Semiclassics in QCD

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1707.08971

Resurgence for QFT

Belief: QFT observables = resurgent transseries in some couplings

$$\mathcal{O}(\lambda) \simeq \sum_n p_n \lambda^n + \sum_c e^{-\frac{S_c}{\lambda}} \sum_k p_{k,c} \lambda^k + \cdots$$

Lots of evidence in special cases:

Integrals with saddles	Stokes, Dingle, Berry, Howls …
matrix models	Marino, Schiappa, Weiss …
topological strings	Aniceto, Hatsuda, Marino, Schiappa, Vonk, …
QM (d=1 QFT)	Basar, Dunne, Kawai, Misumi, Nitta, Sakai, Takei, Sulejmanpasic, Unsal, Zinn-Justin …
some SUSY theories	Aniceto, Dorigoni, Hatsuda, Honda, Russo, Schiappa, …

What about realistic QFTs with asymptotic freedom like QCD?

Basic resurgence questions for QCD

(1) What is a **useful** expansion parameter λ?

$$\mathcal{O}(\lambda) \simeq \sum_n p_n \lambda^n + \sum_c e^{-\frac{S_c}{\lambda}} \sum_k p_{k,c} \lambda^k + \cdots$$

(2) What kind of transseries should we expect?

In simpler cases studied so far, transmonomials restricted to

$\{\lambda,\ 1/\lambda,\ \log \lambda,\ e^{-1/\lambda}\}$

Turns out in QCD we also need

$$e^{-e^{+1/\lambda}}$$

as well, at the least!

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Outline

QCD coupling λ changes with energy scale. Most interesting observables are "low-energy" ones.

Relevant coupling isn't small.

Big difference from QM and special QFT/string examples.

Challenge: construct a useful semiclassical expansion for QCD + its cousins.

useful = first few orders already give good guide to expected behavior.

1. Explain currently best-understood approach, giving its motivation and why we believe it works.

2. Highlight physical and mathematical lessons.

Semiclassics requirements for QCD

Need coupling to be small at long distance

Keep as many features of QCD as many possible:

1. asymptotic freedom
2. quark confinement
3. chiral symmetry breaking
4. Lorentz invariance
5. No microscopic scalars

Turns out we can **almost** have it all.

Lorentz-invariant route to weak coupling:

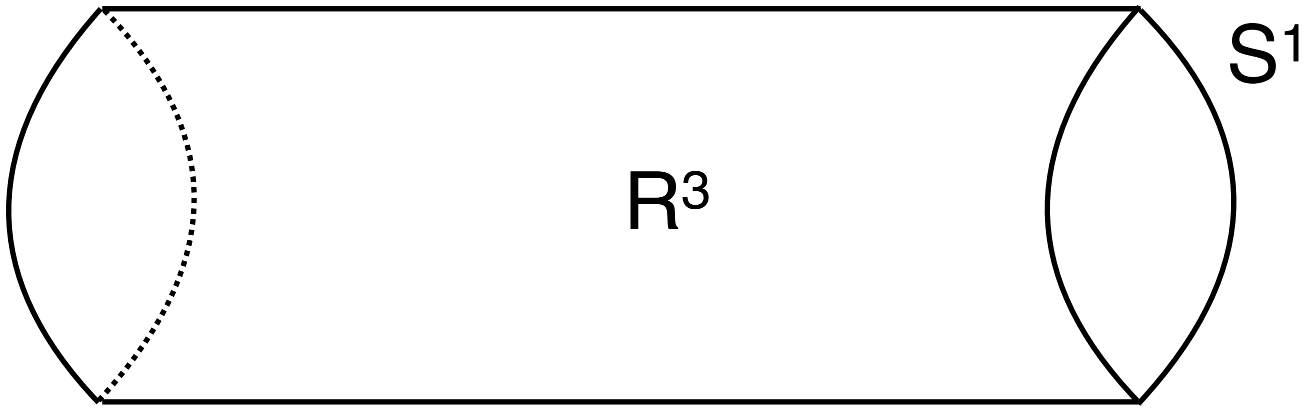
Scalar fields $\Rightarrow$ Higgs mechanism $\Rightarrow$ weak coupling ?

Works in EW part of SM + many SUSY theories, but **not in QCD**.

Adiabatic compactification

Unsal, Yaffe, Shifman, … 2008-onward

Break 4D Lorentz, but as little as possible!



If circle size L is small, get weak coupling by asymptotic freedom

NB: still have Lorentz invariance in R^3 directions.

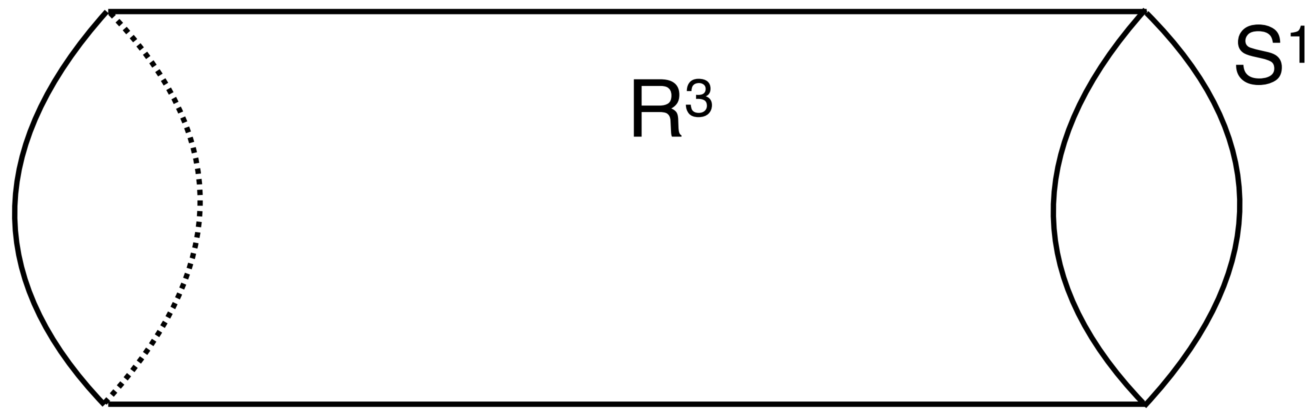
$3 > 2 \Rightarrow$ symmetries can break spontaneously.

Large L: some symmetries preserved, others spontaneously broken!

Need $d \geq 3$ non-compact directions to avoid Coleman-Mermin-Wagner thm.

Want **same** pattern at small L for our expansion to be useful.

Self-Higgsing



When YM compactified on S^1, Polyakov loop becomes an observable

$$\mathrm{Tr}\ \Omega = \mathrm{Tr}\ \mathcal{P} e^{i \oint A_4} \simeq \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_{\dots}} \end{pmatrix}$$

Eigenvalues = classical moduli space

Non-coincident eigenvalues ⇒ breaking $SU(N) \rightarrow U(1)^{N-1}$

VEV of "A_4" produces a (compact) adjoint Higgs mechanism!

But we don't get to choose eigenvalues: theory picks own vacuum

Adiabatic compactification for YM

Unsal + friends, 2008-onward

Confined phase at large L $\Longleftrightarrow$ $< \mathrm{tr}\,\Omega > \approx 0$; related to “center symmetry”.

But at small L in pure YM dynamics force $A_4 = 0 \Longleftrightarrow < \mathrm{tr}\,\Omega > \neq 0$

Idea: add something that leaves large L theory
the “same”, but makes small L limit smooth

Options: (a) add 1 heavy Dirac adjoint fermion with *periodic* BCs
(b) add appropriate double-trace deformation

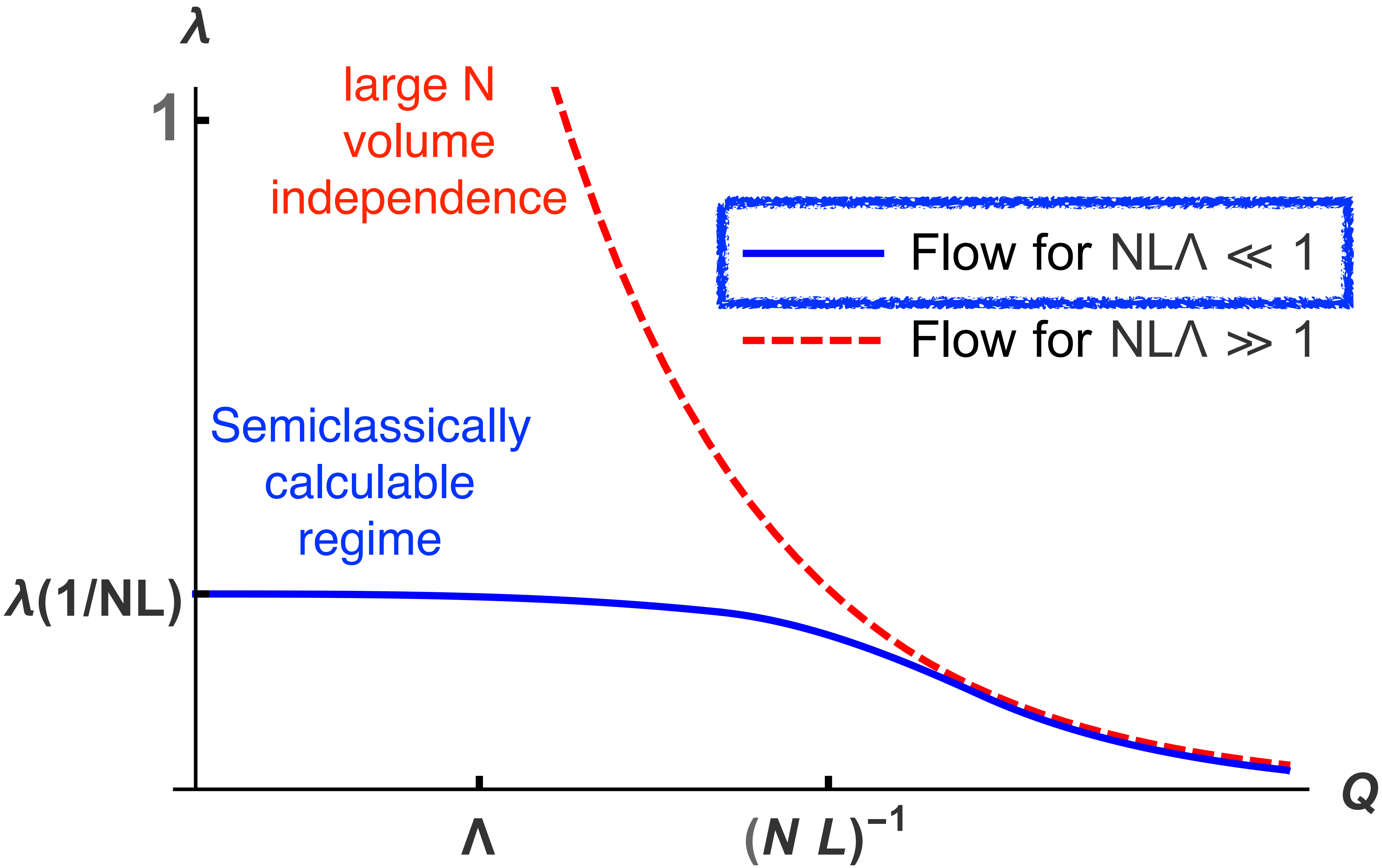
Resulting theories remains center-symmetric at small L

Non-coincident eigenvalues of $\Omega \Rightarrow$ breaking $SU(N) \rightarrow U(1)^{N-1}$

Adjoint Higgs mechanism driven by VEV of “A_4” !

W-boson mass scale is $m_W = 2\pi/NL$

Coupling flow with adiabatic compactification



The NLΛ << 1 regime gives a weakly-coupled theory at all scales!

Small L limit in perturbation theory

At long distances ℓ >> N L ~ 1/m_W

$$SU(N) \to U(1)^{N-1}$$

due to the center-symmetric background holonomy.

N^2 - N W-bosons with masses m_W ~ 1/NL; ignore for now.

The light fields are N_c - 1 "Cartan gluons"

$$F^j_{\mu\nu} = \frac{1}{N}\sum_{p=0}^{N-1} e^{2\pi i j p/N}\,\mathrm{tr}\left(\Omega^p F_{\mu\nu}\right)$$

(added fictitious p=0 mode for notational simplicity; it decouples exactly.)

Small-L physics easiest to describe using 3D Abelian duality

Small L limit in perturbation theory

N - 1 Cartan gluons are classically gapless.

$$F^i_{\mu\nu} = g^2/(2\pi L)\epsilon_{\mu\nu\alpha}\partial^\alpha\sigma^i$$

$$S_\sigma = \int d^3x \, \frac{g^2}{8\pi^2 L}(\partial_\mu\vec{\sigma})^2.$$

σ^i shift symmetry $\Longleftrightarrow$ conservation of magnetic charge.

But there are no magnetic monopoles in perturbation theory.

σ^i are massless to all orders in perturbation theory.

Non-perturbative mass gap

Unsal,
Yaffe, Shifman, Poppitz,
Sulejmanpasic,
…

Due to $SU(N) \rightarrow U(1)^{N-1}$, instantons break up into N 'monopole-instantons'

Associated 't Hooft amplitudes look like

$$\mathcal{M}_i \sim e^{-8\pi^2/\lambda} e^{i(\sigma_i - \sigma_{i+1})}$$

Magnetic-charge-carrying M_i events produce potential for σ^i

$$V(\sigma) = -m_W^3 e^{-8\pi^2/\lambda} \sum_i \cos(\sigma_i - \sigma_{i+1})$$

So the dual photons get a non-perturbative mass gap:

$$m_p \sim m_W e^{-4\pi^2/\lambda} |\sin(\pi p/N)|$$

Here p = 1, … , N-1.

Weak coupling confinement

Unsal,
Yaffe, Shifman, Poppitz,
Sulejmanpasic,
…

Magnetic-charge-carrying M_i events produce potential for σ^i

$$V(\sigma) = -m_W^3 e^{-8\pi^2/\lambda} \sum_i \cos(\sigma_i - \sigma_{i+1})$$

Have mass gap, preserved center symmetry

Also have finite string tension ~ $m_W e^{-1/\lambda}$
for test quarks, independent of N.

Poppitz, Erfan Shalchian T., 1708.08821

We really have confinement!

Compare to e.g. Seiberg-Witten
confinement, where tension ~ $1/N^2$!

Quantum number issue

Aitken, AC,
Poppitz, Yaffe,
1707.08971

Expect a non-perturbative gap at large L as well - time to celebrate?

First, let's dig into meaning of 'p' in

$$m_p \sim m_W e^{-4\pi^2/\lambda} |\sin(\pi p/N)|$$

'color' index 'j' is discrete Fourier transform of holonomy winding number 'p'

$$F^j_{\mu\nu} = \frac{1}{N} \sum_{p=0}^{N} e^{2\pi i j p/N} \mathrm{tr}\left(\Omega^p F_{\mu\nu}\right)$$

Dual photon mass term looks like

$$-\cos(\sigma_j - \sigma_{j+1}) \sim (\sigma_j - \sigma_{j+1})^2$$

Need to use inverse Fourier transform to diagonalize.

Quantum number issue

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$$m_p \sim m_W e^{-4\pi^2/\lambda} |\sin(\pi p/N)|$$

'p' is tied to the winding number of the Polyakov loop holonomy!

So actually the dual photons are center symmetry eigenstates:

$$\mathcal{S} : \sigma_p \to e^{2\pi i p/N} \sigma_p$$

p = 1, … , N-1; so no neutral states.

Ooops. The light sector is all charged under center symmetry?

That's alarming!

Large L extrapolation

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center p-charged states = p-winding string states.

At small fixed L, their energy is small.

But as L → ∞ , energy ~ L * p * (string tension) → ∞

What are the lightest states that have the right quantum numbers to extrapolate to finite energy states at large L?

Answer: bound states of dual photons!

Light bound states

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Set N = 2 to keep it simple.

$$S_{3\text{D}} = \int d^3x \left[\tfrac{1}{2}(\partial_\mu \sigma)^2 + \tfrac{1}{2} m_\gamma^{\,2}\, \sigma^2 - \tfrac{2}{3}\epsilon\, m_\gamma\, \sigma^4 + \frac{16}{45}\epsilon^2\, \sigma^6 - \frac{32}{315}\epsilon^3\, m_\gamma^{-1}\, \sigma^8 + \cdots \right],$$

$m_\gamma \sim m_W\, e^{-4\pi/\lambda}$, $\epsilon \sim e^{-4\pi/\lambda}$

center symmetry: $\sigma \to -\sigma$

3D relativistic power counting: σ^4 is relevant, σ^8 marginal, σ^8 irrelevant.

May seem mass gap + $\epsilon \ll 1$ should ensure one can ignore interactions.

Indeed, they are completely ignored in literature.

But they produce bound states!

Non-relativistic effective field theory

Binding energy small compared to m_γ, so take non-relativistic limit

$$\sigma = (2m_\gamma)^{-1/2} e^{-im_\gamma t}\,\Sigma + (\mathrm{h.c.})$$

$$S_{3\mathrm{D,NR}} = \int dt\, d^2x \left[\Sigma^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_\gamma}\right)\Sigma + \frac{\epsilon}{m_\gamma}\,(\Sigma^\dagger)^2\,\Sigma^2 - \frac{8\epsilon^2}{9m_\gamma^3}\,(\Sigma^\dagger)^3\,\Sigma^3 + \cdots\right].$$

Non-relativistic power counting

[t] = -2 , [x] = -1 , [Σ] = (d-1)/2 , [m] = 0

Marginal interactions in QM

1d: none , 2d: Σ^6, 3d: Σ^4, 4d: $\Sigma^{10/3}$, 5d: Σ^3, …

3d power counting: $|\Sigma|^6$ is irrelevant, $|\Sigma|^4$ is marginal!

Non-relativistic effective field theory

Binding energy small compared to m_γ, so take non-relativistic limit

$$\sigma = (2m_\gamma)^{-1/2} e^{-im_\gamma t}\, \Sigma + (\mathrm{h.c.})$$

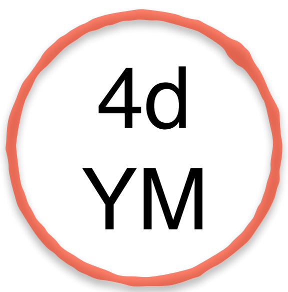
$$S_{3\mathrm{D,NR}} = \int dt\, d^2x \left[\Sigma^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_\gamma} \right) \Sigma + \frac{\epsilon}{m_\gamma} (\Sigma^\dagger)^2\, \Sigma^2 - \frac{8\epsilon^2}{9m_\gamma^3} (\Sigma^\dagger)^3\, \Sigma^3 + \cdots \right].$$

Coefficient of $|\Sigma|^4$, ϵ , runs with energy scale:

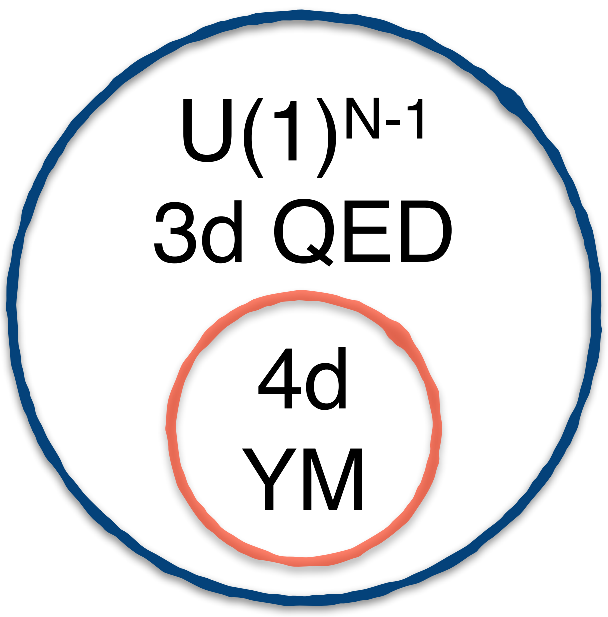
$$\mu \frac{d\,\epsilon(\mu)}{d\mu} = -\frac{1}{\pi}\, \epsilon(\mu)^2$$

Asymptotic freedom, and hence “strong coupling” at long distances!

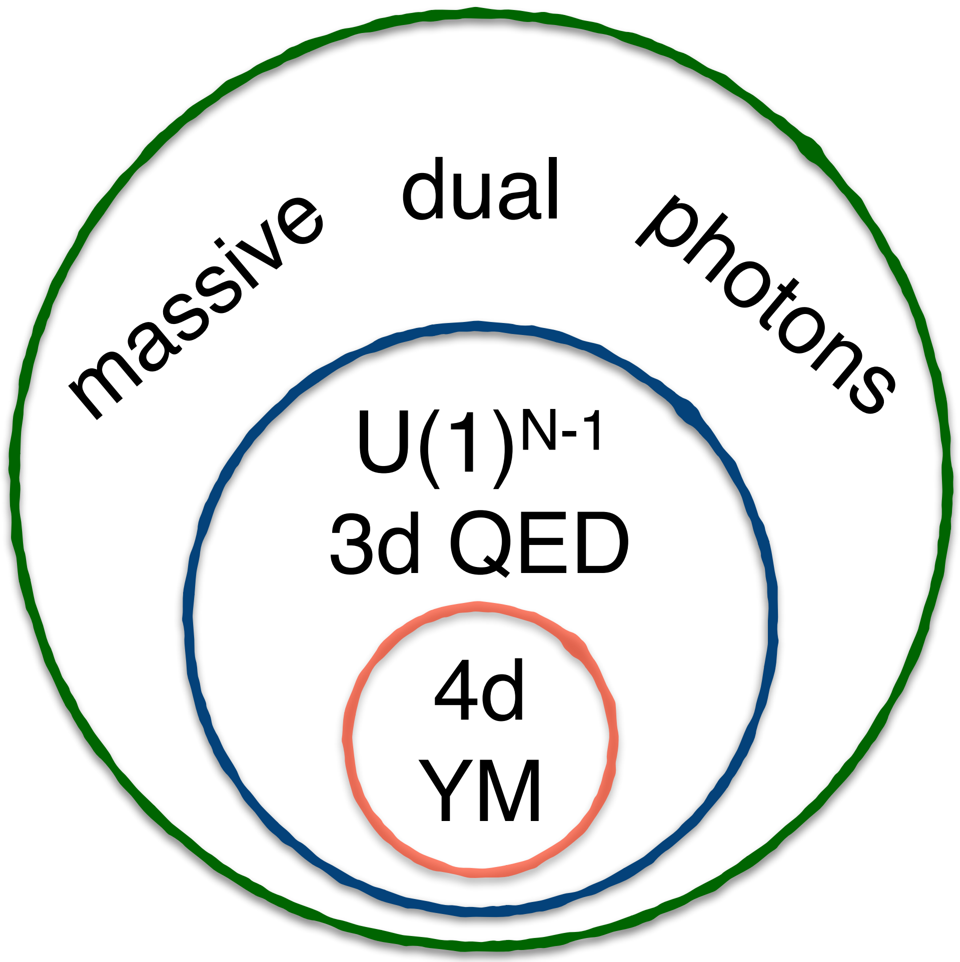
Effective field theory matryoshka doll



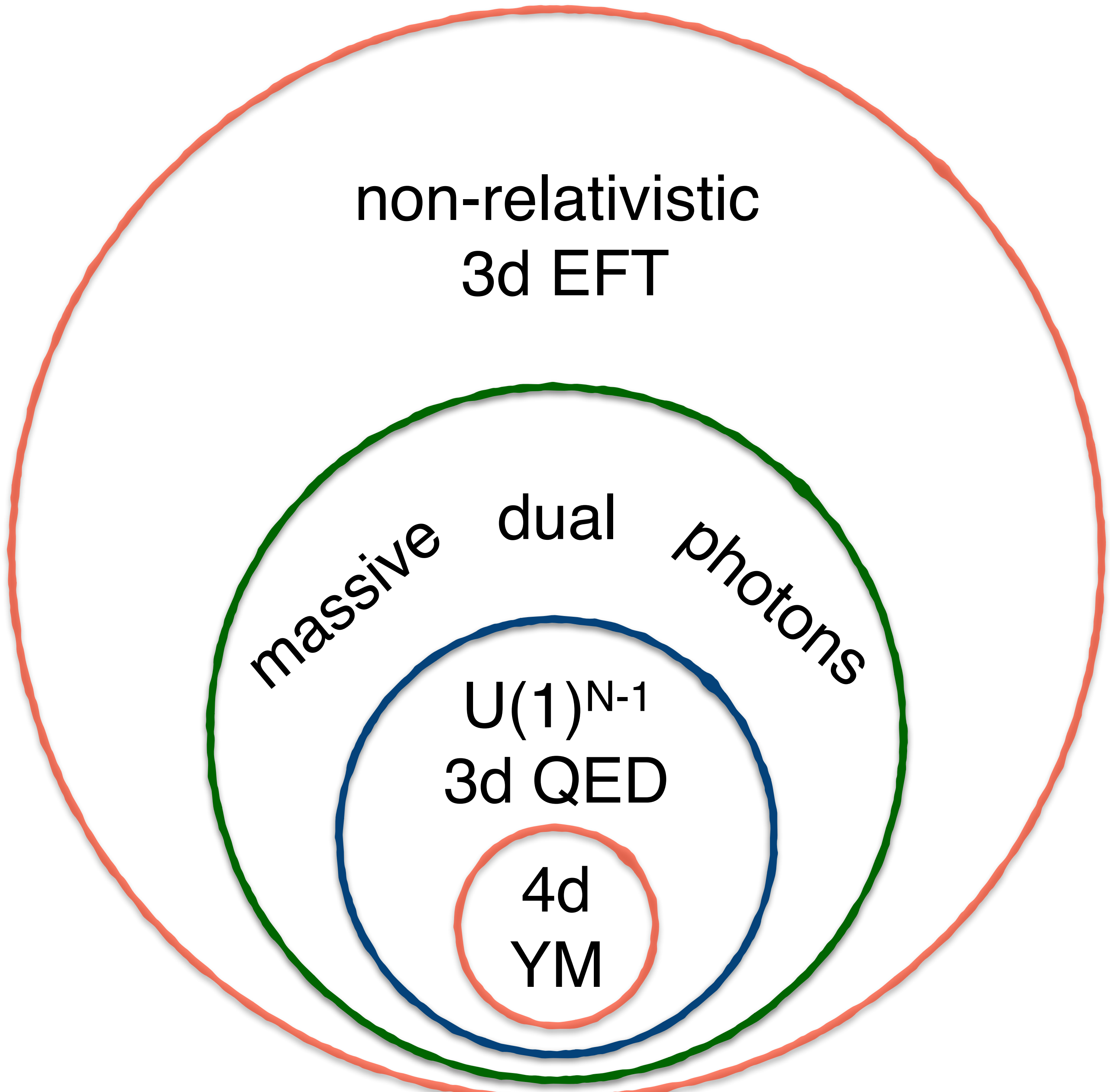
Effective field theory matryoshka doll



Effective field theory matryoshka doll



Effective field theory matryoshka doll



Light bound states

Strong coupling in the IR would be a disaster in a relativistic QFT - couldn't continue to calculate.

But here we have a non-relativistic QFT!

In two-particle sector we can actually sum all the Feynman diagrams exactly, and solve for the bound states.

In fact this is how Schrodinger equations are derived from QFT.

But even though here asymptotic freedom is harmless, it's quite funny that it appears!

Light bound states

Binding energies ~ (strong-coupling momentum)^2/m_γ

$$\Lambda_{\mathrm{IR}} = \mu_{\mathrm{UV}} e^{-\frac{\pi}{\epsilon(\mu_{\mathrm{UV}})}}\,,\ \mu_{\mathrm{UV}} \sim m_\gamma\,,\ \epsilon(\mu_{\mathrm{UV}}) \sim e^{-\frac{4\pi^2}{\lambda}}$$

$$\frac{\Delta E_2}{m_\gamma} \sim -\exp\left(-C\,\lambda^{5/2}\,e^{4\pi^2/\lambda}\right),$$

When N = 2, two-photon bound state is center neutral!

Analogous results for any N. Always get some center-neutral bound states.

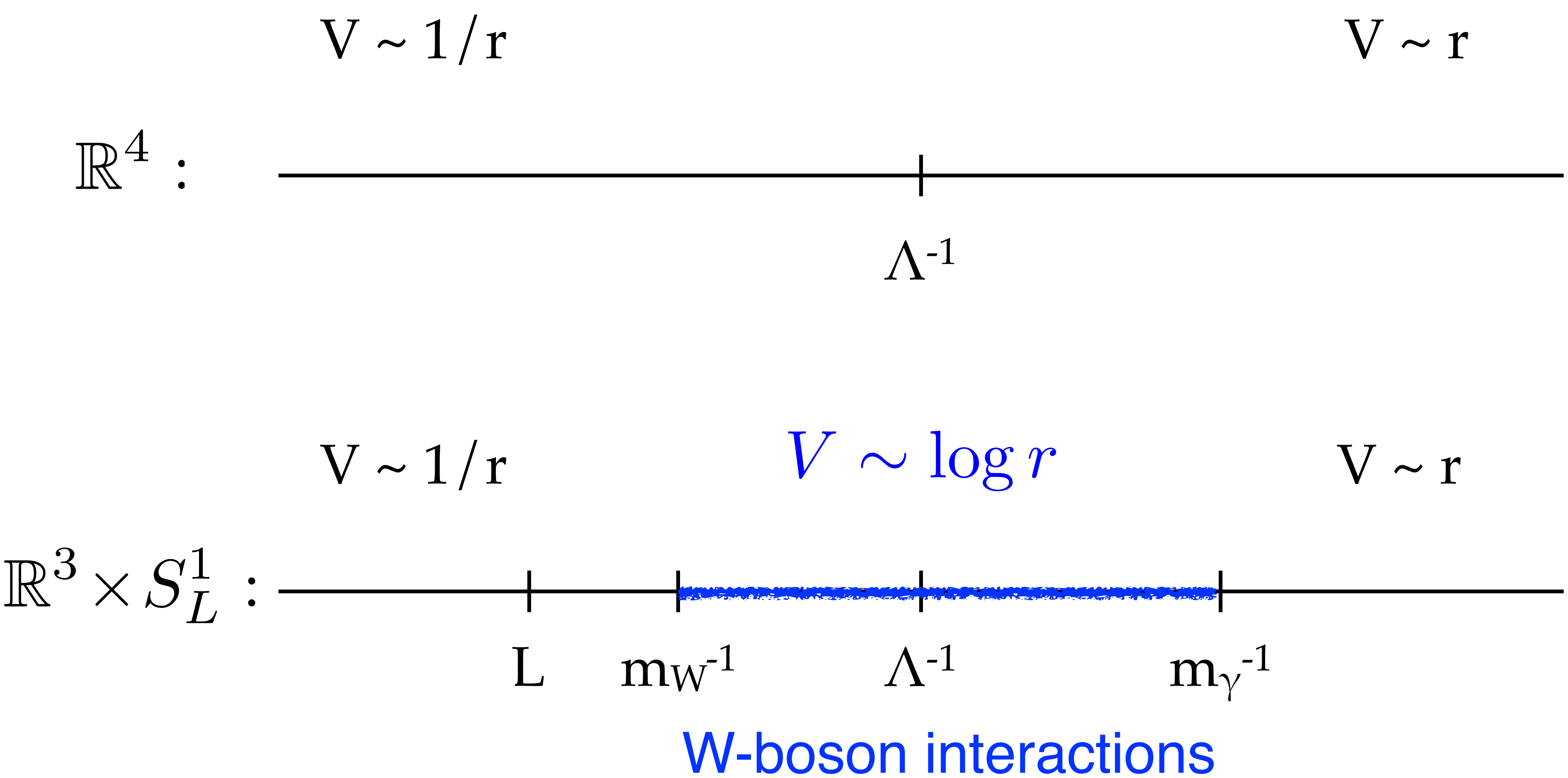
Dual photon mass scale is indeed associated with single-particle states which can extrapolate smoothly to large L!

$$\log\left[\langle \mathrm{tr}\,F^2(x)\mathrm{tr}\,F^2(0)\rangle\right] = \ldots + e^{-e^{+1/\lambda}}(\cdots) + \ldots$$

Should include iterated exponentials in transseries of YM.

Heavy state interactions

Larger mass states, involving W-bosons, described by 3d $U(1)^{N-1}$ QED EFT



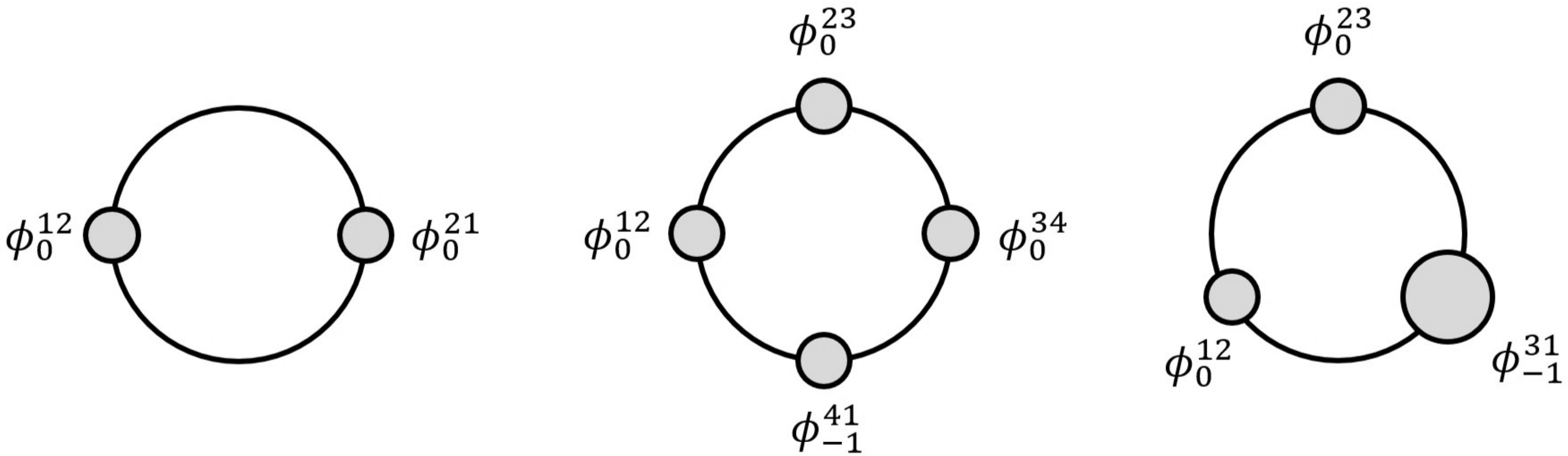
$$H_{WW} \sim \frac{p^2}{m_W} + \frac{\ell^2 - 1/4}{m_W r^2} + \lambda m_W \log(r/m_W)$$

Heavy bound states

Note: in 3D log r is a confining force. Can't isolate individual W bosons.

Physical states must have vanishing charge

spectrum = `closed string' W-boson bound states



Indeed, the 4D-variable interpolating operators for these states are color singlets.

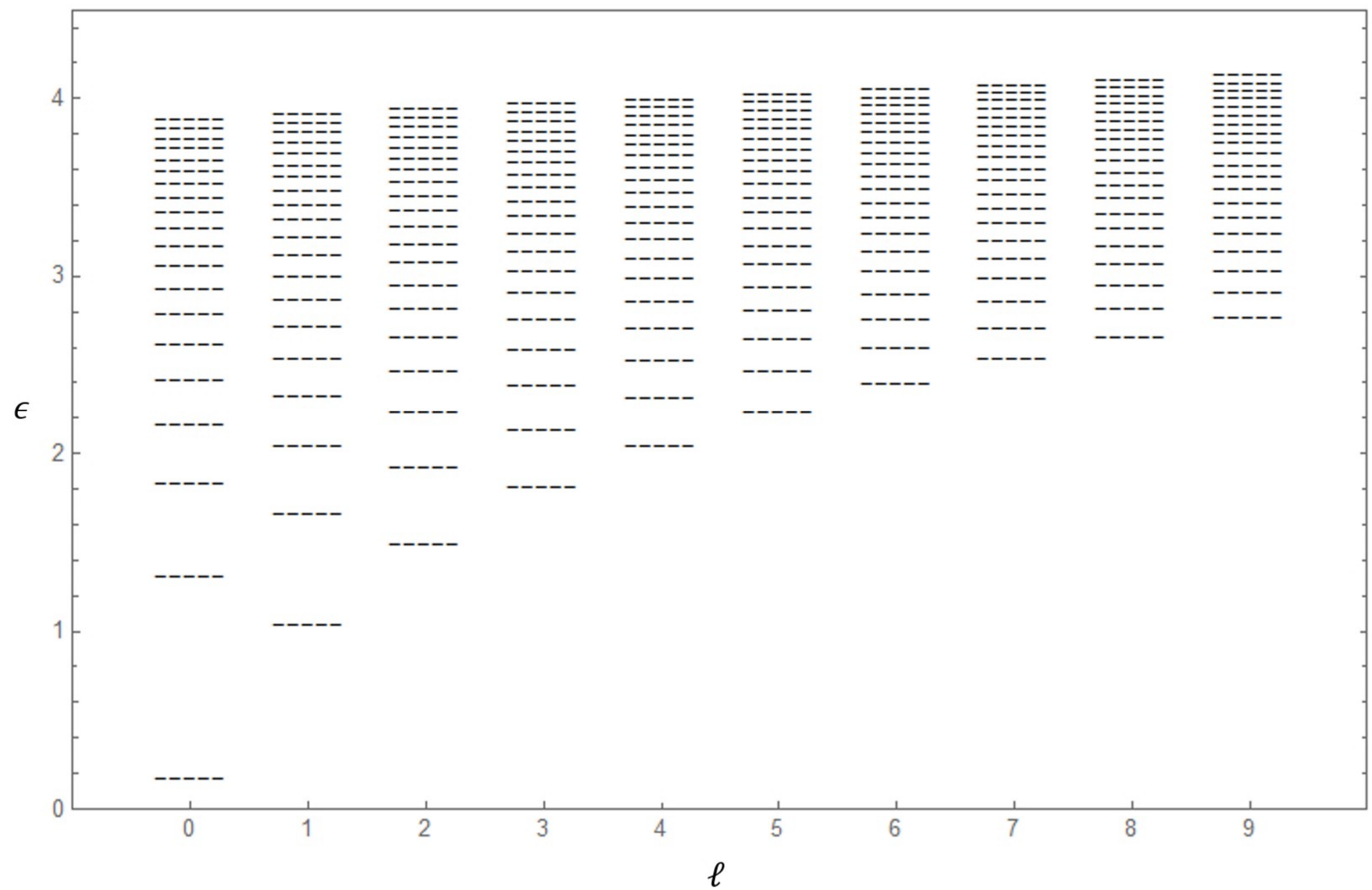
Exactly what we should from confinement and match to large L

Hagedorn behavior

Aitken, AC, Poppitz, Yaffe, 1707.08971

Energy-dependence of W-W bound states is interesting

$$\frac{E_{\rm WW}}{m_W} \sim \lambda \left[\log(2n+1) - \frac{1}{2}\log(\lambda) + {\rm const}\right]$$



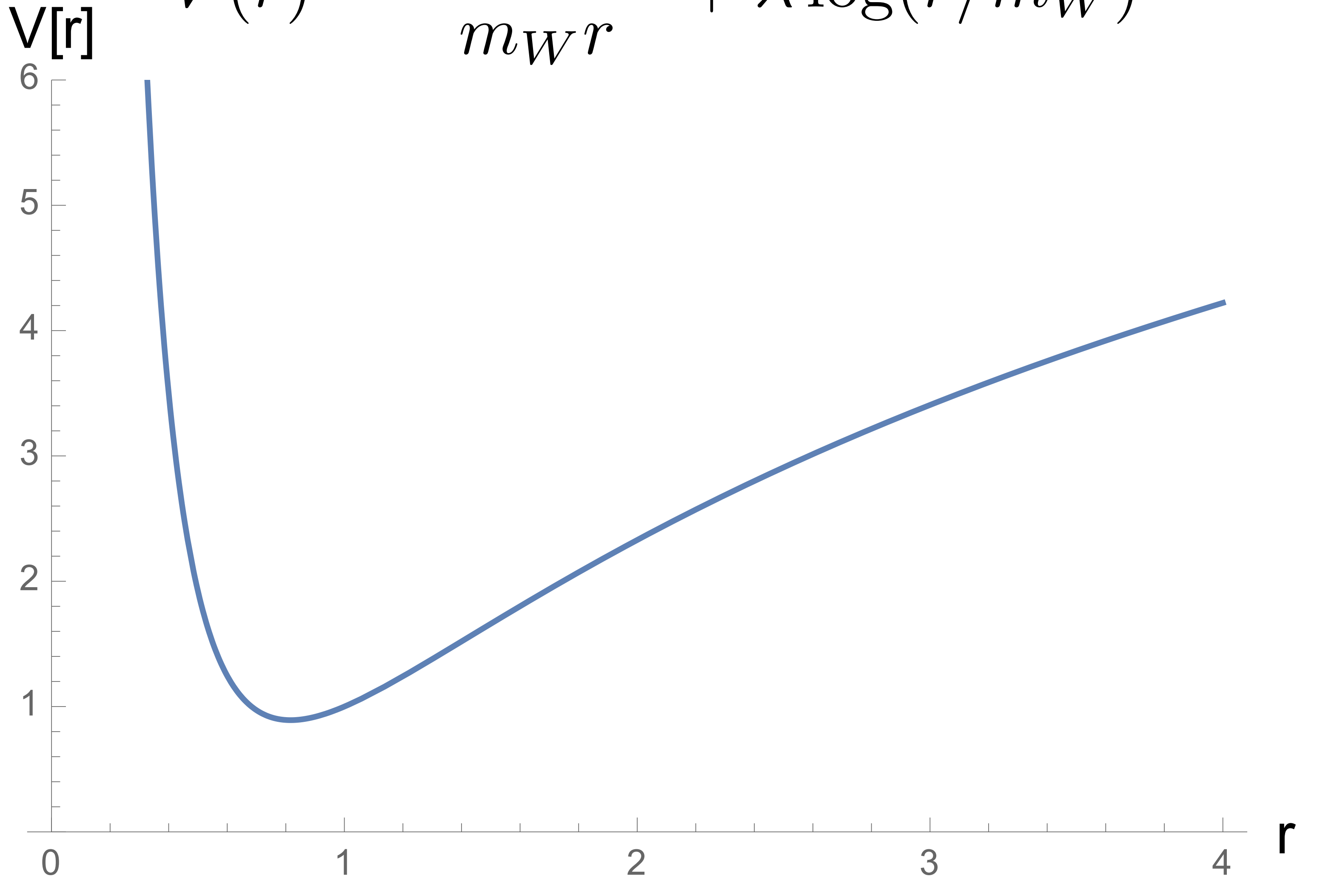
Hagedorn behavior

Aitken, AC,
Poppitz, Yaffe,
1707.08971

Energy-dependence of W-W bound states is interesting

$$\frac{E_{\mathrm{WW}}}{m_W} \sim \lambda \left[\log(2n+\ell+1) - \frac{1}{2}\log(\lambda) + \mathrm{const}\right]$$

$$V(r) \sim \frac{\ell^2 - 1/4}{m_W r} + \lambda \log(r/m_W)$$



Hagedorn behavior

Aitken, AC, Poppitz, Yaffe, 1707.08971

Energy-dependence of W-W bound states is interesting

$$\frac{E_{\mathrm{WW}}}{m_W} \sim \lambda \left[\log(2n + \ell + 1) - \frac{1}{2}\log(\lambda) + \mathrm{const}\right]$$

⇒ need λ log(λ) in transseries!

(Contrast with $e^{-1/\lambda}$ log(λ) from instanton physics)

Number of states grows exponentially with energy:

Rumer 1961

$$N(E/m_W) \sim e^{+2E/m_W}$$

Folk theorem 1: "Hagedorn comes from confinement." True here, but bizarrely already comes from quantum-mechanics analysis.

Folk theorem 2: "Hagedorn comes from weakly-coupled string theory". Not true here - happens even when N is finite!

From YM to QCD

To get to QCD, add n_F color-fundamental fermions.

If fermions mass m = 0, get symmetry

$$SU(n_F)_L \times SU(n_F)_R \times U(1)_Q$$

Spontaneous chiral symmetry breaking

$$SU(n_F)_{V=L+R} \times U(1)_Q$$

Spectrum has $n_F^2 - 1$ massless Nambu-Goldstone bosons

This is the story on R^4.

Continuity for QCD

Idea: to keep chiral symmetry breaking for any L, take "center-symmetric" BCs for quarks.

$$\psi_a(x_4 + L) = (\Omega_F)_{ab}\psi_b(x_4)$$

$$\Omega_F \sim \mathrm{diag}(1, e^{2\pi i/n_F}, \cdots, e^{2\pi i(n_F-1)/n_F})$$

If $d = \gcd(N, n_F) > 1$, keep Z_d "color-flavor center" subgroup of Z_N color center and Z_{nF} cyclic flavor permutation symmetries

AC, Sen, Wagman, Unsal, Yaffe 2017

Kashiwa, Kouno, Misumi, Takahashi, Yahiro, …, 2012-now

related ideas in Shimizu, Yonekura 2017

When $n_F < N$, preserving Z_d leads to XSB

AC, Schafer, Unsal 2016

Fits with heuristic idea that confinement is tied up with XSB

Large L expectations

Twisted BCs break flavor symmetry to

$$U(1)_L^{n_F-1} \times U(1)_R^{n_F-1} \times U(1)_Q$$

N_F-1 'pions' remain gapless, all others pick up positive gaps $E^2 \gtrsim 1/L^2$

So expect N_F -1 gapless NGBs at small L.

Twisted BCs = holonomies for background flavor gauge fields
= real (chiral-symmetry-preserving) mass terms for quarks

Can ensure all 3d matter fields have non-vanishing real masses by $U(1)_Q$ twist

Necessary to make sure 3d QED EFT stays weakly coupled.

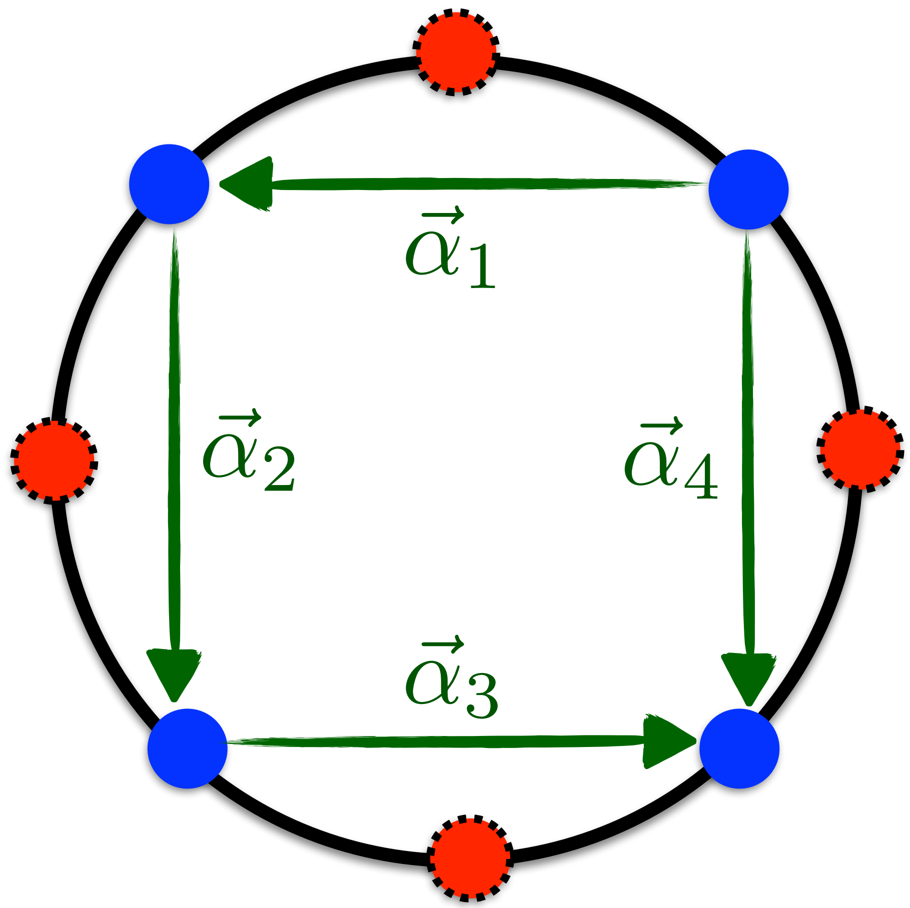
Fermion zero modes

With $\mathbb{Z}_{N_f}$ twist

AC, Schafer, Unsal, 2016

using index theorem of Poppitz+Unsal 2008

$N_c = N_F = 4$



$$\mathcal{M}_i = e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} (\bar{\psi}_{L,i}\psi_{R,i}), \qquad i = 1, \ldots N_f$$

This drives chiral symmetry breaking!

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

Look at symmetries preserved by

$$e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_k\cdot\vec{\sigma}}(\bar{\psi}_{L,k}\psi_{R,k})$$

$[U(1)_V]^{Nf-1} x U(1)_Q$ is obvious. What about axial transformations?

$$U(1)_A^{N_F-1}: \quad (\bar{\psi}_{L,k}\psi_{R,k}) \to e^{i\epsilon_k}(\bar{\psi}_{L,k}\psi_{R,k})$$

Monopole-instanton vertex invariance **requires**

$$\begin{aligned} (\bar{\psi}_{L,k}\psi_{R,k}) &\to e^{i\epsilon_k}(\bar{\psi}_{L,k}\psi_{R,k}), \\ e^{i\vec{\alpha}_k\cdot\vec{\sigma}} &\to e^{-i\epsilon_k} e^{i\vec{\alpha}_k\cdot\vec{\sigma}}. \end{aligned}$$

(Can also see it from Abelian duality and how background chiral gauge fields enter mixed Chern-Simons terms.)

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

So N_F - 1 'dual photons' pick up an exact shift symmetry.

They remain exactly massless, even at non-perturbative level.

All topological molecules have uncompensated fermi zero modes. No "magnetic bions" exist here.

chiral symmetry breaking = VEV for dual photon fields.

In fact dual photon action can be written as

$$S_\sigma = L\int d^3x \left[\frac{f_\pi^2}{4}\mathrm{Tr}\ \partial_\mu \Sigma' \partial^\mu \Sigma'^\dagger - c\,\mathrm{Tr}\ (M_q^\dagger \Sigma' + \mathrm{h.c.})\right]$$

Σ' is usual chiral field restricted to maximal torus, and here f_π is **calculable**:

$$f_\pi^2 = \left(\frac{g}{\pi L\sqrt{6}}\right)^2 = \frac{N_c\,\lambda\, m_W^2}{24\pi^4}$$

Mesons and baryons

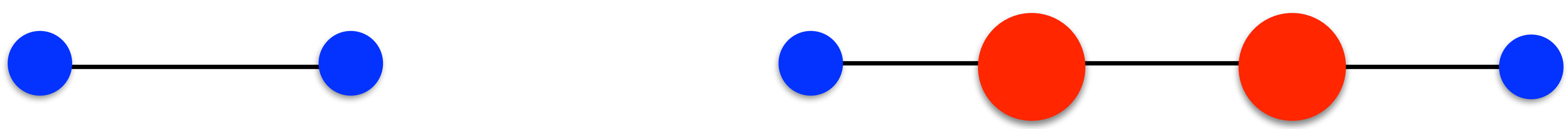
So we have χ-SB and pions. What about other hadrons?

All the usual suspects are there!

Quarks pick up real masses ~ m_W from color and flavor holonomies
+ small ~ $(m_W\, m_q)^{1/2}\, e^{-4\pi^2/\lambda}$ chiral-breaking masses from X-SB.

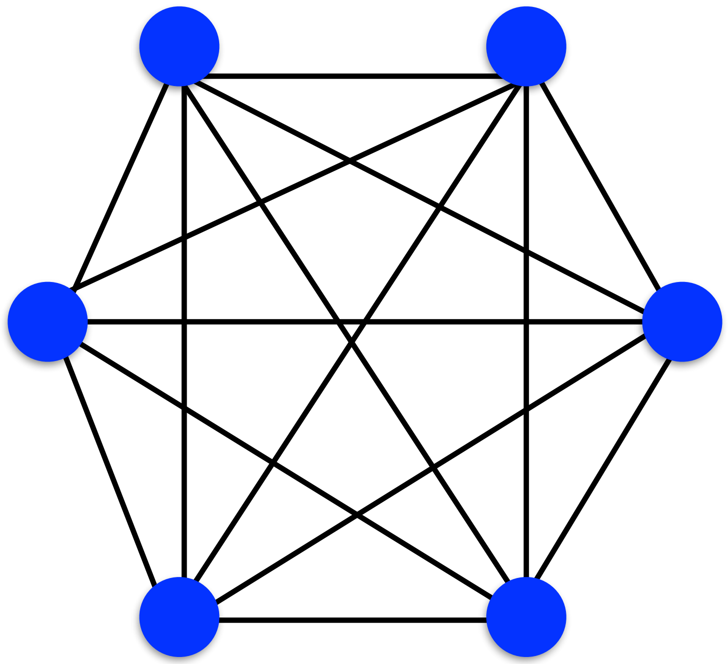
So get mesons which look like non-relativistic bound states

Circular quiver $U(1)^{N-1}$ QED produces `open string' meson states



Mesons and baryons

Also get baryons:



baryon mass ~ N m_W

Baryon masses calculable.

In progress: baryon interactions such that nuclear physics exists.

Conclusions

∃ small L limit of YM/QCD with unbroken center/CFC symmetries, as well as broken chiral symmetry.

Strong evidence for continuity in L, and hence for existence of useful semiclassical expansion for QCD-type theories.

Surprises:

4D XSB at weak coupling is possible.

Hagedorn scaling arising from non-relativistic confining dynamics.

Transseries structure not the one assumed so far:

$$\{\lambda,\ 1/\lambda,\ \log\lambda,\ e^{-1/\lambda},\ \exp(-\exp[+1/\lambda]),\ \ldots\ \}$$

Future: better understanding of large N limit. Very surprising features.

AC, Poppitz, 2016