J \sim \mu_B \sim \sqrt{\tau_1 P_{J_T}}, \qquad \mu_S \sim \tau_1$$

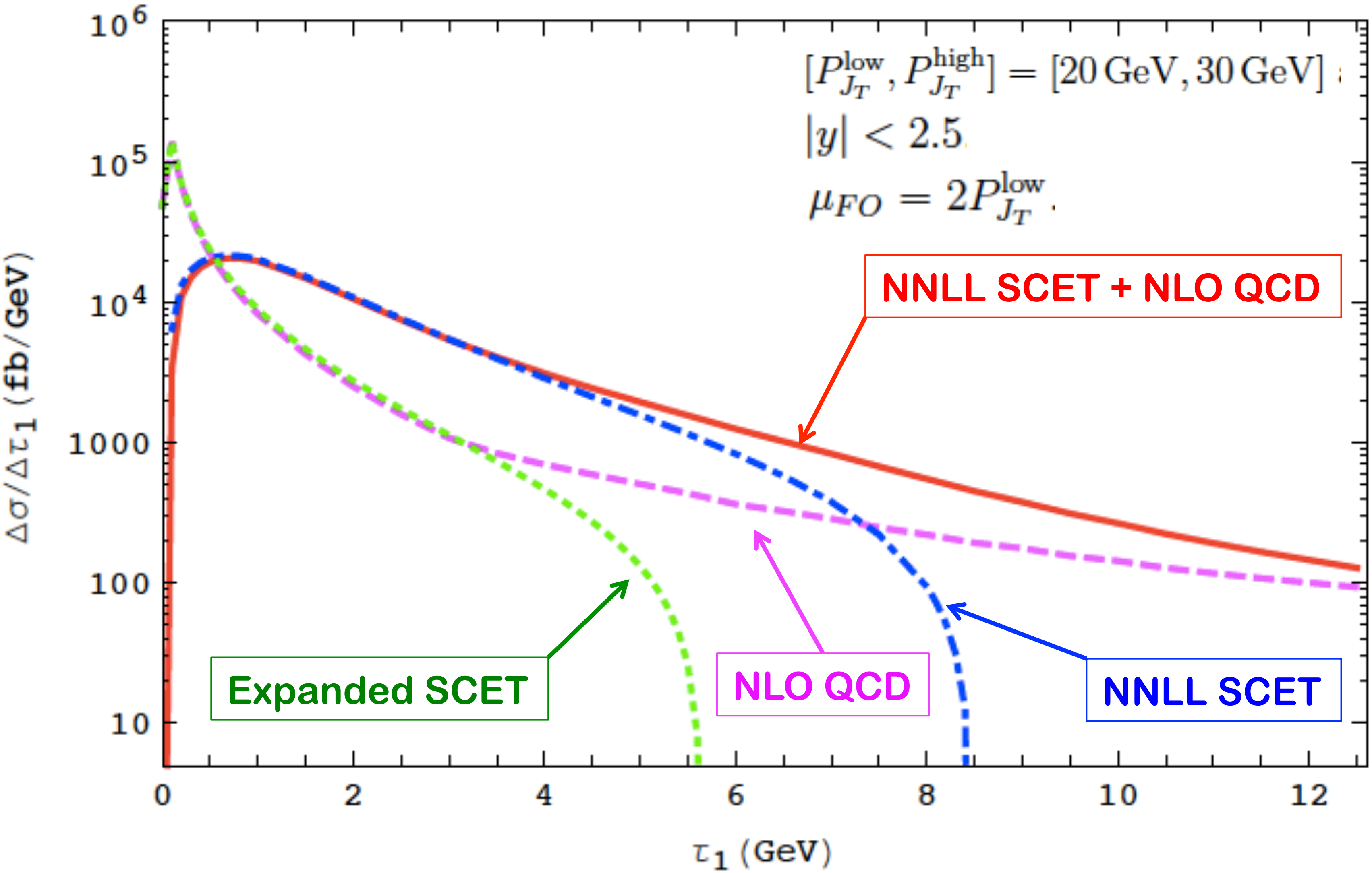
$$d\sigma^{FO}_{\rm resum} = d\sigma_{\rm resum}(\mu = \mu_H = \mu_J = \mu_B = \mu_S)$$

$$d\sigma^{FO} \sim \int dPS\ \hat{\mathcal{F}}_{\rm meas.}([PS])\, |\mathcal{M}|^2 \otimes f$$

Matching from low τ to high τ

- Full spectrum in τ_1 on proton:

Kang, Liu, Mantry, 1312.0301



Summary

- Event shapes based analysis can be a useful tool to probe gluon shower and induced radiation
- Allows for jet shape analysis, and also gives information on wide-angle soft radiation – complementary to jet x-section
- Probe nuclear dynamics through distributions in multiple dimensional space on various nuclear targets:

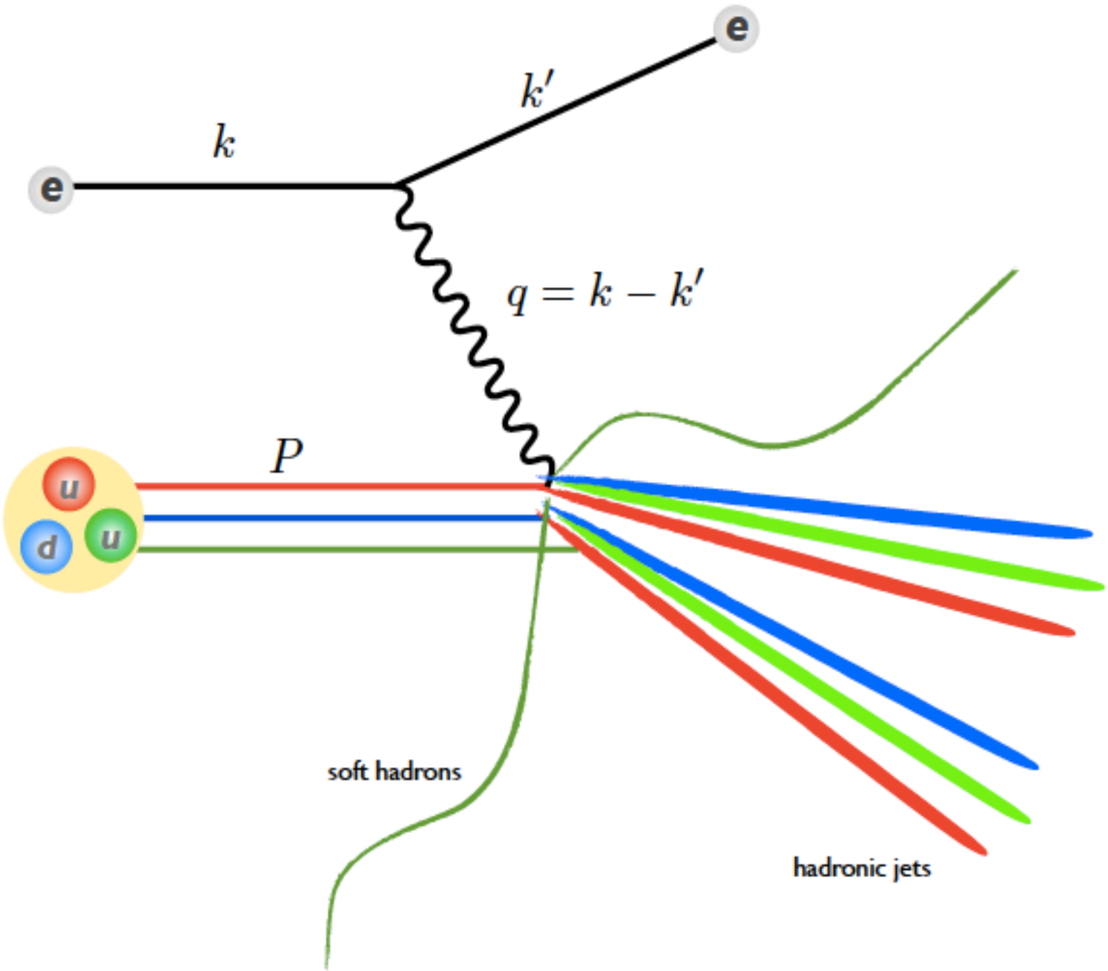
$$\{A, Q_e, P_{J_T}, y, \tau_1\}$$

- SCET can be a natural tool to take care of the resummation of large logarithms from various scales
- Many directions can be pursued with event shape analysis …

Thank you!

BACKUP SLIDES

DIS Kinematics



$s = (k+P)^2$ squared center-of-mass energy

$Q^2 = -q^2$ momentum transfer

$x = \frac{Q^2}{2P\cdot q}$ Björken scaling variable

$y = \frac{P\cdot q}{P\cdot k}$ lepton energy loss in proton rest frame

$Q^2 = xys$

$p_X = q + P$ total momentum of final hadronic state

$p_X^2 = \frac{1-x}{x}Q^2$ invariant mass of final hadronic state

Limit $x \to 1$ corresponds to single collimated jet in final state

We will look away from *x* = *1* at *two*-jet like final states

Kinematics

- Electron momentum:

$$p_e^\mu = (p_e^0, \vec{p}_e)$$

- Nucleus momentum:

$$P_A^\mu = A(p_e^0, -\vec{p}_e)$$

- Electron energy:

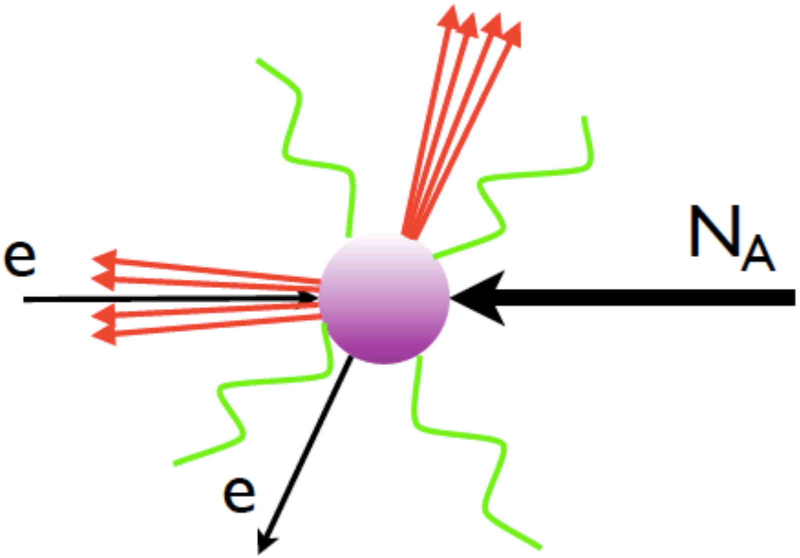
$$p_e^0 = |\vec{p}_e| = \frac{Q_e}{2}$$

- Center of mass energy squared:

$$s = (p_e + P_A)^2 = A\,Q_e^2$$

- Center of mass energy per nucleon:

$$Q_e$$

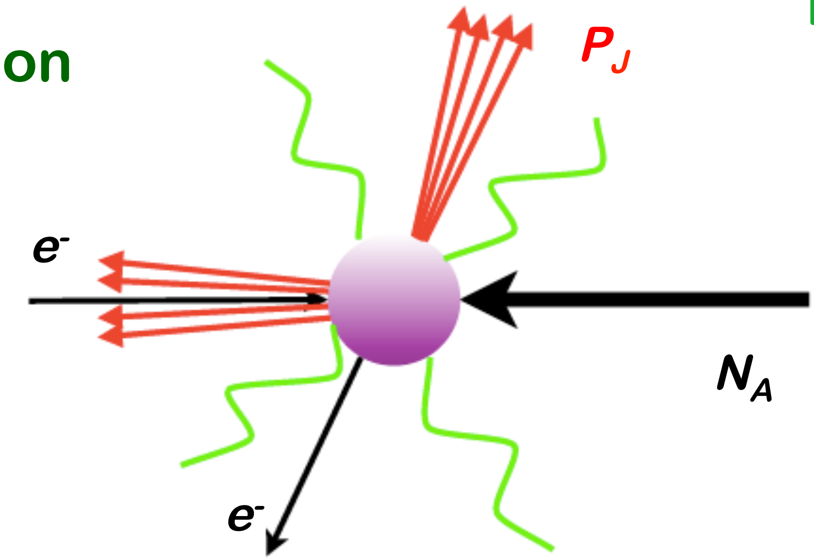


$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

Jets in e+A collisions (at EIC)

Kang, Metz, Qiu, Zhou, 2011

- A clean calibration and a lot more:



- Study jet distributions in e-A collisions.
- Probe of nuclear PDFs at leading twist.
- Higher twist correlations.
- Parton propagation through cold nuclear matter.
- Energy loss mechanisms.
- Nuclear medium effects.
- ...

Power corrections

- Many different sources of power corrections.
- Dominant nuclear-dependent power corrections come from the OPE of the beam function

$$B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} \mathcal{I}^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B) + \mathcal{O}\left(\frac{Q_s^2(A)}{t_a}\right)$$

- Size of power corrections controlled by

$$\frac{Q_s^2(A)}{t_a} \sim \frac{A^\alpha \Lambda_{QCD}^2}{\tau_1 P_{J_T}} \longrightarrow$$

-Higher twist correlations

-Nuclear medium effects: energy loss, multiple scattering,...

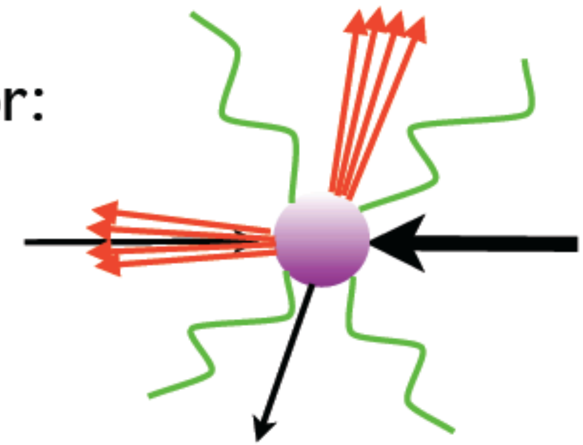
- Power corrections can be studied as a function of: $\{A, \tau_1, P_{J_T}\}$

Non-perturbative region

- Hierarchy of scales:

$$\frac{d^3\sigma}{dy\,dP_{JT}\,d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S}, \qquad P_{J_T} \gg \sqrt{\tau_1 P_{J_T}} \gg \tau_1$$

- Soft function becomes non-perturbative for:



$$\tau_1 \;\sim\; \Lambda_{\rm QCD}$$

- Soft function model

(Ligeti, Stewart, Tackmann)

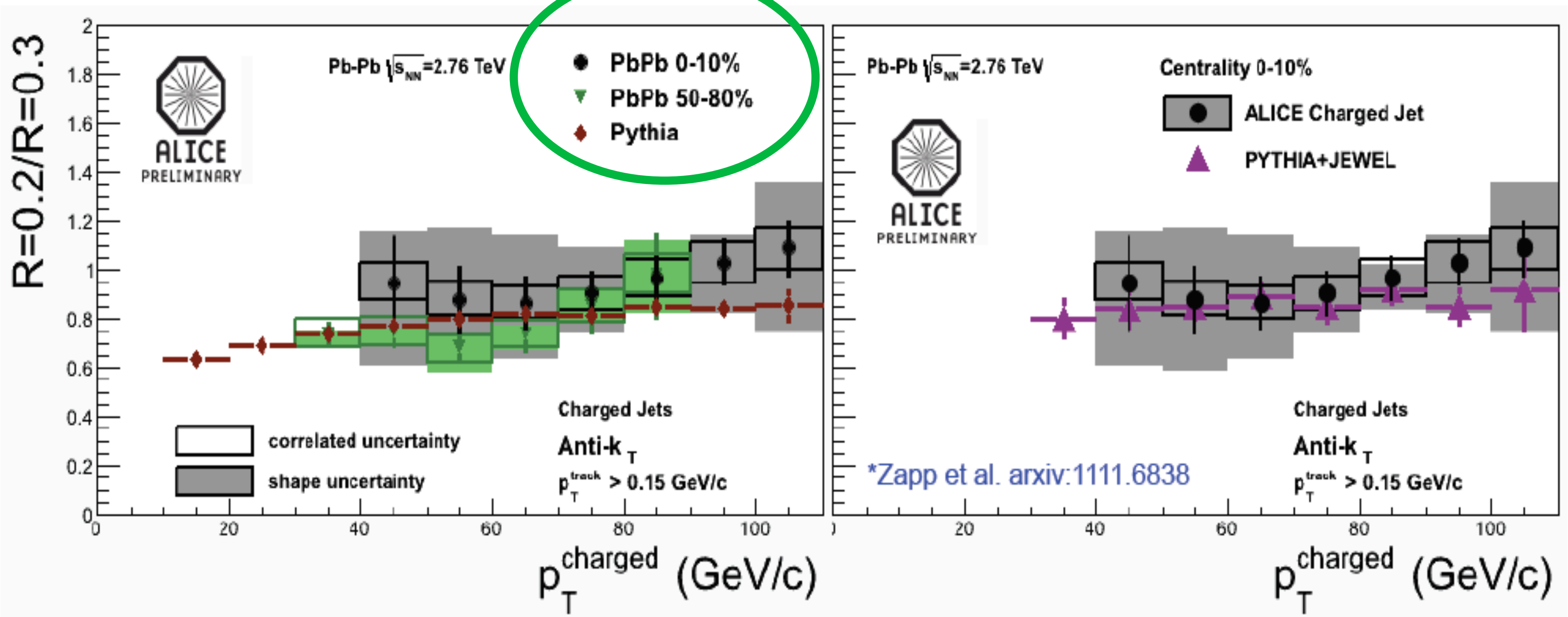
$$\mathcal{S}(k_a, k_J, \mu_S) \;=\; \int dk'_a \int dk'_J\, \mathcal{S}_{\rm part.}(k_a - k'_a, k_J - k'_J, \mu_S)\, S_{\rm mod.}(k'_a, k'_J).$$

perturbative soft function

model

Role of Jet's cone size

❑ Cone size dependence of Jet quenching:



Ratio is consistent with vacuum jets for peripheral and central collisions

Multiple scattering ➔ radiation ➔ energy loss ➔ cone size ➔ …