Philosophy of Mathematics (P.O.M):

The study of the philosophical concepts and theories which govern mathematics. It is especially concerned with the relationship between logic and mathematics, and the concept of realism and how it applies to math as we understand it. [1]

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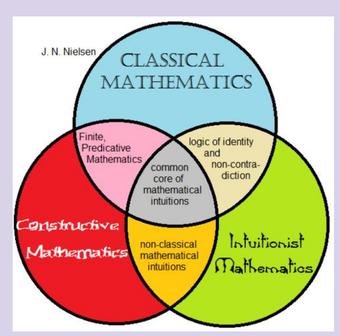
**Realism, in this context, refers to the question of whether mathematical objects are real

Logicism: The idea that mathematical concepts are logical concepts

- Gottlob Frege used Hume's Principle to formulate his idea (equinumerosity) [7]
- Second Order Logic [1/7]
- Numbers have qualities that differentiate them from other objects [7]
- Hume's Principle can't be a definition of numbers, neither can Frege's program [7]
- Russell found the contradiction [1]

Intuitionism: The idea that numerical systems, and the rules of math are imagined and not concrete

- L.E.J. Brouwer
- Contradiction to classical mathematics
- Mathematical statement = mental construction
- Rejects the idea of infinity
- Rejects non-constructive proofs (no example for the proof)
- Depends on time



[2]

Formalism: The idea that mathematical theories and statements can be formalized and proven free of contradictions

- Hilbertian Formalism
- Formalization: axiomatized theory is expressed using appropriate first order language
- Five types of first order language are variables, connectives, equal sign, and "For all" and "there exist"
  - Undefined terms
- Formalizing an axiomatized theory allows fundamental questions to be answered mathematically
- Avoid accidental contradictions
- Gödel found the contradiction
- Zermelo and Fraenkel (ZF) set theory-9 axioms [9]

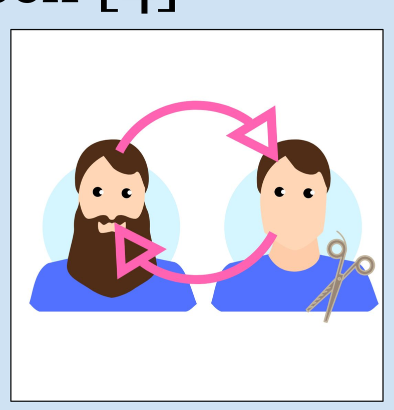
Naturalism: Exists as a sort of intermediary between intuitionism and classical math

- States that the only possible authorities in math are
  - Natural sciences
  - Math itself
  - Combination of the two
- Often used as an argument for why intuitionistic math should not replace classical math
- Philosophy is not "better" than science, thus, scientific truths should = philosophical truth
- Math and science have the strongest methodologies

[6]

Naive Set Theory and Russell's Paradox:

- Russell's Paradox appears within naive set theory [4]
- Naive Set Theory: Considers the set of all sets, can be in the set iff it's not a member of itself [4]
- "Barber Paradox" simplifies the idea [4]
- The set cannot include itself [3]

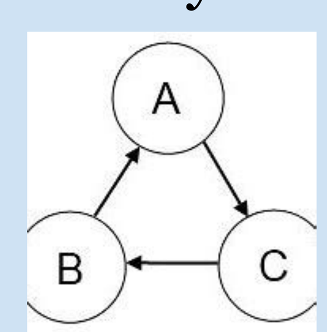


Why We Should Care and Further Questions:

- Knowing the origin of a concept often aids in understanding why the concept works
- Has implications in many other branches of math (set theory, computational math (CM), etc.)
- Our prior knowledge is limited, meaning that a broad topic has many implications whose full impact we cannot quite understand
- Vocabulary is also limited for this particular area, thus, deciphering academic sources was difficult
- Game formalism, ZF Set Theory, Gödel & CM

Gödel's Incompleteness Theorems:

- Completeness: within the system, every statement or its negation can be proven
- Consistency: only a statement *or* its negation can be derived, but not both
- Theorem 1: Consistent (sufficiently complex) formal systems are incomplete
- Theorem 2: Within a consistent (sufficiently complex) formal system, the consistency of the system cannot be proven within itself [8]



Sources: [1] Horsten, Leon, "Philosophy of Mathematics", The Stanford Encyclopedia of Philosophy (Spring 2019 Edition). Edward N. Zalta (ed.), <https://plato.stanford.edu/archives/spr2019/entries/philosophy-mathematics/>.

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[8] Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Spring 2021 Edition), Edward N. Zalta (ed.), <https://plato.stanford.edu/archives/spr2021/entries/goedel-incompleteness/>.

[9] Snapper, Ernst. "The Three Crises in Mathematics: Logicism, Intuitionism and Formalism." *Mathematics Magazine*, 1979,Three_Crises_in_Mathematics.pdf (psu.edu)