

### Hadron-Nucleus Interactions

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#### Hadronic showers: many particle species, wide energy range



# The FLUKA hadronic Models

Hadron-nucleus: PEANUT



### Hadron-nucleon interaction models



Elastic, charge exchange and strangeness exchange reactions:

- Available phase-shift analysis and/or fits of experimental differential data
- At high energies, standard eikonal approximations are used

### Nonelastic hN interactions at intermediate energies

•  $N_1 + N_2 \rightarrow N'_1 + N'_2 + \pi$  threshold at 290 MeV, important above 700 MeV, •  $\pi + N \rightarrow \pi' + \pi'' + N'$  opens at 170 MeV.

Anti-nucleon -nucleon open at rest!



Dominance of the  $\Delta$  resonance and of the N\* resonances

 $\rightarrow$  isobar model

 $\rightarrow$  all reactions proceed through an intermediate state containing at least one resonance.

FLUKA: ≈ 60 resonances, and ≈ 100 channels

Resonance energies, widths, cross sections, branching ratios from data and conservation laws, whenever possible. Inferred from inclusive cross sections when needed

 $\begin{array}{ll} N_{1} + N_{2} \rightarrow N_{1}^{''} + \Delta(1232) & \rightarrow N_{1}^{'} + N_{2}^{'} + \pi \\ \pi + N & \rightarrow \Delta(1600) & \rightarrow \pi' + \Delta(1232) & \rightarrow \pi' + \pi'' + N' \\ N_{1+} N_{2} & \rightarrow \Delta_{1}(1232) + \Delta_{2}(1232) & \rightarrow N_{1}^{'} + \pi_{1} + N_{2}^{'} + \pi_{2} \end{array}$ 

# Inelastic hN at high energies: (DPM, QGSM, ...)

- $\hfill\square$  Problem: "soft" interactions  $\rightarrow$  QCD perturbation theory cannot be applied.
- Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string)
- Interactions treated in the Reggeon-Pomeron framework
- □ each of the two hadrons splits into 2 colored partons  $\rightarrow$  combination into 2 colourless chains  $\rightarrow$  2 back-to-back jets
- each jet is then hadronized into physical hadrons

### Inelastic hN interactions at high energies (DPM, QGSM)

Problem: "soft" interactions  $\rightarrow$  no perturbation theory

Solution! Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string )

- Interactions treated in the Reggeon-Pomeron framework
- > At sufficiently high energies the leading term corresponds to a Pomeron (P) exchange (a closed string exchange)
- > Each colliding hadron splits into two colored partons  $\rightarrow$  a combination into two color neutral chains  $\rightarrow$  two back-to-back jets
- $\triangleright$  Physical particle (Reggeon,  $\mathbb{R}$ ) exchanges produce single chains at low energies
- > Higher order contributions with multi-Pomeron exchanges important at E<sub>lab</sub>>> 1 TeV

#### Nonelastic hN at high energies ( DPM )

Parton and color concepts, Topological expansion of QCD, Duality



#### Hadron-hadron collisions: chain examples



Leading two-chain diagram in DPM for p-p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities

Leading two-chain diagram in DPM for pbar-p scattering. The color (red, antired, blue, antiblue, green, and antigreen) and quark combination shown in the figure is just one of the allowed possibilities

#### Hadron-hadron collisions: chain examples II



Single chain (s-channel) diagram for  $\pi^+$ -p scattering. The color (red, antired, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities Leading two-chain diagram in DPM for  $\pi^+$ -p scattering. The color (red, antired, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities



### Dual Parton Model<sup>#</sup> and hadronization

#### From DPM:

- > Number of chains
- > Chain composition
- > Chain energies and momenta
- > Diffractive events

#### Almost No Freedom

\*Chain formation and "decay" (hadronization) processes are assumed to be decoupled Does it sound familiar?

#For a review: Physics Report 236

#### Chain hadronization

- > Assumes chain universality\*
- Fragmentation functions from hard processes and e<sup>+</sup>e<sup>-</sup> scattering
- Transverse momentum from uncertainty considerations
- Mass effects at low energies

The same functions and (few) parameters for all reactions and energies

### Inelastic hN interactions: examples



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#### FLUKA (PEANUT) modeling of nuclear interactions



### ... INC, a bit like snooker...

11.

The projectile i nucleus. The proand protons and light fragments senting the neutrons 1d a few

0:16

...it is in this phase that if energy is chough extra "balls" (new particles) are produced (contrary to snooker). The target "balls" are an way protons and neutrons, so further collisions will mostly knock out p's and ns

0.04

# "Classical" IntraNuclearCascade (INC) model:

50 MeV nucleon:  $\lambda = \hbar/p = 0.64$  fm, MFP ~ 1.2 fm at  $\rho$ ~0.08 200 MeV nucleon:  $\lambda = \hbar/p = 0.31$  fm, MFP ~ 4 fm at  $\rho$ ~0.08

Both Mean Free Path's without accounting for Pauli blocking which would increase them by a significant factor (~2 at 50 MeV)

Hence at intermediate energies, nucleon-nuclear reactions can be described as the passage of the incoming nucleon through the nucleus, undergoing individual nucleon-nucleon collisions (IntraNuclear Cascade).

#### Main assumptions:

- > Target nucleons occupy states of a cold Fermi gas;
- > Incoming nucleon follows a classical (straight) trajectory;
- Given a nucleon-nucleon interaction cross section and N, Z and density profile of the target nucleus, one can evaluate the mean free path (MFP) of the incoming nucleon
- The nucleon trajectory can be simulated as subsequent nucleon-nucleon collisions between straight-line trajectory segments, governed by the calculated MFP
- Collision products must be above the Fermi level ("Pauli blocking") and can either escape or get "captured" if their energy is insufficient versus the binding or Coulomb barrier
- "Captured" nucleon energies above E<sub>F</sub> and the holes in the Fermi gas both contribute to a residual excitation to be spent through the statistical model

### Sketch\* of IntraNuclearCascade (INC):



### Evaporation:

After many conucleus is left can be describ of "droplets", from a "boiling

The process is leftover nucle **energies** ~ **A** enough to allo neutrons have **favoured**.



#### Fermi gas model: Nucleons = Non-interacting Constrained Fermions

The observed central/saturation density of nuclei,  $\rho \approx 0.17$  fm<sup>-3</sup> (1.7×10<sup>38</sup> nucleons/cm<sup>3</sup>), implies:

 $K_F = 1.36 \, \text{fm}^{-1}$   $E_F = 38 \, \text{MeV}$ 

These are called the Fermi momentum and Fermi Energy

The probability distribution for the momentum/energy of a nucleon are therefore given by:

$$P(K)dK = \frac{K^2}{3K_F^3}dK \qquad P(E_k)dE_k = \frac{2\sqrt{E_k}}{3E_F^3}dE_k$$

In nuclei with  $N \neq Z$ , two different values of the Fermi energy can be defined:

$$\rho_{p}(r) = \frac{Z}{A}\rho = \frac{1}{3\pi^{2}} \left(K_{F}^{p}\right)^{3} \qquad \rho_{n}(r) = \frac{N}{A}\rho = \frac{1}{3\pi^{2}} \left(K_{F}^{n}\right)^{3}$$

The so defined Fermi energies are kinetic energies, counted from the bottom of a potential well that in this model must be input from outside. This gives an average potential depth of about **38+8=46 MeV**. The Fermi energy can be made radius-dependent in a straightforward way, through the so called *local density approximation*:

$$\rho(r) = \frac{2}{3\pi^2} K_F^3(r)$$

# Nuclear potential for n/p: schematic drawing



# (Generalized) IntraNuclear Cascade

- Primary and secondary particles moving in the nuclear medium
- Target nucleons motion and nuclear well according to the Fermi gas model
- Interaction probability
  - $σ_{free}$  + Fermi motion × ρ(r) + exceptions (ex. π)
- Glauber cascade at higher energies
- $\Box$  Classical trajectories (+) nuclear mean potential (resonant for  $\pi$ )
- $\hfill\square$  Curvature from nuclear potential  $\rightarrow$  refraction and reflection
- Interactions are incoherent and uncorrelated
- $\square$  Interactions in projectile-target nucleon CMS  $\rightarrow$  Lorentz boosts
- **Multibody absorption for**  $\pi$ ,  $\mu^-$ , K<sup>-</sup>
- Quantum effects (Pauli, formation zone, correlations...)
- Exact conservation of energy, momenta and all addititive quantum numbers, including nuclear recoil

#### hA at high energies: Glauber-Gribov cascade with formation zone

#### Glauber cascade

- > Quantum mechanical method to compute Elastic, Quasi-elastic and Absorption hA cross sections from Free hadron-nucleon scattering + nuclear ground state
- > Multiple Collision expansion of the scattering amplitude

#### Glauber-Gribov

- Field theory formulation of Glauber model
- ➤ Multiple collisions ↔ Feynman diagrams
- > High energies: exchange of one or more Pomerons with one or more target nucleons (a closed string exchange)

#### Formation zone (=materialization time)

#### From one to many: Glauber cascade

At energies below a few GeV hA interactions can be described by a single primary collision hN (elastic or non-elastic), followed by reinteraction of the secondary particles (INC).

At higher energies, the **Glauber** calculus predicts explicit multiple primary collisions



Due to the relativistic length contraction and the uncertainty principle, at high energy most of the newly produced particles escape the nucleus without further reinteraction

#### Gribov interpretation of Glauber multiple collisions

The absorption cross section can be shown to be just the integral in the impact parameter plane of the probability of getting at least one non-elastic hadron-nucleon collision

and the overall average number of collision integrated over the impact parameter space is given by

$$\langle \nu \rangle = \frac{Z\sigma_{hpr} + N\sigma_{hnr}}{\sigma_{hA\,abs}}$$

- Glauber-Gribov model = Field theory formulation of Glauber model
- $\square Multiple collision terms \Rightarrow Feynman graphs$
- At high energies : exchange of one or more pomerons with one or more target nucleons
- □ In the Dual Parton Model language: (neglecting higher order diagrams): Interaction with *n* target nucleons  $\Rightarrow$  2*n* chains
  - > Two chains from projectile valence quarks + valence quarks of one target nucleon =>valence-valence chains
  - > 2(n-1) chains from sea quarks of the projectile + valence quarks of target nucleons  $\Rightarrow$  2(n-1) sea-valence chains

#### Glauber-Gribov: chain examples



Leading two-chain diagrams in DPM for *p-A* Glauber scattering with 4 collisions. The color (red blue green) and quark combinations shown in the figure are just one of the allowed possibilities Leading two-chain diagrams in DPM for  $\pi^{+-}A$ Glauber scattering with 3 collisions.



Condition for possible reinteraction inside a nucleus:

\* J.Ranft applied the concept, originally proposed by Stodolski, to hA and AA nuclear interactions

$$\Delta x_{for} \le R_A \approx r_0 A^{\frac{1}{3}}$$

# Effect of Glauber and Formation Zone



Rapidity distribution of charged particles produced in 250 GeV  $\pi^+$ collisions on Gold Points: exp. data (Agababyan et al., ZPC50, 361 (1991)). ( rapidity  $\approx -\ln(tg(\theta/2))$ )

Large Effects on:
Multiplicity
Energy distribution
Angular distribution





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## Double differential pion production



#### **Pions: nuclear medium effects** $\implies$ Non resonant channel

Free  $\pi$  N interactions  $\Rightarrow$ 

#### P-wave resonant $\Delta$ production

 $\Delta$  in nuclear  $\Rightarrow$  decay  $\Rightarrow$  elastic scattering, charge exchange  $\stackrel{\text{medium}}{\longrightarrow} \text{reinteraction} \stackrel{\longrightarrow}{\longrightarrow} \text{Multibody pion absorption}$ 

Assuming for the free resonant  $\sigma_{res}^{Free} = \frac{8\pi}{p_{cms}^2} \frac{M_{\Delta}^2 \Gamma_F^2(p_{cms})}{\left(s - M_{\Delta}^2\right)^2 + M_{\Delta}^2 \Gamma_F^2(p_{cms})}$ 

An `` in medium'' resonant  $\sigma$  ( $\sigma^{A}_{res}$ ) can be obtained adding to  $\Gamma_{F}$  the imaginary part of the (extra) width arising from nuclear medium

 $\frac{1}{2}\Gamma_{T} = \frac{1}{2}\Gamma_{F} - \operatorname{Im}\Sigma_{\Delta} \quad \Sigma_{\Delta} = \Sigma_{qe} + \Sigma_{2} + \Sigma_{3} \quad \text{(Oset et al., NPA 468, 631)}$ quasielastic scattering, two and three body absorption

The in-nucleus  $\sigma_t^A$  takes also into account a two-body s-wave absorption  $\sigma_s^A$ derived from the optical model

$$\sigma_t^A = \sigma_{res}^A + \sigma_t^{Free} - \sigma_{res}^{Free} + \sigma_s^A \quad \sigma_s^A(\omega) = \frac{4\pi}{p} \left(1 + \frac{\omega}{2m}\right) \operatorname{Im} B_0(\omega) \rho$$

# Pion absorption

Pion absorption cross section on Gold and Bismuth in the  $\triangle$  resonance region (multibody absorption in PEANUT)

Emitted proton spectra at different angles , 160 MeV  $\pi^+$  on <sup>58</sup>Ni Phys. Rev. C41,2215 (1990) Phys. Rev. C24,211 (1981) Proton spectra extend up to 300 MeV



Particle energy (MeV)

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# Preequilibrium emission

For E >  $\pi$  production threshold  $\rightarrow$  only (G)INC models At lower energies a variety of preequilibrium models == share the excitation energy among many nucleons/holes

#### The quantum-mechanical multistep The semiclassical exciton model model: Statistical assumptions Very good theoretical background Simple and fast Complex, difficulties for multiple Suitable for MC emissions

#### Two leading approaches

Statistical assumption:

any partition of the excitation energy  $E^*$  among N, N = N<sub>h</sub> + N<sub>p</sub>, excitons has the same probability to occur Step: nucleon-nucleon collision with  $N_{n+1}=N_n+2$  ("never come back approximation) Chain end = equilibrium =  $N_n$  sufficiently high or excitation energy below threshold  $N_1$  depends on the reaction type and cascade history



Angle-integrated <sup>90</sup>Zr(p,xn) at 80.5 MeV

The various lines show the **total**, **INC**, **preequilibrium** and **evaporation** contributions

Experimental data • from M. Trabandt et al., Phys. Rev. C39, 452 (1989)



### Coalescence



High energy light fragments are emitted through the coalescence mechanism: "put together" emitted nucleons that are near in phase space.

Example : double differential t (<sup>3</sup>H) production from 542 MeV neutrons on Copper

Warning: coalescence is OFF by default Can be important, expecially for . residual nuclei. To activate it:

PHYSICS 1.

COALESCE

If coalescence is on, switch on Heavy ion transport and interactions (see later)

#### Equilibrium particle emission (evaporation, fission and nuclear break-up)

From statistical considerations and the detailed balance principle, the probabilities for emitting a particle of mass m<sub>j</sub> spin S<sub>j</sub>ħ and energy E, or of fissioning are given by\*: (i, f for initial/final state, Fiss for fission saddle point)



Neutron emission is strongly favoured because of the lack of any barrier Heavy nuclei generally reach higher excitations because of more intense cascading Weisskopf-Ewing approach

# Equilibrium particle emission in Fluka

- Evaporation: Weisskopf-Ewing approach
  - ~600 possible emitted particles/states (A<25) with an extended (heavy) evaporation/fragmentation formalism</li>
  - Full level density formula with level density parameter A,Z and excitation dependent
  - Inverse cross section with proper sub-barrier
  - Analytic solution for the emission widths (neglecting the level density dependence on U, taken into account by rejection
  - Emission energies from the width expression with no. approx.
- Fission: past, improved version of the Atchison algorithm, now
  - $\Gamma_{fis}$  based of first principles, full competition with evaporation
  - Improved mass and charge widths
  - Myers and Swiatecki fission barriers, with exc. en. dependent level density enhancement at saddle point
- Fermi Break-up for A<18 nuclei
  - ~ 50000 combinations included with up to 6 ejectiles
- γ de-excitation: statistical + rotational + tabulated levels

### Thick target examples: neutrons



# Residual nuclei

Experimental and computed residual nuclei mass distribution for Ag(p,x)X at 300 GeV

Data from Phys. Rev. C19 2388 (1979)



The heavy evaporation/fragmentation model ("New FLUKA") has much improved the FLUKA predictions

Also for A-A interactions

Warning: heavy evaporation/fragmentation is OFF by default, because it is a cpu-eater. It is NECESSARY to activate it for activation studies:

PHYSICS 3.

EVAPORAT

If fragmentation is on, switch on Heavy ion transport and interactions (see later)

# Residual Nuclei

- The production of residuals is the result of the last step of the nuclear reaction, thus it is influenced by all the previous stages
- Residual mass distributions can be very well reproduced
- Individual residuals near to the compound mass are usually well reproduced
- The production of specific isotopes may be influenced by additional problems which have little or no impact on the emitted particle spectra (Sensitive to details of evaporation, Nuclear structure effects, Lack of spinparity dependent calculations in most MC models)



# Example of fission/evaporation



### **Isotope production for** <sup>nat</sup>Fe(p,x):



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### Gamma De-excitation in Fluka

**\Box** At the end of evaporation : cascade of  $\gamma$  transitions

At high excitation: assume continuous level density and statistical emission:

$$P(E_{\gamma})dE_{\gamma} = \frac{\rho_f(U_f)}{\rho_i(U_i)} \sum_{L} f(E_{\gamma}, L) \quad \text{L= multipo}_{\rho = \text{level der}}$$

f = strength from single
particle estimate (c)+ hindrance (F)

/ - -

L= multipole order p=level density at excitation energy. U

• At low excitation: through discrete levels  $f(E_{\gamma}, L) = c_L F_L(A) E_{\gamma}^{(2L+1)}$ 

- database of known levels and transitions taken from RIPL-3 (IAEA)
- Rotational approximation outside tabulations
- Discrete level treatment extended to evaporation stage
- > Same for residuals from ion-ion interactions

Examples of prompt photon predictions for therapy monitoring in the Fluka outlook, last day

# **Prompt photons**

- Application: On-line monitoring during hadrotherapy
- prompt γ follow dose profile (data: IEEE TNS 57 (2009))
- Need collimation and time cut





Benchmark: γ spectrum from 160 MeV
 p in PMMA FLUKA+experimental
 resolution red line, data black line
 (J.Smeets et al., ENVISION WP3)

# **Real and Virtual Photonuclear Interactions**

**Photonuclear reactions** (*PHOTONUC* card, off by default)

- □ Giant Dipole Resonance interaction (special database)
- Quasi-Deuteron effect
- Delta Resonance energy region
- Vector Meson Dominance in the high energy region
- □ INC, preequilibrium and evaporation via the PEANUT model
- Possibility to bias the photon nuclear inelastic interaction length to enhance interaction probability (LAM-BIAS card, see manual)

#### Virtual photon reactions

- □ Muon photonuclear interactions (MUPHOTON card, on by default)
- □ Electromagnetic dissociation (PHYSICS card with SDUM=EM-DISSO, off by default)

### Electromagnetic dissociation



... nuclear and, mostly, ElectroMagneticDissociation collisions on machine elements or at IP's produce a variety of (excited), possibly radioactive, fragments in flight <u>م</u> 15 °° Pb ions on various targets Cross section (b) 158A GeV Pb-Pb total EMD total 10 Alice: 1n √s<sub>nn</sub>=2. nuclear Symbols: exp. Data 8 TeV 2n 4 Lines: Fluka 5 10 EMD total nuclear 200 50 100 150 n  $10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}$ Α, Total EMD and nuclear cross section Total charge changing cross section as a function of atomic mass as a function of the effective  $\gamma$  factor

however ...

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# Thanks for your attention!

#### hA at high energies: Glauber-Gribov cascade with formation zone

- Glauber cascade
  - Quantum mechanical method to compute Elastic, Quasi-elastic and Absorption hA cross sections from Free hadron-nucleon scattering + nuclear ground state
  - Multiple Collision expansion of the scattering amplitude
- Glauber-Gribov
  - Field theory formulation of Glauber model
  - Multiple collisions ↔ Feynman diagrams
  - High energies: exchange of one or more Pomerons with one or more target nucleons (a closed string exchange)
- Formation zone (=materialization time)

Glauber formalism/cascade (R. Glauber, 2005 Physics Nobel prize) Quantum mechanical method to compute all relevant hadron-nucleus cross sections from hadron-nucleon scattering:  $S_{\mu N}(\vec{b},s) = e^{i\chi_{\mu N}(\vec{b},s)} = \eta_{\mu N}(\vec{b},s)e^{2i\delta_{\mu N}(\vec{b},s)}$ and nuclear ground state wave function  $\Psi_i$  $\sigma_{hAT}(s) = 2\int d^2 \vec{b} \int d^3 \vec{u} \left| \Psi_i \left( \vec{u} \right)^2 \right| 1 - \prod_{i=1}^A \operatorname{Re} S_{hN} \left( \vec{b} - \vec{r}_{j\perp}, s \right) \right|$ Total Elastic  $\sigma_{hAel}(s) = \int d^2 \vec{b} \int d^3 \vec{u} |\Psi_i(\vec{u})|^2 \left[ 1 - \prod_{i=1}^A S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right]^2$ Scattering  $\sigma_{hA\Sigma f}(s) \equiv \sum_{f} \sigma_{hAfi}(s) = \int d^2 \vec{b} \int d^3 \vec{u} |\Psi_i(\vec{u})|^2 \left[ 1 - \prod_{i=1}^{A} S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right]^2$ These relations are derived from the expression of the nuclear scattering amplitude.

For "Scattering" we mean all the channels where the particles in the initial and final state are the same, with different momenta and excitation energies

From these basic quantities we can define other relevant cross sections:

#### Glauber: quasi-elastic and absorption cross sections

 $\sigma_{hAr}(s) \equiv \sigma_{hAT}(s) - \sigma_{hAr}(s)$ Reaction: Quasi-elastic (incoherent-elastic):  $\sigma_{hAae}(s) \equiv \sigma_{hA\Sigma f}(s) - \sigma_{hAel}(s)$ 

**Particle production** (alias absorption) cross section (the fundamental formula)

 $\sigma_{hAabs}(s) \equiv \sigma_{hAT}(s) - \sigma_{hA\Sigma f}(s)$  $=\sigma_{hAr}(s)-\sigma_{hAge}(s)$  $= \int d^{2}\vec{b} \int d^{3}\vec{u} |\Psi_{i}(\vec{u})|^{2} \left\{ 1 - \left\{ \prod_{i=1}^{A} 1 - \left[ 1 - \left| S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right|^{2} \right\} \right\} \right\}$ 

Absorption probability over a given b and nucleon configuration

That can be written in a synthetic way by defining the function 
$$\mu$$
 to replace the integral on nuclear coordinates

$$\sigma_{hA\,abs}(s) \equiv \int d^2 \vec{b} \ \mu_{hA\,abs}(\vec{b},s)$$

#### Glauber: continued

 $\sigma_{hA abs}$  can be interpreted in terms of multiple collisions of the projectile:

From the impact parameter representation of the hadron-nucleon reaction cross section

$$\sigma_{hNr}(s) = \int d^2 \vec{b} \left[ 1 - \left| S_{hN}(\vec{b}, s)^2 \right] \right]$$

And from the thickness function for non-elastic reactions,  $T_{rj} =$  contribution of the j-th target nucleon to the amount of nuclear matter seen by the incident hadron traveling along the impact parameter *b* when folded with its profile function

 $\sigma_{hNr}T_{rj}(b) \equiv P_{rj}(b)$ 

Is the probability to have an inelastic reaction on the j-th target nucleon

assuming that all nucleons are equal  $\mu_{\mu}$  we can write

$${}_{hA\,abs}(b) = 1 - \left[1 - \sigma_{hN\,r} T_r(b)\right]^A = \sum_{\nu=1}^A \binom{A}{\nu} \left[\sigma_{hN\,r} T_r(b)\right]^\nu \left[1 - \sigma_{hN\,r} T_r(b)\right]^{A-\nu}$$

Therefore

$$\mu_{hAabs}(b) = \sum_{\nu=1}^{A} \binom{A}{\nu} P_{r}^{\nu}(b) [1 - P_{r}(b)]^{A-\nu} = \sum_{\nu=1}^{A} P_{r\nu}(b)$$

#### Glauber: continued

$$P_{rv}(b) = \begin{pmatrix} A \\ v \end{pmatrix} P_r^v(b) [1 - P_r(b)]^{A-1}$$

Since  $P_r(b)$  is the probability of getting one specific nucleon hit and there are A possible trials,  $P_{rv}(b)$  is exactly the binomial distribution for getting v successes out of A trials, with probability  $P_r(b)$  each

Therefore the absorption cross section is just the integral in the impact parameter plane of the probability of getting at least one non-elastic hadron-nucleon collision

$$\sigma_{hA\,abs}(s) \equiv \int d^2 \vec{b} P_{rv}(b)$$

The average number of non-elastic hadron-nucleon collisions for a given impact parameter b is given by

 $\langle v(b) \rangle = A P_r(b)$ 

and the overall average number of collision is given by

$$\langle \nu \rangle = \frac{Z\sigma_{hpr} + N\sigma_{hnr}}{\sigma_{hAabs}}$$



#### Heavy ion interaction models in FLUKA

DPMJET-III

DPMJET (R. Engel, J. Ranft, S. Roesler<sup>1</sup>): Nucleus-Nucleus interaction model. Used in many Cosmic Ray shower codes. Based on the Dual Parton Model and formation zone Glauber cascade, like the highenergy FLUKA h-A event generator.

Modified and extended version of rQMD-2.4 rQMD-2.4 (H. Sorge et al.<sup>2</sup>) Cascade-Relativistic QMD model. Successfully applied to relativistic A-A particle production.

BME (BoltzmannMasterEquation) FLUKA implementation of BME from E.Gadioli et al (Milan)

**FLUKA Evaporation**fissionfragmentation module handles fragment deexcitation **Tested** and benchmarked in h-A reactions

(Projectile-like evaporation is responsible for the most energetic fragments)

<sup>1</sup>proc. MC2000, p 1033 (2001) <sup>2</sup>NPA 498, 567c (1989), Ann.Phys. 192,266 (1989), PRC 52, 3291 (1995)

### Photonuclear int.: example

Reaction:  $^{208}Pb(\gamma, x n)$   $20 \le E\gamma \le 140 \text{ MeV}$ 

Cross section for multiple neutron emission as a function of photon energy, Different colors refer to neutron multiplicity  $\ge n$ , with  $2 \le n \le 8$ 

Symbols: exp data (NPA367, 237 (1981) ; NPA390, 221 (1982) )

Lines: FLUKA

