22. BEYOND THE BLACK HOLE

John Archibald Wheeler

The Sibyline Strangenesses of the Landscape

Arthur Wellesley, Duke of Wellington, in the long years of activity in England that followed Waterloo, from time to time for relaxation would take a companion along for a carriage ride of hours through a distant countryside unfamiliar to them both. The Iron Duke was accustomed to draw his companion into his favorite game. From the look of the terrain up to this moment, predict what new panorama will be seen as the carriage tops the next long hill. Wellington generally produced the winning forecast of the lay of the land. Einstein traveled through a different countryside. His ability to sense ahead of time the upcoming landscape of physics is well known.

Today we find ourselves traversing a new realm. It contains such strange features as the black hole, the gauge or phase field, and complementarity. What lies beyond, over the hill?

If all strangenesses of a landscape made for Wellington the best indicators of the new terrain, the same was true, we know, for Einstein and the same surely holds for physics now. There is no hope of progress, we often say, until we are in possession of a central paradox, a difficulty, a contradiction. However, in our hearts we know it takes more. We need two paradoxes. Only then can we play off one against the other to locate the new point.

Two strangenesses stand out with special prominence in the landscape of the physics of our day: one is the Bounds of Time; the other, the Quantum.

Of the bounds of time the black hole[1] is the one most immediately accessible; then, beyond, the big bang[2] and — if the universe, as Einstein argued,[3] is closed and therefore collapses[4] in time to come — the big crunch.[5] The bounds of time tell us that physics comes to an end. Yet physics has always meant that which goes on its eternal way despite all surface changes in the appearance of things. Physics goes on, but physics stops; physics stops, but physics goes on. That is paradox number one, strange feature number one in the landscape we survey.

Paradox number two, the quantum principle[6] thrusts upon us. In every elementary quantum process the act of observation, or the act of registration,[7] or the act of observer-participancy,[8] or whatever we choose to call it, plays an essential part in giving "tangible reality" to that which we say is happening. Paradox number two is this: The universe exists "out there" independent of acts of registration, but the universe does not exist out there independent of acts of registration.

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If these are no small paradoxes, they suggest no small questions about what lies over the hill. How can one possibly believe that the laws of physics were chiseled on a rock for all eternity if the universe itself does not endure from everlasting to everlasting? If law, field, and substance come into being at the big bang and fade out of existence in the final stage of collapse,[9] how can a change so all-encompassing take place except through a process, the elementary mechanism of which has already made itself known? In what other way does an elementary quantum phenomenon become a phenomenon except through an elementary act of observer-participancy? To what other foundation then can the universe itself owe its existence except billions upon billions of such acts of registration?[10] What other explanation is there than this for the central place of the quantum principle in the scheme of things, that it supplies the machinery by which the world comes into being?[11]

Laws Derived from Symmetry Considerations but They Hide the Machinery Underlying Law[12]

Before we inspect more closely the two sibyline strangenesses of the landscape, let us look at the laws of physics themselves to recognize how little guidance they give us in forecasting what lies over the hill. Nothing in all the great achievements of science is more beautiful than Maxwell's electromagnetism, Einstein's geometric theory of gravitation, and the Yang–Mills theory of the quark-binding field.[13] Each expressible in a single line, these three theories are the yield of decades of research by hundreds of investigators performing thousands of experiments. However, the more we learn, the more we learn how little we have learned.[14]

Are not these three great theories of our time about to suffer the reappraisal already undergone by the great theory of elasticity of an earlier century?[15] Look at one of the good old textbooks on that subject. In the first chapter or two the laws of elasticity are deduced from elementary symmetry considerations. To say that the energy of deformation of a homogeneous isotropic material goes as the square of the deformation is to be confronted with two alternatives. One must either take the trace of the tensor of deformation and square it, or square the tensor of deformation and take its trace. More generally one makes a linear combination of these two expressions with two disposable constants of proportionality to construct the general expression for the energy of elastic strain. From this reasoning — that there are two constants of elasticity and no more — one goes on to build up all the rest of the great treatise on elasticity of one hundred years ago, complete with theorems, methods of analysis, applications and all kinds of beautiful problems for the student to solve at the end of each chapter. Likewise in our own time we have textbooks on electromagnetism,[16] gravitation,[17] and the Yang–Mills quark-binding field[18] or, more generally, on "gauge" fields, or "phase" fields, as Professor Yang suggests we call them, again complete with symmetry-argument foundations, theorems, applications, and problems for the student.

When we look back to elasticity, however, we recall that the most important fact about the subject — where the forces come from, molecular interactions between dozens of different atoms and molecules, multiplied by appropriate direction cosines — was not revealed one bit by these laws of elasticity. One hundred years of the study of elasticity would not have revealed atomic and molecular forces.[19] Neither would one hundred years of the study of atomic and molecular forces have revealed that these forces went back for their foundations to Schroedinger's equation and the motions of individual electrons and nothing more. We had to learn that we should explain, not electronic motions in terms of elasticity, but elasticity in terms of electronic motions. The very considerations of symmetry that had allowed one to master elasticity so early, taken by themselves, would have hidden from view forever the mechanism of elasticity.

The considerations of symmetry that reveal law hide the mechanism that underlies law. This lesson out of elasticity we today see afresh in electromagnetism, gravitation, and the dynamics of the Yang–Mills field, thanks to considerations of Hojman, Kuchař, and Teitelboim.[20] They consider a spacelike hypersurface σ_1 slicing through space-time (Fig. 22.1). They think of the field in question as given on all points of that hypersurface, along with the initial time rate of change of that field or, equivalently, the "field momentum." They ask: How does one go about predicting what the field will have for values at the points on a later spacelike hypersurface σ_2? The general marching his troops forward from one river to another may move the front ahead faster first on the right and later on the left, or alternatively order the line to advance more rapidly first on the left and then on the right, ending up, however, in the same stance on the same river. So the analyst of the field with his computer calculations can calculate ahead from instant to instant the successive configurations of the field either on the dashed or upon the dotted sequence of intermediate spacelike hypersurfaces in the diagram. He, unlike the general, ordinarily will arrive at different results for the field on σ_2 by the two maneuvers, the two alternative slicings of space-time, the two foliations: two incompatible predictions for one future. The fault, when there is one, is wrong choice of the particular Hamiltonian law assumed to govern the evolution of the field from instant to instant.

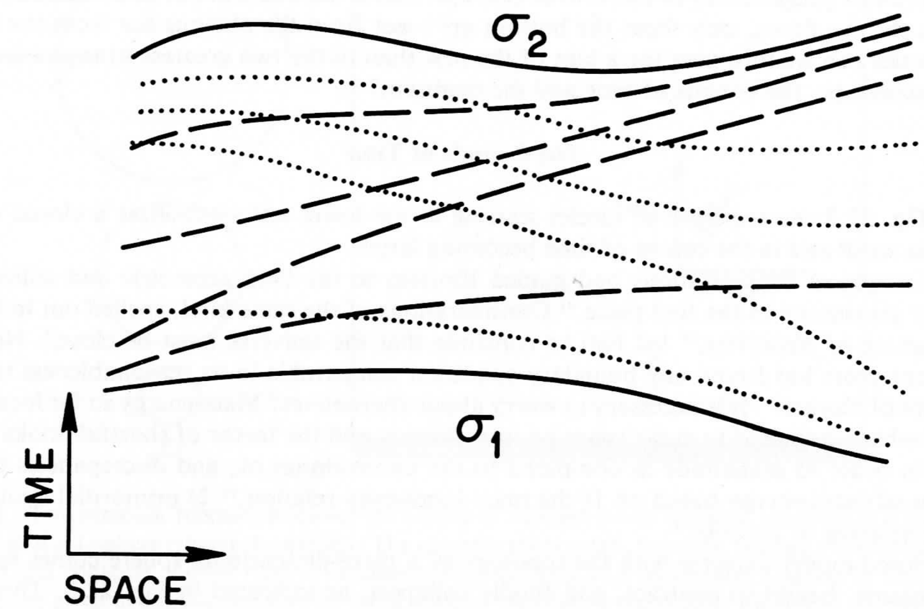

Fig. 22.1 The two alternative ways to calculate physics forward step by Hamiltonian step from the spacelike hypersurface σ_1 to the spacelike hypersurface σ_2 have to give the same result, the central point of the Hojman–Kuchař–Teitelboim "embeddability requirement." This simple demand leads straight to Maxwell electrodynamics, Einstein geometrodynamics, and the Yang–Mills theory of the quark-binding field.

When the field in question is a vector field and we restrict attention to Hamiltonians of the second order, there is only one option that is compatible with consistency. It is Maxwellian electrodynamics. When the field is a tensor field — the metric measuring the distance from point to point on the spacelike hypersurface — the requirement of consistency leads uniquely to Einstein's general relativity theory of gravitation. Any other Hamiltonian conflicts with the requirement that different ways of figuring ahead should fit into, be embeddable in, one and the same space-time manifold. Finally, when we impose this Hojman–Kuchař–Teitelboim demand of embeddability on a vector field that has an internal spin degree of freedom, we get the Yang–Mills theory of the quark-binding field.[21]

All three great theories of physics fall straight out of the utterly elementary demand for embeddability, as epitomized in Fig. 22.1. One does not have to recall Einstein's now abandoned dream of a geometrical unification of the forces of nature.[22] One does not have to have followed the exciting rebirth of this dream within the framework of that new and wider concept of geometry that is forced on us by the discovery in nature[23] — and in mathematics[24] — of "gauge" or "phase" fields, fields possessing at each point of space an "internal spin" degree of freedom. It is enough for the theoretical physicist to demand embeddability to deduce in a few hours what it took great men years of work to establish. Again, the more we learn the more we learn how little we have learned.

Fields in the end remind us more than ever of elasticity: modes of "vibration" of something; modes of a structure quite different from anything that shows on the surface; modes of a substrate, call it pregeometry or call it what one will, that is not and will not be revealed by reasoning from the top down, only from the bottom up[25]; not from the obvious but from the strange. Where better can we turn now for a hint of the new than to the two greatest strangenesses of the present landscape, the bounds of time and the quantum?

The Bounds of Time

In Fig. 22.2 the sequence of circles starting at the lower left symbolizes a closed universe beginning small and in the course of time becoming larger.

Philosophical considerations had guided Einstein to his 1915 geometric and still-standard theory of gravitation in the first place.[26] Considerations of the same kind, spelled out in his book *The Meaning of Relativity,*[27] led him to conclude that the universe must be closed. No one in subsequent years has found any boundary condition comparable in its reasonableness to the requirement of closure.[28] Is it necessary to worry about alternatives? Mass-energy so far located falls short of what is required to curve space up into closure, and the factor of shortfall looks big, but not big in order of magnitude as compared to the uncertainties of, and discrepancies between, estimates of mass-energy based on 1) the mass-luminosity relation,[29] 2) primordial deuterium,[30] and 3) clustering of galaxies.[31]

A closed-model universe with the topology of a three-dimensional sphere comes to a maximum volume, begins to contract, and finally collapses, as indicated in Fig. 22.2. Therefore it might seem that all the particles gather together in a common place at the start of time. However, the phrase "common place" we know to be a bad phrase and we know how to see that it is bad. We "unroll the space"; or in our symbolic model for the space, we unroll the circle from north pole to south pole. Then the separation between two particles measures itself not in miles but in degrees. The space-time history of the particles then allows itself to be displayed, though it is not

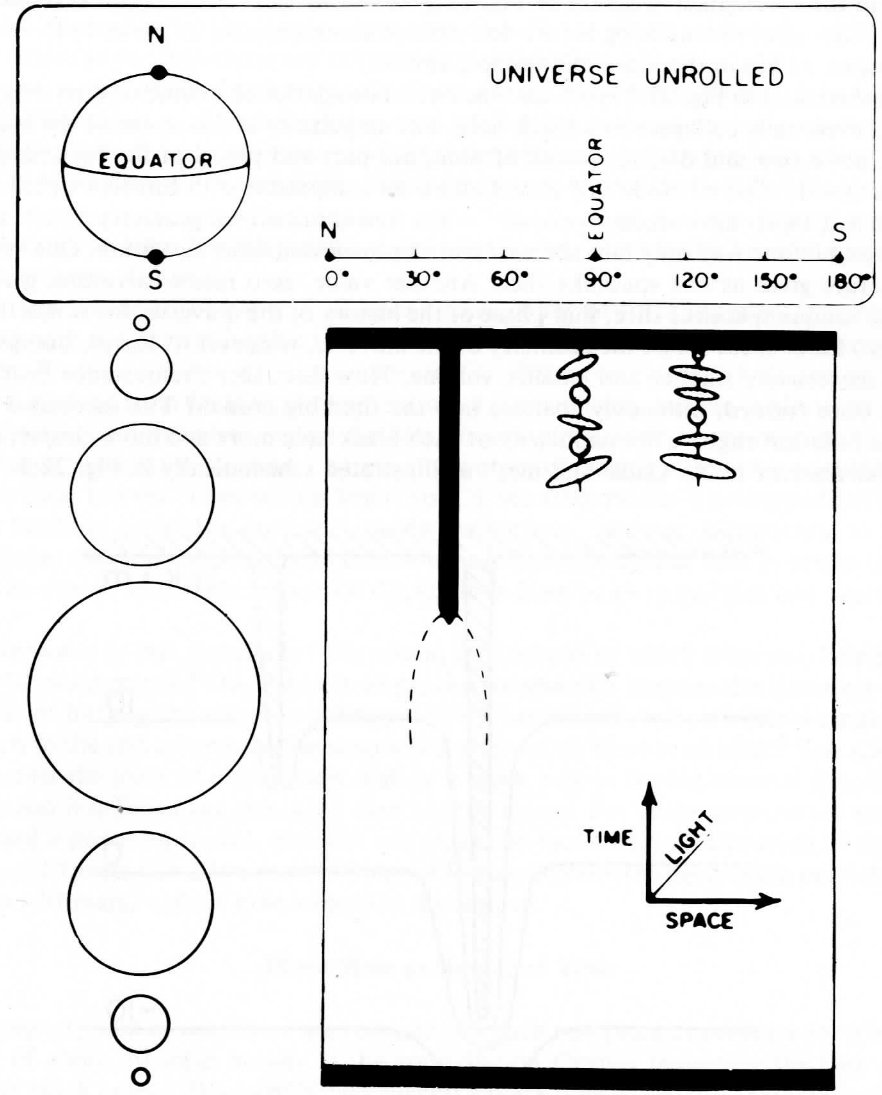

Fig. 22.2 Two symbolic representations of the history of a closed-model universe, idealized as a 3-sphere but depicted as if a 1-sphere (circle). *Lower left:* The universe starts small, expands, reaches a maximum volume, contracts and collapses. *Lower right:* The same history depicted in a rectangular diagram in which particle positions are given in angular measure by the scheme of translation sketched at the top of the page. Time is plotted on such a scale that light rays run at $\pm 45°$, being bounded however by the big bang at the bottom of the rectangle and the big crunch at the top. Two particles separated by 30° at the time of the big bang have to wait $\sim 10^8$ to $\sim 10^9$ years before a signal from the one gets to the other. The simple 45° algorithm is modified when there are inhomogeneities, such as the black hole "spike hanging from the roof" or the symbolically represented "mixmaster oscillations" of the geometry in the final stages of gravitational collapse.

displayed, in the rectangular diagram of Fig. 22.2 as two vertical lines. It can take hundreds of millions of years after the big bang before one particle communicates its presence to another particle that began its life in the same microscopic fireball.

The dashed lines in Fig. 22.2 symbolize the outer boundaries of a cloud of dust that gradually shrinks and eventually collapses to a black hole. The singularity at the center of the black hole is seen to be, not a new and distinct bound of time, but part and parcel of the big crunch.[32]

In a very wide class of models of closed universes compatible with Einstein's field equations — Marsden and Tipler have recently proved[33] — the four-dimensional geometry admits a foliation in one way and in one way only into slices of constant mean extrinsic curvature. One value of the mean curvature gives us one spacelike slice. Another value, zero mean curvature, gives us that instant, that unique spacelike slice, that phase of the history of the universe, for which the volume is the largest. Later slices depict the geometry of the universe, whatever its lumps, bumps, and ripples, with successively smaller and smaller volume. How does this circumstance bear on black holes that, once formed, ultimately coalesce into the final big crunch? The successive hypersurfaces of the foliation englove the singularity of each black hole more and more closely, according to recent calculations by A. Qadir and me,[34] as illustrated schematically in Fig. 22.3. The "last

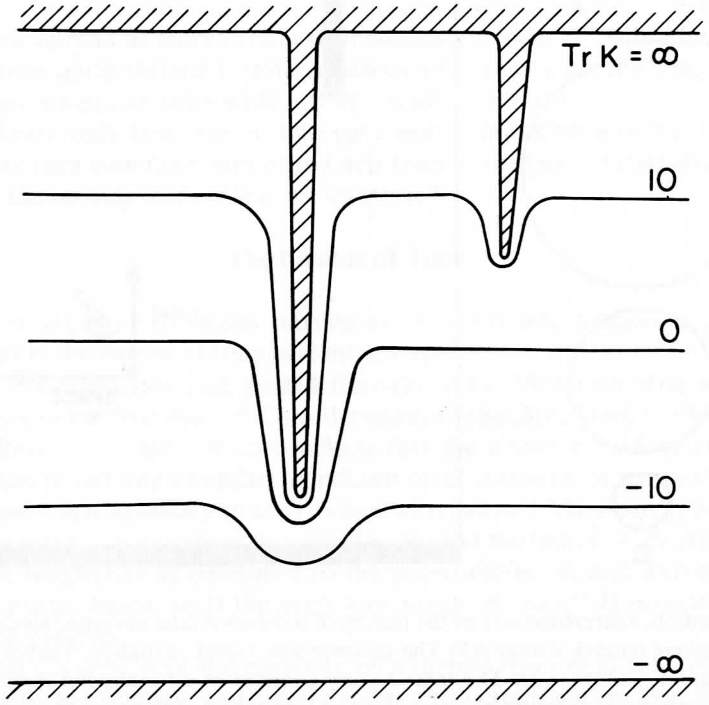

Fig. 22.3 Black hole and big crunch seen as part and parcel of the final singularity. The closed model universe is uniquely foliated by a sequence of spacelike hypersurfaces distinguished one from another by the value — constant over any one hypersurface — of the mean extrinsic curvature; that is, the trace of the tensor of extrinsic curvature or the fractional rate of decrease of volume per second.

hypersurface," the one of infinite mean extrinsic curvature, "establishes contact" simultaneously all along its front with the black hole singularity and the big crunch singularity. No better way could one desire to see that those are not two singularities but one.

The generic way of approach to the final singularity, if Belinsky, Khalatnikov, and Lifshitz are right,[35] proceeds through so-called mixmaster oscillations in the geometry, with the amplitude, phase, and direction of the principle axes of the space deformation varying from point to point of the spacelike hypersurface. Therefore also for the approach to the singularity of the physical black hole, as distinguished from the ideal Schwarzschild "dead" — or Reissner–Nordstrøm charged[36] or Kerr rotating[37] or Kerr–Newman charged *and* rotating[38] — black hole, it is not unreasonable to expect a mixmaster character.

More than one hundred papers[39] of recent years, many of them beautiful in method and in results, deal with the physics outside the "horizon" of a black hole, but almost none with conditions inside. Thanks to this work we have learned in what sense "a black hole has no hair."[40] A "hair," a departure from ideality, a perturbation in the geometry outside the horizon associated with irregularities and turbulence when the black hole formed, washes out by a factor of $1/e = 1/2.718$ in each "characteristic time," a time of the order of magnitude of 10^{-4} seconds for a black hole of ten solar masses. Thus such a black hole, 1 sec after matter has stopped falling in, has attained a fantastic perfection outside.[41] Inside the horizon, however, it is natural to expect the direct opposite: small initial departures from ideal symmetry as matter falls in across the horizon leading to enormous mixmaster curvature fluctuations from point to point as one approaches the singularity.[42]

How far away is that singularity? My watch, the baryons of which came into being at the big bang, has 10 more years of life. When it stops, can we spare its baryons the ignominy of further use? Instead of burying the watch or melting it down can we obliterate it? Can we make those 10 years stretch to the end of time, to the singularity after which there is no after? Yes. Can we even choose whether the place of obliteration shall be a black hole or the big crunch? Yes, if there *is* a big crunch and if it lies at the estimated time[43] in the future. For either purpose we must put the watch aboard a powerful rocket, one that will make the factor of time dilatation of the order of 10^{11} years per 10 years if in 10 years of life we would have it reach the big crunch; or of the order of 10^4 years per 10 years, a black hole located in this galaxy.

Black Hole as Bound of Time

How near the end of time is we are reminded by each new piece of evidence for a black hole, the object of about 10 solar masses in the constellation Cygnus remaining the best studied of presumptive black holes.[44] Bursts of X rays suggest a black hole of 100 to 1,000 solar masses at the center of each of five clusters of stars in our own galaxy.[45] Charles Townes and his colleagues, as well as Jan Oort, and others give considerations arguing for a black hole of about 4×10^6 solar masses at the center of the Milky Way.[46] The Lick Observatory group and others find evidence[47] pointing to the possible existence of a black hole of about 5×10^9 solar masses at the center of the violently active galaxy M87.

Is it clear that the center of a black hole offers obliteration only, not the chance to emerge somewhere else in space? In favor of such a possibility for space travel there exists not the slightest evidence. On the contrary, if at any time there ever were a wormhole or tunnel of appreciable diameter (as distinguished from the dimensions of quantum fluctuations), it would collapse with the speed of light.[48] The matter that falls in does not reappear somewhere else. All its details fade away, but its gravitational attraction remains. Any planet once in circumambient orbit stays in orbit. Mass-energy, an exterior property, remains; matter, an interior property, is obliterated.

Figure 22.4 reminds us that all details of whatever is dropped in are washed away. Provided that nature has no other long-range charge-conserving field than electromagnetism, we have to conclude that the resulting black hole is fully characterized by its mass, charge, and angular momentum, and nothing more. Of course mass implies energy, and therefore also the possibility of another property for a black hole, momentum.[49] However, we think of this momentum, not as an independent feature of the black hole, but as a consequence of our choice of reference frame. There is another feature of the black hole, Claudio Teitelboim tells us,[50] its spinor spin, that — like momentum — can be given one value or another depending on our choice of reference frame, except that now the frame of reference that comes into consideration is not the Lorentz frame but the spinor reference frame. It does not matter for this reasoning whether we use the theory of supergravity as originally developed by Freedman, van Nieuwenhuizen, and Ferrara, and by Deser and Zumino, or whether we follow Teitelboim's beautiful procedure of taking "the Dirac square root" of Einstein's general relativity, for by these two very different routes we come to the same theory with the same "internal spin-$\frac{3}{2}$" or "phase" degree of freedom.[51]

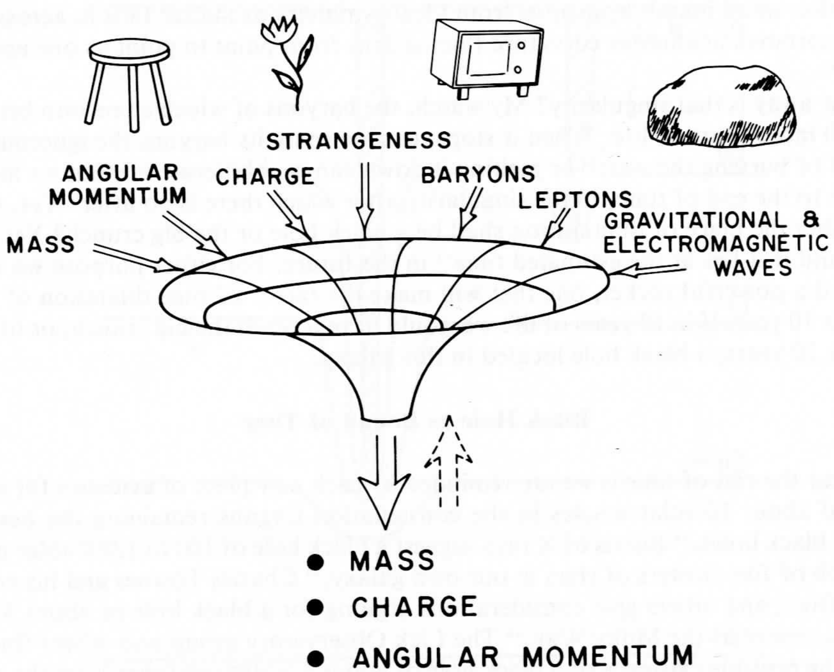

Fig. 22.4 Details of all objects dropped into a black hole are obliterated. The resulting system, according to available theory, is characterized by its linear momentum, angular momentum, mass, and charge and by no other parameter. No detail inside its "horizon," or surface of no return, can be probed from outside. At its center sits the singularity of final crunch.

Of baryon number, lepton number, and strangeness not a trace is left, if present physics is safe as guide.[52] Not the slightest possibility is evident, even in principle, to distinguish between three black holes of the same mass, charge, and angular momentum, the first made from baryons and leptons, the second made from antibaryons and antileptons, and the third made primarily from pure radiation.[53] This circumstance deprives us of all possibility to count, or even define, baryon and lepton number at the end and compare them with the starting counts.[54] In this sense the laws of conservation of baryon and lepton number are not violated; they are transcended.

Up the Staircase of Law and Law Transcended to Mutability

Figure 22.5 pictures the development of physics as a staircase.[55] Each step symbolizes a new law or discovery. Each riser marks the attainment of conditions so extreme as to overcome the usefulness of that law, or transcend it.

Archimedes, discovering how to measure density,[56] could regard it as a constant of nature. However later ages achieved pressures great enough to bring about measurable alterations in density.[57] The concept of valence[58] brought into order the major facts of chemistry, but today we

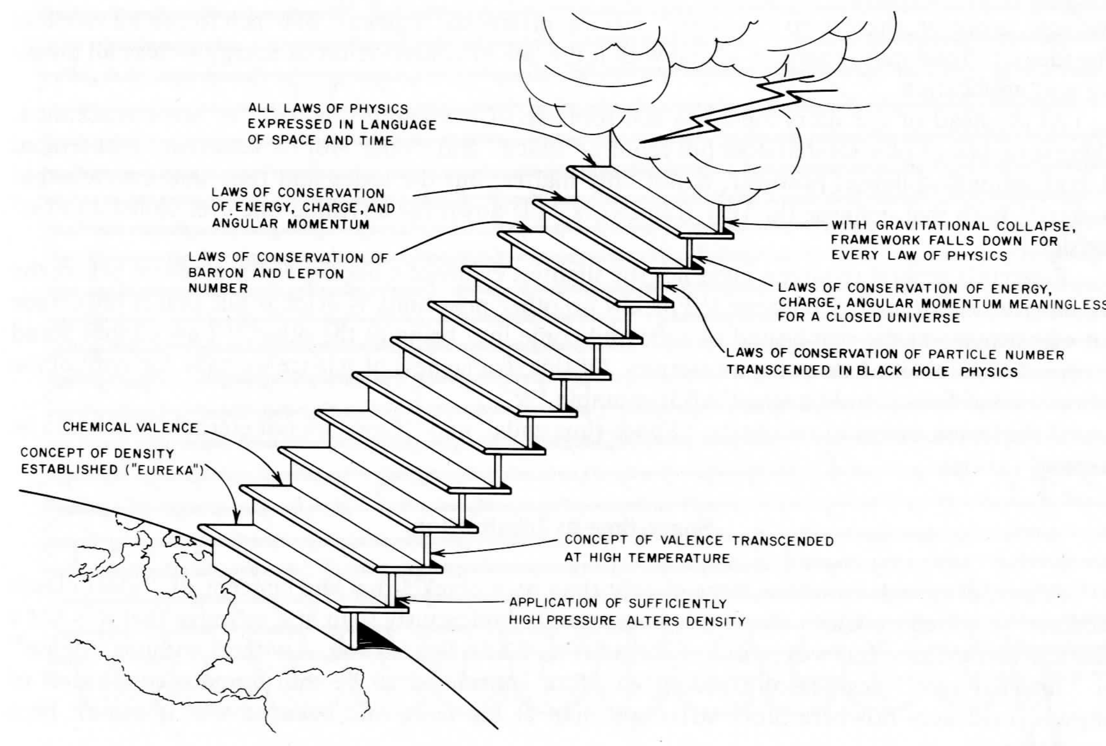

Fig. 22.5 The staircase of law and law transcended. Each step symbolizes the discovery of a new regularity or constancy of nature; each riser, the discovery of a technical means or a natural condition so extreme as to overcome or transcend that regularity.

know we have only to go to very high temperatures to outrun traditional valence considerations.[59] Later came the discovery that every atomic nucleus admits rigid classification by its charge number and its mass number;[60] but the advent of nuclear transmutations[61] destroyed that rigidity. The laws of conservation of baryon number and lepton number are indispensible in accounting for the wealth of experience in elementary particle physics,[62] but they have no application in black hole physics.[63] There they are not violated, but transcended.

In the end can we not at least say that the black hole has mass and therefore mass-energy? And does not the law of conservation of energy stand up against arbitrarily extreme conditions? In an asymptotically flat space, yes; in a closed universe, no. There total energy is not even defined.[64] Thus the local law of conservation allows one to express the total energy in a bounded region as an integral over the two-dimensional frontier of that region. The larger the region subsumed in counting up the energy, the larger at first is the boundary. However, as more and more volume is swept for energy, the boundary pushes on over the great bulge of the universe and begins to shrink. As we complete the sweep through "the other half of space," we push this surface down to extinction. The law of conservation of energy degenerates to the identity $0 = 0$. This lesson of the mathematics physics can be put into other words. To measure the mass-energy of a moon, a planet, a star, or larger system, it says, put a satellite in orbit about it. Measure the period of revolution, apply Kepler's "1-2-3 law" of motion[65] and obtain the mass. In the case of the closed universe, however, there is no "outside," no circumferential highway, in which to orbit a satellite. The idea of "total mass-energy" — and with it the law of conservation of energy — lose all meaning and application.

At the head of the stairs there is a last footstep of law and a final riser of law transcended. There is no law of physics that does not require "space" and "time" for its statement. Obliterated in gravitational collapse, however, is not only matter, but the space and time that envelop that matter.[66] With that collapse the very framework falls down for anything one ever called a law of physics.

Einstein's general relativity gives not the slightest evidence whatsoever for a before before the big bang or an after after collapse.[67] For law no other possibility is evident but that it must fade out of existence at the one bound of time and come into being at the other.[68] Law cannot stand engraved on a tablet of stone for all eternity. Of this strangeness of science we have for symbol the staircase; and for central lesson, "All is mutable."[69]

If the lesson comes in two parts, "Space-time ends" and "Laws are not eternal," each can be pursued further.

Space-time as Idealization

A crystal reveals nowhere more clearly than at a crack[70] that the concept of "ideal elastic medium" is a fiction. Cloth shows nowhere more conspicuously than at a selvedge that it is not a continuous medium, but woven out of thread (Fig. 22.6). Space-time — with or without "phase" or "internal spin" degrees of freedom — often considered to be the ultimate continuum of physics, evidences nowhere more strikingly than at big bang and collapse that it cannot be a continuum.[71]

There is an additional indication that space cannot be a continuum. Quantum fluctuations of geometry and quantum jumps of topology are estimated[72] and calculated[73] to pervade all space at the Planck scale of distances and to give it a foamlike structure.

"Space is a continuum." So bygone decades supposed from the start when they asked, "Why does space have three dimension?"[74] We, today, ask instead, "How does the world manage to give the impression it has three dimensions?" How can there be any such thing as a space-time continuum except in books? How else can we look at "space" and "dimensionality" except as approximate words for an underpinning, a substrate, a "pregeometry,"[75] that has no such property as dimension?

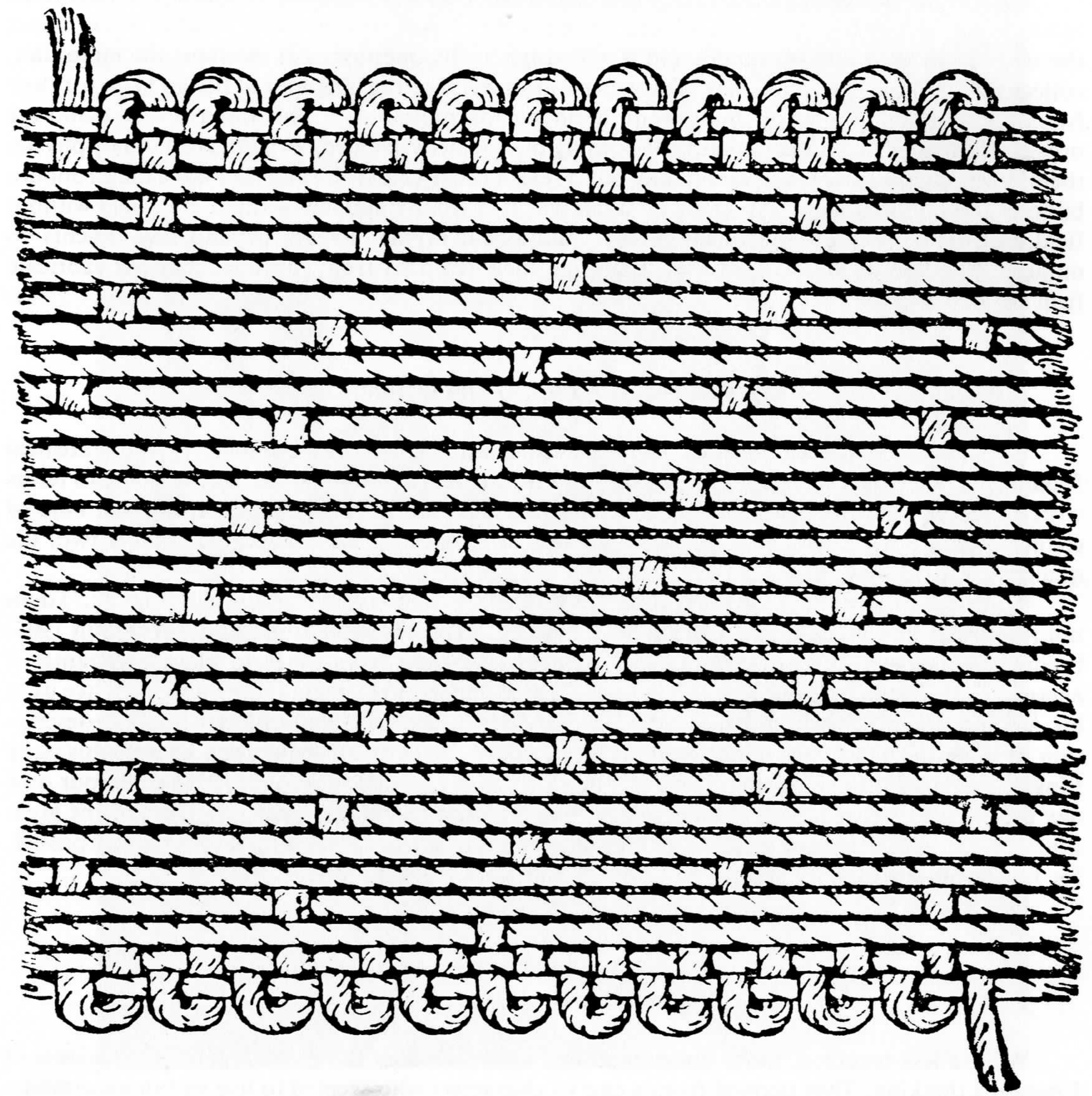

Fig. 22.6 It is disclosed more clearly at the selvedges than anywhere that what is woven is not a continuum.

Law without Law

"Physical space-time is not mathematical space-time" is the one lesson of mutability; the other, "Physical law is not ideal mathematical law." Law that comes into being at the beginning of time and fades away at the end of time cannot be forever 100 percent accurate. Moreover, it must have come into being without anything to guide it into being.

It is not new for a regularity to develop unguided. Thermodynamics, we know, rests upon the random motions of billions upon billions of molecules.[76] Ask any molecule what it thinks about the second law of thermodynamics and it will laugh at the question. All the same the molecules, collectively, uphold the second law. The genera and species of the kingdom of life go back for their foundation to billions upon billions of accidents of mutation.[77] The fantastically elaborate organization of plants and animals is of nothing but higgledy-piggledy origin. The laws of physics themselves, coming into being and fading out of existence: in what else can they have their root but billions upon billions of acts of chance? What way is there to build law without law, field without field, substance without substance except "Individual events. Events beyond law. Events so numerous and so uncoordinated that, flaunting their freedom from formula, they yet fabricate firm form?"'[78]

Strangeness Number Two: Quantum and Chance

We have been led to consider chance events, astronomical in number, as the statistical foundation of all the regularities of physics, and this in default of any other way to come to terms with mutability and the bounds of time, strangeness number one of the landscape. What kind of chance event? For a clue it is not clear where else to look except at strangeness number two, the quantum, "God plays dice."

"I cannot believe that God plays dice." Who that has known or read Einstein (Fig. 22.7) does not remember him arguing against chance in nature?[79] Yet this is the same Einstein who in 1905, before anyone, explained that the energy of light is carried from place to place as quanta of energy,[80] accidental in time and space in their arrival; and in 1916, again before anyone, gave us in his A's and B's, his emission and absorption coefficients, the still standard mathematical description of quantum jumps as chance events.[81] How could the later Einstein speak against this early Einstein, against the evidence and against the views of his greatest colleagues? How can our own day be anything but troubled to have to say "nay" to one teaching, "yea" to others of the great Einstein, the man who gave us in his geometric account of gravitation[82] a model, still unsurpassed, for how a physical theory should be founded and what it should do?

Was Einstein's Thinking Constrained by His Philosophical Antecedents?

We are less troubled, more understanding, when we recall the philosophical antecedents of Einstein's thinking. They derived from a cast of characters who seemed to live within his cranium and counsel with him as he spoke: Leibniz and Newton, Hume and Kant, Faraday and Helmholtz, Hertz and Maxwell, Kirchhoff and Mach, Boltzmann and Planck; but above them all Benedictus

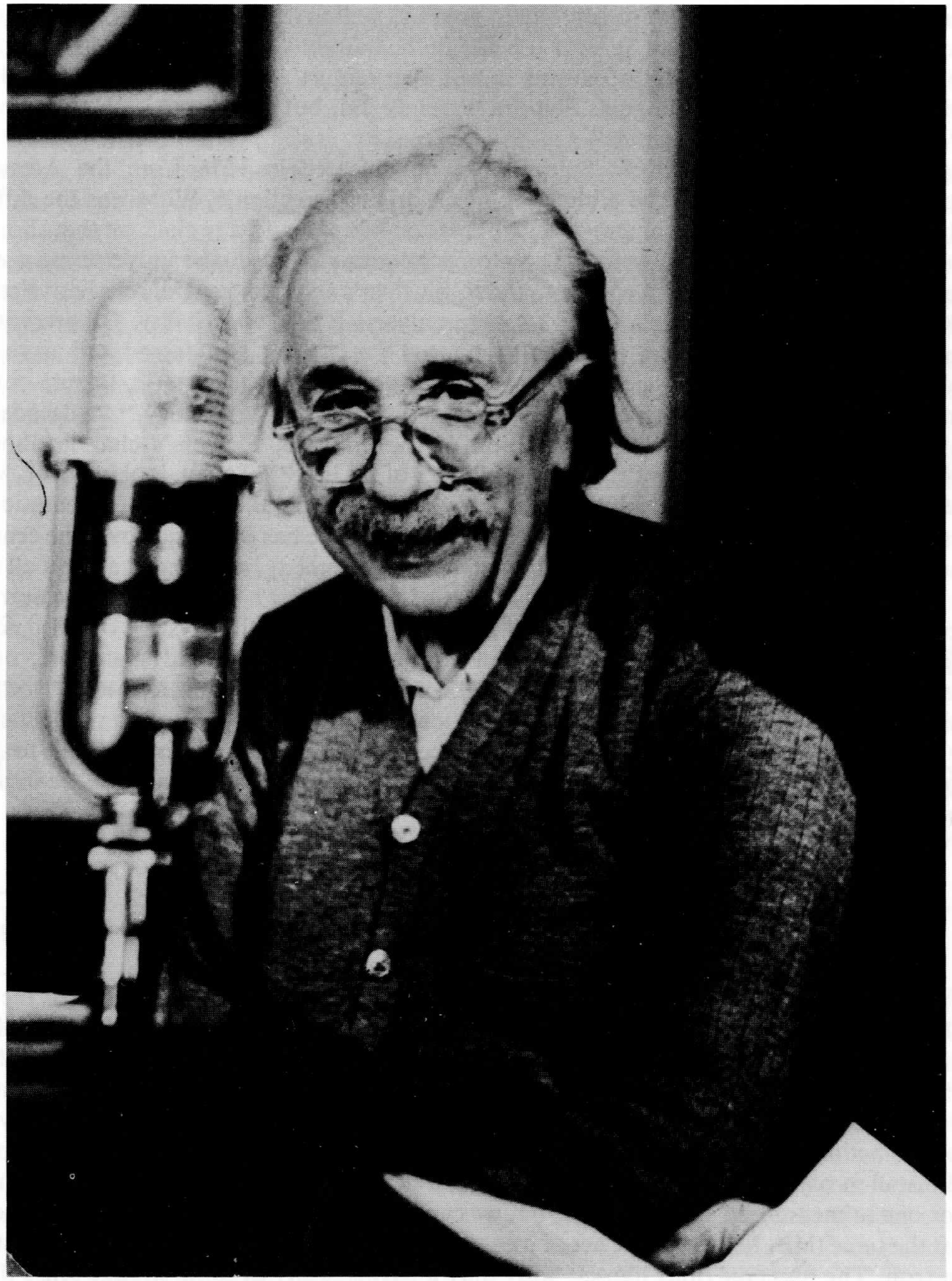

Fig. 22.7 Albert Einstein photographed at Princeton during broadcasting by Popperfoto (reproduced with the kind permission of the photographer — copyright reserved by the photographer).

de Spinoza, hero and role-creator to Einstein in youth as well as later life.[83] In earlier centuries no one expressed more strongly than Spinoza a belief in the harmony, the beauty, and — most of all — the ultimate comprehensibility of nature; in our own century, no one more than his admirer, Einstein. Guide Einstein to high goals Spinoza certainly did; but did he not — Hans Küng suggests[84] — on two occasions misguide?

Why was twenty-four-year-old Spinoza excommunicated in 1656 from the Amsterdam synagogue? Because he denied the bible story of an original creation.[85] What was the difficulty with the teaching? In all the nothingness before creation where could any clock sit that should tell the universe when to come into being! Therefore, Spinoza reasoned, the universe must endure from everlasting to everlasting. In contrast, and to Einstein's surprise, general relativity already in its first two years predicted that a static 3-sphere universe is an impossibility. Of necessity it is dynamic. Consequently Einstein reluctantly changed the theory and introduced a so-called cosmological term with the sole point and purpose to hold the universe static, to rule out what Alexander Friedmann later showed was a big-bang-to-big-crunch cosmology.[86] A decade later, when Edwin Hubble established the expansion of the universe, Einstein's chagrin about the cosmological term is well known: "the biggest blunder of his life."[87] Today, looking back, we can forgive him his Spinoza-inspired blunder and give him credit for the theory of gravitation that predicted the expansion. Of all the great predictions that science has ever made over the centuries, was there ever one greater than this, to predict, and predict correctly, and predict against all expectation a phenomenon so fantastic as the expansion of the universe? When did nature ever grant man greater encouragement to believe he will someday understand the mystery of existence?

Spinoza's influence on his thinking about cosmology Einstein could shake off, but not Spinoza's deterministic outlook. Proposition XXIX in *The Ethics* of Spinoza states, "Nothing in the universe is contingent, but all things are conditioned to exist and operate in a particular manner by the necessity of divine nature."[88] Einstein accepted determinism in his mind, his heart, his very bones. What other explanation is there for his later-life position against quantum indeterminacy than this "set" he had received from Spinoza?

No Elementary Phenomenon Is a Phenomenon until It Is a Registered Phenomenon

From Einstein's discomfort we turn to today's assessment of the central lesson of the quantum. In Fig. 22.8 the left-hand view symbolizes the concept of the universe of the old physics. Galaxies, stars, planets, and everything that takes place can be looked at, as it were, from behind the safety of a one-foot-thick slab of plate glass without ourselves getting involved. The right-hand view reminds us that the truth is quite different. Even when we want to observe, not a galaxy, not a star, but something so miniscule as an electron, we have in effect to smash the glass so as to reach in and install measuring equipment. We can install a device to measure the position x of the electron, or one to measure its momentum p, but we cannot fit both registering devices into the same place at the same time. Moreover the act of measurement has an inescapable effect on the future of the electron. The observer finds himself willy-nilly a participator. In some strange sense this is a participatory universe.[89]

A story may symbolize what it means for the observer to find himself a participator.[90] We had been playing the familiar game of twenty questions. Then my turn came, fourth to be sent from

the room, so that Lothar Nordheim's other fifteen after dinner guests could consult in secret and agree on a difficult word. I was locked out unbelievably long. On finally being readmitted, I found a smile on everyone's face, sign of a joke or a plot. I nevertheless started my attempt to find the word. "Is it animal?" "No." "Is it mineral?" "Yes." "Is it green?" "No." "Is it white?" "Yes." These answers came quickly. Then the questions began to take longer in the answering. It was strange. All I wanted from my friends was a simple yes or no. Yet the one queried would think and think, yes or no, no or yes, before responding. Finally I felt I was getting hot on the trail, that the word might be "cloud." I knew I was allowed only one chance at the final word. I ventured it: "Is it cloud?" "Yes," came the reply, and everyone burst out laughing. They explained to me there had been no word in the room. They had agreed not to agree on a word. Each one questioned could answer as he pleased — with the one requirement that *he* should have a word in mind compatible with his own response and all that had gone before. Otherwise if I challenged he lost. The surprise version of the game of twenty questions was therefore as difficult for my colleagues as it was for me.

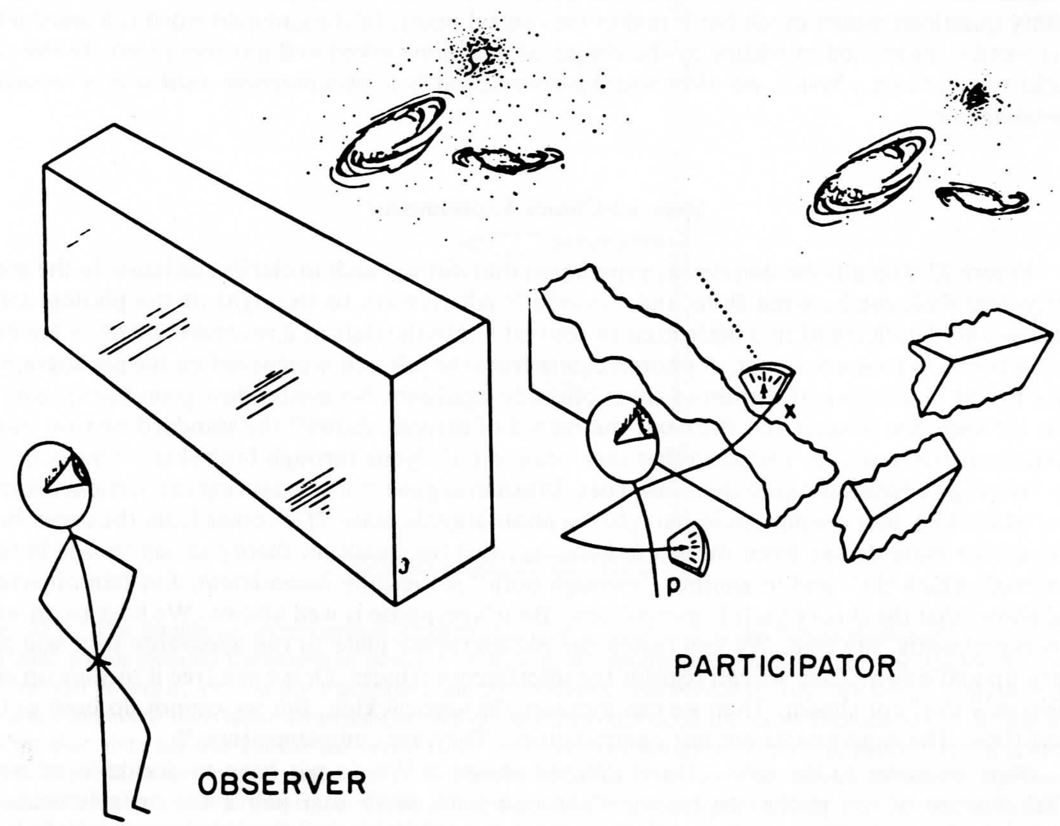

Fig. 22.8 Quantum mechanics evidences that there is no such thing as a mere "observer (or register) of reality." The observing equipment, the registering device, "participates in the defining of reality." In this sense the universe does not sit "out there."

What is the symbolism of the story? The world, we once believed, exists "out there" independent of any act of observation. The electron in the atom we once considered to have at each moment a definite position and a definite momentum. I, entering, thought the room contained a definite word. In actuality the word was developed step by step through the questions I raised, as the information about the electron is brought into being by the experiment that the observer chooses to make; that is, by the kind of registering equipment that he puts into place. Had I asked different questions or the same questions in a different order I would have ended up with a different word as the experimenter would have ended up with a different story for the doings of the electron. However, the power I had in bringing the particular word "cloud" into being was partial only. A major part of the selection lay in the "yes" and "no" replies of the colleagues around the room. Similarly the experimenter has some substantial influence on what will happen to the electron by the choice of experiments he will do on it, "questions he will put to nature"; but he knows there is a certain unpredictability about what any given one of his measurements will disclose, about what "answers nature will give," about what will happen when "God plays dice." This comparison between the world of quantum observations and the surprise version of the game of twenty questions misses much but it makes the central point. In the game no word is a word until that word is promoted to reality by the choice of questions asked and answers given. In the real world of quantum physics, *no elementary phenomenon is a phenomenon until it is a recorded phenomenon.*[91]

Delayed-Choice Experiments[92]

Figure 22.9 recalls the double-slit experiment that did so much to clarify the issues in the great thirty-year dialogue between Bohr and Einstein.[93] All features to the right of the photographic plate — and the slicing of that plate itself to convert it into the slats of a venetian blind[94] — are new and to be postponed a moment. A photon enters from the left and is recorded on the photographic plate by the blackening of a grain of silver bromide emulsion. No matter how great the spacing in time between one photon and the next, the record of arrivals shows[95] the standard two-slit interference pattern, basis for deducing that each photon has "gone through both slits." One can also tell "through which slit" each quantum goes, Einstein argued,[96] by measuring the vertical component of the kick that the photon imparts to the photographic plate. If it comes from the upper hole it kicks the plate down; from the lower hole, up. But for quantum theory to say in one breath "through which slit" and in another "through both" is logically inconsistent, Einstein objected, and shows that the theory itself is inconsistent. Bohr's response is well known. We have to do with two experiments, not one. We can fasten the photographic plate to the apparatus so it will not move up and down. Then we can register the interference fringes. Or we can free it to slide up and down in a slot, not shown. Then we can measure the vertical kick. But we cannot do both at the same time. The experiments are not contradictory. They are complementary.[97]

Now we come to the new feature: delayed choice.[98] We do not have to decide in advance which feature of the photon to record, "through both slits" that pierce the metal screen, or "through which slit." Let us wait until the quantum has *already* gone through the screen before we — at our free choice — decide whether it *shall have* gone "through both slits" or "through one."

We use a carefully timed source. We know when the photon has definitely passed through the metal screen and is on the last lap of its journey toward the photographic plate. At this moment we

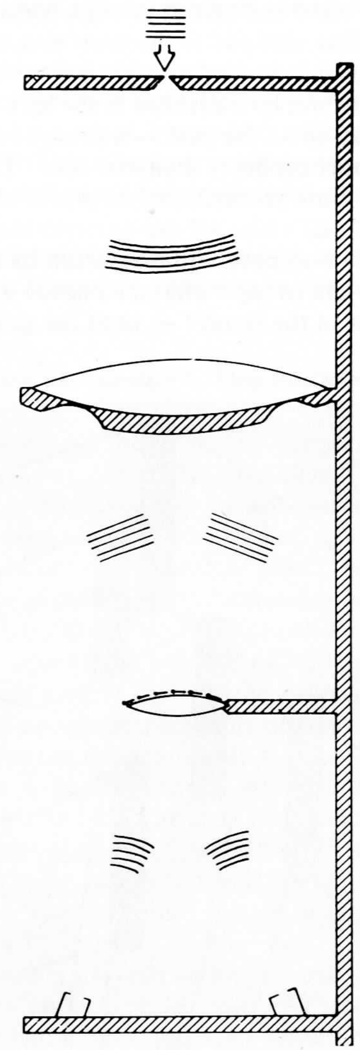

Fig. 22.9 The double-slit experiment both in the familiar version and in the "delayed-choice" version. The familiar layout includes the source of photons at the top, the entering slit, the first lens, the doubly slit metal screen that covers it, and the photographic plate that registers interference fringes. We secure delayed choice by supplements to this classic arrangement. We replace the continuous source of illumination at the top by a source that gives off one photon per timed flash. We slice the photographic plate to make it into a venetian blind. We make a last-minute choice, after the photon has *already* traversed the doubly slit screen, whether to open this blind or close it. Closed, as shown, it registers on a blackened grain of silver halide emulsion the arrival of that photon "through *both* slits." Opened, it allows the light to be focused by the second, or L. F. Bartell, lens on the two photon counters. There being only one photon, only one counter goes off. It tells "through *which* slit" the photon came. In this sense we decide, after the photon has *passed* through the screen, whether it *shall* have passed through only one slit or both.

make our choice: open the venetian blind and record through which slit the photon came; or close the blind, use it as a photographic plate, and add to the interference-pattern record that testifies to photons all going through both slits.

In the delayed-choice experiment we, by a decision in the here and now, have an irretrievable influence on what we will want to say about the past — a strange inversion of the normal order of time. This strangeness reminds us more explicitly than ever that ''The past has no existence except as it is recorded in the present''; or more generally, in the words of Torny Segerstedt, ''Reality is theory.''[99] What we call ''reality,'' that vision of the universe that is so vivid in our minds, we plaster in (Fig. 22.10) between a few iron posts of observation by an elaborate labor of imagination and theory. We have no more right to say ''what the photon is doing'' — until it is registered — than we do to say ''what word is in the room'' — until the game of question and response is terminated.

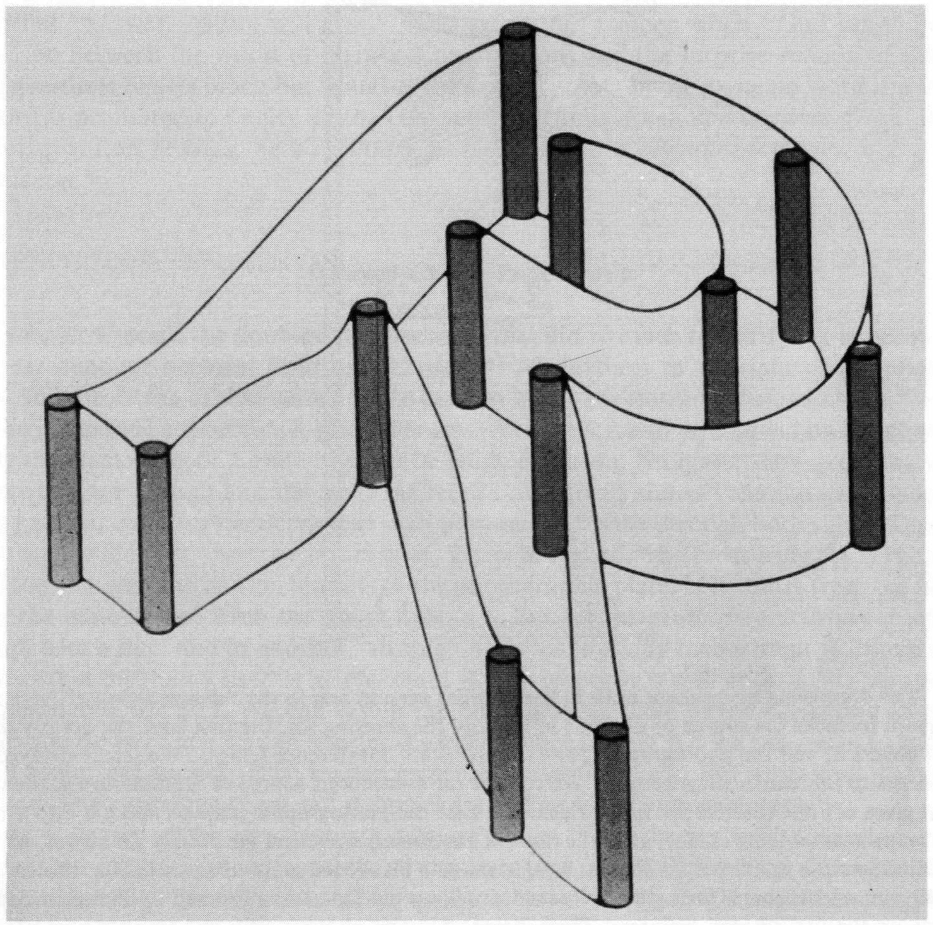

Fig. 22.10 What we call ''reality,'' symbolized by the letter R in the diagram, consists of an elaborate papier-mâché construction of imagination and theory filled in between a few iron posts of observation.

The Central Lesson of the Quantum

''No elementary phenomenon is a phenomenon until it is a registered phenomenon.''[100] This summary of the central lesson of the quantum takes its two key words from Bohr. ''Registered'' as Bohr uses it means ''brought to a close by an irreversible act of amplification'' and ''communicable in plain language.''[101] This adjective, equivalent in most respects to ''observed,'' has a special feature as compared to that more frequently seen word. It explicitly denies the view that quantum theory rests in any way whatsoever on ''consciousness.''[102] The critical word, ''phenomenon,'' Bohr found himself forced to introduce in his discussions with Einstein to stress how different ''reality'' is from Einstein's ''any reasonable conception of reality.''[103]

The Building of ''Reality'': This Participatory Universe

What lies over the hill? What are we to project ahead out of the present landscape's two greatest strangenesses? Of these, one, the ''bounds of time,'' argues for mutability, law without law, law built on the statistics of multitudinous chance events, events that — undergirding space and time — must themselves transcend the categories of space and time. What these primordial chance events are, however, it does not answer; it asks. Unasked and unwelcomed, the other strangeness, the quantum, gives us chance. In ''elementary quantum phenomenon'' nature makes an unpredictable reply to the sharp question put by apparatus. Is the ''chance'' seen in this reply primordial? As close to being primordial as anything we know. Does this chance reach across space and time? Nowhere more clearly than in the delayed-choice experiment. Does it have building power? Each query of equipment plus reply of chance inescapably do build a new bit of what we call ''reality.'' Then for the building of all of law, ''reality'' and substance — if we are not to indulge in free invention, if we are to accept what lies before us — what choice do we have but to say that in some way, yet to be discovered, they all must be built upon the statistics of billions upon billions of such acts of observer-participancy? In brief, beyond the black hole, past the two great strangenesses of the landscape and over the hill, what other kind of universe can we expect to see than one built as ''phenomenon'' is built, upon query of observation and reply of chance, a *participatory* universe?

If the concept of a participatory universe seems to make the world a never-never land, we can recall Samuel Johnson's remark on kicking the stone. Whatever the theory of reality, the pain in the toe made the stone real enough to him. In recent decades we have judged solid matter no less solid for being made up of electrons, nuclei, and mostly emptiness. It will make the stone no less real to regard it as entirely emptiness.

The Example of Mathematics

Mathematics also is emptiness without emptiness. A familiar theorem tells us that the sum of the three interior angles of a plane triangle is 180°. However, when we review all the definitions, postulates, and axioms that go into proving that theorem, we find that the statement reduces in the end to an identity, equivalent to ''0 = 0.'' No identity? Then no theorem! It may take 300 pages of computer paper to spell out all of the foundation pieces of a theorem, the customary journal proof of which requires only two pages.[104] But packaged as all the parts of the theorem are, and useful as the theorem is, it is still in the end packaged identity. Like the structure of rope in Fig. 22.11, it has only to be pulled on to fall apart into nothingness.

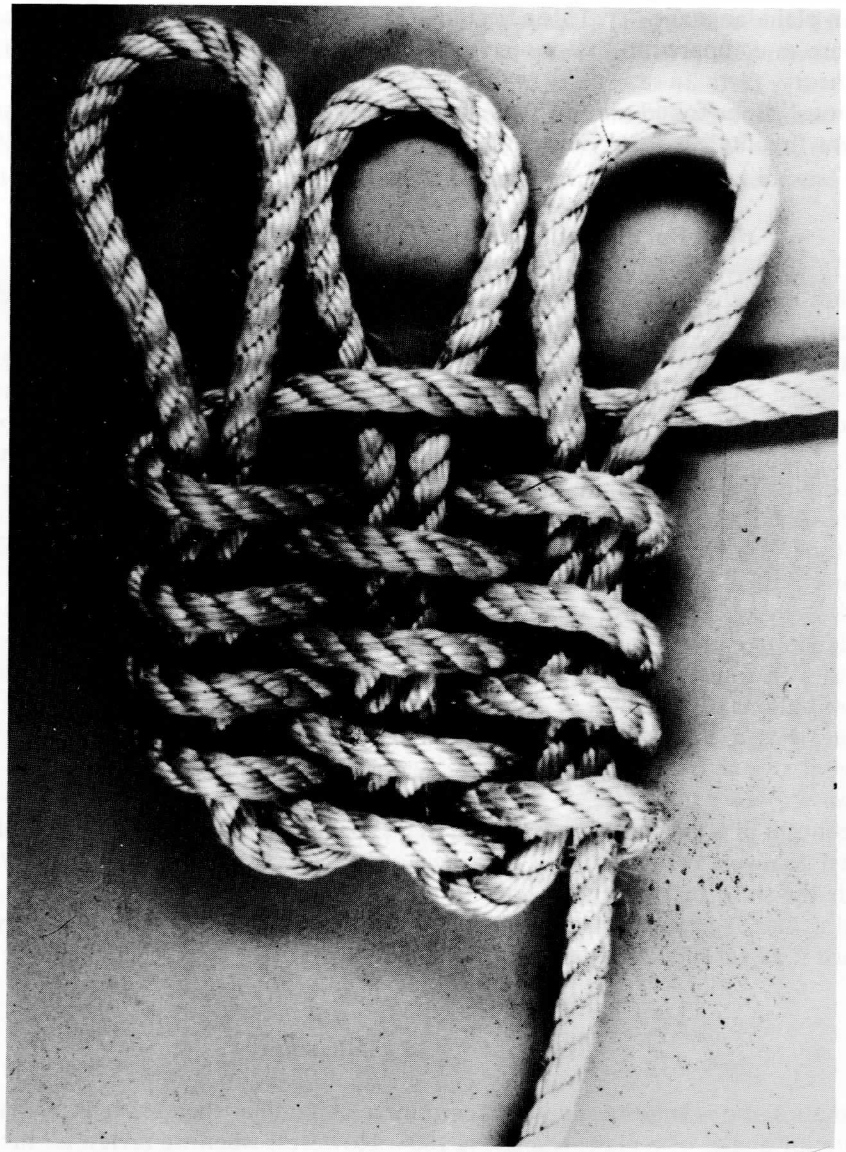

Fig. 22.11 The construction of rope that falls apart on being pulled symbolizes the elaborate theorem that. conformant to the inexorable demand of mathematics, dissolves into an identity on being analyzed. The author thanks William Wootters for engineering this photograph.

Participatory Universe as Self-Excited Circuit

Looking at an empty courtyard, we know that the game will not begin until a line has been drawn across the court to separate the two sides. Where, is not very important; but whether, is essential. ''Elementary phenomena'' are impossible without the distinction between observing equipment and observed system[105]; but the line of distinction can run like a maze (Fig. 22.12), so convoluted that what appears from one standpoint to be on one side and to be identified as observing apparatus, from another point of view has to be looked at as observed system.

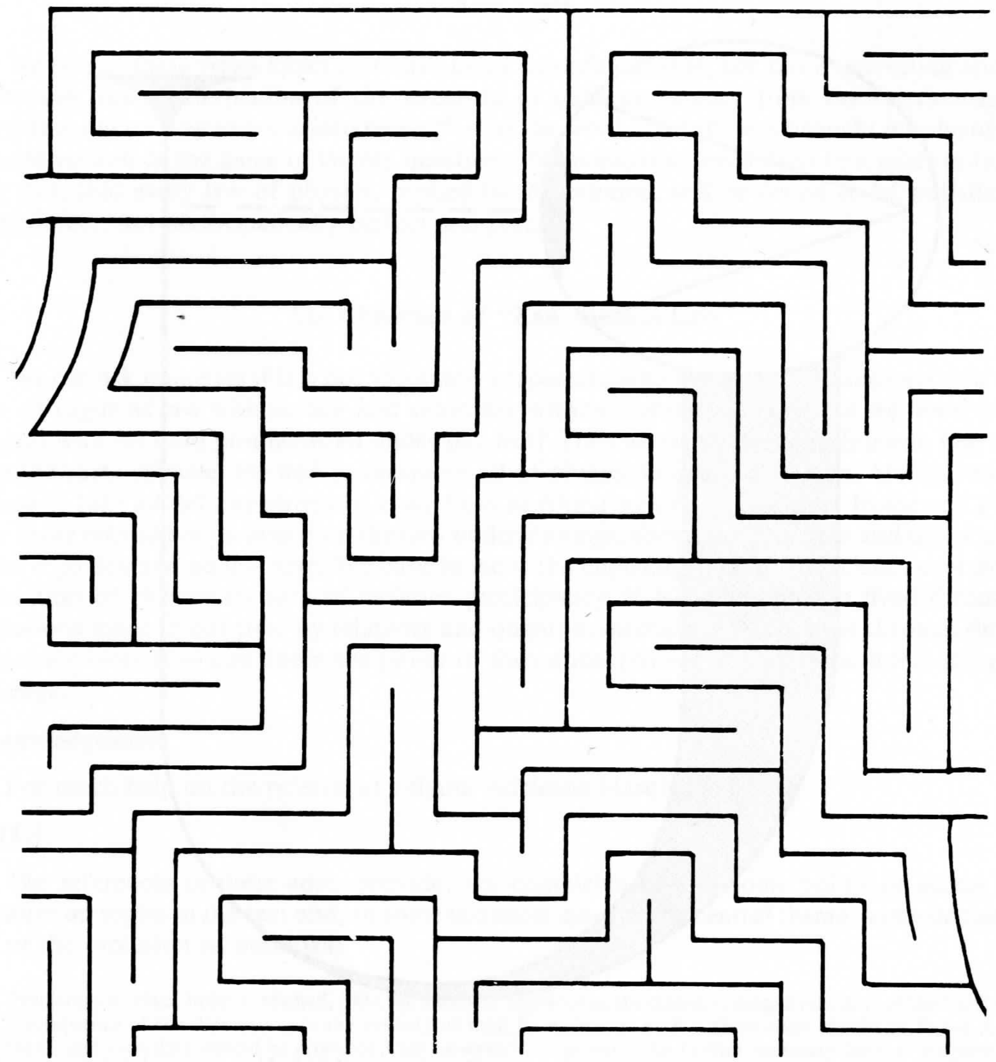

Fig. 22.12 Maze symbolic of the tortuous course through nature of the interface between observing equipment and system observed.

From "nothingness ruled out as meaningless,"[106] to the line of distinction that rules it out; from this dividing line to "phenomenon"; from one phenomenon to many; from the statistics of many to regularity and structure: these considerations lead us at the end to ask if the universe is not best conceived as a self-excited circuit[107] (Fig. 22.13): Beginning with the big bang, the universe expands and cools. After eons of dynamic development it gives rise to observership. Acts of observer-participancy — via the mechanism of the delayed-choice experiment — in turn give tangible "reality" to the universe not only now but back to the beginning. To speak of the universe as a self-excited circuit is to imply once more a participatory universe.

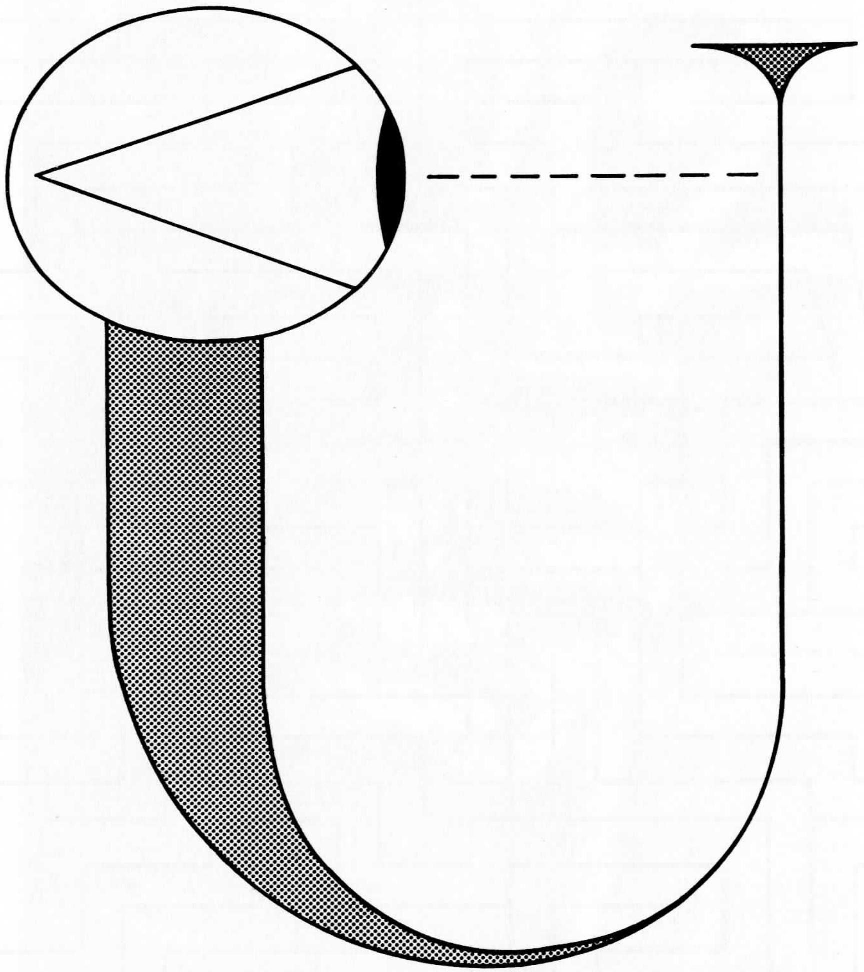

Fig. 22.13 The universe viewed as a self-excited circuit. Starting small (thin U at *upper right*), it grows (loop of U) and in time gives rise (*upper left*) to obsever-participancy — which in turn imparts "tangible reality" (cf. the delayed-choice experiment of Fig. 22.9) to even the earliest days of the universe.

If the views that we are exploring here are correct, one principle, observer-participancy, suffices to build everything. The picture of the participatory universe will flounder, and have to be rejected, if it cannot account for the building of law; and space-time as part of law; and out of law substance. It has no other than a higgledy-piggledy way to build law: out of the statistics of billions upon billions of acts of observer-participancy each of which by itself partakes of utter randomness.

Two Tests

No test of these views looks more like being someday doable, nor more interesting and more instructive, than a *derivation* of the structure of quantum theory from the requirement that everything have a way to come into being[108] — as the word "cloud" was brought into being in the surprise version of the game of twenty questions. No prediction lends itself to a more critical test than this, that every law of physics, pushed to the extreme, will be found to be statistical and approximate, not mathematically perfect and precise.

The Challenge of "Law without Law"

We can ask ourselves if it is not absolutely preposterous to put into a formula anything at first sight so vague as law without law and substance without substance. How can we hope to move forward with no solid ground at all under our feet? Then we remember that Einstein had to perform the same miracle. He had to reexpress all of physics in a new language. His curved space seemed to take all definite structure away from anything we can call solidity. In the end physics, after being moved bodily over onto the new underpinnings, shows itself as clear and useful as ever. We have to demand no less here. We have to move the imposing structure of science over onto the foundation of elementary acts of observer-participancy.[109] No one who has lived through the revolutions made in our time by relativity and quantum mechanics — not least through the work of Einstein himself — can doubt the power of theoretical physics to grapple with this still greater challenge.

Acknowledgement

For much help on the references I thank Adrienne Harding.

NOTES

The references in these notes provide, not completeness, but some points of access to the literature on topics in the text and, in some instances close to the central theme, some documentation of the evolution of outlooks.

1. Prehistory of black hole: J. Michell, "On the means of discovering the distance, magnitude, & c. of the fixed stars, in consequence of the diminution in the velocity of their light, in case such a diminution should be found in any of them, and such data should be procured from observations, as would be further necessary for that purpose," Phil. Trans. [Roy. Soc. London] **74,** 35–37 (1784) (read 27 Nov. 1783), cited and discussed in S. Schaffer, "John Michell and Black Holes," J. Hist. Astron. **10,** 42–43 (1979). "Un astre lumineux de même densité que la terre, et dont le diamètre serait deux cents cinquante fois plus grand que celui du soleil, ne laisserait en vertu de son attraction,

parvenir aucun de ses rayons jusqu'a nous; il est donc possible que les plus grands corps lumineux de l'univers, soient par cela meme, invisibles." [P.-S. Laplace, *Exposition du systeme du monde,* vol. 2 (Cercle-Social, Paris, 1795), p. 305.] Laplace gives the calculations underlying this statement in *Allgemeine geographische Ephemeriden,* edited by F. X. von Zach, IV, Band I St. (Einer Gesellschaft Gelehrten, Weimar, 1799), May 1798, p. 603; translated, "Proof of the theorem, that the attractive force of a heavenly body could be so large, that light could not flow out of it," in S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973), pp. 365–68.

First treatment of collapse within the framework of general relativity: "The total time of collapse for an observer comoving with the stellar matter is finite . . .; an external observer sees the star asymptotically shrinking to its gravitational radius." [J. R. Oppenheimer and H. Snyder, "On Continued Gravitational Attraction," Phys. Rev. **56,** 455–59 (1939).]

Coming to terms with gravitational collapse: B. K. Harrison, M. Wakano, and J. A. Wheeler, 'Matter-energy at High Density; End Point of Thermonuclear Evolution," in Onzième Conseil de Physique Solvay, *La structure et l'évolution de l'univers,* (Stoops, Brussels, 1958), pp. 124–46. In particular: white dwarfs and neutron stars shown for the first time to be two sectors of one continuous family giving "the absolutely lowest state possible for an A-nucleon system under the dual action of nuclear and gravitational forces," pp. 137–38; the equilibrium state of "cold matter ideally catalyzed to the end point of thermonuclear evolution," p. 138; "What is the final state of an A-nucleon system under gravitational forces when A is large? Perhaps there is no equilibrium state when A is large: this is the proposal of Oppenheimer and Snyder," pp. 139–40. "If we are to reject as physically unreasonable the concept of an indefinitely large number of nucleons in equilibrium in a finite volume of space, it seems necessary to conclude that the nucleons above a critical number convert themselves to a form of energy that can escape from the system: radiation . . . [C]onditions of superdensity would seem to be particularly favorable for altering the number of nucleons in the universe" (a proposed 1958 rejection of complete gravitational collapse in favor of an as then undiscovered mechanism of radiation; Y. B. Zel'dovich [Zh. eksp. teor. Fiz. **42,** 641 (1962)], English translation ["The Collapse of a Small Mass in the General Theory of Relativity," Soviet Physics JETP **15,** 446 (1962)] notes, "By prescribing a sufficiently large density we can obtain for any given number N of particles a configuration with mass as close to zero as we please, and clearly less than the mass of the static solution. Such a solution obviously cannot go over into the state of equilibrium (into the static solution), and consequently can only contract without limit." J. A. Wheeler, "Geometrodynamics and the Issue of the Final State," *Relativity, Groups and Topology,* edited by C. DeWitt and B. DeWitt, pp. 315–520 (Gordon and Breach, New York, 1965); in particular, "Thus there exists a second crushing point, the 'Landau-Oppenheimer-Volkoff crushing point,' with central density $\sim 10^{16}$ g/cm^3, and mass $\sim 0.7\ M_\odot$. One cannot add matter to the system without initiating collapse. . . . Cannot one save the day by assuming that matter becomes incompressible at a sufficiently high density? No!" The relativistic equation of hydrostatic equilibrium "has the remarkable feature that it provides a mechanism for multiplying pressure . . . ('divergent chain reaction')," p. 321; "No matter how small the number of nucleons that one starts with, in principle they can be pressed from outside with enough pressure to initiate collapse," p. 325; gravitational collapse of a toroidal bundle of magnetic lines of force, pp. 445–49; Schwarzschild and geon geometry as unstable with respect to gravitational collapse, p. 500–501; and a collapsing "cloud of matter may be of dust and so dilute that its density is 10^{-3} g/cm^3 or less at the moment when its radius decreases to the order of the Schwarzschild value. Therefore no details of any equation of state can save it from gravitational collapse," pp. 502–503. Also, "It is difficult to escape the conclusion that the creation or destruction of matter goes on in regime IV [where quantum effects dominate]. At issue here is not the familiar process of a positive electron annihilating a negative electron, or an antiproton disappearing by union with a proton. Instead, one is concerned about a process in which ordinary matter — composed of protons, neutrons and electrons — is crushed out of existence, or brought into being, by a mechanism intimately connected with gravitation and with the curvature of space. . . . [P]rocesses of baryon creation or destruction would seem unavoidable," pp. 513–16, discussion of relation between the quasi-stellar objects discovered in 1963 and gravitational collapse. "[No] escape is now known . . . from a new physical process. In this process baryons disappear . . . [G]ravitational collapse must occur for a subcritical mass as well as for a supercritical mass [via] a quantum mechanical tunneling process. . . . [A]ll matter must manifest, however weakly, a new form of radioactivity, in which baryon number changes." [B. K. Harrison et al. *Gravitation Theory and Gravitational Collapse* (University of Chicago Press, Chicago, Illinois, 1965), p. vii, viii.] Name "black hole": J. A. Wheeler, "Our Universe: The Known and the Unknown," address before the American Association for the Advancement of Science, New York, 29 Dec. 1967 in Am. Scholar **37,** 248–74 (1968) and Am. Scientist **56,** 1–20 (Spring 1968). See also R. Ruffini and J. A. Wheeler, "Introducing the Black Hole," Phys. Today **24,** 30–36 (1971), and "The Black Hole," in *Astrophysics and Gravitation: Proceedings of the Sixteenth Conference on Physics at the University of Brussels, September 1973* (Editions de l'Université de Bruxelles, 1040 Bruxelles, Belgium, 1974), pp. 279–316. In black hole physics the laws of conservation of particle number are transcended; see J. A. Wheeler, "Transcending the Law of Conservation of Leptons," in *Atti del Convegno Internazionale sul Tema: The Astrophysical Aspects of the Weak Interactions; Quaderno N.157* (Accademia Nazionale dei Lincei, Roma, 1971), pp. 133–64. Gravitational collapse implies that

"there is no law except the law that there is no law" [J. A. Wheeler, "From Relativity to Mutability," in *The Physicist's Conception of Nature,* edited by J. Mehra (Reidel, Dordrecht, Holland, 1973), pp. 202–47].

Proof that gravitational collapse is inescapable under assumptions more and more elementary: A. Avez, "Propriétés globales des espace-temps périodiques clos," Acad. des Sci., Paris, Comptes Rend. **250,** 3583–87 (1960); R. Penrose, "Gravitational Collapse and Spacetime Singularities," Phys. Rev. Lett. **14,** 57–59 (1965); S. W. Hawking, "The occurrence of singularities in cosmology," Proc. Roy. Soc. London **A294,** 511–21 (1966); R. P. Geroch, "What is a Singularity in General Relativity?" Ann. Phys. (U.S.A.) **48,** 526–40 (1960); S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-time* (Cambridge University Press, Cambridge, England, 1973); J. E. Marsden and F. J. Tipler, "Maximal Hypersurfaces and Foliations of Constant Mean Curvature in General Relativity," preprint, Mathematics Department, University of California at Berkeley, 1979.

Theorems on the uniqueness of the geometry around a black hole: B. Carter, "An Axisymmetric Black Hole Has Only Two Degrees of Freedom," Phys. Rev. Lett. **26,** 331–36 (1970); papers by B. Carter and others in *Black Holes,* Proceedings of 1972 session of École d'été de physique théorique, edited by C. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1973).

Quantum aspects of the black hole: "wormholes" continually being produced and annihilated at the Planck scale of distances, giving rise to a "foam-like structure" of space [J. A. Wheeler, "On the Nature of Quantum Geometrodynamics," Ann. Phys. **2,** 604–14 (1957)]; calculation of same by the method of sum over histories [G. W. Gibbons and S. W. Hawking, "Action Integrals and Partition Functions in Quantum Gravity," Phys. Rev. **D15,** 2752–57 (1977)]; surface area and surface gravity of black hole not merely analogous to, but identical with, entropy and temperature [J. Bekenstein, "Black Holes and Entropy," Phys. Rev. **D7,** 2333–46 (1973)]; thermal radiation associated with this effect calculated by S. W. Hawking, ["Particle Creation by Black Holes," Comm. Math. Phys. **43,** 199–220 (1975)].

2. A. Friedmann, "Über die Krummung des Raumes," Z. f. Phys. **10,** 337–86 (1922); E. P. Hubble, "A Relation between Distance and Radial Velocity among Extragalactic Nebulae," Proc. Nat. Acad. Sci. U.S. **15,** 169–73 (1929); R. A. Alpher, H. A. Bethe, and G. Gamow, "The Origin of Chemical Elements," Phys. Rev. **L 73,** 803–804 (1948); R. H. Dicke et al., "Cosmic-black-body Radiation," Astrophys. J. **142,** 414–19 (1965); A. A. Penzias and R. W. Wilson, "A Measurement of Excess Antenna Temperature at 4080 *Mc/s,*" Astrophys. J. **142,** 419–21 (1965).

3. "Thus we may present the following arguments against the conception of a space-infinite and for the conception of a space-bounded universe [(1) simplicity (2) Machian] [A. Einstein, *The Meaning of Relativity,* 3rd ed. (Princeton University Press, Princeton, N. J., 1950), pp. 107–108]; "In my opinion the general theory of relativity can only solve this problem [of inertia] satisfactorily if it regards the world as spatially self-enclosed," [A. Einstein, *Essays in Science* (Philosophical Library, New York, 1934), translated from *Mein Weltbilde,* (Querido, Amsterdam, 1933)]; J. A. Wheeler, "Conference Summary: More Results Than Ever in Gravitation Physics and Relativity," in *General Relativity and Gravitation,* edited by G. Shaviv and J. Rosen (Wiley, New York, 1975), pp. 299–344, especially status of the "mystery of the missing mass," lens effect and its difficulties as a way to check on closure, difficulties with alternatives to closure [pp. 320–24].

4. Mathematical investigation from which one concludes there are not any periodic closed model universes — an indirect argument that a closed universe necessarily collapses [A. Avez, op. cit. in n. 1]; S. W. Hawking and R. Penrose, "The Singularities of Gravitational Collapse and Cosmology," Proc. Roy. Soc. London **A314,** 529–48 (1969); all "W model universes" are closed and have an upper limit to the time from big bang to big crunch [J. E. Marsden and F. J. Tipler, op. cit. in n. 1].

5. J. R. Gott, III, et al. "Will the Universe Expand Forever?" Sci. Am. **234,** 62–79 (March 1976), esp. p. 69 for term "big crunch"; see also J. R. Gott, III, et al. "An Unbound Universe?" Astrophys. J. **194,** 543–53 (1974). Principle that the universe must have a way to come into being and to fade out of existence proposed, not as a deduction from cosmology, but as a requirement for cosmology [C. M. Patton and J. A. Wheeler, "Is Physics Legislated by Cosmology?" in *Quantum Gravity: An Oxford Symposium,* edited by C. J. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1975)].

6. N. Bohr, *Atomic Theory and the Description of Nature* (Cambridge University Press, Cambridge, England), 1934; *Atomic Physics and Human Knowledge* (Wiley, New York, 1958); and *Essays 1958–1962 on Atomic Physics and Human Knowledge* (Wiley, New York, 1963).

7. N. Bohr, "Discussion with Einstein on Epistemological Problems in Atomic Physics," in *Albert Einstein: Philosopher-Scientist,* edited by P. A. Schilpp (Library of Living Philosophers, Evanston, Ill., 1949), pp. 201–41, "registration," p. 238; "irreversible amplification," p. 88, and "closed by irreversible amplification," p. 73, in Bohr, *Atomic Physics and Human Knowledge,* op. cit. in n. 6.

8. J. A. Wheeler, "Genesis and Observership," *Foundational Problems in the Special Sciences,* edited by R. E. Butts and K. J. Hintikka (Reidel, Dordrecht, 1977), pp. 3–33, esp. "direct involvement of observership in genesis," p. 26;

J. A. Wheeler, *Frontiers of Time* [North-Holland, Amsterdam (for the Società Italiana di Fisica, Bologna), 1979]; also appears from the same two houses in *Problems in the Foundations of Physics,* edited by G. Toraldo di Francia (1979), "billions upon billions of acts of observer-participancy," p. 5ff; add 394 for pagination in latter.

9. Come into being, fade out of existence: Patton and Wheeler, op. cit. in n. 5; Wheeler, op. cit. in n. 8; in gravitational collapse the framework falls down for everything one ever called a law [J. A. Wheeler, "From Relativity to Mutability," in *The Physicist's Conception of Nature,* edited by J. Mehra, Reidel, Dordrecht, 1973), pp. 202–47]; to be contrasted with "Beyond the End of Time," in *Gravitation,* by C. W. Misner, K. S. Thorne, and J. A. Wheeler (Freeman, San Francisco, 1973), ch. 44, pp. 1196–1217, where gravitational collapse was envisaged as precipitating a reprocessing of the universe, except for a penultimate paragraph foreshadowing the concept of genesis through observer-participancy.

10. "Billions," op. cit. in n. 8, n. 9.

11. Challenge to derive the quantum principle from the requirements that the universe should have a way to come into being: Patton and Wheeler, op. cit. in n. 5, p. 564; Wheeler, "Genesis and Observership," op. cit. in n. 8, p. 29; Wheeler, *Frontiers of Time,* op. cit. in n. 8, p. 8.

12. Derivation from symmetry principle hides the machinery underlying physical law: Wheeler, "Genesis and Observership," op. cit. in n. 8, pp. 15–16; Wheeler, *Frontiers of Time,* op. cit. in n. 8, sec. 4

13. J. C. Maxwell, "A Dynamical Theory of the Electromagnetic Field," Trans. Roy. Soc. London **155,** 459 ff. (1865), and *A Treatise on Electricity and Magnetism,* 1873; 3rd ed. (Clarendon, Oxford, 1892); A. Einstein, "Die Feldgleichungen der Gravitation," Preuss. Akad. Wiss. Berlin, Sitzber., 844–47 (1915); C. N. Yang and R. L. Mills, "Conservation of Isotopic Spin and Isotopic Gauge Invariance," Phys. Rev. **96,** 191–95 (1954).

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15. A. E. H. Love, *The Mathematical Theory of Elasticity,* (Cambridge University Press, London, 1892); 4th ed. (Dover, New York, 1944).

16. J. D. Jackson, *Classical Electrodynamics,* 2nd ed. (Wiley, New York, 1975); L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1960).

17. S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972); Misner, Thorne, and Wheeler, op. cit. in n. 9.

18. L. D. Faddeev and A. A. Slavnov, *Gauge Fields: Introduction to Quantum Theory,* translated by D. B. Pontecorvo (Benjamin/Cummings, Advanced Book Program, Reading, Mass., 1980).

19. The discussion here comes from Wheeler, "Genesis and Observership," op. cit. in n. 8, p. 16, and Misner, Thorne, and Wheeler, op. cit. in n. 9, pp. 1206–1208.

20. S. Hojman, K. Kuchař, and C. Teitelboim, "New Approach to General Relativity," Nature Phys. Sci. **245,** 97–98 (1973); C. Teitelboim, "How Commutators of Constraints Reflect Spacetime Structure," Ann. Phys. **79,** 542–57 (1973), and "The Hamiltonian Structure of Spacetime," doctoral dissertation, unpublished, Princeton University, 1973, available from University Microfilms, Ann Arbor, Mich.; K. Kuchař, "Canonical Quantization of Gravity," in *Relativity, Astrophysics and Cosmology,* edited by W. Israel (Reidel, Dordrecht, Holland, 1973), pp. 238–88, and "Geometrodynamics regained: a Lagrangian approach," J. Math. Phys. **15,** 708–15 (1974); C. Teitelboim, "Surface Deformations, Spacetime Structure and Gauge Invariance" in *Relativity, Fields, Strings and Gravity: Proceedings of the Second Latin American Symposium on Relativity and Gravitation SILARG II held in Caracas, December 1975,* edited by C. Aragone (Universidad Simon Bolivar, Caracas, 1976); S. A. Hojman, K. Kuchař, and C. Teitelboim, "Geometrodynamics Regained," Ann. Phys. **76,** 88–135 (1976).

21. Teitelboim, op. cit. in n. 20.

22. A. Einstein, "Autobiographical Notes," pp. 2–95 in *Albert Einstein: Philosopher-Scientist,* op. cit. in n. 7, where Einstein discusses his attempts to find a unified field theory [pp. 89–95]; M. A. Tonnelat, *La Théorie du Champ Unifié d'Einstein et Quelque-uns de ses Developpments* (Gauthier-Villars, Paris, 1955), and *Les Théories Unitaires de l'Électromagnetisme et de la Gravitation* (Gauthier-Villars, Paris, 1965).

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Gauge theory of gravitation step by step recognized as equivalent to the metric-plus-torsion theory of gravitation originally urged by Élie Cartan on geometrical grounds: "Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion," Acad. Sci., Paris, Compt. Rend. **174,** 593–95 (1922), for more on the development of which see *Élie Cartan–Albert Einstein Letters on Absolute Parallelism 1929–1932,* edited by R. Debever (Princeton University Press, Princeton, N. J., 1979). The history and bibliography of this growth in understanding to what is today known as the Einstein-Cartan-Sciama-Kibble (or U_4) theory of gravitation, derivable in its spin-torsion parts from a Lorentz gauge, is recounted by A. Trautman, "On the Einstein-Cartan Equations," Bull. Acad. Polon. Sci., Ser. Sci. Mat. Ast. et Phys. **20,** 185–90, and in "Theory of Gravitation," in *The Physicist's Conception of Nature,* edited by J. Mehra (Reidel, Dordrecht, Holland, 1973), pp. 179–98; also see F. W. Hehl et al., "General Relativity with Spin and Torsion: Foundations and Prospects," Rev. Mod. Phys. **48,** 393–416 (1976), who clarify the gauge associated with translations; for a still more recent perspective, see Y. Ne'eman, "Gravity Is the Gauge Theory of the Parallel-Transport Modification of the Poincaré Group," in *Differential Geometrical Methods in Mathematical Physics II,* edited by K. Bleuler, H. R. Petry, and A. Reetz (Springer, New York, 1978), pp. 189–215.

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25. Misner, Thorne, and Wheeler, op. cit. in n. 9, p. 1207, Fig. 44.2.

26. "Last Lecture of Albert Einstein," as recorded by J. A. Wheeler, in *Albert Einstein: His Influence on Physics, Philosophy and Politics,* edited by P. C. Aichelburg and R. U. Sexl (Vieweg, Braunschweig/Wiesbaden, 1979), pp. 207–11; Einstein, "Autobiographical Notes," pp. 2–95 in *Albert Einstein, Philosopher-Scientist,* op. cit. in n. 7; A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen," Jahrb. Radioakt. **4,** 411–62 (1908); and A. Einstein, letter to Ernst Mach, 26 June 1913, reproduced as Fig. 21.5 in Misner, Thorne, and Wheeler, op. cit. in n. 9, pp. 544–545; for this and other correspondence between Einstein and Mach in the original German, see "Die Beziehungen zwischen Einstein und Mach, dokumentarisch dargestellt," in F. Herneck, *Einstein und sein Weltbild,* (*Der Morgen,* Berlin DDR, 1979), pp. 109–15.

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28. "Mach's principle" interpreted as the requirement that the universe be closed, thus to provide a boundary condition to separate allowable solutions of Einstein's equation from physically inadmissible solutions: J. A. Wheeler in Onzième Conseil de Physique Solvay, op. cit. in n. 1, pp. 49–51; H. Hönl, in *Physikertagung Wien,* edited by E. Brüche (Physik Verlag, Mosbach/Baden, 1962); J. A. Wheeler, "Geometrodynamics and the Issue of the Final State," in DeWitt and DeWitt, eds., op. cit. in n. 1, pp. 363–399, 411–425, esp. p. 425, "every so-called 'asymptotically flat' geometry taken up hereafter will be considered to be a part of a closed universe"; J. A. Wheeler, "Mach's Principle as Boundary Condition for Einstein's Equations," pp. 303–349 in *Gravitation and Relativity,* edited by H.-Y. Chiu and W. F. Hoffmann (W. A. Benjamin, New York, 1964). "Mach's Principle and the Origin of Inertia," sec. 21.12, pp. 543–49 in Misner, Thorne, and Wheeler, op. cit. in n. 9. J. Isenberg and J. A. Wheeler, "Inertia Here Is Fixed by Mass-Energy There in Every W Model Universe," in *Relativity, Quanta, and Cosmology in the Development of the Scientific Thought of Albert Einstein* (in English: Johnson Reprint, New York, 1979), pp. 267–93, and in *Astrofisica e Cosmologia Gravitazione, Quanti Relatività negli Sviluppi del Pensiero Scientifico di Albert Einstein* (in Italian: Giunti Barbera, Firenze, 1979), pp. 1099–1136, both edited by M. Pantaleo and F. De Finis.

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33. Marsden and Tipler, op. cit. in n. 1.

34. Op. cit. in n. 32.

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52. Carter, "Black Hole Equilibrium States," in *Black Holes,* op. cit. in n. 1, esp. Sec. 12, pp. 205–209; also, see the references cited in the "No hair" paragraphs of n. 40.

53. Lepton number of black hole not measurable: see references in third "No hair" paragraph of n. 40 and, also, the references cited under "Theorems on the uniqueness of the geometry around a black hole" in n.1. See the latter as well as the fourth "No hair" paragraph of n. 40 on the subject of immeasurability of baryon number of black holes.

54. J. A. Wheeler, "Transcending the Law of Conservation of Leptons," op. cit. in n. 1., and "From Relativity to Mutability," op. cit. in n. 9, esp. p. 202: "Baryon number and lepton number are well defined quantities for a normal star; but when this star collapses to a black hole, the well established laws of conservation of particle number lose all applicability."

55. The staircase is described in Wheeler, "From Relativity to Mutability," op. cit. in n. 9, p. 241.

56. The method of determining density of an object by weighing it, first in air, then under water is reputed to have been found in answer ("*Eureka*" — I have found it) to the question of Hieron, king of Syracuse, whether his "gold" crown did not contain an admixture of silver. [Archimedes of Syracuse (287–212 B.C.), *Peri ochoumenon* (On Floating Bodies)].

57. Fixity of density transcended: J. A. Morgan, "The Equation of State of Platinum to 680 GPa," High Temperature-High Pressure **6,** 195–201 (1974), which reports density approximately doubled via use of gun; H. K. Mao et al., "Specific Measurements of Cu, Mo, Pd, and Ag and Calibration of Ruby R_1 Fluorescence Pressure Gauge from 0.06 to 1 *M* Bar," J. Appl. Phys. **49,** 3276–83 (1978), for use of nuclear explosion to go to extreme density; C. E. Ragan, III, M. G. Silbert, and B. C. Diven, "Shock Compression of Molybdenum to 2.0 TPa by Means of a Nuclear Explosion," J. Appl. Phys. **48,** 2860–70 (1977), for achievement of pressures in the 30 megabar range and determination of increase of density by a *measured* factor of about 3; L. V. Al'tshuler et al., "Shock Adiabats for Ultrahigh Pressures," Soviet Phys. JETP **45,** 167–71 (1977), which reports the extreme of published pressures, 50 megabars, where *any* measurements have been made. Thanks are expressed here to William Deal for guidance to this literature.

58. The first table of atomic weights, and idea that chemical combination takes place between atoms of different weights, law of combination in multiple proportions, and atomic theory, are included in outline form by J. Dalton's consent (6 Sept. 1803) in T. Thomson, *System of Chemistry,* 3rd ed. (1807) and in the first volume of Dalton's own *New System of Chemical Philosophy* (S. Russell, Manchester, England, 1808). Under identical physical conditions equal volumes of gas contain the same number of molecules: A. Avogadro, "Essai d'une manière de déterminer les masses relatives des molécules élémentaires des corps, et des proportions selon lesquelles elles entrent dans les combinaisions," J. de Phys. (1811). Simplest molecular formulas for organic compounds and systematic use of "rational" chemical formulas: C. F. Gerhardt, *Introduction a l'étude de la chimie par le systéme unitaire* (1848). Distinction between molecular and atomic weights and deduction of atomic weights from vapor density or specific heat: S. Cannizzaro, *Sunto di un corso di filosofia chemica* (1858). Atom of each elementary substance can combine only with a certain limited number of the atoms of other elements, foundation of theory of valency: E. Frankland, "On the Dependence of the Chemical Properties of Compounds upon the Electrical Characters of Their Constituents," Roy. Inst. Proc. **1,** 451–54 (1852). Carbon tetravalence with some of the affinities of the generic carbon atom bound by atoms of other kinds, some bound by other carbon atoms: F. A. Kekulé, "Über die Constitution und die Metamorphosen der chemischen Verbindungen und über die Natur der Kohlenstoffs," Liebig, Annal. **106,** 129–59 (1858); result also obtained independently and reported by A. S. Cooper, Acad. des Sci., Paris, Comptes Rend. (1858).

59. Carbon was found to have valence 3 in the first free radical discovered, triphenyl methyl [M. Gomberg, "An Instance of Trivalent Carbon," J. Am. Chem. Soc. **22,** 757–71 (1900)], which opened way to the realization that valence can be transcended in a few cases at room temperature and for every atom at a sufficiently high temperature.

60. Mass number and isotopes: F. Soddy, "Radio Elements and the Periodic Law," Chem. News **107,** 97–99 (1913), elucidates the chemical identity discovered by B. B. Boltwood in 1906 and by H. N. McCoy and W. H. Ross in 1907 between radioactively distinct ionium, radiothorium, and thorium, and discovered in other cases by Soddy himself and other workers in the subject.

Charge number: H. G. J. Moseley, "High-Frequency Spectra of the Elements," Phil. Mag. **26,** 1024–34 (1913), and same title, II, Phil. Mag. **27,** 703–13 (1914), reported determination of number of elementary charges on the nucleus, and the place of the corresponding element in the periodic table, by measuring the wavelengths of the strongest lines in the X-ray spectrum of the atom and applying Bohr's theory of the atom.

61. Discovery of natural radioactivity: H. A. Becquerel, "Sur les radiations émises par phosphorescence," Acad. Sci., Paris, Compt. Rend. **122,** 420–21 (1896). First artificial transmutation: E. Rutherford, "Collision of α Particles with Light Atoms. IV. An Anomalous Effect in Nitrogen," Phil. Mag. **37,** 581–87 (1919).

J. D. Cockroft and E. T. S. Walton, "Experiments with High-Velocity Positive Ions. Part II. Disintegration of Elements by High-Velocity Protons," Proc. Roy. Soc. London **137,** 229–42 (1932), and M. A. Tuve and L. R. Hafstad, "The Emission of Disintegration-Particles from Targets Bombarded by Protons and by Deuterium Ions at 1200 Kilovolts," [Letter] Phys. Rev. **45,** 651–53 (1934).

62. Baryon conservation: F. Reines, C. L. Cowan, Jr., and M. Goldhaber, "Conservation of the Number of Nucleons," Phys. Rev. **96,** 1157–58 (1954); F. Reines, C. L. Cowan, Jr., and H. W. Kruse, "Conservation of the Number of Nucleons," Phys. Rev. **109,** 609–10 (1957); G. N. Flerov et al., "Spontaneous Fission of Th^{232} and the Stability of Nucleons," Sov. Physics Doklady **3,** 79–80 (1958); G. Feinberg and M. Goldhaber, "Microscopic Tests of Symmetry Principles," U.S. Nat. Acad. Sci., Proc. **45,** 1301–12 (1959); G. Feinberg and M. Goldhaber, "Experimental Tests of Symmetry Principles," Science **129,** 1285 (1959); H. S. Gurr et al., "Experimental Test of Baryon Conservation," Phys. Rev. **158,** 1321–30 (1967). J. Learned, F. Reines, and A. Soni, "Limits on Nonconservation of Baryon Number," Phys. Rev. Lett. **43,** 907–909 (1979), report nucleon lifetime greater than about 10^{30} years at 90 percent confidence level.

Baryon conservation seriously questioned: S. Weinberg, "Cosmological Production of Baryons," Phys. Rev. Lett. **42,** 850–53 (1979).

Lepton conservation: M. Goldhaber, "Weak Interactions: Leptonic Modes — Experiment," in *Proceedings of the 1958 Annual International Conference on High Energy Physics at CERN,* pp. 233–50; V. R. Lazarenko, "Double Beta Decay and the Properties of the Neutrino," Usp. Fiz. Nauk **90,** 601–22 (1961) [English translation in Sov. Phys. Uspekhi **9,** 860–73 (1967)]; B. Pontecorvo, "Neutrino Experiments and the Problem of Conservation of Leptons," Sov. Phys. JETP **26,** 984–88 (1968); K. Boher et al., "Untersuchung über die Erhaltung der μ-Leptonenzahl," Helv. Phys. Acta **43,** 111–32 (1970).

R. I. Steinberg et al., "Experimental Test of Charge Conservation and Stability of the Electron," Phys. Rev. **D12,** 2582–86 (1975), reported mean life of electron against decay into nonionizing particles greater than 5.3×10^{21} years.

63. Transcended, see n. 54.

64. Energy not defined in a closed universe: "There is no such quantity as total energy, for example, in a closed universe; there the integrated conservation laws reduce to the trivial identity, zero equals zero" [J. Weber and J. A. Wheeler, "Reality of the Cylindrical Gravitational Waves of Einstein and Rosen," Rev. Mod. Phys. **29,** 509–15 (1957), p. 512].

The key point for defining mass is the existence of a region where the geometry goes over asymptotically to the Schwarzschild character. When there is no such region, then it is not known how to give an unambiguous

meaning to the term "mass." This is particularly the case for a closed universe. There is no asymptotically flat region in which to measure the pull of the system by the bending of light or by the periods of planetary orbits and their precession. If there is no experimental way to *measure* mass for a closed universe, and no theoretical way to *define* mass, this is happily compatible with the circumstance that no one knows any *use* for the concept of the mass of a closed universe. Therefore it would appear appropriate to reject this phrase as being physically meaningless as well as being subject to misunderstanding.

[J. A. Wheeler, "Geometrodynamics and the Issue of the Final State," op. cit. in n. 1, pp. 434–35.]

Misner, Thorne, and Wheeler, op. cit. in n. 9, pp. 457–58.

65. Misner, Thorne, and Wheeler, op. cit. in n. 9, p. 450: (mass of center of attraction)[1] = (2π/orbital period)^2 (semi-major axis of ellipse)^3.

66. "The dimensions of the collapsing system in a finite proper time are driven down to indefinitely small values. The phenomenon is not limited to the space occupied by matter. It occurs also in the space surrounding the matter." [Wheeler, "Superspace and the Nature of Quantum Geometrodynamics," op. cit. in n. 48, p. 254].

67. No before, no after: "[A]t small distances and in the final phase of collapse" 'spacetime' is nonexistent, 'events' and the 'time ordering of events' are without meaning, and the question 'what happens after the final phase of gravitational collapse' is a mistaken way of speaking" (Ibid., p. 254]. "*[T]here is no such thing as spacetime in the real world of quantum physics* . . . complementarity forbids. [S]uperspace leaves us space but not spacetime and therefore not time. With time gone the very ideas of 'before' and 'after' also lose their meaning." [Wheeler, "From Relativity to Mutability," op. cit. in n. 1, p. 227.] "Nowhere more clearly than in the ending of spacetime are we warned that time is not an ultimate category in the description of nature" [Wheeler, *Frontiers of Time,* op. cit. in n. 8, p. 6]. " 'Before' and 'after' don't rule everywhere, as witness quantum fluctuations in the geometry of space at the scale of the Planck distance. Therefore 'before' and 'after' cannot legalistically rule anywhere. Even at the classical level, Einstein's standard closed-space cosmology denies all meaning to 'before the big bang' and 'after the big crunch.' Time cannot be an ultimate category in the description of nature. We cannot expect to understand genesis until we rise to an outlook that transcends time" [Ibid., p. 20]. "Not the slightest warrant does Einstein's equation give for thinking there can be any such thing as a 'before' before the big bang or an 'after' after the big crunch or after the collapse of a star to a black hole. These three processes mark three 'gates of time' " [Ibid., p. 75]. "Little escape is evident from these words: there is no 'before' before the big bang and no 'after' after the big crunch. Time ends with spacetime. The universe does not endure from everlasting to everlasting. Everything came from 'nothing' " [Ibid., p. 85].

68. "There never was a law of physics that did not require space and time for its statement. With collapse the framework falls down for everything one ever called a law. The laws of physics were not installed in advance by a Swiss watchmaker, nor can they endure from everlasting to everlasting. They must have come into being. They could not always have been accurate. They are derivative and superficial, not primary and revelatory." [Wheeler, *Frontiers of Time,* op. cit. in n. 8, p. 20]. This position, based on the conclusion that the category of "time" is itself not primordial, but secondary, derivative and approximate, differs in that respect from the position of Peirce, who tacitly accepted the primordiality of time: "May they [these forces of nature] not have naturally grown up?" *The philosophy of [Charles S.] Peirce: Selected Writings,* edited by J. Buchler (Routledge and Kegan Paul, London, 1940); available also as a paperback reprint under the title *Philosophical Writings of Peirce* (Dover, New York, 1955). The quotation is from p. 358 in the paperback; see further on that page; also see pp. 335–37 and p. 353.

69. Mutability: see the references in n. 9.

70. Crack in crystal: A. Joffe, "On the Cause of the Low Value of Strength," pp. 72–76, and "On the Mechanism of Brittle Rupture," pp. 77–80 in *International Conference on Physics, London 1934. A Joint Conference Organized by the International Union of Pure and Applied Physics and the Physical Society. Papers and Discussions in Two Volumes. Vol. 2. The Solid State of Matter* (Cambridge University Press and The Physical Society, Cambridge, England, 1935).

71. Collapse inevitable: see paragraph entitled "Proof that gravitational collapse is inescapable . . ." in n. 1. Theorem on inescapability of singularity: Hawking and Penrose, op. cit. in n. 4. Consideration of details of approach to singularity in the generic case: see the references listed in n. 35.

72. Quantum fluctuations in topology and geometry predicted and estimated at the Planck scale of distances: Wheeler, "On the Nature of Quantum Geometrodynamics," op. cit. in n. 1; Wheeler, "Superspace and the Nature of Quantum Geometrodynamics," op. cit. in n. 48; and Misner, Thorne, and Wheeler, op. cit. in n. 9, pp. 1190–94.

73. These fluctuations calculated: Gibbons and Hawking, op. cit. in n. 1. Further calculations: M. J. Perry, S. W. Hawking, and G. W. Gibbons, "Path Integrals and the Indefiniteness of the Gravitational Action," Nucl. Phys. **B 138,** 141–50 (1978).

74. Three dimensions of space (as other laws of nature) as a "precondition for the possibility of phenomena" [I. Kant, *Critique of Pure Reason,* translated by F. M. Muller (Anchor, Garden City, N. Y., 1966), p. 24]; see, however, A. Grünbaum, *Philosophical Problems of Space and Time* (Knopf, New York, 1963), ch. 11.

Space as analyzed, not metrically, but via analysis situs (topology in the large) shows itself to be three-dimensional: H. Poincaré, *Mathematics and Science: Last Essays,* translated by J. W. Bolduc (Dover, New York, 1963), ch. 3, pp. 27–28.

Only in three-dimensional space is sense made by the laws of gravitation and planetary motion, the duality of 1) translation and rotation, of 2) force and pair of forces, and of 3) electric field and magnetic field: P. A. Ehrenfest, "In What Way Does It Become Manifest in the Fundamental Laws of Physics That Space Has Three Dimensions?", Proc. Amsterdam Acad. **20,** 200–209 (1917).

Only in a space with an odd number of dimensions "will darkness follow the extinction of a candle"; gauge invariance holds only for three dimensions; other considerations and reference to others who have asked, "Why three dimensions?": H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton University Press, Princeton, N. J., 1949; paperback reprint, Atheneum, New York, 1963), p. 36.

In three-dimensions Einstein's field equation requires space to be flat, thereby making geodesics be straight lines; but gravitation nevertheless shows itself in the global equivalent of curvature produced by conelike singularities. A. Staruszkiewicz, "Gravitation Theory in Three-Dimensional Space," Acta Phys. Polonica **24,** 735–40 (1963).

75. Pregeometry:

> [T]he number of dimensions should not be assumed in advance; it should be *derived* to be four. . . . [A]ny derivation of the 4-dimensionality of spacetime can hardly start with the idea of dimensionality. . . . [O]ne can imagine probability amplitudes for the points in a Borel set to be assembled into manifolds with this, that and the other dimensionality. . . . [D]efine an action principle over a collection of points of undefined dimensionality. One might also wish to accept to begin with the idea of a distance, or edge length, associated with a pair of these points, even though this idea is a very great leap, and one that one can conceive of later supplying with a foundation of its own. . . . [T]here must be a connection in the appropriate action principle between *every* point and every other point. . . . Try therefore a propagator of the form
>
> $$\sum_{\text{diagram}} \exp i S_{\text{diagram}}$$
>
> Here the sum goes over all conceivable ways of connecting the given number of vertices up into nearest neighbors, whatever the dimensionality or lack of dimensionality of these "wiring diagrams." How this phase depends upon the topology of the diagram is to be deduced — in whole or in part — from natural combinatorial principles.

[J. A. Wheeler, "Geometrodynamics and the Issue of the Final State," pp. 315–520 in *Relativity, Groups and Topology,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964), pp. 495–99.]

Pregeometry, including discussion of "pregeometry as the calculus of propositions": Misner, Thorne, and Wheeler, op. cit. in n. 9, pp. 1203–12. "[The] concept of 'ideal mathematical geometry' is too finalistic to be final and must give way to a deeper concept of structure [- . . .] 'pregeometry' " [C. M. Patton and J. A. Wheeler, "Is Physics Legislated by Cosmogony?" pp. 538–605 in *Quantum Gravity: An Oxford Symposium,* edited by C. J. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1975), p. 573]. "[W]e have to give up the idea that pregeometry is the calculus of propositions, or the statistics of propositions, or the mathematical machinery of any formal axiomatic system" [Report on the Search for Pregeometry, February-March-April 1974, op. cit., Appendix B, pp. 589–91].

Pregeometry viewed as the statistics of billions upon billions of acts of observer-participancy: J. A. Wheeler, "Pregeometry: Motivations and Prospects," in *Quantum Theory and Gravitation,* edited by A. R. Marlow (Academic Press, New York, 1980).

76. Thermodynamics rests upon the random motions of billions upon billions of molecules: for key selections from the original literature see S. G. Brush, *Kinetic Theory,* Vol. 1, *The Nature of Gases and of Heat,* and Vol. 2, *Irreversible Processes* (Pergamon, Oxford, 1965, 1966).

77. C. Darwin, *Origin of Species by Means of Natural Selection* (London, 1859); T. Dobzhansky, *Genetics and the Origin of Species* (New York, 1937); M. Eigen, "The Origin of Biological Information," *The Physicists' Conception of Nature,* edited by J. Mehra (Reidel, Dordrecht, 1973), pp. 594–632; M. Eigen and R. Winkler, *Das Spiel: Naturgesetze steuern den Zufall* (Piper, Munich, 1975).

78 Wheeler, *Frontiers of Time,* op. cit. in n. 8, p. 6.

79. "The statistical character of the present theory would then have to be a necessary consequence of the incompleteness of the description of the systems in quantum mechanics, and there would no longer exist any ground for the supposition that a future basis of physics must be based upon physics" [Einstein, "Autobiographical Notes," in *Albert Einstein, Philosopher-Scientist,* op. cit. in n. 7, p. 87]. "[I]t [quantum mechanics] seems to make the world quite nebulous unless somebody, like a mouse, is looking at it" ["Last Lecture of Albert Einstein," op. cit. in n. 26].

80. A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," Ann. d. Phys. **17,** 132–48 (1905).

81. A. Einstein, "Strahlungs-emission und -absorption nach der Quantentheorie," Deutsche physikalische Gesellschaft, Verhandlungen **18,** 318–23 (1916); "Quantentheorie der Strahlung," Physikalische Gesellschaft, Zürich, Mitteilungen **16,** 47–62 (1916).

82. A. Einstein, "Die Feldgleichungen der Gravitation," op. cit. in n. 13.

83. "Ich habe keinen besseren Ausdruck als den Ausdruck [religiös] fur dieses Vertrauen [von Spinoza] in die vernünftige und der menschlichen Vernunft wenigstens einigermassen zugängliche Beschaffenheit der Realität" [A. Einstein, letter to Maurice Solovine, 1 January 1951]. "Wo stünden wir wenn Leute wie Giordano Bruno, Spinoza, Voltaire und Humboldt so gedacht und so gehandelt hätte?" [A. Einstein, letter to Max von Laue, May 1933, regarding the capitulation of intellectuals to the advent of gangsterism]. The latter two letters are reproduced in F. Herneck, *Einstein und sein Weltbild* (Der Morgen, Berlin, DDR, 1979), p. 35 and p. 87. "[I]gnoramuses who use their public positions of power to tyrannize over professional intellectuals must not be accepted by intellectuals without a struggle. Spinoza followed this rule when he turned down a professorship at Heidelberg and (unlike Hegel) decided to earn his living in a way that would not force him to mortgage his freedom" [Einstein, letter to *The Reporter,* published 5 May 1955, a few days after his death]. "Just in this appears the moral side of our nature — that internal striving towards the attainment of truth, which under the name *amor intellectualis* was so often emphasized by Spinoza" [Einstein statement, Forum **83,** 373–374 (1930)].

84. Appreciation is expressed here to Professor Hans Küng for emphasizing in June 1978 at Tübingen the influence of Spinoza on Einstein's outlook.

85. "But Spinoza rejected the idea of an external Creator suddenly, and apparently capriciously, creating the world at one particular time rather than another, and creating it out of nothing" ["Spinoza," in *Encyclopaedia Brittanica,* Vol. 21 (Chicago, Ill., 1959), p. 235].

86. A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," Preuss. Akad. Wiss., Berlin, Sitzber. 142–52 (1917); Friedmann, "Über die Krummung des Raumes," op. cit. in n. 2.

87. Hubble, "A Relation between Distance and Radial Velocity among Extragalactic Nebulae," op. cit. in n. 2; Einstein as quoted by G. Gamow, *My World Line* (Viking, New York, 1970).

88. B. de Spinoza, *Ethics,* finished at The Hague 1675 and circulated privately; English translation by H. White and A. H. Stirling (1899).

89. Participatory: see n. 8.

90. Story of twenty questions: in Wheeler, *Frontiers of Time,* op. cit. in n. 8.

91. Phenomenon: Introduced by N. Bohr to meet and overcome the objections of Einstein, op. cit. in n. 7, p. 230. Preliminary account of stages in Bohr's evolution of this term: A. Petersen, *Quantum Mechanics and the Philosophical Tradition* (M.I.T. Press, Cambridge, Mass., 1968). "No phenomenon is . . . until it is . . . ," used by J. A. Wheeler in Varenna lectures of 1977; revised in printed version, *Frontiers of Time,* op. cit. in n. 8, to read "No elementary phenomenon . . ." to exclude macroscopic phenomena. The ending used there, "until it is an observed phenomenon" is revised here to "until it is a registered phenomenon" to exclude any suggestion that quantum mechanics has anything whatsoever directly to do with "consciousness" and to recall Bohr's point that an irreversible act of amplification is required to bring an elementary phenomenon to a close.

92. J. A. Wheeler, "The 'Past' and the 'Delayed-Choice' Double-Slit Experiment," in *Mathematical Foundations of Quantum Theory,* edited by A. R. Marlow (Academic, New York, 1978).

93. N. Bohr, op. cit. in n. 7; chapters on the Bohr–Einstein dialogue in M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974).

94. This "venetian blind" and other experimental arrangements, alternative to that depicted in n. 92, have been devised and generously communicated to the author by Professor L. F. Bartell of the University of Michigan at Ann Arbor.

95. "No significant change in the correlation was observed over separations of up to 2.5 m" [A. R. Wilson, J. Lowe, and D. K. Butt, "Measurement of the Relative Planes of Polarization of Annihilation Quanta as a Function of Separation Distance," J. Phys. G: Nuc. Phys. **2,** 613–24 (1976)] is a distinct but related finding.

96. N. Bohr, op. cit. in n. 7.

97. Ibid. Complementarity defined: N. Bohr, *Atomic Theory and the Description of Nature* (Cambridge University Press, Cambridge, England, 1934), p.5.

98. Wheeler, op. cit. in 92.

99. "Past" exists only in the present: Wheeler, *Frontiers of Time,* op. cit. in n. 8, p. 21; T. Segerstedt as quoted in ibid.

100. Cf. n. 91.

101. Closed by irreversible amplification: Bohr, *Atomic Physics and Human Knowledge,* op. cit. in n. 6, pp. 73, 88. Unambiguously communicable in plain language: *Essays 1958–1962 . . .,* op. cit. in n. 6, pp. 3, 6, 7.

102. E. P. Wigner, "Are We Machines?" Proc. Am. Philos. Soc. **113,** 95–101 (1969), p. 97, and "The Philosophical Problem," *Foundations of Quantum Mechanics,* edited by B. d'Espagnat (Academic, New York, 1971), p. 3.

103. N. Bohr, n. 91; A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Phys. Rev. **47,** 777–80 (1935).

104. See for example D. Gorenstein, "The Classification of Finite Simple Groups. I. Simple Groups and Local Analysis," Bull. Am. Math. Soc. **1,** 43–199 (1979), esp. ch. 1, sec. 3, "Why the Extreme Length?": "There exists an often expressed feeling in the general mathematical community that the present approach to the classification of simple groups must be the wrong one — no single theorem can possibly require a 5,000 page proof!"; also the discussion on pp. 50–52 of problems and progress in completing the classification.

105. For a study of this distinction between and interaction of observed system and observing equipment see especially M. M. Yanase, "Optimal Measuring Apparatus," Phys. Rev. **123,** 666–68 (1961); E. P. Wigner, "The Problem of Measurement," Am. J. Phys. **31,** 6–15 (1963), "Interpretation of Quantum Mechanics," 93 pages of mimeographed notes of lectures delivered at Princeton University in 1976, on deposit in Fine Library, Princeton University, Princeton, N. J.; A. Peres, "Can We Undo Quantum Measurements?" 1979 preprint, Center for Theoretical Physics, The University of Texas at Austin, to appear in Phys. Rev. D (1980).

106. "Nothingness" ruled out as meaningless: "There are three ways of research, and three ways only. Of these, one asserts '*It is not,* and there must be not-being.' This is utterly forbidden: what is not cannot even be thought of. A second way [is] that of mortals without wisdom, who say of what is that 'it is and is not,' 'is the same and not the same.' In contrast to them the way of truth starts from the proposition '*It is,* and not-being is impossible.' " [Parmenides of Elia, poem (~502 B.C.) *Nature,* part 2 "Truth," as summarized in article "Parmenides, p. 327–28 in *Encyclopaedia Brittanica,* Vol. 17 (Chicago 1959), p. 327.]

107. Universe as a self-excited circuit: in Wheeler, "Is Physics Legislated by Cosmogony," in op. cit. in n. 75, p. 565 and in *Frontiers of Time,* op. cit. in n. 8, p. 11.

108. "Towards the finding of this 'pregeometry' no guiding principle would seem more powerful than the requirement that it should provide the universe with a way to come into being. It is difficult to believe that we can uncover this pregeometry except as we come to understand at the same time the necessity of the quantum principle, with its 'observer-participator,' in the construction of the world" [Patton and Wheeler, op. cit. in n. 75, p. 575]. "No test of these views looks more like being someday doable, nor more interesting and more instructive, than a derivation of the structure of quantum theory from the requirement that everything have a way to come into being out of nothing" [Wheeler, *Frontiers in Time,* op. cit. in n. 8, p. 28]. See also J. A. Wheeler, "Pregeometry: Motivations and Prospects," and W. K. Wooters, "Information Is Maximized in Photon Polarization Measurements," in A. R. Marlow (ed.), op. cit. in n. 75.

109. Move over onto the new "foundation of elementary acts of observer-participancy." For three steps towards this development see R. M. F. Houtappel, H. Van Dam, and E. P. Wigner, "The Conceptual Basis and Use of the Geometric Invariance Principles," Rev. Mod. Phys. **37,** 595–632 (1965), esp. secs. 4.1–4.5 on pp. 610–16; W. Wooters, "Information Is Maximized in Photon Polarization Measurements," in A. R. Marlow (ed.), op. cit. in n. 75; and A. Peres, op. cit. in n. 105.