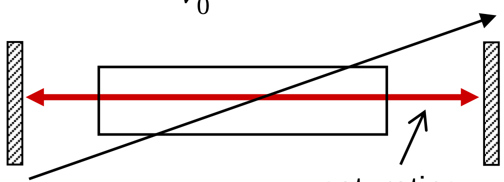
Gain Saturation – Inhomogeneous Broadening

Evolution of oscillation

Laser beam interacts with the following group velocity:

$$v_z^* = \pm \frac{c}{v_0}(v_L - v_0)$$



The dip occurs when the probe beam interacts with the same *vz i.e. at:

$$v_p - v_0 = \frac{v_0}{c} v_z^* = \pm (v_L - v_0)$$

$$v_{p1} = v_L \qquad v_{p2} = v_0 - (v_L - v_0)$$

(v_L has to be a cavity mode v_q !)

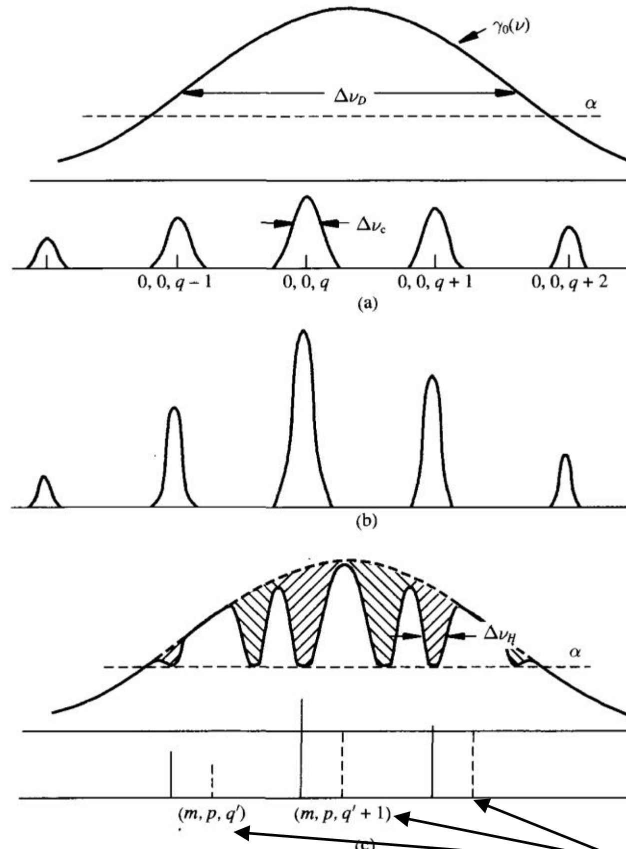


FIGURE 8.12. Evolution of oscillation in a Doppler-broadened transition.

Doppler effect

$$v' = v_0\left(1 \pm \frac{v_z}{c}\right)$$

The Lamb dip

$\nu_L = \nu_0$

Then the "+z" and the "-z" waves interact with the same $v_z^* = 0$ so that these atoms "feel" a stronger field => stronger saturation

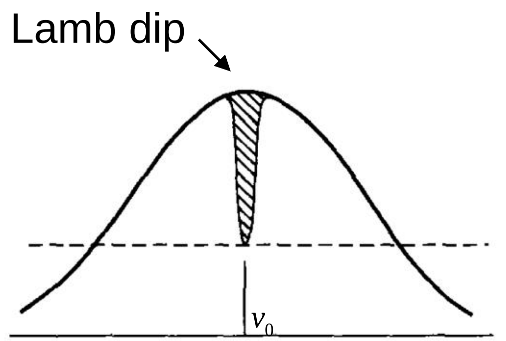


FIGURE 8.13. Lamb dip. (Courtesy of Spectra-Physics.)

(output power for $\nu_L = \nu_0$
The output power has a dip)

ν - Light frequency

f - resonant frequency of moving atoms

$p(f)df$ - fraction of atoms with center frequency $\left[\int p(f)df = 1\right]$

$$\gamma(\nu, I_\nu) = \int \frac{\gamma_0(\nu, f)\,p(f)}{1 + \left(I_\nu / I_S\right)\bar{g}(\nu, f)}\,df$$

Doppler

$$f \rightarrow \nu_0 + \frac{\nu_0}{c} v_z$$

$$\gamma(\nu, I_\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\int_0^\infty \frac{p(f)g_h(\nu, f)}{1+\left(I_\nu/I_S\right)\bar{g}_h(\nu, f)}df$$

$\bar{g}_h(\nu, f)$ - The Lorentzian normalized to 1 at $f = \nu$

$$\bar{g}_h(\nu, f) = \frac{\pi\Delta\nu_h}{2}g(\nu, f) = \frac{\left(\Delta\nu_h/2\right)^2}{(f-\nu)^2 + \left(\Delta\nu_h/2\right)^2}$$

Only those atoms whose center frequency f is within $\Delta\nu$ will contribute to the gain, and can be saturated by I_ν

$$p(f) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[-4\ln 2\left(\frac{f-\nu_0}{\Delta\nu_D}\right)^2\right]$$

$$\gamma(\nu, I_\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\int_0^\infty \frac{\left\{\left(\frac{2}{\pi\Delta\nu_h}\right)\left(\frac{\Delta\nu_h}{2}\right)^2 \Big/ \left[(\nu-f)^2 + \left(\frac{\Delta\nu_h}{2}\right)^2\right]\right\}p(f)df}{1+\left(I_\nu/I_S\right)\left(\frac{\Delta\nu_h}{2}\right)^2 \Big/ \left[(\nu-f)^2 + \left(\frac{\Delta\nu_h}{2}\right)^2\right]}$$

$$g = \frac{2}{\pi\Delta\nu_h}\bar{g}_h$$

$$\gamma(\nu, I_\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\left(\frac{\Delta\nu_h}{2\pi}\right)\int_0^\infty \frac{p(f)df}{\left[(\nu-f)^2 + \left(\frac{\Delta\nu_h}{2}\right)^2\right] + \left(I_\nu/I_S\right)\left(\frac{\Delta\nu_h}{2}\right)^2}$$

$$\gamma(v, I_v) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\frac{\Delta v_h}{2\pi}\int_0^\infty \frac{p(f)df}{(v-f)^2 + \underbrace{\left(\frac{\Delta v_h}{2}\right)^2\left(1+ I_v\big/ I_S\right)}_{\left(\frac{\Delta v_H}{2}\right)^2}}$$

Intensity dependent width

$$\exp\left[-\left(\frac{f-v}{\Delta v_D}\right)^2 4\ln 2\right]$$

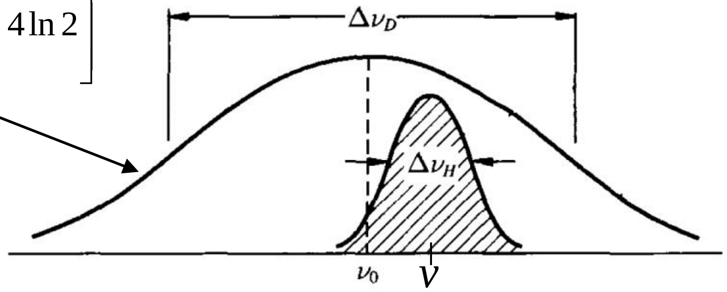


FIGURE 8.16. Saturation of an inhomogeneously broadened amplifier: (a) intermediate intensity;

$$\gamma(v, I_v) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\frac{\Delta v_h}{\Delta v_H}\int_0^\infty p(f)\frac{\left(\Delta v_H/2\pi\right)}{(v-f)^2 + \left(\Delta v_H/2\right)^2}df$$

$$\boxed{\Delta v_H = \Delta v_h\left(1+ I_v\big/ I_S\right)^{1/2}}$$

numerical evaluation
(or approximate)

Strongly inhomogeneous broadening

$$(\Delta\nu_D >> \Delta\nu_H)$$

$$\frac{\Delta\nu_H/2\pi}{(\nu-f)^2+\left(\Delta\nu_H/2\right)^2} \to \delta(f-\nu)$$

$$\gamma(\nu,I_\nu)=\left[A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0-N_1^0\right)p(\nu)\right]\frac{\Delta\nu_h}{\Delta\nu_H}$$

$\gamma_0(\nu)$

$$\boxed{\gamma(\nu,I_\nu)=\frac{\gamma_0(\nu)}{\left(1+\frac{I_\nu}{I_S}\right)^{1/2}}}$$

for homogeneous broadening:

$$\gamma(\nu,I_\nu)=\frac{\gamma_0(\nu)}{1+\bar{g}(\nu)\frac{I_\nu}{I_S}}$$

$$I_S=\frac{h\nu}{\tau_2}\frac{1}{\sigma(\nu_0)} \qquad (\tau_2 >> \tau_1) \quad \text{“resonant” saturation}$$

$\sigma(\nu_0)$ depends on $\Delta\nu_h$ (not $\Delta\nu_D$!!)

(1) Saturates more slowly than homogeneous line (width broadens as the hole deepens)

(2) Saturation factor is frequency independent (provided $\Delta\nu_H << \Delta\nu_D$)

For $\Delta\nu_H >> \Delta\nu_D$ ($p(f)$ centered at $f = \nu_0$)

$$\gamma(\nu, I_\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + \left(\frac{\Delta\nu_h}{2}\right)^2}\int_0^\infty p(f)df$$

$$\gamma(\nu, I_\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2^0 - \frac{g_2}{g_1}N_1^0\right)\frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + \left(\frac{\Delta\nu_H}{2}\right)^2}$$

$$\left(\frac{\Delta\nu_H}{2}\right)^2 = \left(\frac{\Delta\nu_h}{2}\right)^2\left(1 + \frac{I_\nu}{I_S}\right)$$

$$\boxed{\gamma(\nu, I_\nu) = \frac{\gamma_0(\nu)}{1 + \frac{I_\nu}{I_S}\bar{g}_h(\nu)}}$$

- recovered previous result

$$\bar{g}_h(\nu, f) = \frac{\left(\Delta\nu_h/2\right)^2}{\left(\nu-\nu_0\right)^2 + \left(\frac{\Delta\nu_h}{2}\right)^2}$$