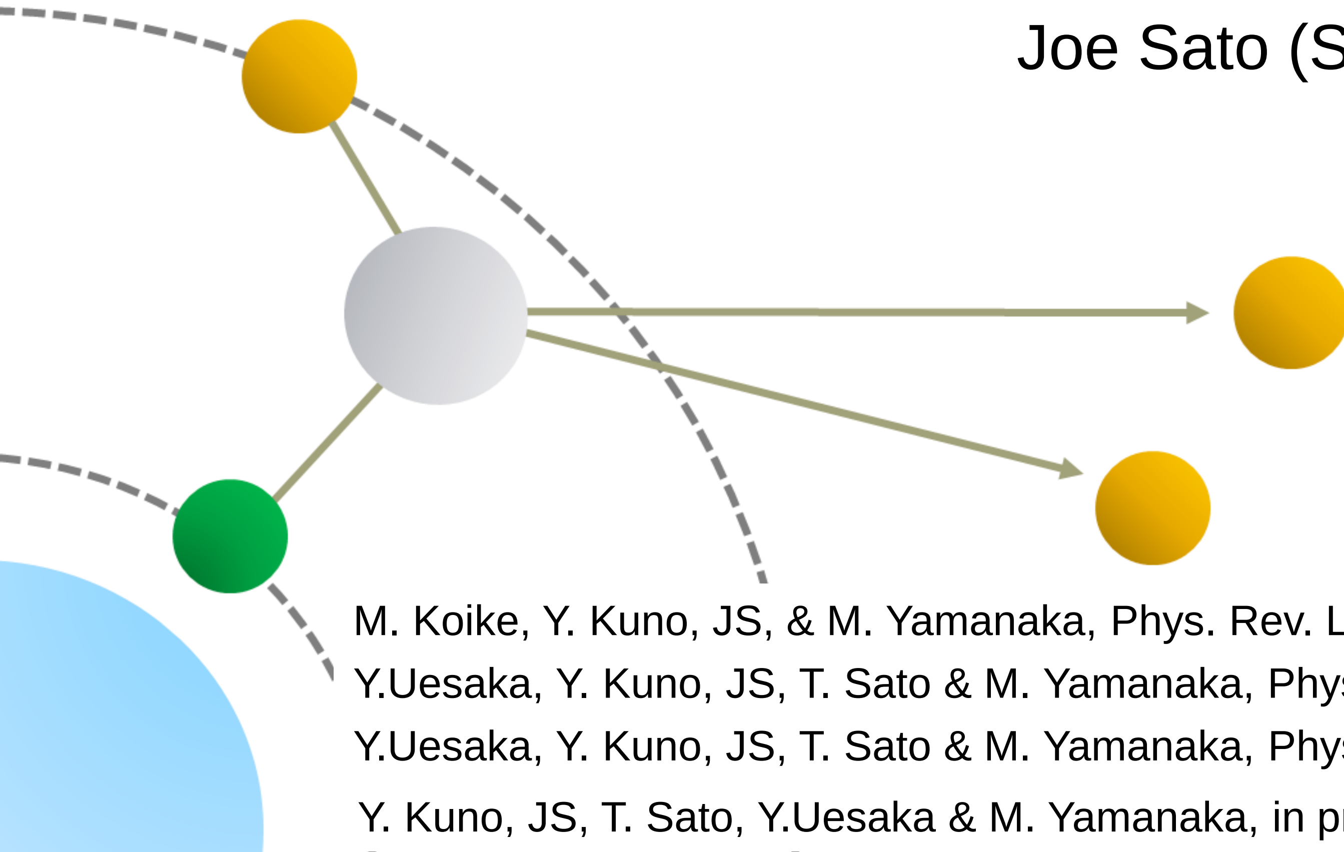
Nufact18 @ Virginia Tech

Distinguishing muon LFV effective couplings using $\mu^- e^- \rightarrow e^- e^-$ in a muonic atom

Joe Sato (Saitama University)



M. Koike, Y. Kuno, JS, & M. Yamanaka, Phys. Rev. Lett. **105**, 121601 (2010).
Y.Uesaka, Y. Kuno, JS, T. Sato & M. Yamanaka, Phys. Rev. D **93**, 076006 (2016).
Y.Uesaka, Y. Kuno, JS, T. Sato & M. Yamanaka, Phys. Rev. D **97**, 015017 (2018).
Y. Kuno, JS, T. Sato, Y.Uesaka & M. Yamanaka, in preparation

Contents

1. Introduction
   - Charged Lepton Flavor Violation (CLFV)
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   - Energy-angular distribution of emitted electrons
   - Asymmetry of emitted electrons by polarizing muon
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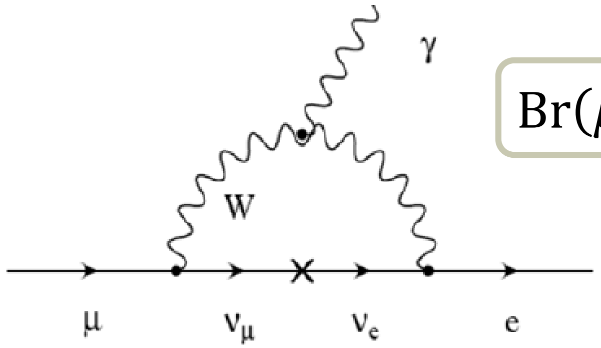
Charged Lepton Flavor Violation (CLFV)

- A probe for new physics -

◆ lepton flavor violation for charged lepton = CLFV

- forbidden in SM
- contribution of lepton mixing → very small



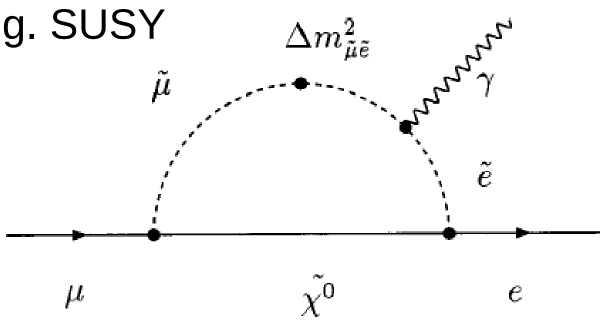
$\mathrm{Br}(\mu \to e\gamma) \lesssim 10^{-54}$

cf. current experimental upper limit $\mathrm{Br} < 4.2 \times 10^{-13}$

✓ cannot be observed by current technology

- enhanced in many theories beyond SM

e.g. SUSY



✓ Searches for CLFV can access high energy physics with little SM backgrounds.

CLFV searches in muon rare decay

Advantages of muon

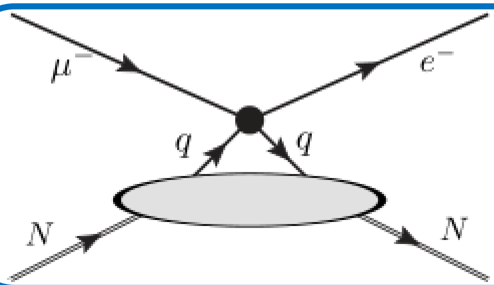
1. high intensity
2. long lifetime

➢ current bounds

L. Calibbi & G. Signorelli, arXiv:1709.00294 [hep-ph].

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006

μ^--e^- conversion



✓ CLFV search using μ-atom

✓ exploring μeqq interaction

New experiments for "$\mu^- - e^-$ conversion" are planned with higher sensitivity than previous ones. (COMET, DeeMe @ J-PARC, Mu2e @ Fermilab)

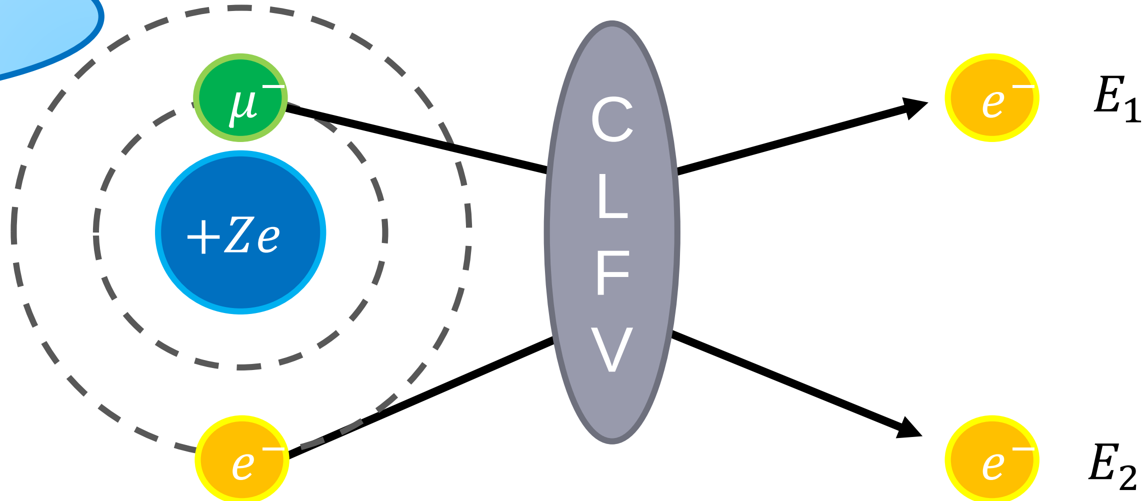
$\mu^- e^- \to e^- e^-$ in a muonic atom

M. Koike, Y. Kuno, J. Sato, & M. Yamanaka,
Phys. Rev. Lett. **105**, 121601 (2010).

New CLFV search using muonic atoms

proposal in **COMET**

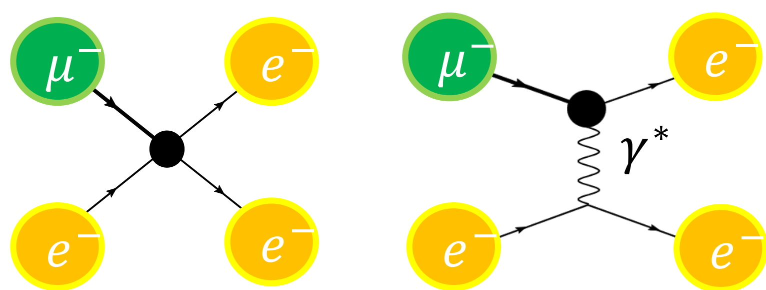
R. Abramishvili et al.,
COMET Phase-I Technical Design Report (2016).



Features

- clear signal : $E_1 + E_2 \simeq m_\mu + m_e - B_\mu - B_e$
- 2 CLFV mechanisms
  - ✓ contact (μeee vertex)
  - ✓ photonic ($\mu e\gamma$ vertex)

  (similar to $\mu^+ \to e^+ e^+ e^-$)

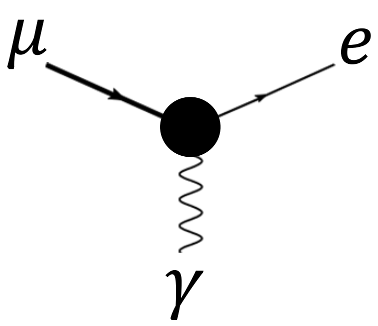


- atomic # Z : large $\Rightarrow$ decay rate Γ : large ($\Gamma \propto (Z-1)^3$)

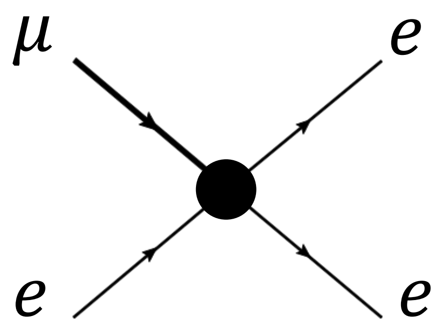
Comparison to other muonic CLFV

Typical effective CLFV interactions

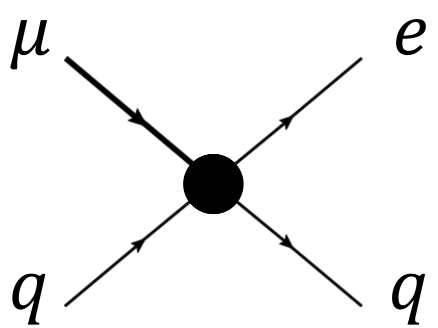
1. $\mu e \gamma$



2. μeee



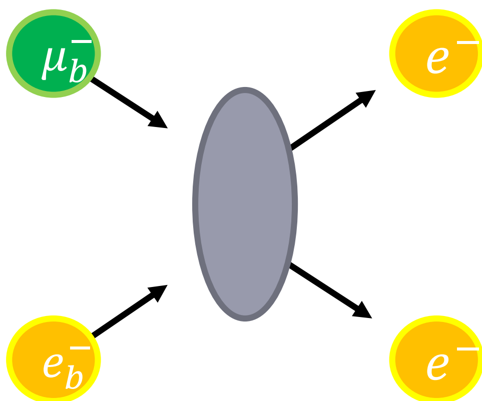
3. μeqq



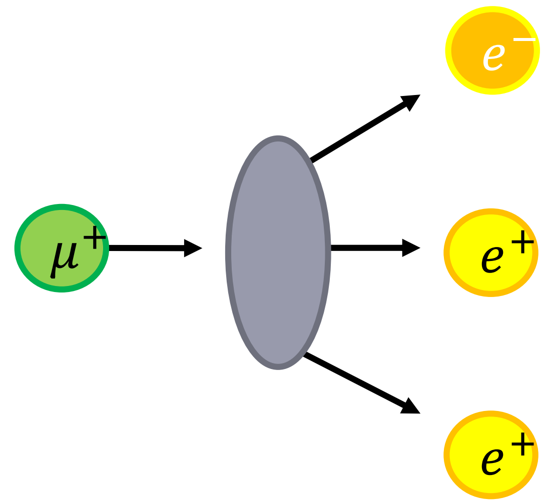
	1. $\mu e \gamma$	2. μeee	3. μeqq
$\mu^+ \to e^+ \gamma$	✓	-	-
$\mu^+ \to e^+ e^- e^+$	✓	✓	-
μ^--e^- conv.	✓	-	✓
$\mu^- e^- \to e^- e^-$	✓	✓	-

Comparison to $\mu^+ \to e^+ e^+ e^-$

$\mu^- e^- \to e^- e^-$ in a muonic atom



$\mu^+ \to e^+ e^+ e^-$



difference 1 : signal

2 e^-s

1 e^- & 2 e^+s

difference 2 : interference among CLFV couplings

$C^Z_{XY}(\bar{e}\Gamma_Z P_X \mu)(\bar{e}\Gamma_Z P_Y e)$

$$\Gamma_{\mu e \to ee} \propto \left|C^S_{RR} + 4C^V_{LL}\right|^2 + \left|C^S_{LL} + 4C^V_{RR}\right|^2 + 4\left|C^V_{RL} - C^V_{LR}\right|^2$$

$$\Gamma_{\mu \to eee} \propto \left|C^S_{RR}\right|^2 + \left|C^S_{LL}\right|^2 + \left|C^V_{RR}\right|^2 + \left|C^V_{LL}\right|^2 + 8\left|C^V_{RL}\right|^2 + 8\left|C^V_{LR}\right|^2$$

(Rough) Estimation of decay rate

"flux"

$$\Gamma = \sigma v_{\rm rel} \int \mathrm{d}V \rho_\mu \rho_e$$

Suppose nuclear Coulomb potential is weak,

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\rm rel} |\psi_{1S}^e(0)|^2$$

(sum of two $1S$ e^-s)

Phys. Rev. Lett. **105**,121601 (2010).

σ : cross section of $\mu^- e^- \to e^- e^-$ (free particles')

$v_{\rm rel}$: relative velocity of μ^- & e^-

$$\psi_{1S}^e(\vec{r}) = \sqrt{\frac{(m_e(Z-1)\alpha)^3}{\pi}} \exp(-m_e(Z-1)\alpha|\vec{r}|)$$

: wave function of $1S$ bound electron (non-relativistic)

$$\Rightarrow \Gamma \propto (Z-1)^3$$

(the same Z dependence in the both contact & photonic cases)

Branching ratio of CLFV decay

How many muonic atoms decay with CLFV, compared to created # ?

$$\mathrm{BR}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$

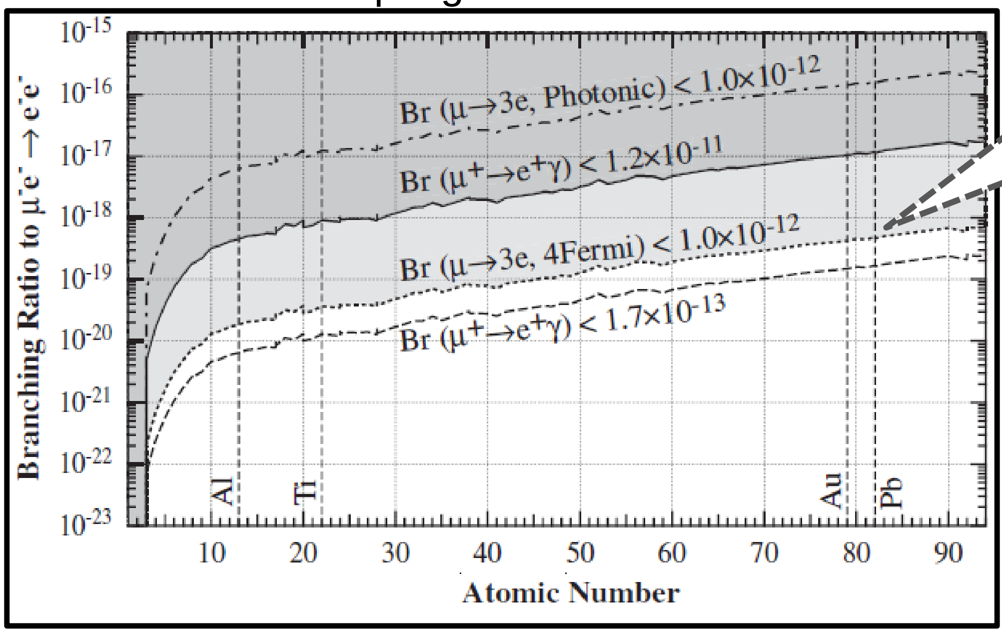
$\Gamma \propto (Z-1)^3$

due to existence prob. of bound e^- at the origin

$\tilde{\tau}_\mu$: lifetime of a muonic atom

cf. $2.2\mu\mathrm{s}$ for a muonic H ($Z = 1$)
$80\mathrm{ns}$ for a muonic Pb ($Z = 82$)

BR with CLFV coupling fixed on allowed maximum



Phys. Rev. Lett. **105**,121601 (2010).

e.g. $\mathrm{BR} < 5.0 \times 10^{-19}$ for Pb ($Z = 82$)

if contact process is dominant

➢ BR increases with atomic # Z.

Using muonic atoms with large Z is favored to search for $\mu^- e^- \to e^- e^-$.

To improve calculation for decay rate

✓ previous formula of CLFV decay rate by Koike *et al.*

$$\Gamma_{\mu^- e^- \to e^- e^-} = 2\sigma v_{\mathrm{rel}} |\psi_{1S}^{e}(0)|^2 \propto (Z-1)^3$$

Note

- ➢ "Z dependence" comes from only $|\psi_{1S}^{e}(0)|^2$ (always $\Gamma \propto (Z-1)^3$)
- ➢ emitted e^-s are expected to be back-to-back with equal energies

used approximations ($Z\alpha \ll 1$)

- spatial extension of bound lepton $\gg$ wave length of emitted e^-
- bound lepton : non-relativistic
- emitted e^- : plane wave

In atoms with large Z,

- ← *small orbital radius*
- ← *relativistic (especially, e^-)*
- ← *Coulomb distortion*

More quantitative estimation is needed ! (important for large Z)

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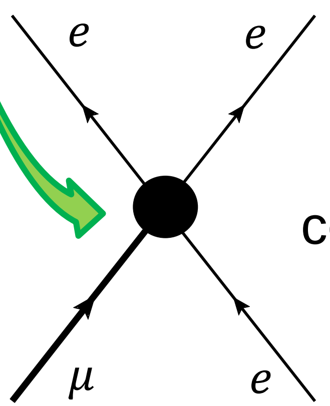
Effective Lagrangian for $\mu^- e^- \to e^- e^-$

$$\mathcal{L}_I = \mathcal{L}_{contact} + \mathcal{L}_{\mu\to e\gamma}$$

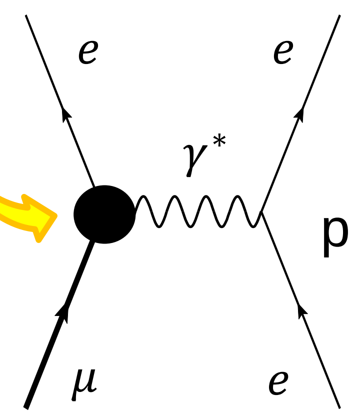
$$\mathcal{L}_{\text{contact}} = -\frac{4G_F}{\sqrt{2}}[g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L)$$
$$+g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L)$$
$$+g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R)] + [H.c.]$$

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F}{\sqrt{2}}m_\mu[A_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + A_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu}] + [H.c.]$$



contact interaction

constrained by $\mu \to eee$



photonic interaction

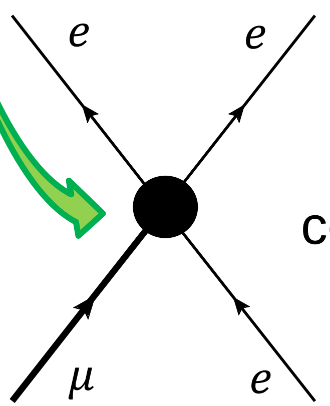
constrained by $\mu \to e\gamma$

Effective Lagrangian for $\mu^- e^- \to e^- e^-$

$$\mathcal{L}_I = \underline{\mathcal{L}_{contact}} + \underline{\mathcal{L}_{\mu\to e\gamma}}$$

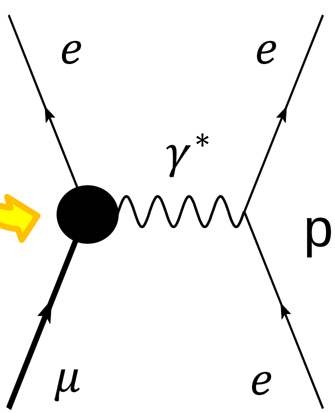
$$\mathcal{L}_{\text{contact}} = -\frac{4G_F}{\sqrt{2}}\left[g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L) + g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L) + g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R)\right] + [H.c.]$$

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F}{\sqrt{2}} m_\mu\left[A_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + A_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu}\right] + [H.c.]$$



contact interaction

constrained by $\mu \to eee$



photonic interaction

constrained by $\mu \to e\gamma$

Our formulation for decay rate

$$\Gamma = \sum_{f} \sum_{\bar{\imath}} (2\pi)\delta(E_f - E_i) \left| \left\langle \psi_e^{\boldsymbol{p}_1, s_1} \psi_e^{\boldsymbol{p}_2, s_2} \middle| H \middle| \psi_\mu^{1s, s_\mu} \psi_e^{1s, s_e} \right\rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^{\boldsymbol{p}, s} = \sum_{\kappa, \mu, m} 4\pi \, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y^*_{l_\kappa, m}(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa, \mu}$$

κ : index of angular momentum

get radial functions by solving "Dirac eq. with ϕ" numerically

$$\frac{d g_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{d f_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

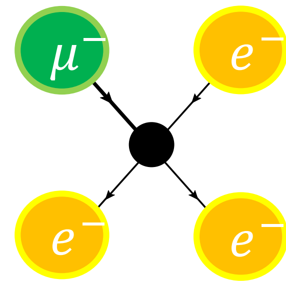
ϕ : nuclear Coulomb potential

$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

Contact process

✓ bound μ^-

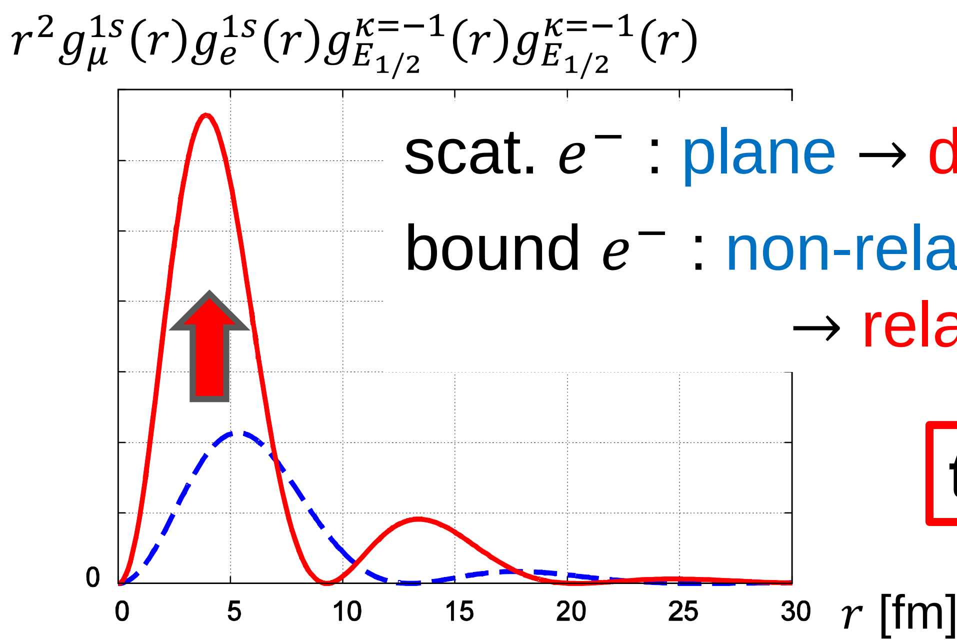
✓ bound e^-



✓ scattering e^-

✓ scattering e^-

◆ overlap of bound μ^-, bound e^-, and two scattering e^-s



scat. e^- : plane → distorted

bound e^- : non-relativistic → relativistic

wave functions shift to the center

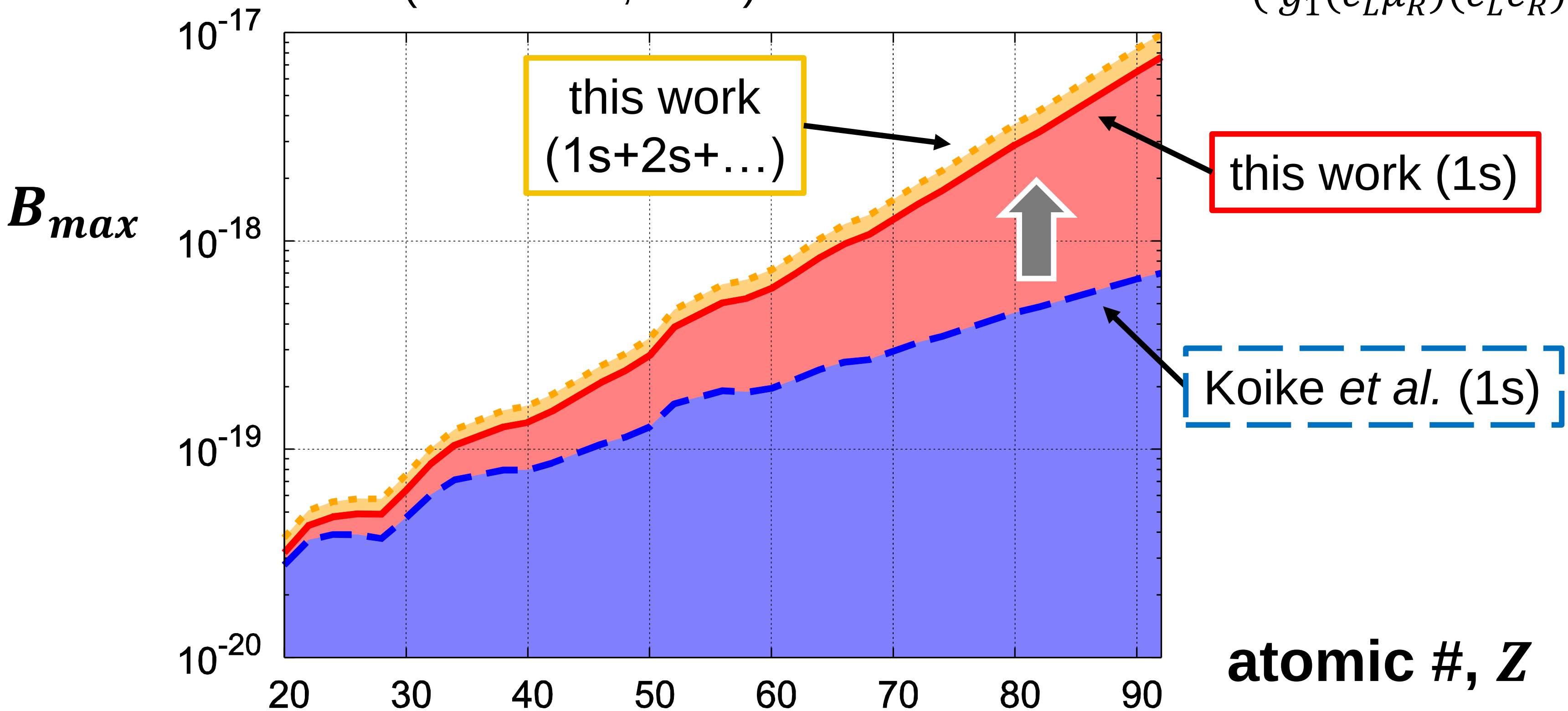
transition rate increases!

Upper limits of BR (contact process)

$BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}$ → $BR(\mu^-e^- \to e^-e^-) < B_{max}$

(SINDRUM, 1988)

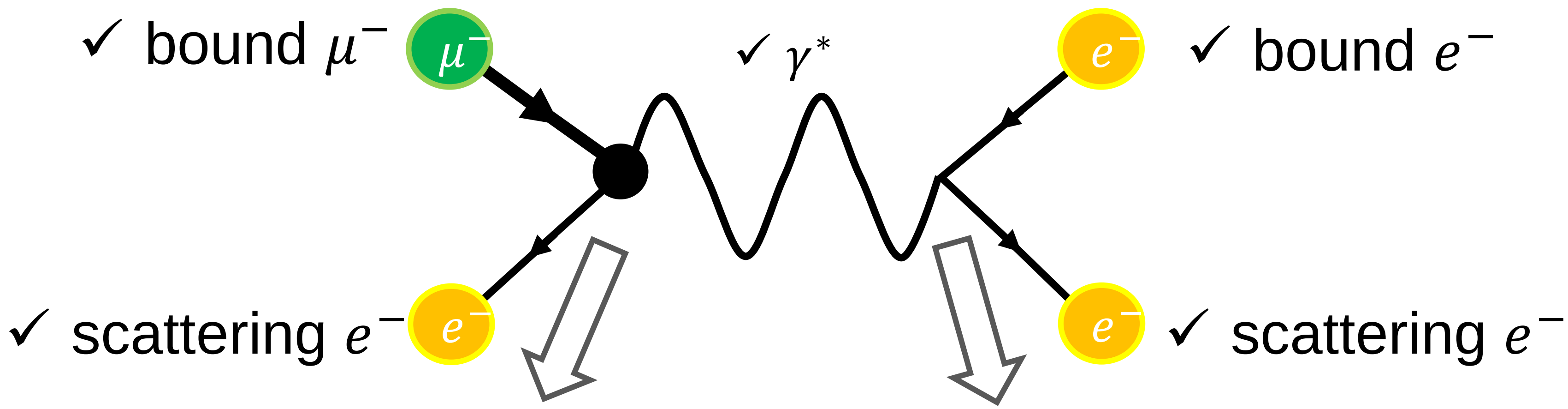
$(g_1(\bar{e_L}\mu_R)(\bar{e_L}e_R))$

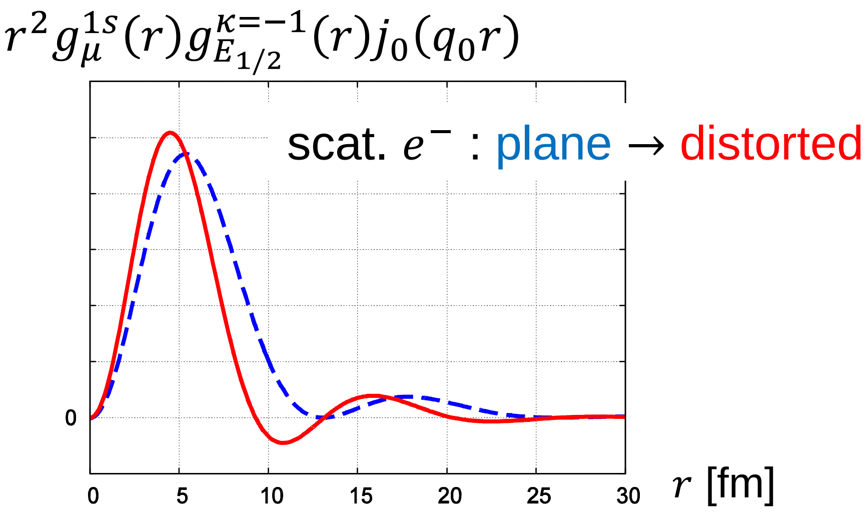


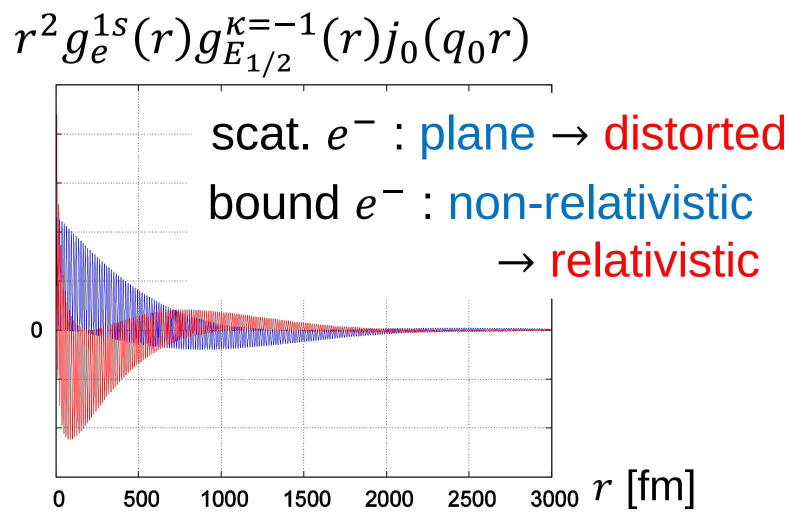
inverse of B_{max} ($Z = 82$)

2.1×10^{18} → 3.0×10^{17}

Photonic process







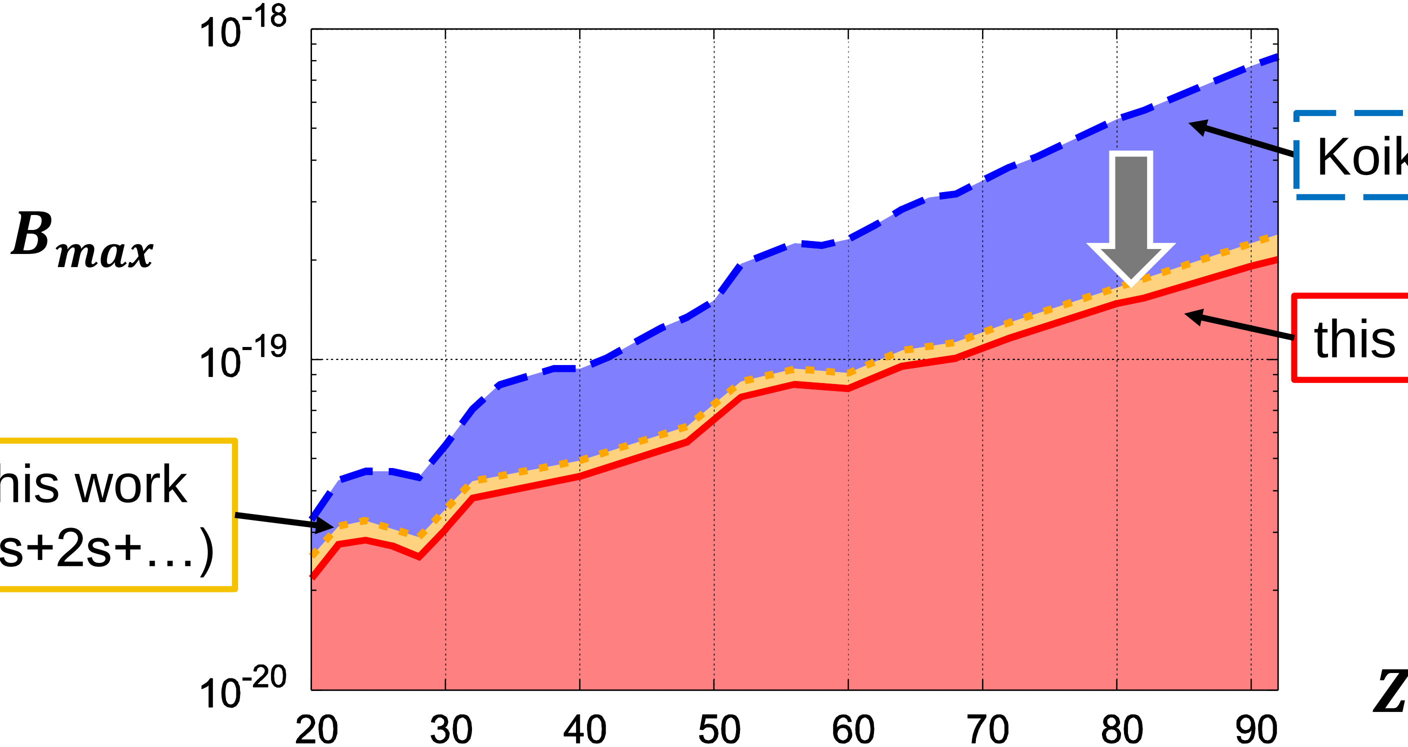
distortion of scattering e^- ➡ overlap integral decreases

Upper limits of BR (photonic process)

$BR(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(MEG, 2016)

$(A_R \overline{e_L} \sigma^{\mu\nu} \mu_R F_{\mu\nu})$



inverse of B_{max} $(Z = 82)$

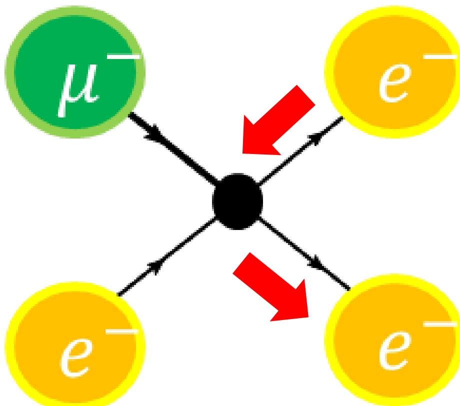
1.8×10^{18} → 6.6×10^{18}

Effect of distortion

scat. e^- : **distorted wave**

(Assuming momentum conservation at each vertex)

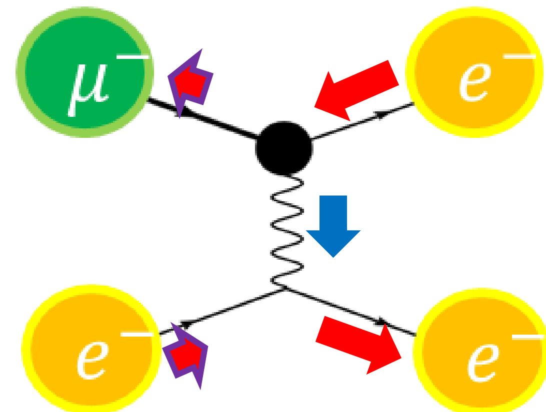
bound μ^- emitted e^-



bound e^- emitted e^-

contact process

bound μ^- emitted e^-



bound e^- emitted e^-

photonic process

- momentum transfers to bound leptons make overlap integrals smaller

Totally (combined with the effect to enhance the value near the origin),

enhanced !!

suppressed…

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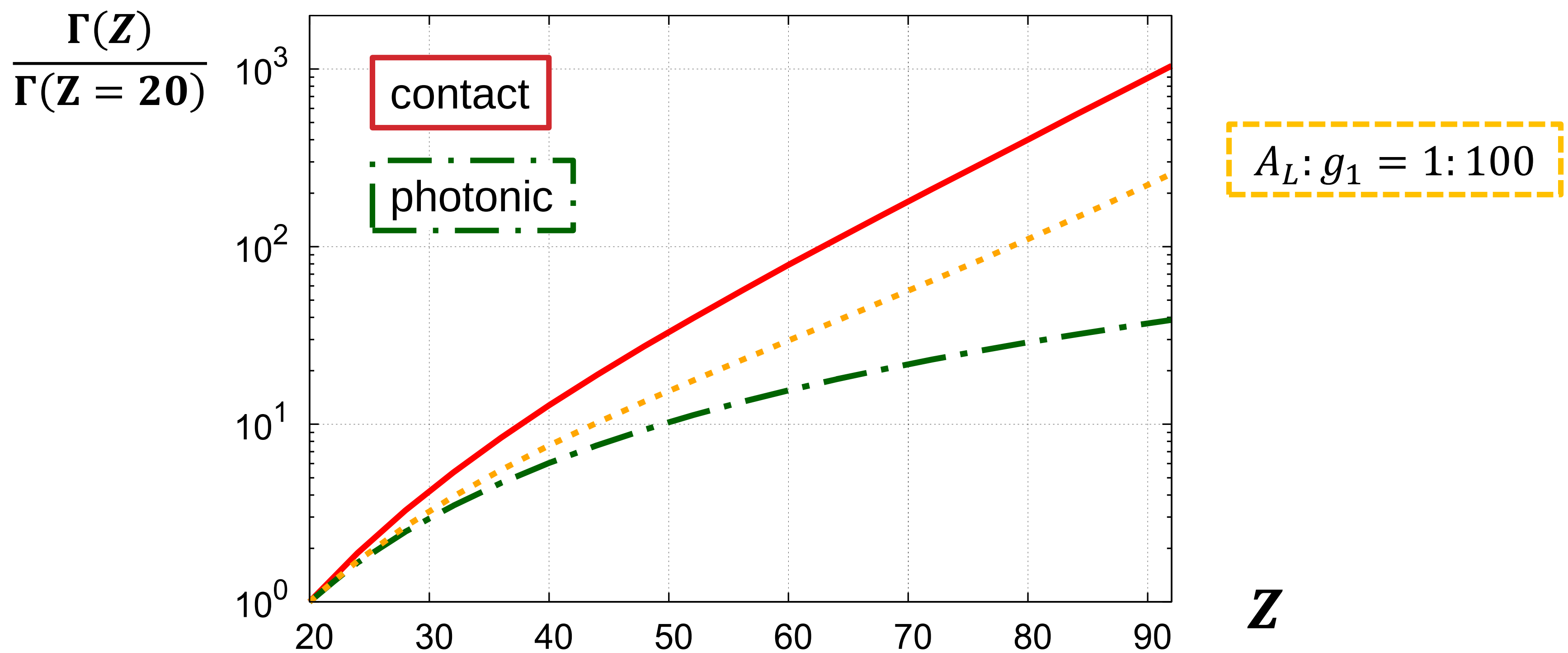
Distinguishing method 1

~ atomic # dependence of decay rates ~

Z dependence of Γ



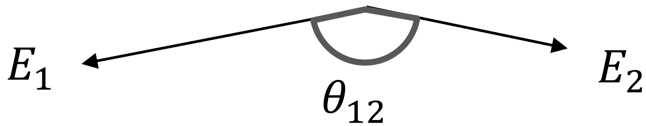
- The Z dependences are different among interactions.
- That of contact process is strongly increasing, while that of photonic process is moderately increasing.

Distinguishing method 2

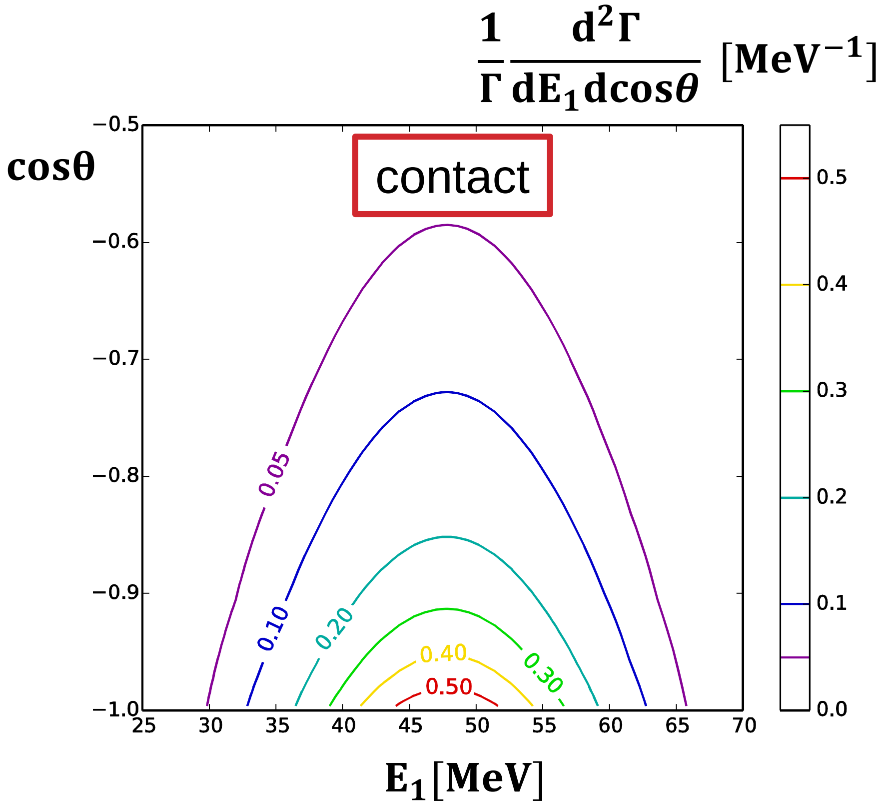
~ energy and angular distributions ~

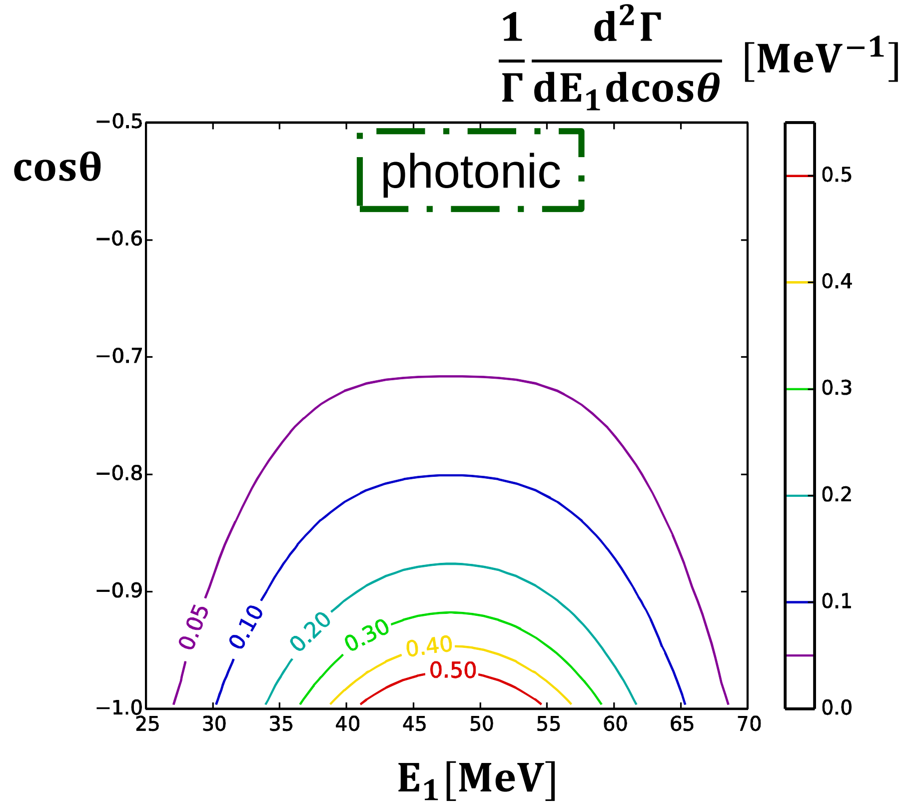
E_1 : energy of an emitted electron

θ : angle between two emitted electrons



$Z = 82$



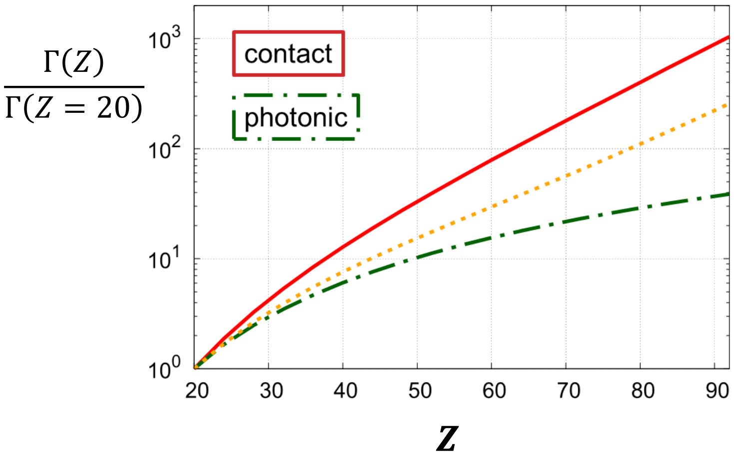


- The distributions are (a little) different among interactions.

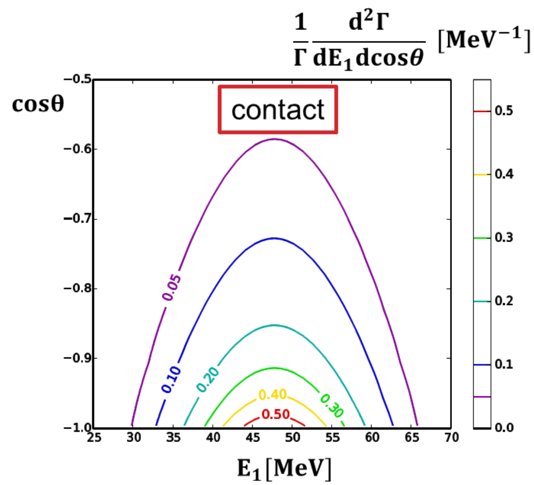
Model distinguishing power

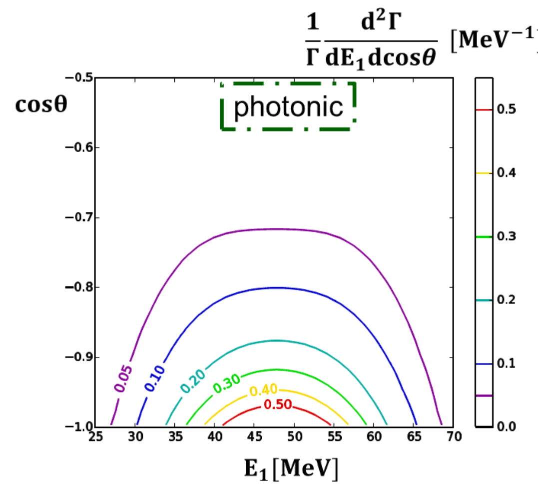
- We can distinguish “contact” or “photonic”.

method 1. Z-dep. of decay rates



method 2. energy-angular distribution



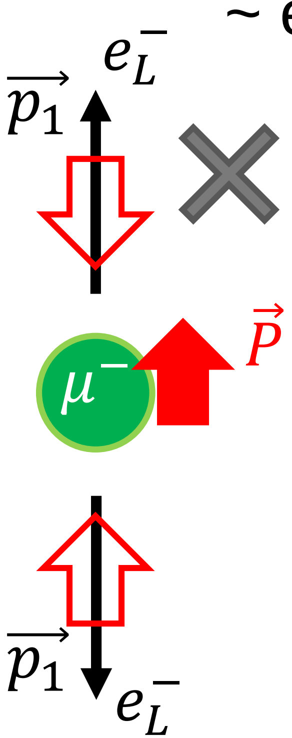


- Can we distinguish “left” or “right” ?

e.g. $g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R)$ & $g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L)$

Distinguishing method 3

~ electron asymmetry from polarized muon ~



$$\mathcal{L}_{CLFV} = (\overline{e_L}\mu_R)(\bar{e}e)$$

In preparation

$$\frac{d\Gamma}{d\cos\theta_1} = \frac{\Gamma}{2}\{1 + \alpha P \cos\theta_1\}$$

$\cos\theta_1 = \hat{P} \cdot \hat{p}_1$

$\alpha < 0$ is expected

($\mathcal{L}_{CLFV} = (\overline{e_R}\mu_L)(\bar{e}e)$ ⇨ $\alpha > 0$ is expected)

Measurement of angular distribution asymmetry

➡ Determination of dominant interaction !?

cf : $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$ with polarized muon

Y. Kuno & Y. Okada, Phys. Rev. Lett. **77**, 434 (1996).

Y. Okada, K. Okumura & Y. Shimizu, Phys. Rev. D **61**, 094001 (2000).

$$\frac{d^5\Gamma}{dE_1 d\Omega_1 d\Omega_2} \propto \left\{1 + F_1 \vec{P} \cdot \hat{p}_1 + F_2 \vec{P} \cdot \hat{p}_2 + F_D \vec{P} \cdot \hat{p}_1 \times \hat{p}_2\right\}$$

- Final state is determined by 4 parameters, say, $(E_1, \theta_1, \theta_2, \theta_{12})$

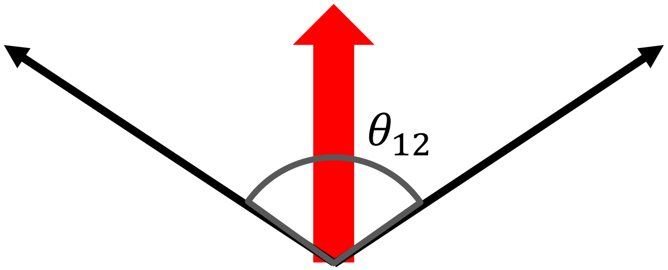
2 are fixed for examples

θ_1 : angle between P, p_1

θ_2 : P, p_2

θ_{12} : p_1, p_2

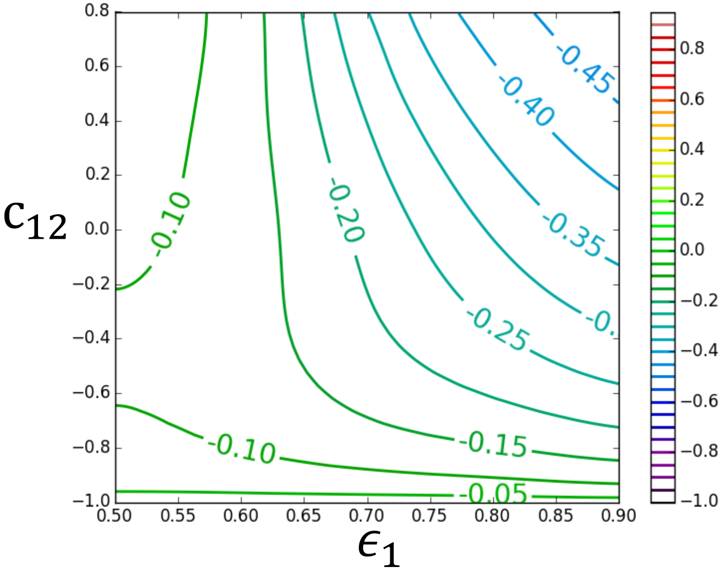
Example 1 : $\theta_1 = \theta_2 = \theta_{12}/2$



$$Asym. = [F(E_1, E_2, c_{12}) + F(E_2, E_1, c_{12})]\cos(\theta_{12}/2)$$
$$= F_S(E_1, E_2, c_{12})$$

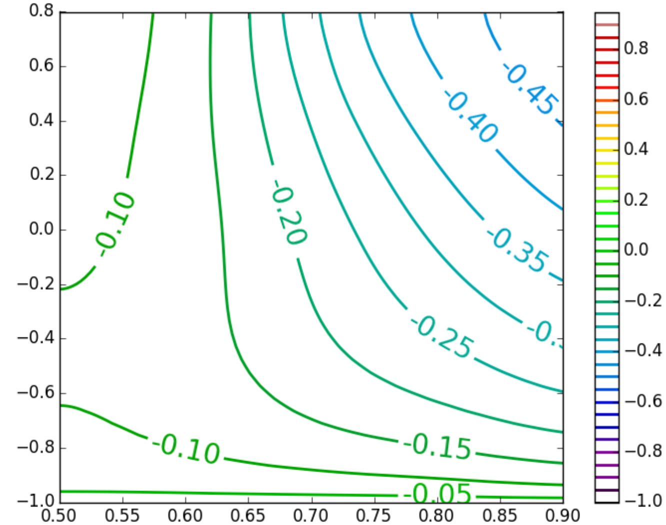
F_S

g_1



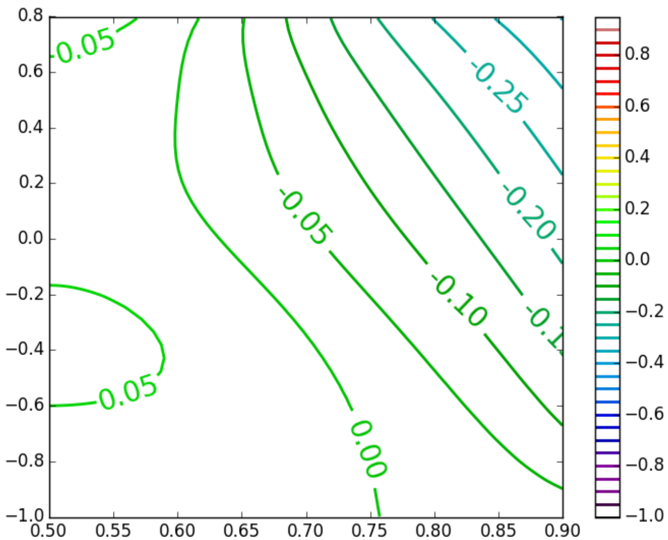
c12
ϵ1
-0.10
-0.20
-0.25
-0.35
-0.40
-0.45
-0.10
-0.15
-0.05

g_3



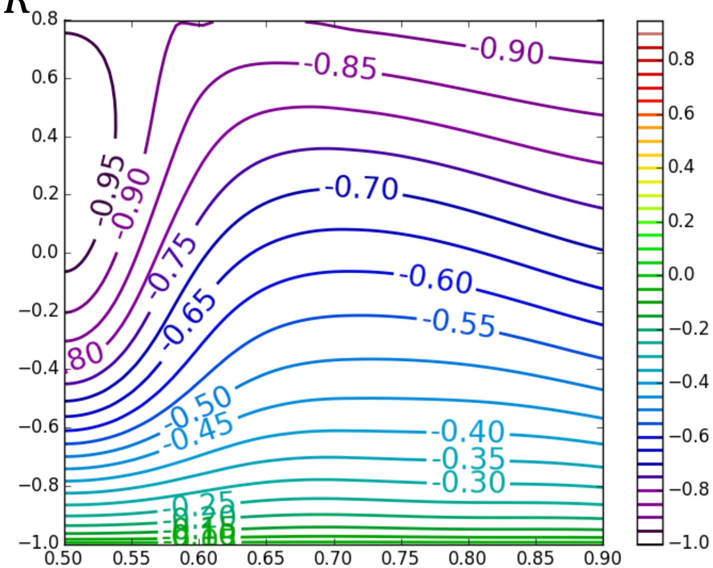
-0.10
-0.20
-0.25
-0.35
-0.40
-0.45
-0.10
-0.15
-0.05

g_5



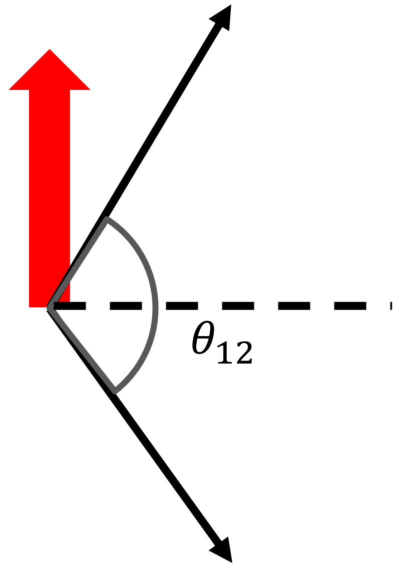
-0.05
-0.05
-0.10
-0.20
-0.25
0.05
0.00

A_R



-0.90
-0.85
-0.95
-0.90
-0.70
-0.75
-0.65
-0.60
-0.55
-0.50
-0.45
-0.40
-0.35
-0.30

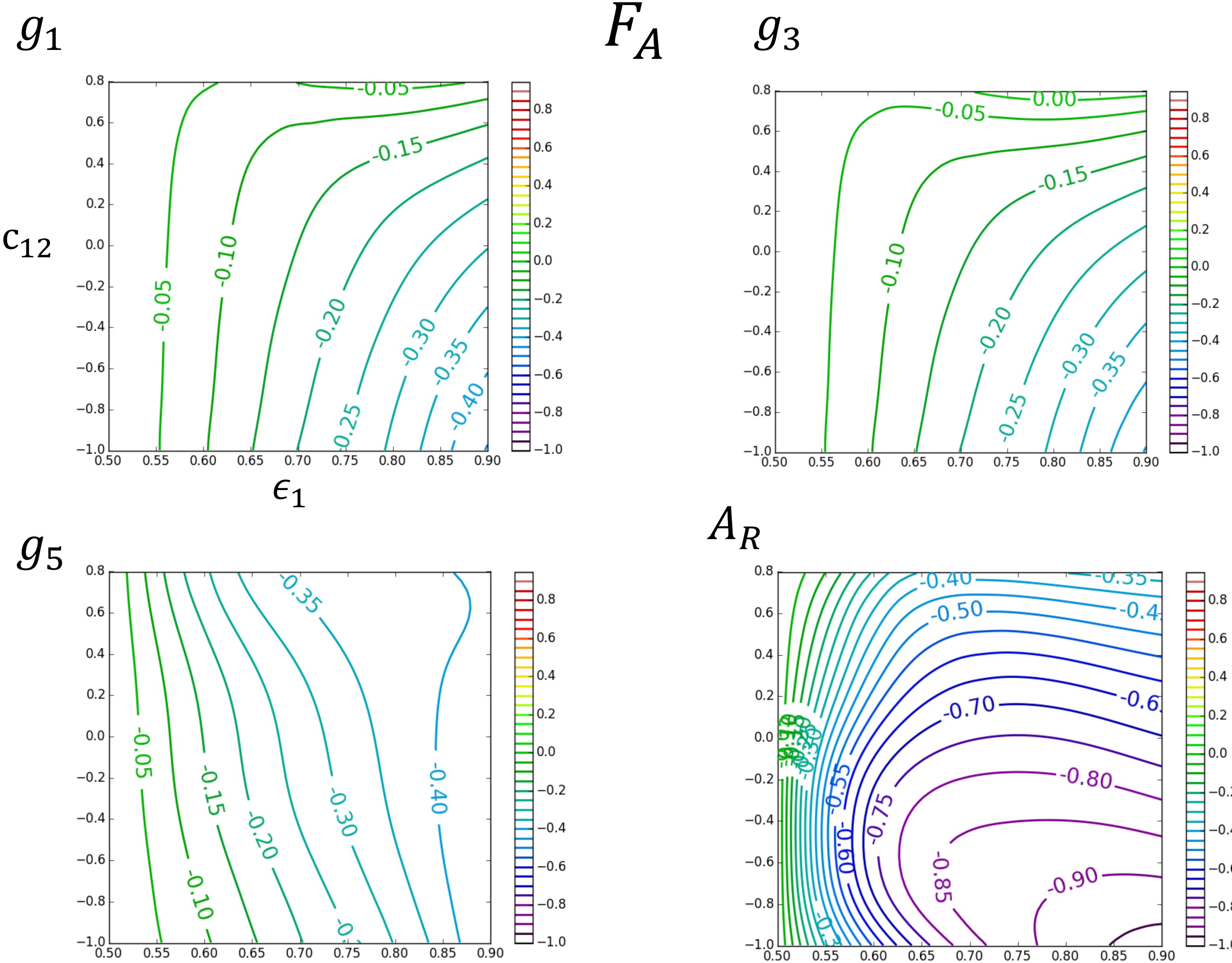
Example 2 : $\theta_1 = \pi - \theta_2 = \frac{\pi}{2} - \frac{\theta_{12}}{2}$



$$Asym. = [F(E_1, E_2, c_{12}) - F(E_2, E_1, c_{12})]\sin(\theta_{12}/2)$$

$$= F_A(E_1, E_2, c_{12})$$

F_A



g1
g3
c12
ϵ1
g5
AR
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
0.00
-0.50
-0.70
-0.75
-0.80
-0.85
-0.90
-0.55
-0.60

- In all cases there is asymmetry

⇒ Useful to determine the parity violation of effective couplings

Especially for photonic interaction

- Relativistic treatment is important
  - g_1 type

    In non-relativistic limit , exactly 0

    Even if relativistic , if nuclear is point like the asymmetry is 0

    $\because$ asymmetry $\propto \int dr\left[g_\mu(r)f_e(r) - f_\mu(r)g_e(r)\right]j_1(pr)$

    - Distortion is very important
  - g_5 type

    In any case , non-zero

- Shape of Assymetry can determine the interaction !?

Contents

1. Introduction
   - Charged Lepton Flavor Violation (CLFV)
   - CLFV searches using muon
   - $\mu^- e^- \to e^- e^-$ in a muonic atom
2. Transition probability of $\mu^- e^- \to e^- e^-$
   - Effective CLFV interactions
   - Distortion of scattering electrons & Relativity of bound leptons
   - Difference between contact & photonic processes
3. Distinguishment of CLFV interaction
   - Atomic # dependence of decay rates
   - Energy-angular distribution of emitted electrons
   - Asymmetry of emitted electrons by polarizing muon
4. Summary

Summary

- $\mu^- e^- \to e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron

    are important in calculating decay rates.

⇩ Distortion makes difference between 2 processes.

- ■ contact process : decay rate Enhanced (7 times Γ_0 in $Z = 82$)
- ■ photonic process : decay rate suppressed (1/4 times Γ_0 in $Z = 82$)

◆ How to discriminate interactions, found by this analyses

- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons
- ✓ asymmetry of electron emission by polarized muon

BACKUP

Coulomb prevents the contact process?

Use the simple Hamiltonian (a muon & an electron in nuclear potential)

$$H = -\sum_{i=\mu,e} \frac{\boldsymbol{\nabla}_i^2}{2m_e} - \sum_{i=\mu,e} \frac{Z\alpha}{|\boldsymbol{r}_i|} + \frac{\alpha}{|\boldsymbol{r}_\mu - \boldsymbol{r}_e|}$$

Assume that the form of the wave function is

$$\psi_{a,b}^{Z_\mu,Z_e}(\boldsymbol{r}_\mu, \boldsymbol{r}_e) = N_{a,b}^{Z_\mu,Z_e} \exp(-m_\mu Z_\mu \alpha |\boldsymbol{r}_\mu|) \exp(-m_e Z_e \alpha |\boldsymbol{r}_e|) \times \{1 - b\exp(-a|\boldsymbol{r}_\mu - \boldsymbol{r}_e|)\}$$

⇩ find the parameter set to minimize the energy

- $Z_\mu \simeq Z$
- $Z_e \simeq Z - 1$
- $b \simeq 0$

⇩

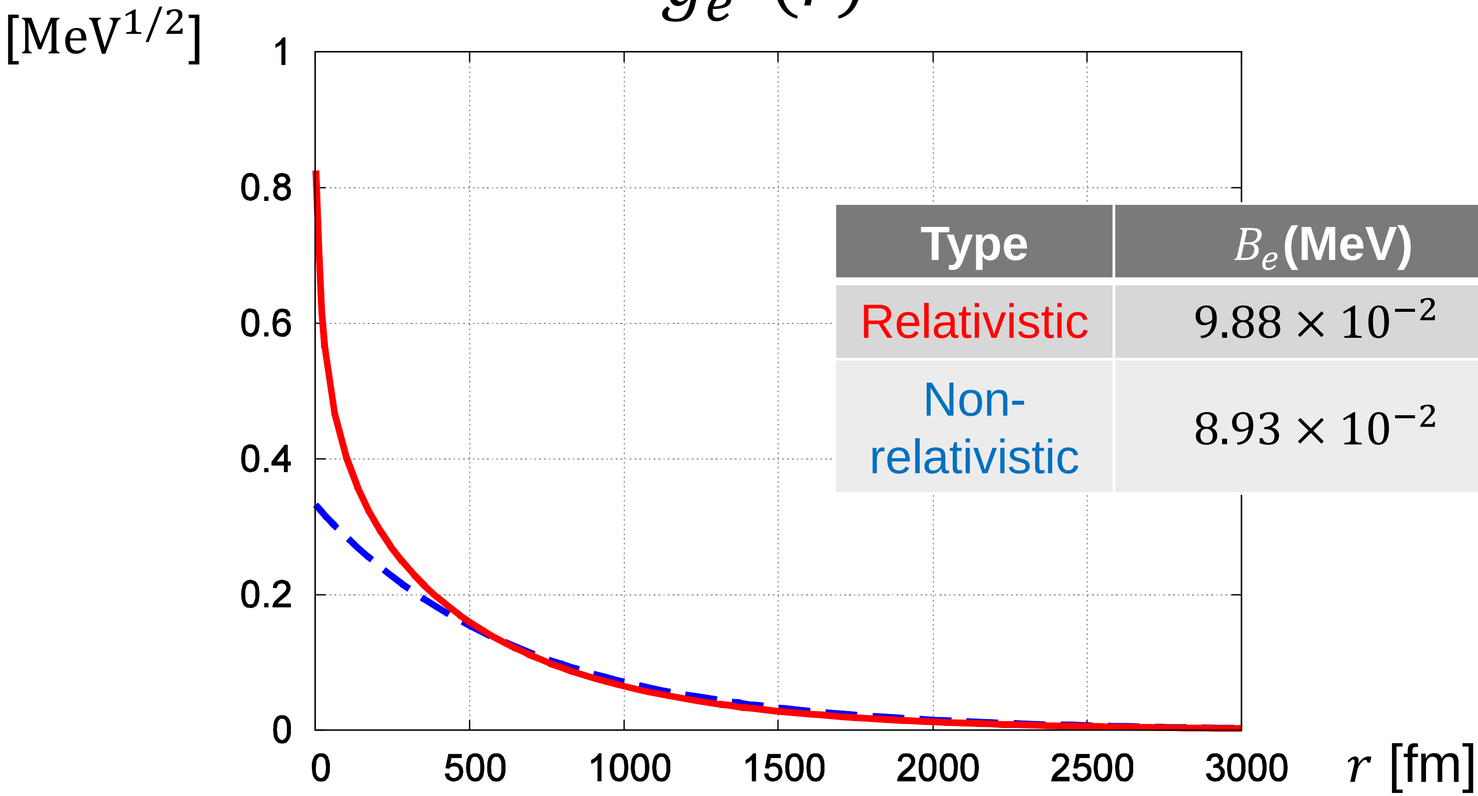
We can safely neglect the additional factor.

Radial wave function (bound e^-)

^{208}Pb case $\quad Z = 81$

(considering μ^- screening)

$g_e^{1s}(r)$



Type	B_e(MeV)
Relativistic	9.88×10^{-2}
Non-relativistic	8.93×10^{-2}

Relativity enhances the value near the origin.

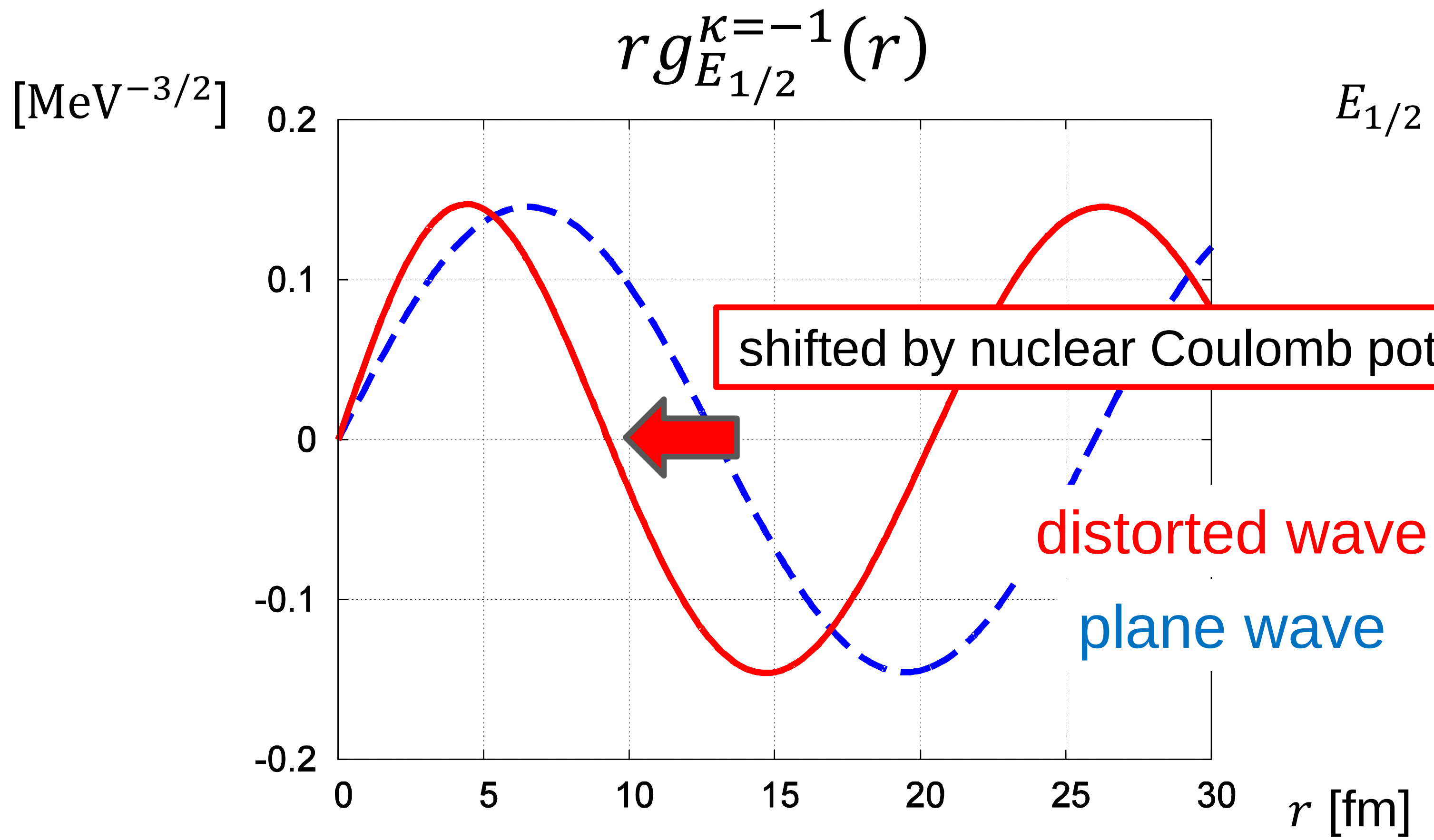
Radial wave function (scattering e^-)

e.g. $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

$E_{1/2} \approx 48$MeV



① enhanced value near the origin

② local momentum increased effectively

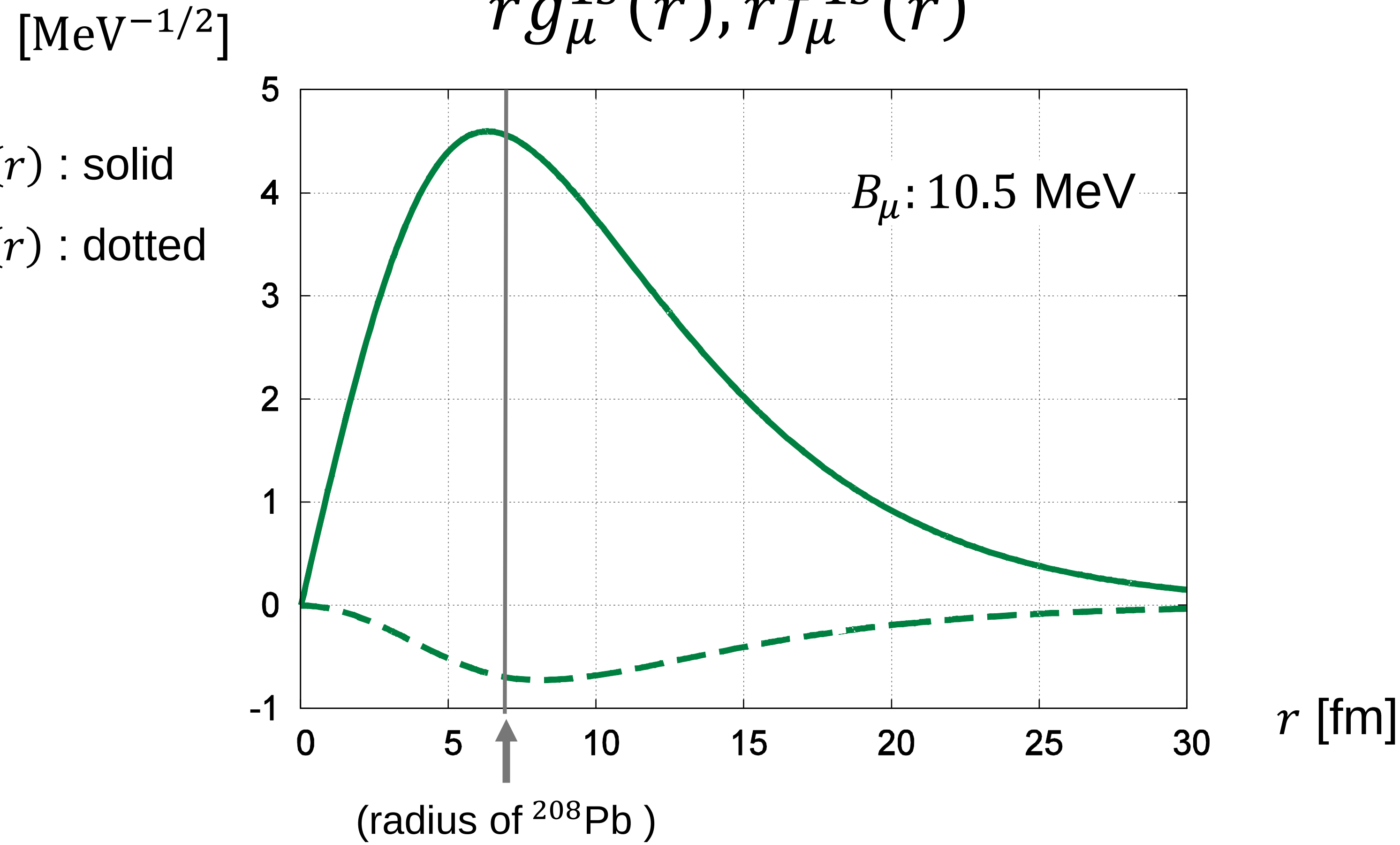
Radial wave function (bound μ^-)

^{208}Pb case $Z = 82$

$rg_\mu^{1s}(r), rf_\mu^{1s}(r)$

$rg_\mu^{1s}(r)$: solid

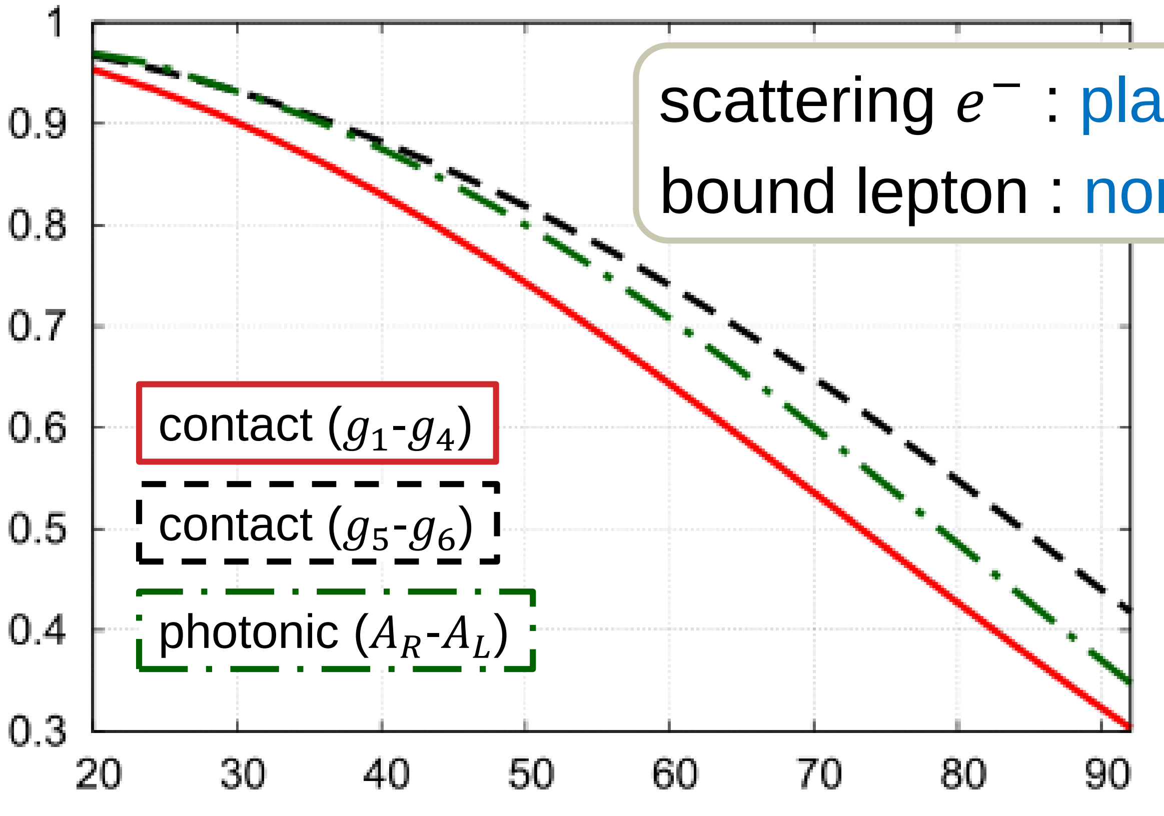
$rf_\mu^{1s}(r)$: dotted



✓ It is important to consider finite nuclear charge radius.

Effect of finite size of muon wave

Γ/Γ^0 (Γ^0 : the rate in the previous approx.)



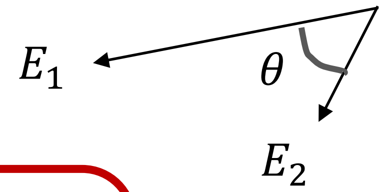
scattering e^- : plane wave

bound lepton : nonrelativistic

Momentum fluctuation of bound muon

→ overlap integral **small**

Decay rate



$$\Gamma(\mu^-(1S)e^-(\alpha) \to e^-e^-) = \frac{1}{2}\int_{m_e}^{m_\mu - B_\mu^{1S} - B_e^\alpha} \mathrm{d}\,E_1 \int_{-1}^{1} \mathrm{d}\cos\theta \, \frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta}$$

E_1 : energy of an emitted electron

θ : angle between two emitted electrons

differential decay rate :

P_l : Legendre polynomial

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1 \mathrm{d}\cos\theta} = \sum_{\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l} M(E_1,\kappa_1,\kappa_2,J) M^*(E_1,\kappa_1',\kappa_2',J) \times w(\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l) P_l(\cos\theta)$$

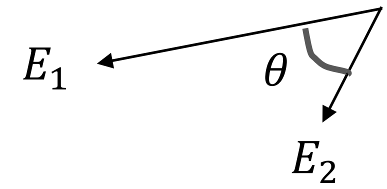
$$M(E_1,\kappa_1,\kappa_2,J) = \underbrace{\sum_{i=1,\cdots,6} g_i\, M_{\mathrm{contact}}^i(E_1,\kappa_1,\kappa_2,J)}_{\text{contact}} + \underbrace{\sum_{j=L,R} g_j\, M_{\mathrm{photo}}^j(E_1,\kappa_1,\kappa_2,J)}_{\text{photonic}}$$

contact

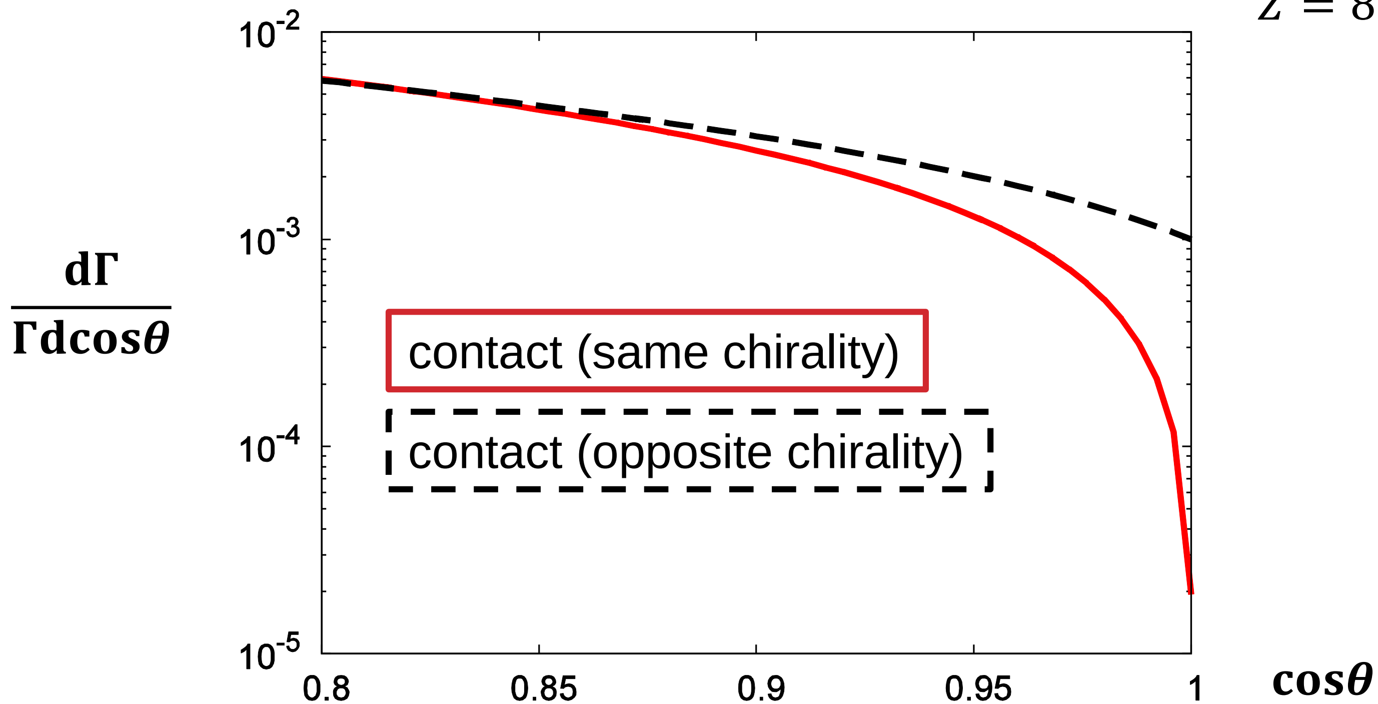
photonic

Discriminating method 2

θ : angle between two emitted electrons



angular distribution ($\cos\theta \approx 1$)



➢ e^- pair has same chirality ⇒ e^- pair cannot emit same momentum

(due to Pauli principle)

Contribution from all bound e^-s

normalize the contribution of $1S$ e^- to 1

contact (g_1)

1S	2S	2P	3S	3P	3D	4S	Total
1	0.17	6.2×10^{-3}	5.1×10^{-2}	3.1×10^{-3}	2.3×10^{-9}	2.1×10^{-2}	1.25

photonic (g_L)

1S	2S	2P	3S	3P	3D	4S	Total
1	0.15	7.3×10^{-3}	4.3×10^{-2}	2.6×10^{-3}	2.4×10^{-5}	1.8×10^{-2}	1.21

- it is sufficient to consider about S electrons for both cases

非対称度の測定

$$\frac{\mathrm{d}^5\Gamma}{\mathrm{d}E_1\mathrm{d}\Omega_1\mathrm{d}\Omega_2} = \frac{1}{8\pi^2}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_1\mathrm{dc}_{12}}\{1 + F(E_1, E_2, \mathrm{c}_{12})\vec{P}\cdot\hat{p}_1 + F(E_2, E_1, \mathrm{c}_{12})\vec{P}\cdot\hat{p}_2\}$$

- 終状態のkinematicsを決めるパラメータは4つ $(E_1, \theta_1, \theta_2, \theta_{12})$

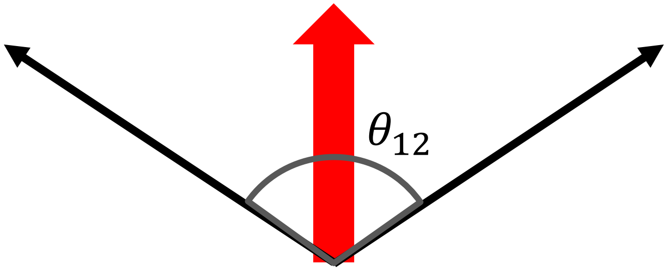
θ_1 : $P - p_1$の角度

θ_2 : $P - p_2$

θ_{12} : $p_1 - p_2$

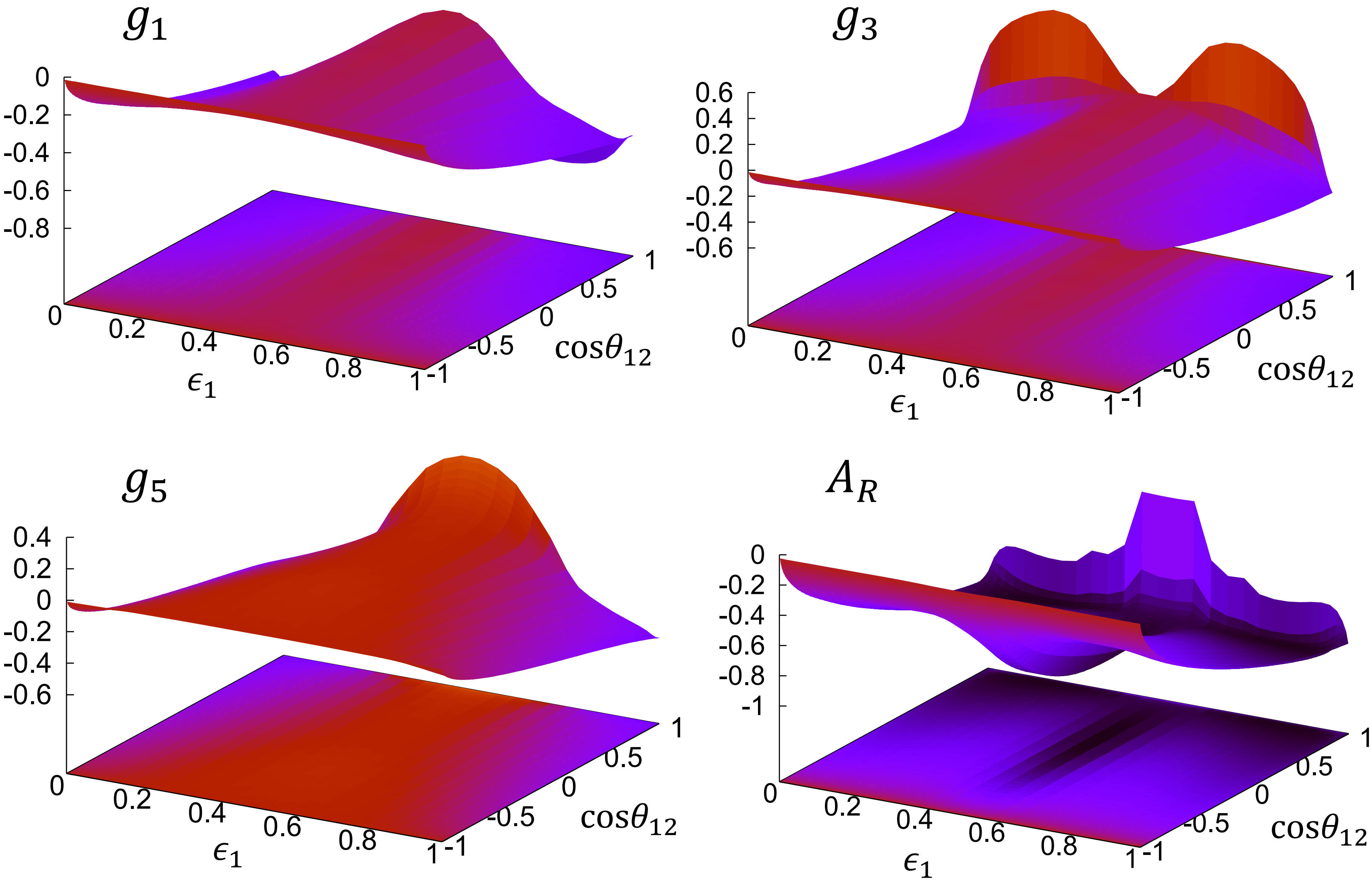
2つを固定して図を作成

例 : $\theta_1 = \theta_2 = \theta_{12}/2$



$$Asym. = [F(E_1, E_2, \mathrm{c}_{12}) + F(E_2, E_1, \mathrm{c}_{12})]\cos(\theta_{12}/2)\ \left(= F_S(E_1, E_2, c_{12})\right)$$

$F_S(E_1, E_2, c_{12})$



$F_A(E_1, E_2, c_{12})$