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GODDARD ASTRONOMIC AND GEODYNAMIC PARAMETERS

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ABSTRACT

A reference set of astronomic and geodynamic parameters to be employed at the Goddard Space Flight Center is presented. The set includes fundamental astronomical constants, gravitational harmonic coefficients, station locations and other geometric quantities. It reflects results of recent research based upon data gathered by means of near-earth satellites and lunar and planetary spacecraft. It is consistent in this context with the IAU System of Astronomical Constants and values given in the Explanatory Supplement to the American Ephemeris and Nautical Almanac. Additional astrodynamic parameters of interest, including those used for manned flight computations, are also presented.

GODDARD ASTRONOMIC AND GEODYNAMIC PARAMETERS

by

Joseph W. Siry

I. INTRODUCTION

A reference set of astronomic and geodynamic parameters for general usc at the Goddard Space Flight Center is presented in this paper. Constants used for certain special purposes such as manned flight operational computing are also listed.

The reference set includes fundamental astronomical constants, gravititional harmonic coefficients, station locations and other geometric quantities.

The following considerations were among those which were kept in mind in making the selections. It is considered that such a set of reference parameters should facilitate the conducting of a variety of orbit determination and analysis activities, including those associated with scientific research and operations, for example. It should be of value in connection with matters relating to accuracy, consistency, and the interpretation and understanding of results. The set should conform to fundamental bases such as the IAU System of Astronomical Constants and material in the Explanatory Supplement to the Astronomical Ephemeris and Nautical Almanac to the greatest extent which is consistent with its purposes (1,2)*. It should also reflect the current state of the art to a reasonable degree, i.e., it should represent reasonably well the relevant observational data. The set considered here consists of several major sub-sets, e.g., those associated with the principal primaries of interest, the Sun, the Earth, and the Moon. Within these subsets there is further structure. Parameters associated with the Earth, for example, include fundamental ones such as the mass and the mean equatorial radius, additional gravitational quantities such as harmonic coefficients of the potential, and other parameters associated with its geometrical figure, such as the flattening and the locations of reference tracking stations.

Each of these subsets should be internally consistent, and the entire ensemble should, as a whole, be consistent, i.e., the major sub-sets should be consistent with one another in an appropriate way. It was considered in this

*Numbers in parentheses will refer to references listed in the last section.

connection to be desirable, in particular, that station locations and harmonic coefficients of the earth's gravitational field form a consistent set, and that the value of GM be consistent with results obtained from the analysis of lunar and deep space probe trajectories. The coupling between station locations and gravitational harmonics was considered to be important since these two types of parameters are frequently determined simultaneously in some sense. Because of this circumstance, station locations and gravitational parameters are not necessarily independent. Changes in certain parameters, such as station coordinates, which are obtained in such a unified solution imply changes in other parameters, such as harmonic coefficients, which it is usually not practical to identify specifically.

The best determinations of GM are currently obtained through the analysis of deep space probe data. The value of GM plays a central role in the fixing of the earth's equatorial radius and hence the scaling factor for such quantities as station location coordinates. It is for this reason that consistency between the station coordinates and the value of GM obtained from deep space trajectory analyses is important.

Other considerations arise which are of a somewhat different nature. It is convenient, from a practical standpoint, for example, if the adopted set is consistent with reference sets used by other institutions which are active in the community. Also, reference sets of parameters employed by the NASA Goddard Space Flight Center should be usable in the open literature.

The parameters of Section XI will be employed as the basis for general orbit determination at Goddard except in the case of manned flight operational computing and certain related network computations where the constants used are chosen jointly by a number of groups including several outside of this center. The system of Section XI may thus, in this sense, be thought of as the Goddard Space Flight Center Adopted Reference Set of Astronomic and Geodynamic parameters. It is sometimes referred to somewhat more briefly as the Goddard Adopted Reference Set of Astrodynamic Parameters. It will often be called simply the Adopted Reference Set. The basis for its selection is discussed in greater detail in the following sections.

Other parameters of interest, including those used for manned flight operations, are presented in additional sections.

II. THE IAU SYSTEM OF ASTRONOMICAL CONSTANTS

The precepts set forth in the Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac are followed in general in matters relating to Fundamental Coordinate Systems. This practice applies, for example, in connection with precession, nutation, time, and related subjects (Cf. references 2 and 3).

The plan for adopting astronomical constants for use at Goddard is organized along the lines laid down in the specification of the International Astronomical Union's System of Astronomical Constants in 1964. (Cf. reference 1.) This system appears in Table I. The actual values chosen for the reference set are those of the IAU System in a great many cases. Alternative selections have been made in certain instances. In general these latter values have been found to give better agreement with significant portions of the body of observational data which has become available in the years since 1964 and are, accordingly, considered to be more suitable for the purposes of meeting our current needs. It is recognized that the present reference set will be modified in due course on the basis of newer findings including, in particular, those brought forward in connection with further deliberations of the IAU.

The alternative choices made here were based upon results from two major streams of investigation, those involving near-earth satellites and those having to do with deep space probes. Considerations underlying these choices are indicated in the present discussion and/or the associated references. The consideration of candidate sets of harmonics of the Earth's field and reference tracking station coordinates, for example, included a review of recent history of work in this area which is indicated in reference (4) and the following discussion.

III. GEOPOTENTIAL HARMONIC COEFFICIENTS AND REFERENCE LOCATION COORDINATES

Determinations of station coordinates and gravitational harmonics in a simultaneous fashion have been made by various groups over the years. Such determinations have been made, for example, by the Smithsonian Astrophysical Observatory, the Naval Weapons Laboratory, the Applied Physics Laboratory, and by W. Kaula. Principal features of the geopotential representations resulting from these solutions and from other determinations which are of interest in this connection are indicated in Table II. These include geopotential solutions obtained by Rapp and Köhnlein using both satellite and gravimetric data, and combined fields obtained by Kaula, and by Murphy and Marsh (5-19).

Kaula examined the first four gravitational fields indicated in Table II by testing them against terrestrial gravimetry (20). He found that the SAO solution was the best. He thought that this was probably due to the fact that the results based on camera data were of a more recent vintage than the others, and that it was entirely possible that a further iteration of the Doppler data analysis could yield a still better solution.

The Naval Weapons Laboratory and the Applied Physics Laboratory have not published determinations of station locations and gravitational harmonics recently.

Table I

THE IAU SYSTEM OF ASTRONOMICAL CONSTANTS

Reference List of Recommended Constants*

Defining constants

1. Number of ephemeris seconds in 1 tropical year (1900)	s = 31 556 925·974 7
2. Gaussian gravitational constant, defining the A.U.	k = 0·017 202 098 95

Primary constants

3. Measure of 1 A.U. in metres	$A = 149\,600 \times 10^6$
4. Velocity of light in metres per second	$c = 299\,792{\cdot}5 \times 10^3$
5. Equatorial radius for Earth in metres	$a_e = 6\,378\,160$
6. Dynamical form-factor for Earth	$J_2 = 0{\cdot}001\,082\,7$
7. Geocentric gravitational constant (units: $m^3\ s^{-2}$)	$GE = 398\,603 \times 10^9$
8. Ratio of the masses of the Moon and Earth	$\mu = 1/81{\cdot}30$
9. Sidereal mean motion of Moon in radians per second (1900)	$n^*_{\text{☾}} = 2{\cdot}661\,699\,489 \times 10^{-6}$
10. General precession in longitude per tropical century (1900)	$p = 5025''{\cdot}64$
11. Obliquity of the ecliptic (1900)	$\epsilon = 23^\circ\ 27'\ 08''{\cdot}26$
12. Constant of nutation (1900)	$N = 9''{\cdot}210$

*The above list was confirmed by the Working Group, adopted as definitive at the Joint Discussion, and endorsed by the Twelfth General Assembly (Resolution no. 4); it now represents the 'IAU System of Astronomical Constants'.

Table I (continued)

THE IAU SYSTEM OF ASTRONOMICAL CONSTANTS

Reference List of Recommended Constants*

Derived constants

13. Solar parallax	$\arcsin(a_e/A) = \pi_\odot = 8''.794\,05\ (8''.794)$
14. Light-time for unit distance	$A/c = \tau_A = 499^s.012$
	$= 1^s/0.002\,003\,96$
15. Constant of aberration	$F_1 k' \tau_A = \kappa = 20''.4958\ (20''.496)$
16. Flattening factor for Earth	$f = 0.003\,352\,9$
	$= 1/298.25$
17. Heliocentric gravitational constant (units: $m^3\ s^{-2}$)	$A^3 k'^2 = GS = 132\,718 \times 10^{15}$
18. Ratio of masses of Sun and Earth	$(GS)/(GE) = S/E = 332\,958$
19. Ratio of masses of Sun and Earth + Moon	$S/E(1 + \mu) = 328\,912$
20. Perturbed mean distance of Moon, in metres	$F_2 (GE(1+\mu)/n_{☾}^{*2})^{1/3} = a_{☾} = 384\,400 \times 10^3$
21. Constant of sine parallax for Moon	$a_e/a_{☾} = \sin \pi_{☾} = 3422''.451$
22. Constant of lunar inequality	$\frac{\mu}{1+\mu}\frac{a_{☾}}{A} = L = 6''.439\,87\ (6''.440)$
23. Constant of parallactic inequality	$F_3 \frac{1-\mu}{1+\mu}\frac{a_{☾}}{A} = P_{☾} = 124''.986$

System of planetary masses

	Reciprocal mass		Reciprocal mass
24. Mercury	6 000 000	Jupiter	1 047·355
Venus	408 000	Saturn	3 501·6
Earth + Moon	329 390	Uranus	22 869
Mars	3 093 500	Neptune	19 314
		Pluto	360 000

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Table I (continued)

THE IAU SYSTEM OF ASTRONOMICAL CONSTANTS

Reference List of Recommended Constants*

Auxiliary constants and factors

k/86400, for use when the unit of time is 1 second	$k' = 1{\cdot}990\ 983\ 675 \times 10^{-7}$
Number of seconds of arc in 1 radian	206 264·806
Factor for constant of aberration[1]	$F_1 = 1{\cdot}000\ 142$
Factor for mean distance of Moon[2]	$F_2 = 0{\cdot}999\ 093\ 142$
Factor for parallactic inequality[3]	$F_3 = 49\ 853''2$

1. Apart from the factor F_1 the constant of aberration is equal to the ratio of the speed of a hypothetical planet of negligible mass moving in a circular orbit of unit radius to the velocity of light; it is conventionally expressed in seconds of arc by multiplying by the number of seconds of arc in one radian. The factor F_1 is the ratio of the mean speed of the Earth to the speed of the hypothetical planet and is given by

$$F_1 = \frac{n_\odot}{k'} \frac{a_\odot}{(1 - e^2)^{1/2}}$$

where $n_\odot$ is the sidereal mean motion of the Sun in radians per second, $a_\odot$ is the perturbed mean distance of the Sun in A.U., and e is the mean eccentricity of the Earth's orbit. Newcomb's values for $n_\odot$, $a_\odot$ and e are of ample accuracy for this purpose. The factor F_1 and the constant of aberration take the following values

	F_1	κ''
1800	1·000 142 7	20·495 83
1900	1·000 142 0	20·495 82
2000	1·000 141 3	20·495 81

The rounded value 20''496 should be used except where the extra figures are required to ensure numerical consistency.

2. The perturbed mean distance of the Moon is the semi-major axis of Hill's variational orbit, and differs from that calculated from Kepler's law by the factor F_2, which depends on the well-determined ratio of the mean motions of the Sun and Moon. (E. W. Brown, Mem. R. astr. Soc., 53, 89, 1897).

3. The constant of the parallactic inequality is defined by the expression given; the coefficient F_3 is consistent with the corresponding quantities in Brown's Tables.

*The above list was confirmed by the Working Group, adopted as definitive at the Joint Discussion, and endorsed by the Twelfth General Assembly (Resolution no. 4); it now represents the 'IAU System of Astronomical Constants'.

Table II

GENERAL DETERMINATIONS OF GEOPOTENTIAL COEFFICIENTS

Author or Designation	Year	Reference Number	Basis			Field Characteristics	
			Satellite Data		Other Information		
			Number of Satellites	Tracking Systems		Last term in Complete Portion	Number of Coefficients
1. NWL 5E-6	1965	6	3	Doppler		7, 6	64
2. APL 3.5	1965	7	5	Doppler		8, 8	84
3. SAO M-1	1966	8	16	Optical		8, 8	122
4. Kaula K-8	1966	9	12	Optical, Doppler		7, 5	99
5. Kaula C	1966	9			Determinations 1-4	7, 2	
6. Rapp	1967	10	16	Optical	Gravimetry	14, 14	219
7. Köhnlein	1967	11	16	Optical	Gravimetry	15, 15	250
8. Kaula UCLA	1967	12	9	Optical, Doppler	Gravimetry	8, 8	
9. Rapp	1968	13	16	Optical	Gravimetry	14, 14	
10. SAO COSPAR	1969	14	24	Optical, Range, Range Rate		14, 14	280
11. SAO B6.1	1969	15	24	Optical, Range, Range Rate		16, 16	
12. SAO B 13.1	1969	16	24	Optical, Range, Range Rate	Gravimetry	16, 16	314
13. SAO SF	1969	17	24	Optical, Range, Range Rate	Gravimetry	16, 16	316
14. GSFC 1.70 C	1970	18			Determinations 1-9, 11, 12	15, 15	249
15. SAO 69 (II)	1970	19	21	Optical, Range, Range Rate	Gravimetry	16, 16	316

The major ongoing activity which has yielded and is yielding results of this type appropriate for our use is the one which has been in progress at the Smithsonian Astrophysical Observatory. The effort in the Mission and Trajectory Analysis Division is getting underway and we all look forward to its fruits. It is anticipated that results from this program will provide a basis for reference sets of constants and station locations when they are available. The SAO results appear to be useful at this time for the purpose of establishing reference values of such parameters as station location coordinates and gravitational harmonic coefficients.

The Smithsonian Astrophysical Observatory had completed a determination of a consistent set of station location coordinates and gravitational harmonics in 1966. This set of parameters has been referred to as the 1966 Smithsonian Institution Standard Earth (8).

Actually, more than one set of station locations was specified in connection with the results which formed the basis for this model. Each of these sets was equivalent to the other to within a scale factor which is specified in terms of the Earth's mean equatorial radius. The selection of this parameter and hence of a particular set of station locations from among these results will be taken up in a subsequent portion of the discussion.

More than one set of values for the harmonic coefficients has been published by SAO on the basis of the 1966 results. The original set was published in 1966. It is often referred to as the SAO M-1 Model for the Earth's gravitational field. A modification was published in 1967 which included results of a new determination for the values of certain of the harmonic coefficients with which two satellites, including GEOS I, were resonant (21). This model for the Earth's gravitational field is sometimes referred to as the SAO Modified M-1 Model.

The Smithsonian Astrophysical Observatory has recently been pursuing a program to generate a new set of constants intended to reflect effects of satellite tracking data gathered in the intervening years. Results of a number of iterations in this program have become available (Cf. references 14-17, 19). Station coordinate results were changed less often, generally, than gravitational field results, although more than one combined solution involving corrections to both types of quantities has been generated.

The situation at this writing with respect to the testing and use of various SAO models here is as follows. The SAO Modified M-1 field has been compared with other fields by Lerch and Marsh. The results are presented in reference (22). They indicated the suitability of this gravity field for a number of our purposes. In addition, this model has been used for a variety of other purposes at Goddard in recent years. These several uses have produced, as a by-product, a body of information which indicated the utility of the SAO Modified M-1 Model for many of our applications.

This program of testing and using the 1966 SAO Modified M-1 Model was reemphasized and extended when the newer SAO findings began to become available for comparison last year. Differences between corresponding sets of station coordinates in the recent SAO sets and the 1966 SAO Modified M-1 Model set are of the order of ten to twenty meters. A considerable amount of testing has already been conducted using several models in the current series which were presented in 1969 (Cf. reference 23). One of these models has also been employed in connection with station location studies conducted at Goddard (Cf. reference 24). These models appear, on the basis of these tests, to represent improvements over the 1966 results. The final set of parameters resulting from the recent SAO constant determination program has now become available (19). It is referred to as the 1969 Smithsonian Standard Earth (II). There has been only a limited amount of testing here, yet, of this particular model. This model differs but slightly, however, from the model of reference (17), for example, which has tested well here. The SAO has examined its 1969 Standard Earth (II) from various standpoints and found it to have good properties (19).

On the basis of these considerations the reference tracking station coordinates and gravitational harmonic coefficients given in reference 19 which are specified in the second and third pages of Section XI, 3, and in Sections XI, 4, and 5 of this paper are selected at this time as part of the Adopted Reference Set. These quantities form a set of values for gravitational harmonic coefficients and primary station locations which is consistent. This is true in the sense that it was determined, through simultaneous differential correction solutions, that this set agrees well with the body of precise optical observations and some additional laser and electronic range and range rate observations of twenty-one satellites which are well distributed in inclination.

This field represents gravitational resonance effects associated with the satellites whose data were employed in its derivation. It may be that some other satellites, while not entering directly in the solution, are nevertheless represented reasonably well for resonance purposes by the satellite data upon which the results are based. It is likely, however, that in many cases, satellites not involved in the adjustment will be subject to resonance effects which are not modelled suitably by the resulting general field. In such cases, if accurate orbits are sought, values for appropriate resonant harmonic coefficients must be found. Results of this type referred to earlier fields have been derived by Wagner, Yionoulis, Gaposchkin and Veis, Murphy and Cole, and Douglas and Marsh (25-29). It is planned, if such needs arise in the future, to determine the resonant coefficients in the context of the adopted general field.

IV. ASTRONOMICAL CONSTANTS AND PARAMETERS OF THE EARTH-MOON SYSTEM

Values for the mass of the Earth, and parameters associated with the Moon and the planetary system have been derived at the Jet Propulsion Laboratory on the basis of the analysis of data from a number of spacecraft and from direct observation of the radar distances, i.e., the light times, to the planet Venus (30). Values for several of these parameters have been considered to be improvements over the earlier IAU values. These are the measure of the light time in one Astronomical Unit, the value of GM for the Earth, and the ratio of the masses of the Earth and Moon (30). These values of the light time for one AU and for the ratio of the masses of the Earth and the Moon are selected for the Adopted Reference Set. They appear in Section XI, 1.

The adoption of the 1969 Smithsonian Standard Earth (II) geopotential coefficient values as a set implies, in particular, the adoption of the value of J_2 from this set. This parameter is also referred to as the dynamical form factor in the IAU System. The flattening is a function of the dynamical form factor. Cook gives the following relation connecting them,

$$J_2 = \frac{2}{3}\left\{f\left(1 - \frac{1}{2}f\right) - \frac{1}{2}m\left(1 - \frac{2}{7}f + \frac{11}{49}f^2\right)\right\},$$

which is valid to order f^3, where $m = w^2 a_e^3 (1 - f)/GM$, and w and a_e denote the Earth's rotational rate and equatorial radius, respectively. The value of 1/298.257 for the flattening is implied by these expressions and the adopted value of the dynamical form factor (31,32). The value of 298.257 is, accordingly, adopted here for the reciprocal flattening.

Satellite orbital periods are, in effect, observed with relatively good accuracy and for the purposes of the present discussion can be considered to be invariant. In the presence of Kepler's third law,

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM},$$

then, a relative change in GM implies a relative change in the semi-major axis, a, which is one third as large. The orbits observed with the Baker-Nunn cameras are therefore taken to be correspondingly different in geometric size or scale. This implies a corresponding change in the scale of geometrical figures, such as the Earth, which are associated with the Baker-Nunn camera positions determined

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then, a relative change in GM implies a relative change in the semi-major axis, a, which is one third as large. The orbits observed with the Baker-Nunn cameras are therefore taken to be correspondingly different in geometric size or scale. This implies a corresponding change in the scale of geometrical figures, such as the Earth, which are associated with the Baker-Nunn camera positions determined from satellite photographs. On this basis a change in the value of GM from 398603.2 km^3/sec^2 to 398601.3 km^3/sec^2 implies a corresponding change in the value of the Earth's equatorial radius from 6378.165 km to 6378.155 km.

SAO adopted the value of 398601.3 km^3/sec^2 for GM, and the value of 6378.155 km for the Earth's equatorial radius and specified them in connection with the values for the gravitational harmonic coefficients which were given in reference (21). The recent determinations of the gravitational harmonics and the station locations which have been in progress at SAO are also referred to the value of 6378.155 km for the Earth's equatorial radius (19).

The value of 398601.2 km^3/sec^2 was adopted for GM for the Earth by the Jet Propulsion Laboratory. The corresponding value of the Earth's mean equatorial radius in the sense of the preceding discussion is 6378.154 km. From the standpoint of a number of uses of the SAO Earth models, the pairs of values 398601.3 km^3/sec^2 and 6378.155 km, and 398601.2 km^3/sec^2 and 6378.154 km are equivalent. The values 398601.2 km^3/sec^2 and 6378.155 km are consistent with the JPL values for GM, and for near-Earth satellites, they involve a difference of no more than about a meter with respect to the SAO selection. This latter difference is considered acceptably small at the moment. This difference is also well within the accuracy, by an order of magnitude, say, of the procedures and results upon which the value is based. The alternative choices of 398601.3 km^3/sec^2 and 6378.155 km could involve differences of the order of thirty meters at the distance of the Moon, which is less negligible relative to currently achievable observational accuracies.

The values of 398601.2 km^3/sec^2 for the Earth's GM and 6378.155 km for its mean equatorial radius are, accordingly, selected here at this time as part of the Adopted Reference Set.

This completes the selection of values for the primary constants. These are given in Section XI, 1.

The corresponding adopted values of the derived constants are specified in Section XI, 2.

Relatively simple models of the lunar potential have been obtained by Murphy, Felsentreger, Wagner and Ryan who found them to give good representations of data obtained during a number of Apollo orbits (34). They were derived as extensions of an earlier field, known as the L1 Model, which was obtained by H. Compton and R. Tolson of the Langley Research Center (35). The L1 model, in turn, represented an augmentation of a still earlier model determined by R. E. Risdal of the Boeing Company in connection with the Lunar Orbiter program (Cf. reference 36). The best field obtained by Murphy and his colleagues, which is referred to as the Goddard Space Flight Center ML1.2 model, is specified in Section XI, 7 (34). It is selected as part of the Adopted Reference Set.

More detailed models of the lunar gravitational field have been given by Lorell, Michael, Blackshear, Muller and Sjogren (59-62).

Values adopted for the mean lunar radius and the principal axes of the Moon are given in Section XI, 6.

The JPL Lunar Ephemeris No. 4 (LE 4), contained in Export Ephemeris DE19, is used at this time to obtain lunar positions. The lunar distance, i.e., the scaling factor for the lunar ephemeris, is actually specified in terms of a parameter corresponding to the earth's radius, the appropriate value for which is 6378.1492 km in this case (30).

V. TRACKING STATION POSITIONS

It was indicated earlier that the selection of a specific set of values for the coordinates of the reference tracking stations depends on the choice of the value for the Earth's mean equatorial radius which serves as a scale factor. The designation C-5 had been assigned to those values for the Baker-Nunn camera station coordinates which were determined in connection with the 1966 Smithsonian Institution Standard Earth results, and which were referred to the equatorial radius of 6378.165 km. Similarly, the designations C-6 and C-7 had been assigned to those values for the Baker-Nunn station coordinates obtained in that determination which were referred to the equatorial radius values of 6378.155 km and 6378.142 km, respectively. The value of 6378.142 km for the Earth's equatorial radius was obtained by Veis partly on the basis of another, earlier value of GM for the Earth obtained at JPL (21). The choice of the C-6 Baker-Nunn station locations, from among the alternatives available in connection with the 1966 Smithsonian Institution Standard Earth results, follows directly as a corollary of the adoption of the value of 6378.155 km for the Earth's mean equatorial radius.

The locations of a set of reference stations such as the SAO Baker-Nunn sites can serve as the basis to which the locations of other stations can be referred. The referral can be accomplished in more than one way. It can be done readily when the station of interest and at least one of the reference stations are connected by a conventional geodetic datum tie. In such a case one has the coordinates, referred to the local geodetic datum, of both the station of interest and the reference station. The differences between the coordinates of the reference station and the coordinates of this same station referred to the local geodetic datum are also available. These differences, which can be thought of as datum shift increments, can be applied to the coordinates of the station of interest to obtain its position coordinates relative to the reference system. Procedures for making the necessary transformations covering the use of one or more reference stations in a local geodetic datum are described in reference (37). It is recognized that this method must be used with caution when, for example, the calculation involves an extrapolation beyond the geometrical configuration of the reference stations.

This method cannot be used when the station of interest is not geodetically connected to any of the points of the reference set. In such a case, a satellite orbit can provide the basis for the tie. GEOS satellite data were used in this way to find the location of a NASA station on Madagascar, for example. (38)

More recently, the same method has been used by Marsh, Douglas, Martin and Klosko to determine locations for a set of some thirty NASA tracking stations relative to a set of SAO Baker-Nunn station positions using data from GEOS I and II (24, 51). The SAO station locations which were used as reference values in this particular adjustment are included in the list of reference (17).

The locations of a set of Goddard tracking stations which were among those determined in the adjustments described in references 24 and 51 are selected as part of the Adopted Reference Set. They are listed here on the first page of Section XI, 3.

The locations of some ten stations which are additional to those listed in Section XI, 3, were found in both the SAO and the Goddard solutions (4, 17, 19, 24, 39).

The latitudes and longitudes of the two sets of locations agree to within the accuracies of the adjustments. The height differences are systematic and somewhat larger than expected. The intersite distances between pairs of these sites derived from NAD survey positions agree more closely with the distances obtained from the positions of references 24 and 39 than they do with the distances obtained from the positions of reference 19. The study of these results is continuing.

Locations for additional stations found in the manner of the determinations of references 24 and 51 can be viewed as constituting an augmentation of the basic reference set of SAO Baker-Nunn station positions. The use of such an augmented reference set as the basis for locating other stations of interest can be advantageous since, in a number of cases, the nearest reference point will be closer to the station whose location is being found. An augmented reference set consisting of SAO Baker-Nunn and NASA station locations has been used as the basis for locating other stations of interest to NASA. The locations of these stations are being issued in reference (39).

It is planned to use the method of references 24 and 51 to determine the locations of a set of NASA reference stations with respect to a reference system defined in terms of parameters of the Adopted Reference Set, including the fundamental SAO Baker-Nunn station positions specified in the second and third pages of Section XI, 3. The locations of the additional stations of interest to NASA will then be found relative to this new augmented basis.

The locations of NASA stations that are additional to those specified in Section XI, 3, which are issued in reference (39) are selected here at this time as part of the Adopted Reference Set. The locations of NASA sites which are in addition to those listed in the second and third pages of Section XI, 3, referred to station coordinates given there as indicated above, will be adopted when they become available for issuance.

VI. PLANETARY PARAMETERS

Values for additional parameters of interest which are selected here as part of the Adopted Reference Set are specified in Sections XI, 8, 9, 10, and 11. These include the planetary masses and radii. The quantities associated with the Moon and the planets correspond, in a number of instances, to those derived by the Jet Propulsion Laboratory. The masses of Mercury, Venus and Mars are based on analyses of radar and spacecraft observations obtained by the Jet Propulsion Laboratory (30, 40).

The masses of the principal planets are based upon recent discussions by Mulholland, O'Handley, Gill, Duncombe, et al. (30, 41-44). Values of the radii of Mercury and Venus were derived from the analysis of data obtained through the radar tracking of these planets. (45-48). The value derived for the radius of Mars reflected results obtained from the tracking of Mariner IV. (49) The values of the radii of the remaining planets are based upon material provided in the Explanatory Supplement to the American Ephemeris and Nautical Almanac (reference 2). The dynamic form factor and geometric flattening for the outer planets are also derived from material presented in this work. Values of the corresponding quantities for the planet Mars are from recent studies. (49-50) The values shown are the only non-zero values adopted for the harmonic coefficients of the sun and the planetary bodies other than the Earth and the Moon.

Planetary positions are ordinarily obtained from JPL Export Ephemeris DE 19. In some cases, DE 39 is used for this purpose (30).

VII. AUXILIARY QUANTITIES

Certain conversion constants are given in Section XI, 12 for convenience.

VIII. ASTRODYNAMIC PARAMETERS FOR MANNED FLIGHT COMPUTATIONS

Constants used in the manned flight program are chosen by groups at Goddard and in other elements of NASA on the basis of a number of factors including limitations imposed by operational requirements.

Astrodynamic parameters currently used in the manned flight program and certain related network computing at Goddard are listed in Table III, which appears following Section XI (52-54). Fundamental parameter values appear in Part 1. Geopotential coefficients used are given in Part 2. The coefficients shown in Part 4 for the lunar gravitational potential are those of the L1 field. Lunar positions are obtained currently by means of the JPL Lunar Ephemeris No. 4 which is contained in Export Ephemeris DE 19. (Cf. also the last paragraph of Section IV.) Positions employed for the Apollo USB tracking sites are modified from time to time on the basis of station location studies. The values which were used in the Apollo XIII Mission are listed in reference 53.

IX. ADDITIONAL ASTRODYNAMIC PARAMETERS OF INTEREST

Astrodynamic parameters which have been employed at Goddard for definitive orbit determination and for predictions in certain cases prior to the use of the Adopted Reference Set are listed in Table IV which appears following Section XI. Fundamental parameters are given in Part 1. Gravitational harmonic coefficients are listed in Parts 2 and 3 (6, 55).

X. DISCUSSION

All the values in a comprehensive system such as the Adopted Reference Set will not be used in most orbital calculations. Orbit determination for a close earth satellite, for example, will not usually involve the use of parameters of the Jovian planets, even when definitive orbits are being determined. As the accuracy requirements decrease, the number of parameters employed decreases correspondingly. Predictions, in fact, can often be made with adequate accuracy by using only a few zonal harmonics of the earth's gravitational field. In such cases, subsets of the Adopted Reference Set will be employed.

Indications of the uncertainties of many of the parameters of the Adopted Reference Set are contained in the preceding discussion and in the references.

Methods used for general orbit determination at Goddard are described in references 56 and 57. Approaches used in estimating uncertainties associated with the orbital positions in some of these cases are set forth in reference 58.

XI. THE ADOPTED REFERENCE SET

1. Fundamental Constants

Primary Astronomical Constants

Light time for 1 AU in A.1 sec	499.004788
Speed of light in km/sec	299792.5
Equatorial radius for Earth in km.	6378.155
Dynamical form-factor for Earth	0.001082628
Geocentric gravitational constant (units: $km^3\ s^{-2}$)	398601.2
Ratio of the masses of the Earth and Moon	81.3010
Sidereal mean motion of moon in rad/sec (1900)	$2.661699489 \times 10^{-6}$
General precession in longitude per tropical century (1900)	5025."64
Obliquity of the ecliptic (1900)	23°27'08."26
Constant of nutation (1900)	9."210
Gravitational constant ($cm^3\ s^{-2}\ g^{-1}$)	6.6732×10^{-8}

2. Fundamental Constants

Derived Astronomical Constants

Solar parallax	8."79417
Measure of 1 AU in km.	149597893.0
Constant of aberration	20."4955
Flattening factor for Earth	1/298.257
Heliocentric gravitational constant (units: $km^3\ s^{-2}$)	132712499×10^3
Ratio of masses of Sun and Earth	332945.6
Ratio of masses of Sun and Earth plus Moon	328900.1
Perturbed mean distance of Moon, in km	384399.3
Constant of sine parallax for Moon	3422."454
Constant of lunar inequality	6."43987
Constant of parallactic inequality	124."9873
Lunar gravitational constant (units: $km^3\ s^{-2}$)	4902.78

XI. THE ADOPTED REFERENCE SET

3. Station Coordinates

Station Number	X	Y	Z	Station Name
1022	0.807864	-5.651973	2.833500	Fort Myers, Fla.
1024	-3.977265	3.725653	-3.303008	Woomera, Australia
1028	1.769718	-5.044607	-3.468271	Santiago, Chile
1030	-2.357244	-4.646317	3.668311	Mojave, Calif.
1031	5.084795	2.670401	2.768159	Johannesburg, So. Afr.
1032	2.602757	-3.419147	4.697642	St. John's, Newfoundland
1035	3.983099	-0.048512	4.964719	Winkfield, England
1038	-4.447485	2.677168	-3.695081	Orroral, Australia
1043	4.091871	4.434287	-2.064745	Tananarive, Mad.
1122	4.091342	4.434231	-2.065970	Tananarive, Mad.
1126	0.647182	-5.178322	3.656141	Rosman, No. Car.
1128	-2.282508	-1.453372	5.756712	Fairbanks, Alaska
1152	-2.328211	5.299699	-2.669358	Carnarvon, Australia
7050	1.130678	-4.831365	3.994105	Goddard Space Flight Center
7052	1.261550	-4.881571	3.893163	Wallops Island, Va.
7054	-2.328156	5.299647	-2.669494	Carnarvon, Australia

The unit is the megameter.

XI. THE ADOPTED REFERENCE SET

3. Station Coordinates (continued)

Station Number	X	Y	Z	Station Name
7815	4.578370	.457951	4.403134	Haute Provence, France
7816	4.654337	1.959134	3.884366	Stephanion, Greece
7818	5.426329	- .229330	3.334608	Colomb-Bechar, Algeria
7901	-1.535757	-5.166996	3.401042	Organ Pass, N. M.
8015	4.578328	.457966	4.403179	Haute Provence, France
8019	4.579466	.586599	4.386408	Nice, France
9001	-1.535757	-5.166996	3.401042	Organ Pass, N. M.
9002	5.056125	2.716511	-2.775784	Pretoria, S. Africa
9003	-3.983776	3.743087	-3.275566	Woomera, Australia
9004	5.105588	- .555228	3.769667	San Fernando, Spain
9005	-3.946693	3.366299	3.698832	Tokyo, Japan
9006	1.018203	5.471103	3.109623	Naini Tal, India
9007	1.942775	-5.804081	-1.796933	Arequipa, Peru
9008	3.376893	4.403976	3.136250	Shiraz, Iran
9009	2.251829	-5.816919	1.327160	Curacao, Antilles
9010	.976291	-5.601398	2.880240	Jupiter, Fla.
9011	2.280589	-4.914573	-3.355426	Villa Dolores, Argentina
9012	-5.466053	-2.404282	2.242171	Maui, Hawaii

The unit is the megameter.

XI. THE ADOPTED REFERENCE SET

3. Station Coordinates (continued)

Station Number	X	Y	Z	Station Name
9021	-1.936782	-5.077704	3.331916	Mt. Hopkins, Ariz.
9023	-3.977766	3.725102	-3.303035	Island Lagoon, Australia
9025	-3.910437	3.376361	3.729217	Dodaira, Japan
9028	4.903750	3.965201	.963872	Addis Ababa, Ethiopia
9029	5.186461	-3.653856	- .654325	Natal, Brazil
9031	1.693803	-4.112328	-4.556649	Comodoro Rivadavia, Argentina
9050	1.489753	-4.467478	4.287304	Harvard, Mass.
9065	3.923411	.299882	5.002945	Delft, Holland
9066	4.331310	.567511	4.633093	Zimmerwald, Switzerland
9074	3.183901	1.421448	5.322772	Riga, Latvia
9077	3.907421	1.602397	4.763890	Uzghorod, U.S.S.R.
9080	3.920178	- .134738	5.012708	Malvern, England
9091	4.595157	2.039425	3.912650	Dioysos, Greece
9113	-2.450011	-4.624421	3.635035	Rosamund, Cal.
9114	-1.264838	-3.466884	5.185467	Cold Lake, Canada
9115	3.121280	.592643	5.512701	Harestua, Norway
9117	-6.007402	-1.111859	1.825730	Johnston Isl., Pacific

The unit is the megameter.

XI. THE ADOPTED REFERENCE SET

4. Zonal Harmonic Coefficients of the Earth's Potential*

J(2) =	1.08262800E-03	J(3) =	-2.5380E-06
J(4) =	-1.5930E-06	J(5) =	-2.3000E-07
J(6) =	5.0200E-07	J(7) =	-3.6200E-07
J(8) =	-1.1800E-07	J(9) =	-1.0000E-07
J(10) =	-3.5400E-07	J(11) =	2.0200E-07
J(12) =	-4.2000E-08	J(13) =	-1.2300E-07
J(14) =	-7.3000E-08	J(15) =	-1.7400E-07
J(16) =	1.8700E-07	J(17) =	8.5000E-08
J(18) =	-2.3100E-07	J(19) =	-2.1600E-07
J(20) =	-5.0000E-09	J(21) =	1.4400E-07

* Unnormalized. Fully normalized and unnormalized coefficients, denoted by $\overline{C}_{lm}$, $\overline{S}_{lm}$, and C_{lm}, S_{lm}, respectively, are related by expressions of the form

$$C_{lm} = \left\{ \frac{(l - m)!}{(l + m)!} (2l + 1)(2 - \delta_{om}) \right\}^{1/2} \overline{C}_{lm},$$

where, here, δ_{om} denotes Kronecker's delta, δ_{lm}, for the case in which $l = 0$.

XI. THE ADOPTED REFERENCE SET

5. Tesseral Harmonic Coefficients of the Earth's Potential*

ℓ	m	$C_{\ell m}$	$S_{\ell m}$	ℓ	m	$C_{\ell m}$	$S_{\ell m}$
2	2	2.4129E-06	-1.3641E-06	3	1	1.9698E-06	2.6015E-07
3	2	8.9204E-07	-6.3468E-07	3	3	6.8630E-07	1.4304E-06
4	1	-5.2989E-07	-4.8765E-07	4	2	3.3024E-07	7.0633E-07
4	3	9.8943E-07	-1.5467E-07	4	4	-7.9692E-08	3.3928E-07
5	1	-5.3816E-08	-9.7905E-08	5	2	6.1286E-07	-3.5087E-07
5	3	-4.3083E-07	-8.6663E-08	5	4	-2.6693E-07	8.3010E-08
5	5	1.2593E-07	-5.9910E-07	6	1	-9.8984E-08	3.7652E-08
6	2	5.4825E-08	-3.5175E-07	6	3	2.7873E-08	4.4626E-08
6	4	-4.0342E-10	-4.0388E-07	6	5	-2.1143E-07	-5.2264E-07
6	6	8.8693E-08	-7.4756E-08	7	1	2.4142E-07	1.1567E-07
7	2	2.8306E-07	1.5645E-07	7	3	2.0285E-07	-2.3448E-07
7	4	-1.9727E-07	-1.1390E-07	7	5	-8.7024E-10	9.8461E-08
7	6	-2.5847E-07	1.0209E-07	7	7	1.5916E-07	-6.7710E-08
8	1	3.1254E-08	2.5696E-08	8	2	4.8161E-08	8.4140E-08
8	3	-5.7444E-08	1.8086E-08	8	4	-1.5378E-07	7.5264E-08
8	5	-5.6733E-08	6.1636E-08	8	6	-5.3903E-08	2.5930E-07
8	7	3.4390E-08	8.9168E-08	8	8	-7.7364E-08	6.7607E-08
9	1	1.3823E-07	-1.6100E-08	9	2	6.6741E-09	-8.1733E-08
9	3	-9.6463E-08	-1.1817E-07	9	4	5.7125E-08	1.1183E-07
9	5	-6.1435E-09	3.3551E-09	9	6	2.4186E-08	2.2028E-07
9	7	-5.0450E-08	-1.2700E-07	9	8	2.3359E-07	5.7239E-08
9	9	-8.2490E-08	9.2326E-08	10	1	1.1251E-07	-1.0167E-07
10	2	-3.1225E-08	-1.0450E-07	10	3	-2.3346E-08	-1.4137E-07
10	4	-4.8185E-08	-4.3248E-08	10	5	-8.0004E-08	-1.4279E-07
10	6	-3.2486E-08	-2.0153E-07	10	7	5.4961E-08	3.2003E-08
10	8	7.3957E-08	-7.9706E-08	10	9	-6.8563E-09	6.2498E-09
10	10	1.2377E-07	-2.5885E-08	11	1	4.3900E-09	2.9751E-08
11	2	4.8900E-08	-9.1994E-08	11	3	-6.3247E-08	-1.3109E-07
11	4	-3.0193E-08	5.4317E-08	11	5	3.2523E-08	1.3215E-07
11	6	3.7517E-08	6.9005E-09	11	7	4.5726E-08	-1.3862E-07
11	8	6.4546E-08	-1.6993E-08	11	9	1.1750E-07	-9.9451E-09
11	10	-1.1736E-07	-1.8900E-08	11	11	1.1785E-07	-4.0688E-08
12	1	-4.5955E-08	-3.1000E-08	12	2	2.7481E-08	7.5986E-08
12	3	5.8386E-08	5.4784E-08	12	4	-4.3649E-08	-2.2262E-08
12	5	2.3375E-08	4.2637E-08	12	6	-2.3868E-08	-6.6770E-10
12	7	1.4507E-08	9.9784E-08	12	8	-5.7854E-09	3.3752E-08
12	9	-3.2232E-08	4.2898E-08	12	10	-1.8590E-08	-4.8382E-09
12	11	-4.4921E-08	-4.8206E-08	12	12	-1.9407E-08	-5.7771E-08
13	1	-5.6042E-08	2.6288E-08	13	2	-4.7456E-08	1.7367E-08
13	3	2.3833E-08	-2.8930E-08	13	4	-1.9980E-08	5.7030E-08
13	5	9.6637E-08	-4.7760E-08	13	6	-8.3417E-08	5.9782E-08
13	7	-5.2217E-08	-3.2562E-09	13	8	-4.1759E-08	-2.0231E-08
13	9	-2.5623E-08	1.0767E-07	13	10	8.6589E-08	-1.0528E-08
13	11	-3.3749E-08	5.8541E-08	13	12	-1.3229E-09	8.2192E-08
13	13	-7.0288E-08	7.4643E-08	14	1	-2.3090E-08	4.9664E-08
14	2	3.2120E-08	-4.5289E-08	14	3	1.9042E-08	1.1919E-09
14	4	7.8017E-09	-3.7527E-08	14	5	-2.5958E-08	-2.3344E-08
14	6	1.9140E-08	-5.8721E-08	14	7	1.1061E-08	8.4132E-09
14	8	-3.0273E-08	-6.0838E-08	14	9	4.9539E-08	9.2345E-08
14	10	5.3732E-08	-4.3168E-08	14	11	2.7833E-08	-8.1637E-08
14	12	1.2481E-08	-5.7314E-08	14	13	5.1554E-08	4.5453E-08
14	14	-5.2082E-08	-1.2840E-08	15	1	-3.5971E-09	4.0142E-08
15	2	-4.4833E-08	-1.6056E-08	15	3	8.3016E-09	-5.7218E-09
15	4	1.3916E-08	6.6644E-08	15	5	3.1684E-08	1.8250E-09
15	6	7.0020E-08	-1.1872E-07	15	7	1.1856E-07	4.2690E-08
15	8	-9.7657E-08	-3.5710E-08	15	9	2.2064E-08	2.6632E-08
15	10	-2.0648E-08	5.3724E-10	15	11	-3.2585E-08	9.4052E-08
15	12	1.0524E-08	6.8726E-09	15	13	-3.7348E-08	4.0249E-09
15	14	1.2193E-08	-2.6786E-08	15	15	1.4515E-09	-1.4802E-08
16	1	-2.3789E-08	7.6413E-08	16	2	2.1327E-08	3.0669E-08
16	3	-4.7358E-08	3.2610E-08	16	4	-1.1591E-08	4.3001E-08
16	5	-4.4201E-08	3.2230E-08	16	6	-5.8439E-08	-4.2809E-08
16	7	1.0591E-07	8.1008E-09	16	8	-8.4738E-08	-2.4677E-09
16	9	9.0001E-09	-1.0628E-07	16	10	-2.9849E-08	-5.2467E-10
16	11	6.8502E-09	-7.0765E-08	16	12	2.2834E-08	-3.4087E-08
16	13	3.5475E-08	2.0683E-08	16	14	-7.3590E-09	-2.2626E-08
16	15	-3.5485E-08	8.4126E-10	16	16	-2.9522E-08	8.6217E-09
17	12	8.3097E-08	3.5424E-09	17	13	3.2749E-08	4.2920E-10
17	14	-1.6058E-08	2.7286E-08	18	12	1.1662E-08	8.4724E-09
18	13	4.6903E-09	-3.5547E-08	18	14	-2.7446E-08	-4.8376E-08
19	12	6.7115E-08	-8.2623E-09	19	13	3.3201E-08	-6.3128E-08
19	14	-3.9779E-09	-2.3817E-08	20	13	5.8374E-08	3.3320E-08
20	14	1.1130E-08	-1.6183E-08	21	13	3.6928E-09	-1.6288E-08
21	14	5.2067E-08	3.0801E-10	22	14	-8.0549E-09	2.6440E-08

*Fully Normalized.

XI. THE ADOPTED REFERENCE SET

6. Lunar Parameters

Mean radius	1738.09 km
Principal axes	
a	1738.57 km
b	1738.21 km
c	1737.49 km

7. Harmonic Coefficients of the Moon's Potential*

$$C_{20} = -0.207108 \times 10^{-3}$$
$$C_{30} = 0.210 \times 10^{-4}$$
$$C_{22} = 0.20715 \times 10^{-4}$$
$$C_{31} = 0.34 \times 10^{-4}$$
$$C_{32} = 0.1012 \times 10^{-4}$$
$$S_{32} = 0.0679 \times 10^{-4}$$
$$C_{33} = 0.2583 \times 10^{-5}$$
$$C_{41} = -0.1284 \times 10^{-4}$$
$$S_{41} = 0.1590 \times 10^{-4}$$

*Unnormalized.

XI. THE ADOPTED REFERENCE SET

8. Radii of the Sun and the Planets

Body	Equatorial Radius, km
Sun	695992
Mercury	2435
Venus	6052
Mars	3393.4
Jupiter	71372
Saturn	60401
Uranus	23535
Neptune	22324
Pluto	7016

9. Planetary Masses

Body	Reciprocal Mass
Sun	1
Mercury	5,983,000
Venus	408,522
Earth and Moon	328,9[illegible]1
Mars	3,098,700
Jupiter	1047.3908
Saturn	3499.2
Uranus	22930
Neptune	19260
Pluto	1,812,000

XI. THE ADOPTED REFERENCE SET

10. The Geometric Flattening, f, and Corresponding Derived Values for the Dynamical Form-Factor, J_2, for Certain Planets

Body	f, observed	J_2, derived
Mars	0.0105	0.0055
Jupiter	0.0667	0.0161
Saturn	0.105	0.022
Uranus	0.0625	0.023
Neptune	None	

11. The Dynamical Form-Factor, J_2, and Corresponding Derived Values for the Geometric Flattening, f, for Certain Planets

Body	J_2, observed	f, Derived
Mars	0.00197	0.00525
Jupiter	0.0296	0.0878
Saturn	0.027	0.115
Uranus	None	
Neptune	0.0035	0.0177

12. Auxiliary Quantities

Conversion Constants

1 international foot	= 0.3048 meters
1 statute mile	= 1.609344 kilometers
1 new international nautical mile	= 1.852 kilometers
1 pound	= 0.45359237 kilograms

These conversion factors are taken to be exact.

TABLE III

ASTRODYNAMIC PARAMETERS FOR MANNED FLIGHT COMPUTATIONS

1. Fundamental Constants

Speed of light in km/sec	299792.5
Equatorial radius for earth in km	6378.165
Geocentric gravitational constant (units: $km^3 s^{-2}$)	398603.2
Measure of 1 AU in km	149599000
Flattening factor for Earth	1/298.3
Lunar gravitational constant (units: $km^3 s^{-2}$)	4902.778

2. Zonal Harmonic Coefficients of the Earth's Potential*

J(2)	1082.3
J(3)	-2.3
J(4)	-1.8

3. Mean radius of the moon 1738.09 km

4. Harmonic Coefficients of the Moon's Potential†

$$C_{20} = -0.207108 \times 10^{-3}$$
$$C_{22} = 0.20716 \times 10^{-4}$$
$$C_{30} = 0.21 \times 10^{-4}$$
$$C_{31} = 0.34 \times 10^{-4}$$
$$C_{33} = 0.2583 \times 10^{-5}$$

*Unnormalized, in units of 10^{-6}.
†Unnormalized.

TABLE IV

A SET OF ASTRODYNAMIC PARAMETERS WHICH HAS BEEN USED FOR DEFINITIVE ORBIT DETERMINATION

1. Fundamental Constants

Speed of light in km/sec	299792.5
Equatorial radius for Earth in km	6378.166
Geocentric gravitational constant (units: $km^3 s^{-2}$)	398604.6
Ratio of the Masses of the Earth and Moon	81.30
Measure of 1 AU in km	149598600
Flattening factor for Earth	1/298.25
Ratio of masses of Sun and Earth plus Moon	328903
Mean distance of Moon in km	384400

2. Zonal Harmonic Coefficients of the Earth's Potential*

$J(2) = 1082.48$

$J(3) = -2.56$

$J(4) = -1.84$

$J(5) = -0.06$

*Unnormalized, in units of 10^{-6}

TABLE IV (Continued)

A SET OF ASTRODYNAMIC PARAMETERS WHICH HAS BEEN USED FOR DEFINITIVE ORBIT DETERMINATION

3. Tesseral Harmonic Coefficients of the Earth's Potential*

l	m	$\bar{C}_{lm}$	$\bar{S}_{lm}$
2	2	2.45	-1.52
3	1	2.15	0.28
3	2	0.97	-0.91
3	3	0.57	1.65
4	1	-0.50	-0.58
4	2	0.27	0.67
4	3	1.00	-0.17
4	4	-0.47	0.47
5	1	0.03	-0.12
5	2	0.61	-0.31
5	3	-0.30	-0.12
5	4	-0.51	0.13
5	5	0.20	-0.41
6	1	-0.09	0.19
6	2	0.16	-0.48
6	3	-0.02	-0.14
6	4	-0.26	-0.26
6	5	-0.12	-0.74
6	6	-0.43	-0.43
7	1	0.03	0.01
7	2	0.30	-0.20
7	3	0.35	0.07
7	4	-0.47	-0.24
7	5	0.06	0.03
7	6	-0.44	-0.29
13	13	-0.03	0.11
15	13	-0.06	-0.06
15	14	0.01	-0.03

*Fully Normalized.

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