

## 1

**BASIC PROPERTIES AND VARIABILITY**

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**INTRODUCTION TO GIANT AND SUPERGIANT M, S, AND C STARS**

The most diverse of any spectral class, M stars range from tiny objects barely able to burn core H—and some which probably cannot—to dust-enshrouded supergiants. In mass, they range from a few hundredths of a solar mass to several tens of solar masses and, in radius, from hundreds of thousands of km to several AU. Spanning the evolutionary spectrum, M stars range from pre-main-sequence dwarfs to stars ready to end their asymptotic giant branch (AGB) stage with a transition to a supernova or planetary nebula. Atmospheres of M stars likewise include a wide range of conditions and phenomena described in this and subsequent chapters. As unevolved low-mass stars, main-sequence M stars form a class quite distinct from the red-giant stars that occupy most of our attention, and *these dwarfs are treated exclusively in Chapters 9 and 10.*

Because of the unusual variety of structures and processes that appear near the end of nuclear burning and the additional complications of binarity, it is not easy to find common characteristics of the M giant and supergiant stars and their numerous relatives of peculiar chemical composition. The authors prefer to reserve

the term “cool” for these objects, even though that term is also sometimes applied to the F, G, and K stars, which are so different that we would call them by comparison “warm” or “intermediate” stars. Among these cool giant and supergiant stars—distinguished firstly by spectroscopy into the classes M, MS, S, SC, C, R, N-type, CH, Ba, mild (or marginal) Ba, J stars, and more—are objects with a wide range in mass, composition, size, and evolutionary history. Collectively they are called red giants. They are red because they are cool (among the coolest stars known); their photospheres range in effective temperature from perhaps 5000 K for the warmest R stars to less than 2000 K for the coolest Mira-type variables, and some dust-enshrouded stars may be cooler still. Many reasons for their becoming giants have been elaborated in the scientific literature, but most of these have been shown to be wrong, and the discussion of that phenomenon continues (cf. Eggleton and Faulkner, 1981; Renzini, 1984).

Red-giant stars are of considerable interest for several reasons. Their unusual evolutionary states provide both a unique means to test, and a unique reason to generalize, stellar evolutionary theory (cf. Scalo, 1981; Iben and Renzini, 1983; Wood, 1985; Iben, 1984, 1985).

Their chemical composition is important to our understanding of nucleosynthesis and mixing and, coupled with mass loss, to the enrichment of the interstellar medium. Their outer atmospheres contain unusual regimes of temperature and pressure that provide a series of nice challenges to theories of stellar atmospheres, both static and dynamic. Eventually, such processes as grain formation, chromospheric heating, circumstellar chemical reactions, and mass loss must be quantitatively understood; the current state of affairs in these areas is reviewed in Chapters 2, 3, 4, 5, 6, and 8.

Ultimately, one would hope to be able to point to a red-giant star—Betelgeuse or  $\chi$  Cyg or IRC +10216—and describe in detail its characteristics and evolutionary history. Although this desirable goal is presently still out of reach, some broad outlines and details of specific processes and phases are available. Sometime after hydrogen core exhaustion, almost all stars, except the most massive, become red giants or supergiants. Although the exact cause of this transformation is still a matter of some dispute, there is no doubt that such stars develop an inflated atmosphere (including envelope) around a compact core.

Most or all of the red giants of interest in this book are intermediate-mass ( $1 \leq M \leq 9 M_{\odot}$ ) stars of Population I, and these are also the best studied. As core hydrogen is exhausted and the main-sequence stage ends, a compact helium core develops, and stars move rapidly to the right in the HR diagram (toward lower temperatures at approximately constant luminosity) and then upward on the red-giant branch (RGB). For the first time, some mixing to the surface of nucleary processed material occurs in all stars (the “first dredge-up”). Rather specific predictions of the results of this first mixing episode on the observed surface abundance are available (Iben and Renzini, 1983; Iben, 1984), and observations confirm the general correctness of these predictions (Lambert, 1981).

At some epoch, helium ignites in the core, either by a “helium core flash” in the degenerate cores of lower mass stars ( $M$  less than  $2.3$

$M_{\odot}$ ) or quietly in the nondegenerate core of more massive objects. Additional mixing (second dredge-up) may occur upon core helium ignition in more massive (4 to  $9 M_{\odot}$ ) stars. In any case, core helium ignition lifts the degeneracy, if any, and the star moves rapidly leftward on the HR diagram to a “helium burning main sequence.” After core helium is burned and a compact, electron degenerate core of carbon and oxygen is formed, helium burning continues in a shell, and the star again moves coolward and then sharply upward toward higher luminosities—the AGB phase of a star’s life. It is now powered by double-shell burning—an outer shell of hydrogen burning and an inner shell of helium burning. As the helium shell burns outward, it narrows and flickers (pulses), driving intermittent convection and leading to a third dredge-up. Cool S- and N-type carbon stars are presumably formed during this AGB phase (Iben and Renzini, 1983; Iben, 1984; Wood, 1985), which is also marked by dredge-up of heavy metals (s-process elements) in many stars. Inferring the details of these remarkable events by careful studies of elemental and isotopic abundances for all types of peculiar red-giant stars is presently an area of intense interest (Lambert, 1981, 1985; Tsuji, 1985b).

Of more direct connection to this series, mass loss becomes general in AGB stars (see Goldberg, this volume; Dupree, 1986). Finally, part or all of the AGB stars develop a superwind (Iben and Renzini, 1983) that leads to rapid mass loss, and the star becomes a planetary nebula. Exactly which stars become planetaries is, however, presently unclear. After the ejection of a planetary nebula, the remnant compact core is a white dwarf that simply continues to cool (forever), although some energy is released by hydrogen burning in the base of the atmosphere. A small fraction (25 percent) of white dwarfs undergo a final helium pulse and briefly return to the red-giant region of the HR diagram, perhaps as RCB stars (Iben, 1984, 1985).

Certain red-giant stars do not fit the above simple scheme for single intermediate-mass

Population I stars. The most notable mysteries are the barium (or Ba II) stars, so-called because of the enhanced line of barium at 4554 Å in their spectrum, for their low luminosity indicates an insufficiently advanced evolutionary stage to have mixed to the surface the observed excess of carbon and s-process elements. The discovery of their binary nature (McClure, 1983) neatly solved the problem; the barium stars were contaminated with mass thrown onto the star by its present white-dwarf companion at the time that companion was itself an AGB star. But at least some of the white dwarf companions have cooling times longer than the red-giant lifetime of these stars, and the situation is presently unclear. R stars pose a similar but tougher problem (Iben, 1984 and discussion by Wing). As with the barium stars, R stars are insufficiently luminous to have dredged up sufficient carbon to become carbon rich ( $C/O > 1$ ). Yet these show no more than average binarity (McClure, 1985), which would appear to eliminate that otherwise attractive hypothesis. Certain S and MS stars also may prove to be challenges to the present theory (Iben, 1985).

Important questions about red giants are: What is the source of neutrons for heavy element formation? What are the various correlations between these physical properties and others such as their galactic distribution, their spatial velocity, their age, their periods of variability, and the shape of their light curve? Is their mass loss steady, periodic, or sporadic? What mechanisms are responsible? How can mass loss be measured? What are the main criticisms of such measurements? How to progress about the evolution of these various types of stars that are concentrated on the right side of the top of the HR diagram? What fundamental physical data are necessary for improving the model of evolution? Are these data sufficiently accurate? What confidence can we have in such models? What is the place of these stars in present-day astrophysics?

Clearly, answers to all of these questions are not yet available, and of the remainder, we must carefully select our material. In this chapter, after a general overview, we stress the vari-

ability because virtually all the stars that are discussed are variable, and all the more since the variability is due to nonthermal processes, the subject of this series of Monographs. This chapter will mainly discuss *photometric variability*; spectroscopic variability is presented in the following chapter by M. Querci.

Particular questions that are addressed on variability include the following: What are the various characteristic times of the observed variations? Are they correlated with any other characteristic period intrinsic to the given object? To what radial or nonradial motions of matter do such variations correspond? How are the irreversible changes explained? Are the latter a sure sign of the stellar evolution of these stars which suffer them?

What should we recommend to people who want to help to disentangle these fundamental questions? In the present exploratory state of our knowledge of the red giants and supergiants and because the observed variations are so complex, every feature found to be variable must be noticed and analyzed, and all correlations must be tested in an attempt to list better the various motions of matter suffered by the different stellar layers. Simultaneously, we must endeavor to find the pulsational frequencies with methods adapted to the necessarily intermittent ground-based observations. Also, theoretical studies of the periodic phenomena of Miras, which seem to be replaced by chaotic or aperiodic phenomena in the semiregular and irregular variables, are to be pursued. To analyze the variability both observationally by monitoring some characteristic stars and theoretically by developing red-giant hydrodynamics (just beginning) would help to clarify the evolutionary problems raised in this part of the HR diagram, where many apparently diverse objects are found.

Basic properties of the M, S, and C variables—more specifically, abundance classification, luminosity, mass, radius, surface gravity, extension of the outer layers, effective temperature, abundances, space distribution, period or pseudoperiod, and shape of the light

curves and their variations—have been described in the past by many reviews: Bidelman (1956), Blanco (1965), Hack (1967), Beeckmans (1969), Spinrad and Wing (1969), Fujita (1970 and references therein), Vardya (1970), Alksne and Ikaunieks (1971, updated by Baumert in 1981), Baumert (1972), Rybski (1973), Wallerstein (1973), Barbaro and Dallaporta (1974), Boyarchuk (1974), Ikaunieks (1975), Keenan and McNeil (1976), Johnson (1978), Keenan and Boeshaar (1980), de Jager (1980), Fujita (1980), Feast (1981), Scalo (1981), Duerbeck and Seitter (1982), Alksne et al. (1983), McClure (1984), Jaschek (1985), and Wing (1985). Even this list is far from being exhaustive. Some reviews are especially devoted to C stars, and others are restricted to M stars, each author favoring a particular point. A few recent colloquia or workshops are completely devoted to such stars: “Physical Processes in Red Giants,” Erice in 1980; “Cool Stars with Excesses of Heavy Elements,” Strasbourg in 1984; and “Mass Loss in Red Giants,” Los Angeles in 1984.

In the following, we present the various classifications of these red-giant stars, their photometry and colors, their space motion and space distribution, and their intrinsic properties. I must apologize for showing my own biases and for omitting many important studies and reviews.

### Classifications

Much more varied than warmer stars, the M, S, and C stars can be classified in several ways. Classification by *spectra* is the most obvious and most common scheme. For the varied objects found among the red giants, classification by *chemical composition* or key *abundance ratios* is important. These objects can also be classified by *photometry* and *color*. Finally, since almost all red giants are variable in light, they are classified by *type of variability*. All of these must be considered to form a complete picture of a red-giant star.

Even before a clear understanding of their nature was available, red-giant stars were classi-

fied on the basis of their *spectra*. The normal red giants—the M stars—were classified according to the increasing strength of the TiO bands. In contrast, other stars have spectra dominated by such carbon-rich molecules as CN and C<sub>2</sub>. Quite early, it was realized that the tightly bound molecule CO controls the spectra because CO forms in cool-star atmospheres until the less abundant of either carbon or oxygen (which are in turn the most abundant elements after hydrogen and helium) is used up. Whichever of the two elements is more abundant is then left over to form other molecules, which dominate the spectrum. The ratio of these two—the C/O ratio—thus becomes a crucial determinant of red-giant spectra. Other abundance parameters such as metallicity and enhancement of s-process elements (those heavy elements—Sr, Y, Zr, and Ba—produced primarily by slow neutron capture and beta decay) are also key classification criteria easily determined from spectra.

A didactic presentation of the M, S, and C stars and related objects and their classification through *abundance ratios* is given in Jaschek (1985). The relative position of the different groups of stars can be easily visualized in the tridimensional representation (Figure 1-1) based on the C/O ratio, the metal abundances, and the s-process abundances. The C/O abundance ratio is the primary criterion differentiating the oxygen-rich M (C/O < 1) and the S stars (C/O ≈ 0.8 to 1) from the carbon-rich stars (C/O > 1). The second gives the cool stars that are metal-deficient, such as the CH stars or the HdC stars, and the third distinguishes the degree of enhancement in the abundance of s-process elements.

Before giving a description of the main types of stars mentioned in Figure 1-1, let us recall that these stars are located between the Hayashi lines for fully convective stars of various solar masses (see Figure 40 in de Jager, 1980, adapted from published HR diagrams: e.g., from Scalo, 1976). Wood (1985) found an evolutionary sequence up to the AGB in the M → S → C, but the quantitative agreement between the theory of shell-flash mixing and observations is still

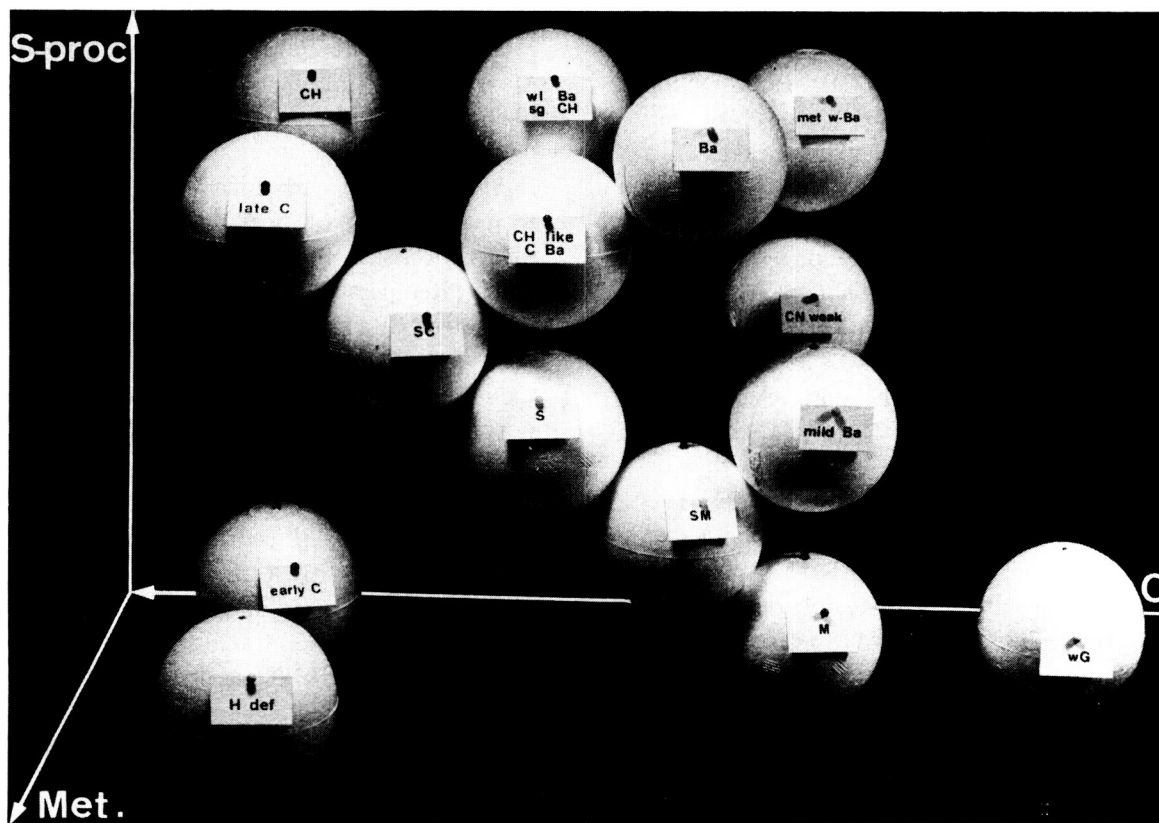


Figure 1-1. Main varieties of red giants, according to the three-dimensional representation described in the text (from Jaschek, 1985).

not satisfactory. Additional insight into the observational constraints on theories of mixing and nucleosynthesis during the advanced red-giant stages of evolution can be gleaned from several recent reviews (Scalo, 1981; Iben and Renzini, 1983; Iben, 1984, 1985). Although we have chosen not to discuss in detail the red giants in other galaxies, conclusions drawn from these stars in the Magellanic Clouds are illuminating, particularly with respect to their evolution, and a later section provides an overview.

The *M stars*, also called oxygen stars in parallel with carbon stars because the composition is basically oxygen-rich (solar  $C/O \approx 0.6$ ), are characterized by molecular absorption bands of such oxygen-rich molecules as TiO and VO in the visible. These bands are very weak in M0 stars; their strength increases with

later (cooler) types. In fact, M stars are classified from M0 to M8 primarily by the increasing strength of the bands of the alpha system of TiO from 4350 to 4950 Å (e.g., Keenan and McNeil, 1976; Wing, 1979; Keenan and Pitts, 1980). A few stars have even been classified M9 or M10. In the infrared, M stars are characterized by CO, OH, SiO, and H<sub>2</sub>O absorption bands. The 10- $\mu$ m region shows the emission of silicate grains, which are discussed in Chapters 2, 4, and 5. Maser lines of CO, OH, H<sub>2</sub>O, and SiO are observed in the coolest M Mira stars.

Among the M variables, some belong to the regularly pulsating group of cool stars, the Miras (see the section on *Variability Types*). Although the Miras change their spectral subtype and magnitude during their light cycle, they occupy a rather distinct zone around the

giant and subgiant tracks. However, nonvariable M giants (see the section *Nonvariable Stars*), such as  $\beta$  And (M0 III), also fall in this region of the HR diagram. The OH/IR stars (i.e., very red objects with double-peaked 1612-MHz OH maser emission) are a continuation of M Mira stars to lower temperatures, larger masses and luminosities, and longer periods of pulsation (Duerbeck and Seitter, 1982). Their evolution is discussed by de Jong (1983); the most luminous ones (8 to 13  $M_{\odot}$ ) are to be identified with the core helium-burning supergiant stars, while those of 3 to 8  $M_{\odot}$  are AGB stars. Some of them are OH objects with undetected IR counterparts (Jones et al., 1982). They pulsate only weakly or not at all, and their mass-loss rate is as large as  $10^{-4} M_{\odot}/\text{yr}$  (Baud and Habing, 1983); they possibly reach the end of their evolution on the AGB (Herman, 1983). This is consistent with the very red colors observed by IRAS (Olson et al., 1984). If our current ideas of stellar evolution are right, these stars soon produce planetary nebulae (Olson et al., 1984).

The *C stars* (carbon stars) are distinguished by the presence of  $C_2$  Swan bands in their spectra (Keenan and Morgan, 1941; Keenan and McNeil, 1976; Fujita, 1980). They are further divided into *R-type stars* (the hotter ones) and *N-type stars* (the cooler ones), the criterion being the strength of the violet flux, which is very weak in N-type stars (Shane, 1928)—weaker than in corresponding M stars and weaker than blackbodies at the effective temperature. Historically, the R and N types were initially used in the HD catalog. The first to observe N stars was Secchi (1868). Later, Pickering (1898) identified a small group of C stars that differed from the N stars in displaying sensible flux shortward of 4500 Å, and Espin (1889, 1898) remarked that none of the stars observed by Secchi were from this group. In 1908, Pickering defined the R subtype (i.e., the present R type) by the extension of the continuum shortward of 4700 Å, the earlier stars having the larger extension. This deficiency of flux in N-type stars continues into the ultraviolet. Finally, the R and N types were

unified by the C<sub>x,y</sub> classification of Keenan and Morgan (1941). Let us note that, in the Keenan-Morgan classification, (C<sub>x,y</sub>), the first figure (0-9) following the “C” is a temperature index obtained from atomic lines; the second figure (0-9) is a carbonicity index obtained from the strength of the  $C_2$  bands. Underlying the C<sub>x,y</sub> classification was the assumption that x and y (temperature and carbonicity) were independent. If so, the C<sub>x</sub> classification would have arrayed the carbon stars in a temperature sequence parallel to that of M stars. Nature is not so simple, however, and the ambiguity of this classification has been amply demonstrated (e.g., Fujita, 1980) both observationally from variability studies (Eggen, 1972b), infrared broadband colors (Mendoza and Johnson, 1965), near-infrared narrowband colors (Baumert, 1972), and infrared spectrophotometry (Goebel et al., 1978) and theoretically from calculations of the effect of carbon enhancement on opacities (Scalo, 1973) and atmospheric structure (Johnson, 1982) and on effective temperatures (Tsuji, 1981c).

The *early R stars* (R0 to R4) are termed “ordinary R stars” because their hydrogen lines have strengths similar to those in normal G9 to K2 giants of the same effective temperature. Vandervort (1958) gives the criteria for the classification of the R stars between R0 and R8.

In the carbon stars, the visible spectrum contains bands of the carbon-rich molecules CN,  $C_2$ ,  $C_3$ , SiC<sub>2</sub>, and CH instead of the oxide bands that characterize the M stars spectra, and these bands dominate the spectra of cooler stars—the late R (R5 to R9) and the N stars. Apart from the CO bands seen in all cool stars (though unexpectedly weak in the R0 to R4 stars), the infrared (1 to 5  $\mu\text{m}$ ) is increasingly chopped up by bands of CN,  $C_2$ , HCN, and  $C_2H_2$  toward later spectral types. Emission at 11  $\mu\text{m}$  due to graphite and SiC grains is often seen. In the radio frequencies, CO, CN, SiO, CS,  $C_2H$ , HNC, HCN,  $C_3N$ , and complicated molecules such as the cyanopolyne family ( $HC_{2n+1}N$ ) or  $CH_3CN$  are detected in the extreme dust-enshrouded C stars.

The s-process elements are stronger in late carbon stars than in M stars (Utsumi, 1970), but the  $^{13}\text{C}$ -rich carbon stars (J stars) apparently show little or no enhancements of s-process elements with respect to Fe (Utsumi, 1985).

The spectra of N-type stars show a stronger absorption in the blue and the ultraviolet than do the R-type spectra. This classic problem, termed the "violet opacity," is discussed further under *Photometric Observations*. It seems that this degree of faintness in the ultraviolet region of N-type spectra is not entirely due to their lower temperature (de Jager, 1980). The absorption by molecular bands (particularly  $\text{C}_3$ ) and graphite and silicon carbide particles could be an important contributor to this observed faintness. Stars redder than  $B-V = 3$  are nearly always carbon stars, of which the reddest are the cool N types with  $B-V \approx 5$  or 6 (Wing, 1985); for comparison, the solar  $B-V$  value is 0.62, and M0 to M6 giants have  $B-V \approx 1.55$  (Johnson, 1966).

The early R stars, extensively described by Dominy (1982, 1984), have an abnormal oxygen/carbon ratio relative to the M stars by 0.7 dex; the nitrogen and oxygen are enhanced by only 0.2 dex. The iron abundance is near the solar value, except in HD 100764, which has an iron abundance 0.6 dex lower than the solar one. The s-process elements such as Y, Zr, Mo, Ba, La, Ce, Nd, and Sm are not enhanced in these hotter carbon stars. A bright member of this class, HD 156074, was well analyzed by several authors: Greene et al. (1973) for the Fe abundance, Wyller (1966), Gordon (1967), and Fujita and Tsuji (1976) for the  $\text{C}^{12}/\text{C}^{13}$  ratio, and Yorka (1981) for the N abundance. The elemental abundances in the early R stars are interpreted as products of mixing during the helium flash (Dominy, 1982; Wood, 1985). On the other hand, due to uncertainties both in observations (mainly in luminosity) and in evolution theory, Tsuji (1981c) remarks that it is difficult to decide whether N stars (N-type Miras are not considered) are produced just after the onset of He-shell flash. The carbon-star evolutionary "mystery," paraphrasing

Iben (1981b), has been studied specifically by this author (Iben, 1984) and by Lucy et al. (1986).

Beside these "classical" carbon stars, a class of stars that are carbon-rich but have the additional peculiarity of hydrogen deficiency—the so-called *hydrogen-deficient carbon (HdC) stars*—group together, following the convenient classification of Richer (1975): (a) the R Corone Borealis (RCB) stars; (b) the helium (He) stars; and (c) a few supergiant R stars (e.g., HD 182040) (see the section *Intrinsic Properties*) that appear to be nonvariable (McKellar and Buscombe, 1948; Bidelman, 1953; Warner, 1963, 1967) and are called the nonvariable HdC stars.

The group of HdC stars is deficient in hydrogen by a factor up to  $10^5$  or more compared to the H/Fe ratio in the Sun. The dominant constituent of their atmosphere is helium, although it is not spectroscopically dominant in all the HdC stars. Because no HdC star is known to be a close binary, the observed abundances can only be due to the ejection of the outer layers during the natural course of evolution. The lifetime of the HdC stage appears to be  $\sim 10^3$  years; if so, secular changes should be observable. It is proposed that these supergiants (see below) quickly evolve to white dwarfs (Iben, 1981a, 1985.).

The *RCB stars* are characterized, at maximum light, by narrow and sharp absorption lines of metals and very strong molecular absorptions such as CN in R CrB and RY Sgr, and  $\text{C}_2$  in S Aps and HV 5637. During minimum light and the subsequent increase, emission lines of helium (e.g., Querci and Querci, 1978) and metals usually appear in the spectrum. These stars have a surplus of carbon (by a factor of 3 to 10 (Warner, 1967)), as well as a surplus of helium, compared to the Sun. Earlier, Ludendorff (1906), Berman (1935), and Herbig (1949) remarked on these peculiar abundances in R CrB, and Bidelman (1948) showed these peculiarities in XX Cam. They are believed to be due to the loss of the hydrogen envelope (Sackmann-Juliana et al., 1974). Some differences exist between them; in XX

Cam, carbon is a little more abundant and hydrogen more deficient than in RY Sgr (Rao et al., 1980)— $\log [(C/H)_{RY\ Sgr} / (C/H)_{XX\ Cam}] \sim -1.16$ . Schönberner (1975) has estimated this ratio to be  $-0.9$ . The abundance in oxygen is also slightly different; it is 3 times more abundant in R CrB than in RY Sgr. Most RCB stars are members of the R type; some of them are known to have earlier type spectra, including the prototype star itself, which is of spectral class cGOep. In the HdC group, they are called cool HdC stars. The RCB are eruptive stars (and not pulsating stars, such as the “classical” C stars; see the section *Types of Variability*) with long periods; moreover, they are the only eruptive stars in which the quiescent state is the state of maximum brightness. Their minima are interpreted by ejections of obscuring matter (soot, particles of graphite, etc.).

*Helium stars* and RCB stars have several common properties, although the He stars have a higher temperature (leading to their being also called hot hydrogen-deficient stars, like BD + 13°3224 (Kilkenny and Lynas-Gray, 1982)), a larger gravity, and a smaller pulsational period than the RCB stars. MV Sgr appears to be a possible evolutionary link between these two classes (Hill, 1967) with its light variations like those of RCB stars and its spectrum like that of the helium star, HD 124448. Tutukov and Iben (1985) show theoretical evolutionary connections between them. Herbig (1964) noted that the strongest lines in the spectrum of MV Sgr are from He I and C II. The latter is somewhat weaker than He I lines, whereas in BD + 10°2179, the lines of the two atoms are of the same strength. Warner (1967) analyzed some nonvariable helium stars and found abundance characteristics like those of the nonvariable HdC stars. Envelopes around the helium stars were suspected by Hill (1964), such as around HD 168476, which appears like an A-type star with many lines of ionized elements of the iron group. This star shows that the excitation temperature for the ionized elements is much lower than that for the other elements, leading Hill to suggest that they are formed in a low-temperature shell. The immediate pro-

genitors of the helium stars are believed to be RCB stars (Schönberner, 1975). However, nothing is really known about their common origin and by what mechanism they have lost their initial hydrogen-rich atmosphere.

As for the *nonvariable HdC stars*, the stars HD 137613, HD 173409, HD 175893, and HD 182040 are hydrogen-deficient by factors of more than  $10^5$  compared to the H/Fe ratio in the Sun. The deficiency is around a factor of 50 in HD 148839; this last star shows C<sub>2</sub> and C I enhancement, while CH and Balmer lines are absent. It represents “an intermediate type between normal C stars and the more extreme hydrogen-deficient stars” (Warner, 1967). In all these stars, the carbon is overabundant by factors of 3 to 10, but C<sup>13</sup> is not observed at all. All elements heavier than oxygen have a roughly solar abundance relative to iron.

Among carbon-rich stars, we note other particular stars such as the *CH stars*, which are G5–K5 giants with strong CH molecular bands (G band at 4300 Å) and enhanced lines of s-process elements in their carbon-rich and metal-poor spectrum. They have high radial velocities and can be considered Population II carbon stars. Stars with a strong G band and low spatial velocity are also observed; they are called *CH-like stars*. Bidelman and McConnell (1973) identified a large number of stars with a very weak or absent G band, the so-called *weak G-band stars*; no correlation was found between the CH faintness and the line strength of metallic lines. The *CN-strong stars* are carbon stars with a metal overabundance and a slight enhancement of carbon (Schmitt, 1971).

Jaschek (1985) stated that “The *S-stars* constitute a natural bridge between M and C stars with all kinds of intermediates.” They are late (K5–M8) giants. These stars contain metals with roughly the same abundances as the M stars, but the enhancement of the heavy elements, including Zr, Y, Sr, Ba, and La, is very strong and consequently the ZrO bands are much more prominent than those of TiO. Oxides of yttrium, lanthanum, and vanadium are affected by the change in C/O between M and C stars.



The C and O atoms are fully trapped in CO; this reduction in the free oxygen supply as carbon is enhanced explains the presence of many atomic hydrides in the S star spectra. Their  $B-V$  colors are between  $-2$  and  $-3$ .

The presence of unstable isotopes such as  $^{99}\text{Tc}$  in the stellar spectrum of some S and C stars (Little-Marenin and Little, 1979; Smith and Lambert, 1985), indicates that the rising to the surface of the interior material has occurred less than about  $10^6$  years ago, which corresponds to a few half-lives of the  $\text{Tc}^{99}$  isotope, the longest lived of the Tc isotopes, under conditions representative of interiors of AGB stars (Iben and Renzini, 1983).

A very few C and S stars (T Sgr, WZ Cas, HR 8714, WX Cyg, T Ara, etc.) present a very strong absorption resonance line of lithium at  $\lambda 6707$  (see the surveys of Torres-Peimbert and Wallerstein, 1966, and Boesgaard, 1969). These stars are named *lithium-rich stars* (LRS). The derived abundance of lithium is up to  $10^5$  times the solar value in WZ Cas. To obtain the Li formation or destruction processes in such stars, two difficulties must be solved (Scalo, 1981): to infer the Li/Fe ratio and to infer the  $^6\text{Li}/^7\text{Li}$  ratio. The first ratio is only tentatively estimated (Merchant-Boesgaard, 1970; Wallerstein, 1977; de la Reza and Querci, 1978) because the  $\lambda 6707$  line is largely blended by CN red-system lines. The knowledge of the second ratio is not easy to obtain through the analysis of the lithium resonance atomic line because, besides the CN blending, the isotopic shift is smaller than the observed Doppler width of the stellar line. Some efforts have been made in the direction of the molecular isotopic shift by the development of computations of molecular abundances of LiH, LiOH, and LiO (Merchant, 1967; Querci and Querci, 1977) and of wavelength measurements in the laboratory (Plummer et al., 1984). Stellar observations of lithium hydride must be made to provide the desired ratio.

Hybrid objects are also observed; for example, the *CS* and *SC* stars have characteristics of both C and S stars (Stephenson, 1967b;

Catchpole and Feast, 1971), and the *MS* stars are intermediate between M and S types. Keenan and Boeshaar (1980) provide reference stars in the sequence MS through S and SC (sometimes called D-line stars (Gordon, 1967)) to C stars with weak  $\text{C}_2$  bands.

We conclude this description of the main types of cool stars by the *Ba II* stars (also named Ba or barium stars) that constitute another family of anomalous giant stars with strong Ba II features (especially the  $\lambda 4554$  line). They share with the early R stars (R0 to R4) the fact of being "warm" or "intermediate-temperature" stars (see the section on *Intrinsic Properties*). Most Ba stars have strengthened CN absorption (Wing, 1985) relative to normal G and K stars, although not as strong as in carbon stars. The s-process elements, especially those with  $38 < Z < 56$ , are enhanced (McClure, 1984; Smith, 1984). An estimate of the strength of the barium lines was added to the spectral type by Warner (1965). This group has no sharp boundaries (Jaschek, 1985). So-called semi-barium stars have s-process element abundances intermediate between those seen in the pure Ba II stars and the M stars. The stars for which the enhancement is not obvious are named "marginal barium stars." To designate the strength of the Ba II characteristics, Keenan and Pitts (1980) give a decimal scale that is often used. Catchpole et al. (1977) and Yamashita and Norimoto (1981) have reobserved the Ba II sample of MacConnell et al. (1972) and conclude that about one-third of these stars are weak-line objects, such as the subgiant CH stars defined by Bond (1974).

The lack of luminosity determinations is the main difficulty in placing the Ba stars on the HR diagram, but they are generally believed to have luminosities of normal giants. Until the recent discovery of their binarity (McClure et al., 1980; McClure, 1983), the evolutionary status of the barium stars was a puzzle because such apparently low-mass stars should not have been able to have mixed to the surface nuclearly processed material. It now appears fairly certain that the abundance anomalies seen in the real Ba stars are the result of mass transfer from

formerly more massive companions when the companions were in the asymptotic giant branch stage of evolution (Smith, 1984; Lambert, 1985). Whether the same holds true for mild or marginal barium stars is, however, still an open question (McClure, 1985). Wood (1985) suggests that the Ba II stars (and CH stars) are probably the result of mass transfer from an N-type star to a white-dwarf companion in a binary system.

To understand better the large variety of these giants and supergiants, which lie on the right side of the HR diagram and cover a large range on absolute visual magnitude (see the section on *Intrinsic Properties*), several other classifications based on criteria other than abundance ratios or temperatures are proposed (Eggen, 1972b; Feast, 1981; Feast et al., 1982). The importance of finding physically meaningful, significant classifications dividing the objects into homogeneous subgroups is stressed. Of the various attempts to classify these stars, the most famous classification, based on type of light variation, is the *General Catalogue of Variable Stars* (GCVS) (Kukarkin et al., 1958, 1976), which essentially divides the late-type variables into Miras, semiregulars (SR), and irregulars (L) according to their light-curve amplitude and profile. This point will be discussed in the section on *Variability Types*. As noted by Eggen (1972b), the GCVS is a poor guide to the type of variation of the N-type red variables, maybe because of their extreme ( $B-V$ ) colors. This author, from an extensive study of red giants, suggests a subdivision of the variables on the basis of their place in the luminosity/temperature plane (i.e.,  $(M_{bol}, R-I)$ -plane, adopting Johnson's (1966) relation between  $R-I$  and  $\log T_e$ ) and from their space motions linked to their age.

Feast (1981) sorts out the variables by a spectroscopic argument—the strength of the emission lines: (1) the variables with strong emission lines, called Xe stars, are Miras (i.e., large-amplitude variables); (2) the variables with weak emission lines at some phases, called X(e), are semiregulars (i.e., small-amplitude variables); and (3) the variables that do not

show emission lines, called X variables, are semiregulars or irregulars, where X denotes the spectral type M, S, C, CS, SC, etc. The basic idea is that the emission-line strength is linked to the extent to which the stellar layers are disturbed by the variability (see M. Querci, this volume). Feast et al. (1982) relate this division to mean infrared colors and define two reddening-free parameters  $b_1, b_2$ . The  $b_1, b_2$  colors suggest an evolutionary sequence M–M(e)–Me that is in fact a sequence of increasing light amplitude and decreasing temperature. The Mira variables that show OH or H<sub>2</sub>O maser emission are at the cold end of the  $b_1 - b_2$  plot, while the carbon stars show a large scatter that is also seen in the infrared photometric indices (Tsuji, 1985). The SC stars form a tight group, whereas the CS stars are scattered more widely. The main sources of infrared opacity, different in each stellar type, evidently have a strong effect on the infrared colors and cause the difference as to the region occupied by each type, as demonstrated by Feast et al. (1982).

The main purpose of establishing such empirical relationships is to provide a norm with which red variables in other galaxies (e.g., the Magellanic Clouds) or in other regions of our Galaxy than in the neighborhood of the Sun (e.g., in the galactic bulge) might be compared.

### Photometric Observations

After the pioneer astronomers who were fascinated by the red stars—let us note Espin, Hale, Kirch, Koch, Maraldi, Pickering, Secchi, Schjellerup, Shane, and Sidgraves—the first sustained observations of visual light curves of red stars began in the late 1930's at the Harvard College Observatory (Campbell, Hughes, Payne, Sterne, etc.). For more than 50 years, amateur astronomers (working together in such groups as AAVSO, AFOEV, GEOS, etc.) have joined their observational efforts to those of the professionals. More recently, the UBV system and its extension into the infrared have been extensively used for determining such physical parameters of the red giants and supergiants as

energy distribution, effective temperature, and infrared excesses and for showing correlations among them at various phases of the light curves (e.g., color/magnitude relation, variations of various indices such as  $(B-V)$ ,  $(I-K)$ , and  $(8\ \mu\text{m}-11\ \mu\text{m})$  indices with spectral class; see below). Astronomers who made important investigations in broadband photometry include: Bahng, Bergeat, Bessel, Bidelman, Blanco, Catchpole, Eggen, Feast, Johnson, Landolt, Leighton, Mendoza, Morgan, Neugebauer, Nicolet, Price, Smak, Stephenson, Walker, Westerlund, and Wood.

Table 1-1 gives broadband magnitudes and colors from the ultraviolet to the infrared for characteristic stars of different spectral and variability types. For  $\chi$  Cyg, an S Mira, it is worth noting that the magnitude variations are much larger in the ultraviolet than in the infrared region. (A specific example of variations in ultraviolet flux for  $\chi$  Cyg is shown in Figure 1-7, and other examples in the visual are given in Figure 1-8.) With the advent of more sensitive infrared detectors, broadband photometry across the entire spectrum became possible, and this material has been published and discussed in detail. Although readers are probably already familiar with these observations and it is impossible to review them here in any case, we note a few of those papers containing fundamental broadband photometric data for M giants (Johnson, 1966; Eggen, 1967; Johnson and Mitchell, 1975), carbon stars (Mendoza and Johnson, 1965; Eggen, 1972; Walker, 1979; Bergeat and Lunel, 1980; Noguchi et al., 1981), and Mira variable stars of all types (Mendoza, 1967; Catchpole et al., 1979). A useful review of photometry of all types of peculiar red giants is that of Wing (1985).

A persistent problem threading its way through the astrophysical literature is the violet flux deficiency of carbon stars. A normal M6 giant has a value of  $(U-V) = 2.43$  (Johnson, 1966), while  $\alpha$  Ori (M2 Iab) has  $(U-V) = 3.96$  (Table 1-1). By contrast, a warmer N-type carbon star such as TX Psc has  $(U-V) = 6.11$  (Table 1-1), and many carbon stars have even larger values. This rapid drop

of flux toward the violet was noticed long ago and became the basis for the distinction between the carbon stars with sensible flux shortward of  $4700\ \text{\AA}$  (R stars) and those with no short wavelength flux (N-type stars) (e.g., Shane, 1928). On the basis that the flux deficiency was due to an unknown violet opacity, the phenomenon is termed the "violet opacity" problem, which is still unsolved. Among several suggestions for the agent responsible, the leading candidates are the molecule  $C_3$  in the photosphere (McKellar and Richardson, 1955; Bregman and Bregman, 1978) or SiC grains in a circumstellar shell (Gilra, 1973; Walker, 1976). Observations with IUE demonstrate that the violet opacity extends into the ultraviolet as well (Querci et al., 1982; Johnson and O'Brien, 1983).

The widths of these broadband filters are too large ( $\geq 1000\ \text{\AA}$ ), however, for the analysis of the time behavior of the various bands of TiO, VO, ZrO, CO, CN,  $C_2$ , CH, and HCN in the visible and the near-infrared, and the observed variations are difficult to interpret quantitatively in terms of energy dissipated in the various stellar layers submitted to acoustic/shock waves. Consequently, intermediate-band photometries (with filter bandwidths in the range 200 to  $500\ \text{\AA}$ ), initially developed for investigations on A to G stars (i.e., with filters adapted to their spectral features), are used for the analysis of the warmer stars among M, S, and C giants and supergiants; such systems are: DDO, Strömngren, Geneva, and Vilnius systems.

In 1966, Wing (1967a) pointed to a spectral band of  $30\ \text{\AA}$  width near  $1.04\ \mu\text{m}$  that was free from serious atomic and molecular blanketing in M, S, and C stars. A new epoch was born: the variations of the continuum of these late-type stars could be followed. Narrowband filters adapted to the M star molecular bands appeared (Wing, 1971; Lockwood and McMillan, 1971). White and Wing (1978) classified M supergiants; Baumert (1972) used the Wing system to measure the CN absorption versus the continuum of C stars; Wing and Stock (1973) identified carbon stars in stellar

Table 1-1  
Broadband Magnitudes for Characteristic Stars

	0.36 $\mu$	0.45 $\mu$	0.56 $\mu$	0.67 $\mu$	0.87 $\mu$	1.20 $\mu$	2.3 $\mu$	3.3 $\mu$	4.8 $\mu$	8.6 $\mu$	10.8 $\mu$	12.2 $\mu$	18.0 $\mu$
<u>M stars:</u>													
R LMi	(1)	11.50	11.12	9.42	5.40	2.24	0.82	-0.48	---	---	---	---	---
Mira, M7e-M8e	(2)	---	---	---	---	---	---	-0.7	-1.4	-1.8	-3.0	-2.8	-3.2
$\alpha$ Ori	(3)	4.36	2.29	0.40	-1.19	-2.47	-2.93	-4.01	---	---	---	---	---
SRc, M1-2lab	(2)	---	---	---	---	---	---	-4.2	-4.5	-4.3	-5.7	-5.5	-5.7
BC Cyg	(3)	16.17	13.23	9.97	6.22	3.44	2.03	0.17	-0.44	---	---	---	---
Lc, M4	(2)	---	---	---	---	---	---	0.4	-0.2	-0.1	-3.2	-2.7	-3.5
<u>C stars:</u>													
U Cyg	(4)	16.30	11.78	8.47	6.32	4.54	3.21	1.05	0.05	---	---	---	---
Mira, C7,2e-C7,9	(2)	---	---	---	---	---	---	1.5	0.7	0.0	-1.5	-1.0	-1.3
V Hya	(5)	---	---	---	---	---	---	---	---	---	---	---	---
SRc, C6,4e	(2)	---	---	---	---	---	---	-0.9	-2.1	-2.5	-4.2	-4.2	-4.2
TX Psc	(6)	11.15	7.65	5.04	3.17	1.82	0.92	-0.56	-0.89	---	---	---	---
Lb, C6,2	(2)	---	---	---	---	---	---	-0.7	-1.2	-0.9	-1.7	-1.3	---
	(7)	---	---	---	---	---	0.73	-0.23	-0.66	-1.02	---	---	---
<u>S stars:</u>													
$\chi$ Cyg	(8)	7.01	6.05	4.23	1.81	0.01	-0.91	-2.43	-2.98	---	---	---	---
Mira, S7le-S10le	(1)	12.53	12.19	10.26	5.55	1.99	-0.05	-1.84	-2.60	---	---	---	---
	(8)	15.49	14.68	13.00	6.90	2.75	0.22	-1.73	-2.32	-2.9	-3.3	-4.3	-4.6
	(2)	---	---	---	---	---	---	-2.5	-3.1	-3.1	-4.5	-4.3	-4.6

(1) Mendoza (1968), (2) Hackwell (1972), (3) Lee (1970), (4) Mendoza and Johnson (1965), (5) Olson and Richer (1975), (6) Johnson et al. (1966), (7) Walker (1980), (8) Mendoza (1967).

fields, Piccirillo (1977, 1980) investigated the S stars with the ZrO and LaO filters, and Yorke (1983) published data on R stars. However, let us note that some of the filters adapted to selected molecules are weakly contaminated by other molecules.

Unfortunately, the observations on the other spectral intervals such as X rays and far-UV, far-IR, and radio frequencies are scarce.

The fingers of one hand suffice to count the Einstein *X-ray* satellite observations of red giants and supergiants (Ayres et al., 1981). For the M supergiants,  $\alpha$  Ori (M2 Iab) and  $\alpha$  Sco (M1 Ib + B), values of  $f_x/f_{bol}$  are respectively  $<0.03 \times 10^{-7}$  and  $<0.02 \times 10^{-7}$ , and for the bright giant,  $\beta$  Peg (M1 II-III), this ratio is  $<0.2 \times 10^{-7}$ . This is consistent with the IUE observations and demonstrates that there is no stellar coronal ( $T \geq 10^6$  K) radiation observed from such stars. In this frame, perhaps the most important result of the observations of soft X rays is the discovery, with the Einstein spacecraft, of a rather sharp dividing line in the HR diagram. For giants, those stars hotter than about  $V-R = 0.80$  (K0 III) emit soft X rays in detectable amounts, while those cooler do not (Ayres et al., 1981). This coronal dividing line (the X rays are presumed to arise from a hot corona) falls close to the temperature dividing line defined by the appearance in the ultraviolet spectra of giant stars warmer than approximately K0 of lines of C IV, indicating the presence of a plasma with  $T \geq 10^5$  K. EXOSAT detections are actually made on Miras with a hot companion (R Aqr and CH Cyg), individual nonvariable M giants ( $\beta$  And), Miras (RS Aqr), and protoplanetary nebulae (V1016 Cyg, HM Sge, and RR Tel). It was demonstrated by Viotti et al. (1985) that the weak X-ray emission at Mira maximum is not correlated with the Mira-type variations. Detection of X-ray emission in any single M, S, and C star would certainly warrant further investigation, since it would clearly be an exception and so little is presently known about the chromospheres of these objects.

Only disparate attempts on *far-UV* flux measurements exist. The Orbiting Astronomi-

cal Observatory (OAO-2) UV spectrometer observed only the M supergiant,  $\alpha$  Ori: no variations were detected on the seven scans (Code and Meade, 1979). On the other hand, Code et al. (1980) gave UV magnitude between 1330 and 4250 Å for five M variable and nonvariable giants and supergiants. Unfortunately, the large bandwidth required for OAO-2 photometry by the rapidly decreasing fluxes toward the ultraviolet and the complex spectra of these stars made the flux determination ambiguous. With the Astronomical Netherlands Satellite (ANS), Wesselius et al. (1982) have observed eight M giants and supergiants, some of them many times. The star  $\alpha$  Ori, registered six times, appears to be variable; however, as the pointing precision was 1 arc-minute, two or more stars could be present in the field of view, and the observed variability cannot be conclusive. On the basis of three observations, the authors conclude that HD 100029 (M0 III) is not variable. A probable hot UV-bright companion was found on HD 216131 (M2 III).

In their comparison between the current UV photometric and spectrophotometric systems used for late-type stars, Kjaergaard et al. (1984) point out: (a) a strong nonlinearity at low flux level for the TD1 2635 Å band; (b) a strong color effect on the TD1 2740 Å band, as well as on the OAO-2 2950 Å band, due to red leaks in the sensitivity functions; and (c) a strong influence of the scattered light of the late-type stars on OAO-2 spectrophotometry. Only the ANS photometry and the International Ultraviolet Explorer (IUE) LWR spectrophotometry are found to be without large systematic errors. Their Table 2 gives several ( $UV-V$ ) indices for stars of various spectral types and luminosity classes. Unfortunately, this table gives no data for giants with  $(B-V) > 1.5$  (K5) and for supergiants with  $(B-V) > 1.8$ ; therefore, it is applicable only for the hotter R and Barium stars, keeping in mind uncertainties in the data as commented by the authors. Moreover, the presence of emission lines such as Si II  $\lambda\lambda$ 1808, 1817 or the Mg II doublet may cause uncertainties in these indices. Only the three M giants or supergiants, HD 6860 (M0 III),  $\alpha$  Ori (M1

Ia), and HD 217906 (M2.5 II-III), are observed with the satellites TD1, OAO-2, and ANS; the accuracy of the measurements does not permit quantitative comparisons.

A new era in research on outer atmospheres and chromospheres dawned with the launch of IUE, for it made possible fairly routine observations of ultraviolet spectra in either a short-wavelength (1150 to 2050 Å) or a long-wavelength (1800 to 3250 Å) band. Most studies of IUE spectra have focused, naturally enough, on the emission lines, and these results are discussed in Chapter 2 (observations) and Chapter 8 (chromospheric modeling). It became clear from the earliest ultraviolet observations that, in the cooler part of the HR diagram, spectra of giants and supergiants were of two distinct types, depending on the presence or absence of lines of stages of high ionization. All stars show emission lines from such easily ionized elements as Mg II and Fe II (the archetypal line being Mg II  $\lambda$ 2800), which are representative of a warm chromosphere (6000 to 15000 K). Those stars to the left of an almost vertical line near K0 also show lines from highly ionized elements (the archetypal line being C IV  $\lambda$ 1550 Å) representative of a hot ( $\sim 10^5$  K) plasma; those to the right of this "temperature" dividing line show no high-ionization lines (Linsky and Haisch, 1979). The asymmetry of the emission lines (Mg II h and k) further indicates an outflow of material in the cooler giants (Stencel and Mullan, 1980), and this "mass-loss" dividing line lies close to the temperature dividing line. As described above, observations with the Einstein spacecraft of X rays reveal a similar dividing line in the HR diagram between stars with (to the left) and without (to the right) soft X-ray emission (Ayres et al., 1981), which X-ray dividing line is also close to the temperature dividing line and the mass-loss dividing line.

These observations accord with and greatly expand previous knowledge that violet emission lines of Fe II (multiplets 1, 6, and 7) are practically ubiquitous in the spectra of M giant stars (Boesgaard and Boesgaard, 1976) and are even seen in N-type carbon stars (Bidelman and Pyper, 1963), but are not seen in warmer stars,

while the high-excitation 10830 Å line of He I is almost universally absent from single red-giant stars (Zirin, 1982). The common interpretation of these results, supported by many observations and based on a fair amount of modeling, is that giant stars warmer than about K0 are solar-like in having warm chromospheres and hot coronae, while cooler giants have warm chromospheres and outflowing matter (Linsky, 1980, 1982; Brown, 1984; Linsky, 1985). Except for binary stars, then, we expect the red giants of interest here, all of which, except perhaps for the warmest R stars, lie clearly to the right of the dividing lines in the HR diagram, generally to show Mg II in emission in some strength and evidence of mass loss but no spectral lines from several-times-ionized elements.

Stickland and Sanner (1981) observed the far-ultraviolet continua of 15 late K and M giant and supergiant stars (as late as M6) with IUE and obtained radiation temperatures at 1850 and 1250 Å. Remarkably, the radiation temperature at 1850 Å of all stars was close to 3400 K. They also demonstrated that this flux originates in the chromosphere. Several studies have found a general decrease in the fraction of ultraviolet flux in both emission lines and continuum, relative to bolometric flux, as one goes to later spectral types among the M giants. IUE has confirmed and delineated this trend down to M6 for stars with angular diameters from lunar occultation (Steiman-Cameron et al., 1985).

Along with numerous G and K giants, several M giants and supergiants have been observed with IUE. Representative spectra of three M giants and one supergiant ( $\alpha$  Ori) are shown in the valuable atlas by Wing et al. (1983). When line profiles of Mg II are available, the usual procedure is to construct a temperature profile through the chromosphere. Even then, no attempt has yet been made to reproduce by any empirical or theoretical model the flux of other lines or the continuum. Average properties of the chromospheres can be deduced from lines of Fe II and C II. A study

of 14 noncoronal G8–M3 giants, for example, leads to temperatures of 7000 to 9000 K and electron densities of  $\sim 10^8 \text{ cm}^{-3}$ , and these appear to be independent of spectral type (Carpenter et al., 1985). Most of the effort in warmer (F, G, and K) stars has been directed toward linking the observed line emission to other stellar parameters such as mass, luminosity, age, or rotational velocity. Although the M, S, and C stars of interest here have often been neglected, progress can best be judged from several recent reviews (Linsky, 1980, 1982, 1985; Brown, 1984; Böhm-Vitense and Querci, 1986).

More attention has been given to Betelgeuse than any other star of interest here; it is often taken as the archetype red giant. Although *spectral lines* are discussed more fully in Chapter 2, we mention here some of the results so far derived. Analyses of the Mg II h and k lines, along with the Ca II H and K lines, leads to a temperature profile of the upper photosphere and lower chromosphere (Basri et al., 1981). Chromospheric densities can be deduced from the relative strengths of the lines in the C II (UV 0.01) multiplet (Stencel et al., 1981; Carpenter, 1984). The extent of the chromosphere can then also be deduced from the absolute intensities of the C II lines. For Betelgeuse, the chromosphere has a temperature of 6000 to 8000 K and a thickness on the order of the radius of the star. The variation of relative velocity within the chromosphere can be obtained from a study of the line shifts of the numerous Fe II lines (Carpenter, 1984).

Ultraviolet observations of other M, S, and C stars we summarize briefly. Barium stars show strong emission lines of Mg II similar to K giants of the same temperature. Early R stars show no ultraviolet chromospheric indicators, while later R stars (R8) have weak emission lines of Mg II and Fe II (Eaton et al., 1985). S stars apparently show a large range in strength of chromospheric emission, but very little has been published. One special use of IUE is to search for hot companions to M, S, and C stars. Several types of these peculiar red giants have been observed in the short-wave-

length region for possible white-dwarf companions, and a few have been discovered (Dominy and Lambert, 1983; Böhm-Vitense and Johnson, 1985).

The first attempt to observe N-type carbon stars was negative (Querci et al., 1982): neither emission lines nor continuous flux were recorded; the observed flux was lower than  $\approx 10^{-16} \text{ erg/cm}^2 \text{ \AA}$  ( $3\sigma$  flux limit). Later observations with IUE detected weak UV spectra, with some continuum down to at least 2850 Å, for seven bright N-type carbon stars (Johnson and O'Brien, 1983; Querci and Querci, 1985; Johnson and Luttermoser, 1986). Longward of 2850 Å, the spectrum is basically an absorption spectrum—apparently the ultravioletward extension of the photospheric spectrum; shortward of 2850 Å, only emission lines (C II, Mg II, Al II, and Fe II) are detected. The continuous flux is, however, much weaker than in M giants, which implies either that the cool C stars have weak chromospheres (low temperatures or densities) or that an important opacity source is located above the emission-line-forming region (see the conclusions obtained in this sense in Chapter 2). Yet the strength of the chromosphere (e.g.,  $L_{Mg}/L_{Bol}$ ) decreases rapidly with spectral type among the M giants. Will M7 or M8 giants, which have effective temperatures as low as those of N-type carbon stars, have chromospheres as weak? Why are chromospheres of carbon stars so weak? Is it due to lower temperatures or different chemical composition or more overlying absorbing material? *Observation time on the Space Telescope* is necessary to observe these stars with enough light signal and at various phases to make decisive progress on this problem.

A violet-ultraviolet color-color diagram (Mg II–V vs. IUE–V) has been developed and used to compare spectrophotometric aspects of IUE–LWR observations for K, M, S, and C stars (Eaton et al., 1985), as is shown in Figure 1-2 (see also the section on *Intrinsic Properties*). As anticipated, red giants and supergiants have low UV continuous fluxes and display lines from only such low ionization stages as Mg II and

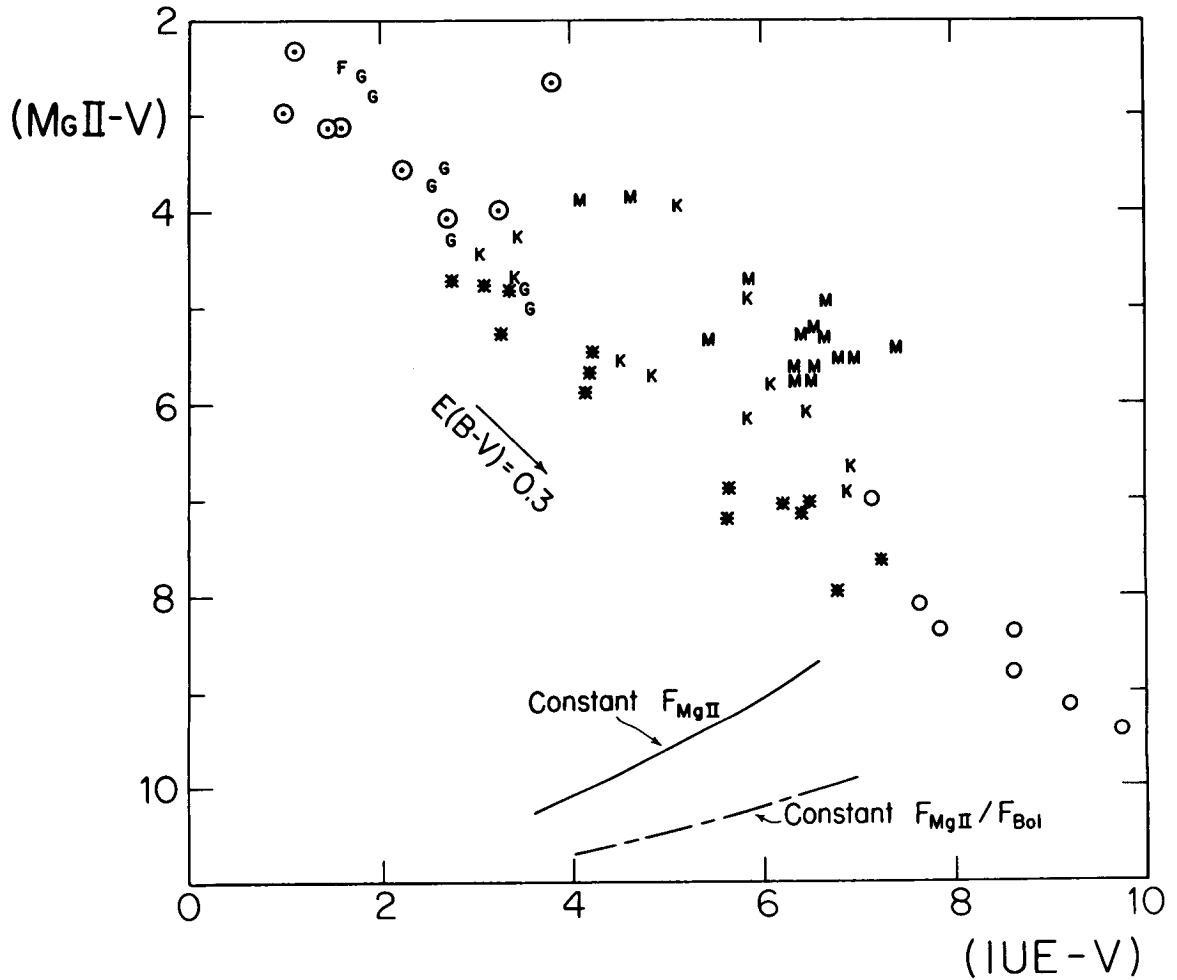


Figure 1-2. A two-color diagram for stars showing an Mg II index (measuring ratio of the flux in a  $30 \text{ \AA}$  band at  $2800 \text{ \AA}$  to the flux at V) against an IUE color (the ratio of flux between  $2585$  and  $3200 \text{ \AA}$  (Mg II omitted) to the flux at V). G and K dwarf stars are shown as circled dots, G, K, and M giants and supergiants as letters, R stars as asterisks, and N stars as open circles (from Eaton et al., 1985).

Fe II (see Chapter 2), indicating the lack of a transition region.

On the low-energy side, (i.e., the far-IR), the opaqueness of the terrestrial atmosphere necessitates the use of satellites. The outstanding of these—the Infrared Astronomical Satellite (IRAS), a joint project of the United States, the Netherlands, and the United Kingdom—produced a wealth of valuable data during 1 year of use (Neugebauer et al., 1984). Another one, the Infrared Space Observatory (ISO), is planned to be launched by the European Space Agency (ESA) in 1992 to perform

imaging, spectroscopic, photometric, and polarimetric investigations in the IR spectral range from  $3$  to  $200 \mu\text{m}$  during an operational lifetime of about 18 months.

The data of IRAS are presently under reduction; the first results on cool giants were presented at the University of California at Los Angeles, meeting, "Mass Loss from Red Giants," held in June 1984, and at the Calgary Workshop in 1986. Physical parameters of the stellar outer layers are deduced. The low-resolution spectrometer (LRS) made spectra



between 8 and 22.5  $\mu\text{m}$  with a spectral resolution varying between 15 and 30. A continuous sequence is found by Olton et al. (1984) from classical Miras to the very red Miras like OH 17.7 - 2.0 or OH 26.5 + 0.6, which are very cool objects (Werner et al., 1980); only the outermost dust shell is visible at all LRS wavelengths, and no absorption features are observed. As it is already known from the radio observations (Smolinski et al., 1977), Stickland (1985) found no contribution to the IR excess from free-free radiations on hypergiants. Analyzing the IRAS data of many RCB stars, Schaefer (1985) finds a bimodal distribution among them (see the section *Eruptive Variables: The RCB Stars*). Maps at 60 and 100  $\mu\text{m}$  around M supergiants are also obtained with IRAS. For  $\alpha$  Ori, "shells" appear that are not centered on the star. Are these shells ejected remnants or parent cloud to  $\alpha$  Ori (Wesselius, 1984)?

Long-term monitoring of a few oxygen-rich Mira variables has already begun to provide information about the temporal variation of the peak intensity and the shape of some radio maser lines (Barcia et al., 1985); their profile and their three-peak structure are in agreement with theoretical calculations (Zhou and Kaifu, 1984). Moreover, the mean intensity ratio between the  $\nu = 1$  and  $\nu = 2$  SiO lines seems to agree qualitatively with the theoretical predictions of Bujarrabal and Nguyen-Q-Rieu (1981). Let us also point out some scarce radio flares detected on  $\sigma$  Cet (Boice et al., 1981), on R Aql (Woodsworth and Hughes, 1973; Bowers and Kundu, 1981), on V Cyg (Querci et al., 1979), and on  $\alpha$  Ori (Kellermann and Pauliny-Toth, 1966; Seaquist, 1967). We stress the importance of monitoring radio, visible, and far-UV continuum for the detection of such nonthermal flares and the analysis of their energy distribution. The radio continuum spectrum of  $\alpha$  Ori obtained with the very large array (VLA) was fitted by a power law  $S_\nu \sim \nu^\alpha$  with  $\alpha = 1.32$  (Newell and Hjellming, 1982). A model of chromosphere has been deduced with an electron temperature of 10000 K and an extension of  $4 R_*$ .

In this brief review, we have summarized the first photometric attempts made in the far-UV and X rays and in the far-IR and radio frequencies to show the relative lack of monitoring in these spectral ranges and the urgency in fostering such observations. The reasons for the paucity of observations are mainly that: (a) the stellar flux is very low, and it can be registered with current instruments for only a few bright stars; and (b) the flux may be strongly variable (why not?), and the phase of the star's maximum has not always been considered.

### Space Motion and Space Distribution

Although detection of M, S, and C stars has mainly been due to objective-prism surveys, new techniques involving photoelectric spectral scanners or photography of star fields through narrowband filters are adopted to identify cool stars in crowded fields such as the fields of rich star clusters (Palmer and Wing, 1982). These filters are selected from Wing's (1971) eight-color photometry previously used to identify carbon stars (e.g., Wing and Stock, 1973) or to classify M supergiants (White and Wing, 1978).

In the solar neighborhood, only 1 percent of all the stars are giants, and the G, K, M, and correlated types of giants make up at most 5 percent of this 1 percent (Jaschek, 1985). The apparent and space distributions in the solar neighborhood are well known through early reviews on the question. M stars are found in large numbers in the galactic center direction (2000 per square degree according to Blanco, 1965), while the C stars avoid this direction. On the other hand, the C stars are strongly concentrated toward the galactic equator, compared to the late M stars. The majority of carbon stars appear to be spiral-arm objects. Among the carbon stars, the R stars have smaller concentration toward the galactic plane than the N stars; moreover, the R stars are found in great numbers where no N stars are present. The dispersion of the early R stars above the galactic plane indicates that they are representative of a population that is older and less massive than the

N stars (Stephenson, 1973). Alksne and Ikaunieks (1971) note an apparent concentration of later (R5–R9) stars toward the galactic equator similar to that for Mira and semiregular C stars, while type Lb (irregular variations of light) variables are concentrated more to the equator. The radial distribution and hydrogen deficiency appear to be correlated: R stars such as HD 100764 have low radial velocity (5 km/s) and do not suffer the severe hydrogen deficiency seen in supergiant R stars such as the high-velocity star, HD 182040, in which the Balmer lines are absent (Sanford, 1944).

The nonvariable HdC stars have strong similarities with the RCB (variable HdC stars): chemical composition, space distribution ( $|\mu_\alpha|$  from 0.008 to 0.024 and  $|\mu_\delta|$  from 0.010 to 0.035). These values of radial velocities and proper motions are the characteristics of high-velocity stars. With space velocities in excess of 400 km/s, these stars are members of the halo population. The He stars (hotter HdC stars) seem to be spread out farther from the galactic plane than the cooler ones, as is also indicated by their somewhat larger radial velocities (Warner, 1967). In summary, all the cool nonvariable HdC stars appear to be closely concentrated to the galactic center, whereas the RCB stars are not noticeably concentrated toward the galactic plane and galactic center. The He stars are widely distributed.

As for the M stars, if the late types (M7–M10) are concentrated in directions close to the galactic center, as quoted above, M5–M6 types are not as concentrated, and M2–M4 types are nearly equal in number in the galactic center and the anticenter directions. Moreover, the early M types present local groupings along the galactic equator. It is concluded by several authors (quoted by Alksne and Ikaunieks, 1971) that half or more of early M-type stars are members of the galactic spiral arms, while late M types belong to the disk population. An illustration is provided by the careful study of 722 M giant stars (with a few S and C stars) in the Perseus spiral arm (Ichikawa, 1981). As pointed out by Gehrz et al. (1981), OH/IR stars are strongly confined to the galactic plane and

are usually found at such large distances (6 to 8 kpc) (Baud et al., 1981; de Jong, 1983) that interstellar extinction at infrared wavelengths cannot be neglected. Space densities ( $n_o$ ) and effective heights of the space distribution ( $z_e$ ) from the galactic plane of late-type M giants in the solar neighborhood observed by various authors are listed by Mikami and Ishida (1981; see their Table 3). These authors compare them with their statistical estimates based on the Two-Micron Sky Survey data (Neugebauer and Leighton, 1969) and find them to be in good agreement; moreover, they determine the space density and distribution of M supergiants (Table 1-2).

S stars are as numerous in the direction of the galactic center (like the late M stars) as they are concentrated in the galactic spiral arms.

Discussing the kinematics and stellar population of the Ba II stars, Eggen (1972a) concludes that their space motions resemble those of an old disk population of 1.0 to 1.5  $M_\odot$  stars. In their sample of Ba II stars, Catchpole et al. (1977) found a difference in velocity dispersion from Ba-strong versus Ba-weak stars, which is explained by Rocca-Volmerange and Audouze (1979) as a reflection of an age-versus-abundance correlation.

Other investigations studied the distribution of late-type red giants in the direction of the central regions of our Galaxy and of other nearby galaxies, the Large and Small Magellanic Clouds, and other galaxies such as M31. Blanco et al. (1978, 1984) survey Baade's Window (that surrounds the globular cluster, NGC 6522), which constitutes a uniquely complete sample of the red-giant population of the galactic nuclear bulge. The striking feature is a very large number of M giants, but no C or S stars were found, even though they should have been detected had they existed. Such a result confirms a previous survey in the Sagittarius I region (Baade, 1951), in a direction closer to the galactic center than Baade's Window. Therefore, the ratio of the number density of carbon stars to the number density of M giants—the C/M ratio—is negligibly small in the galactic bulge.

**Table 1-2**  
**Space Densities and Distributions of Nearby M Giants and Supergiants\***

Spectral Type	Member Density ( $10^{-6}$ stars/pc <sup>3</sup> )	Effective Height (pc)
K0-K4	40.0	170
K5-M1	3.5	200
M2-M4	3.1	310
M5-M6	0.9	390
≥ M7	0.6	380
M supergiants	0.054	210

\*Adapted from Mikami and Ishida (1981).

The distribution of carbon and M-type giants in the Magellanic Clouds (Aaronson, 1984) and nearby galaxies is important for interpreting AGB evolution (e.g., Iben, 1984; Wood, 1985) and for understanding fundamental galactic properties, such as the metal abundance (C/M is strongly correlated with this quantity (e.g., Scalo, 1981)), the distance to the galaxy, and the full AGB luminosity function for the galactic field studied. Outstanding is the lack of late-type M stars in the central regions of the Small Magellanic Cloud (SMC), reminding us of the almost complete lack of C stars in the bulge of our Galaxy (Blanco et al., 1978). Also, differences in the C/M ratio between the SMC and the Large Magellanic Cloud (LMC) are noticeable. A survey of high degree of completeness for carbon stars and M-type giants in the Magellanic Clouds (Blanco and McCarthy, 1983) indicates that the ratio of the surface density of carbon stars and that of giants of type M6 or later in the SMC varies from  $19.2 \pm 0.8$  at the center to  $4.7 \pm 0.4$  in the periphery, while the C/M ratio is  $2.2 \pm 0.1$  throughout the LMC. It is suggested that the mixture of stellar populations is roughly uniform across the LMC, but not in the SMC. Recalling the negligible value of the C/M ratio in the galactic

nuclear regions, Blanco and McCarthy (1983) note that their population includes older and less massive red giants than those observed in the Magellanic Clouds. A recent review of the distribution and motions of red giants in the Magellanic Clouds is by Catchpole and Feast (1985).

A field in M31 has been investigated by Richer, Crabtree, and Pritchett as quoted by Richer (1985) in which C/M is about 0.12, implying a metal abundance higher than that in either the LMC or NGC 300, but lower than that in the galactic center. In addition, there appears to be a deficiency of luminous AGB stars as in the LMC.

To conclude on the spatial distribution, we note that carbon stars have been discovered in all dwarf spheroidal galaxies (Aaronson et al., 1983).

#### Intrinsic Properties

Red giants and supergiants (M, S, C, Ba II, MS, CS, CH, and CN stars, etc.) occur in a large range of stellar masses and ages because they are found in halo, old disk, and young disk populations. The observational data indicate that they have mixed nuclearely processed material from the core to the atmosphere for

luminosities such as  $-10 \leq M_{bol} \leq +3$ , ages from  $\sim 10^7$  to  $10^{10}$  yr, and a broad range of masses from  $\sim 1 M_{\odot}$  to  $\geq 20 M_{\odot}$ , with a mean mass of  $1.2 M_{\odot}$  (Scalo, 1976, 1984). A summary of the physical properties (luminosities, temperatures, masses, abundance of elements, population types, etc.) of red giants of the disk population, as well as their characteristic spectral features, is given in Table 1-3 (from Scalo, 1981).

**Absolute Magnitudes and Colors.** The N-type stars and the various S-, MS-, and CS-type stars belong to luminosities  $-4 > M_{bol} > -6$  (Richer, 1981; Bessel et al., 1983; Lloyd-Evans, 1983). They are asymptotic giant branch (AGB) stars where the helium shell flashing occurs in the intermediate mass stars. Ishida (1960) gives an  $M_v$  of  $-1.5$  to  $-2.0$  for the N-type stars. The R5-R8 stars are also very bright but probably not well classified (Eggen, 1972a). The hotter R stars (R0 to R4 types), the Ba II and CH stars, fall in a fainter range of luminosity ( $+3 \leq M_{bol} \leq -3$ ); they are not so high on the AGB as the N-, S-, MS-, and CS-type stars quoted above. They are probably older, lower mass stars. With the well-known relation among the emission core of Ca II H and K lines and the luminosity of late-type stars (Stratton, 1925; Wilson, 1959; Wilson and Bappu, 1957; Warner, 1969), Richer (1975) obtains the range in  $M_v$  between  $-0.1$  and  $+1.1$  for the hot R stars. Bright members of this class are HD 182040 with  $V = 6.99$ ,  $B-V = 1.07$ ,  $U-B = 0.65$ ,  $R-I = 0.55$ , and  $I-L = 0.44$  and HD 156074 with  $V = 7.61$ ,  $B-V = 1.1$ ,  $R-I = 0.3$ , and  $I-L = 2.2$ .

Although the great majority of these stars are variable, we must keep in mind that some are considered nonvariable, such as the early R stars, which are predominantly nonvariable giants (see the section *Nonvariable Stars*). Distinctions in absolute magnitudes or in colors are noted between variables and nonvariables of the same type or same class. The variable R stars are redder than the latest R nonvariables (Vandervort, 1958). The nonvariable R and N stars both have a continuum

I104 magnitude  $M(104) \geq -1.2$  (Baumert, 1974); they are approximately 1.5 to 5 mag fainter than the variables, while the early nonvariable R stars are 2 mag fainter than the late R nonvariable stars. Baumert (1974) notes that the brightness increases in the sequence nonvariables, irregular Lb giants, semiregular SRb giants, Miras, and semiregular SRa giants. (See the section *Variability Types of Giants and Supergiants* for the definition of these various variables.) No clear correlations were observed between absolute magnitude and CN strength or temperature.

To conclude on the "classical" carbon stars, we stress that the early R stars are physically distinct from the later carbon stars. Such a dichotomy is not only striking from visible indices, as mentioned above, but also from the  $(R-I)$  index (Eggen, 1972b) and from a far-UV index (Eaton et al., 1985). From the latter, the carbon stars seem to be separated into three groups in the color diagram  $(Mg \text{ II-V})/(IUE-V)$  (Figure 1-2). Early R stars fall into two groups among the late G and early K giants; late R stars (R5 to R8) distinctly overlap with the reddest K and M giants, joining smoothly with the N stars at their bluer end. The N sequence extends 2.5 mag farther to the red. On the other side, Figure 1-2 illustrates the relation between color and chromospheric emission for C stars and other giants: the fraction of stellar luminosity radiated in the chromospheric Mg II lines decreases with effective temperature. Low chromospheric emission characterizes the N and R stars (also the G dwarfs) and a few K stars, whereas most of the K and M giants are located above the carbon sequence by up to 2 mag.

All the properties of the HdC stars are well summarized by Warner (1967), Wallerstein (1973), and Hunger (1975).

The atmospheric parameters of the RCB stars indicate high luminosity; they are as luminous as other HdC stars (Warner, 1967). These stars are supergiants (Orlov and Rodriguez, 1977). Herbig (1958) gives  $M_{bol} \sim -4.0$  to  $-6.0$  for the prototype star, R CrB. In his Table 20, Glasby (1968) gives 25 RCB stars

**Table 1-3**  
**Physical Properties and Characteristic Spectral Features of Disk Population Red Giants\*,\*\***

Spectral Type	Distinguishing Spectra	C/O	s-process	Tc	$^{12}\text{C}/^{13}\text{C}$	Li	Relative Numbers	$M_{bol}$ Range	Mass Range ( $M_{\odot}$ )	Mean Mass ( $M_{\odot}$ )
K	----	0.3	N	A	----	--	$7 \times 10^4$	+1 - -3?	1 - 5:	1.2
Ba	Ba II $\lambda$ 4554 CH, C <sub>2</sub> , CN	$\geq 0.6$	+	A	N	N	$10^3$	+1 - -3	1 - 5:	1.2
R0-R4 (C0-C4)	Strong C <sub>2</sub> , CH, CN	$> 1$	N	A	$\sim 10$	N?	$10^3$ :	$\sim 0$	1 - 1.7:	$\sim 1$
M	TiO, VO	0.3 - 0.7?	(+)	+	?	N	$10^3$	-3 - -6	1 - 10?	1.2
MS	TiO, weak ZrO	0.5? - 0.80	+	+	?	N	$10^2?$	-3 - -6	?	$\sim 1.2$ :
S	Weak or absent TiO; ZrO, LaO	0.80 - 0.99	+	+	?	(++)	10	-3 - -6	1 - $> 10$	1.2
SC	Very weak ZrO and C <sub>2</sub>	0.99 - 1.01	+	+	?	(++)	1	?	?	1.2:
N (C5-C9)	Very strong C <sub>2</sub> , CN, CH	$> 1.01 - 2?$	+	+	(--)	(++)	$10^2$	-3 - -6	1 - $> 10$	1.2 - 1.5

\*From Scalo (1981).

\*\*K type is given for comparative purposes.

Symbols: N = K giant normal abundance, A = Absent, + or - = somewhat enhanced or deficient with respect to normal K giants, ++ or -- = strongly enhanced or deficient, ? = unknown or uncertain, () = only a small fraction of the spectral type possess the anomaly.

The straight line across the table delineates the "hot" stars, early R and Ba II stars (above the line) for which  $\sim 4000 < T_{color} < 5500$  K, and the cooler ones (below the line) for which  $\sim 2000 < T_{color} < 3800$  K.

with maximum and minimum magnitudes and spectral types. Alksne and Ikaunieks (1971) show the spectral class, Howarth (1976) shows the maximum magnitude and spectral class, and Sherwood (1976) adds some stars to Glasby's list. Among them, four are observed in the Large Magellanic Cloud: W Men, SY Hyi, HV 5637, and HV 12842, with the absolute magnitude at maximum being  $-4.8$ ,  $-6.6$ ,  $-3.2$ , and  $-4.9$ , respectively. Duerbeck and Seitter (1982) give a list of RCB stars with their spectral type, their  $V$ ,  $B-V$ , and  $U-B$  values, and their pulsational period.

Various estimates have been made concerning the helium stars; they are based on the strength of interstellar lines and proper motions. For MV Sgr, Klemola (1961) gives  $M_v \approx -2.0$  to  $-4.0$ , and consequently,  $M_{bol} \approx -4.0$  to  $-6.0$ ; Herbig (1964) gives a value of  $-5.0$  for  $M_{bol}$ .

The analysis of the interstellar D lines of the typically hydrogen deficient C (HdC) non-variable star, HD 182040, shows that this R star is highly luminous and confirms Warner's (1967) conclusion that the HdC stars are supergiants (Utsumi and Yamashita, 1971). From observations of the Ca II K emission cores and the Wilson-Bappu effect, Richer (1975) gives  $M_v \approx -4.1$  for the three nonvariable HdC stars: HD 137613, HD 173409, and HD 182040. Because their atmospheric parameters indicate high luminosity, they are as luminous as their variable analogs (Warner, 1967). The hydrogen-deficient (supergiant) R stars are physically distinct from the N and "classical" giant R stars. The He, RCB, and nonvariable HdC stars have a common range in values of mean  $M_v$  value  $-2.0$  to  $-5.0$ .

MacConnell et al. (1972) estimate that 1 percent of the G and K giants are Ba II stars; therefore, the Ba II are Population I giants (classical and very weak line objects), and their absolute magnitudes are similar to those of G and K giants (Baumert, 1974). The classical Ba II stars are all binaries, but this may not be true for the mild Ba stars. Bond and Neff (1969) and Gow (1976) showed the excess absorption in the violet region of the Ba II stars: a controversy

developed about the reality and the identity of this feature (McClure, 1984, and references therein). The difficulty in placing the Ba II stars in the HR diagram is mainly one of determining their luminosities and Morgan-Keenan spectral classification. From his analysis of many studies, McClure (1984) concludes that most Ba II stars have "normal or slightly above normal ( $\langle M_v \rangle \approx 0.0$  mag) G and K giant luminosities." However, the classical bright Ba II star,  $\xi$  Cap, has a very high luminosity:  $M_v \approx -3$ . This implies that the star has a large mass or that it could be in a helium shell flashing stage of evolution on the second AGB (McClure, 1984).

For the CH stars, we have  $M(104) = -3.9$  as derived by Baumert (1974), who believes that the CH stars measured by Wing and Stock (1973) are at the extreme tip of the red-giant branch.

Among the brightest stars are early M-type supergiants (M0 to M4 types) with  $-8 \leq M_v \leq -5$ . Stothers and Leung (1971), Humphreys (1978, 1979), and Cowley and Hutchings (1978) collected such bright objects in clusters, nearby galaxies, and the Large Magellanic Cloud. Their  $M_{bol}$  would be  $-7$  to  $-10$  (Glass, 1979), like the M supergiant near 30 Doradus, the carbon stars in NGC 1783 or NGC 2477, or  $\rho$  Cas and VX Sgr (Sargent, 1961). The most luminous of the nearby supergiants is  $\mu$  Cep. With its  $M_v \approx -8$  and its  $M_{bol} \approx -10$ , it is about one million times brighter than the Sun. All these stars are also considered to be super-supergiants, a term introduced by Feast and Thackeray (1956), otherwise called hypergiants (Van Genderen, 1979).

For the M Miras, Clayton and Feast (1969) conclude that the absolute magnitudes vary smoothly within the period:  $M_m = -3.0$  to  $-1.0$  at maximum light and  $M_1 = -1.5$  to  $0.1$  at mean light intensity, with periods varying from  $\sim 180$  to  $\sim 500$  days. The shorter period Miras ( $\sim 130$  days) deviate from this period/luminosity relation. They have  $M_m = -1.6$  and  $M_1 = 0.1$ . Robertson and Feast (1981) show that the stellar luminosity decreases with period for the galactic Miras, and Glass and

Lloyd-Evans (1981) reach the same conclusion for the Miras in the Large Magellanic Cloud.

Information on colors and luminosities are best displayed as an observational HR diagram, and several attempts to construct such diagrams for galactic red giants have been made (e.g., Scalo, 1976; Tsuji, 1981c). A better result comes from the Magellanic Clouds, where the known distance permits absolute luminosities to be determined (Wood et al., 1983). (See the section *Aspects of Evolution of Long-Period Variables (LPV's) in the Magellanic Clouds.*)

**Effective Temperatures.** Effective temperatures can be measured or estimated for at least a few stars from the relation  $F = \sigma T_{\text{eff}}^4$ , where  $F$  is the integrated energy flux at the stellar surface. This flux can in turn be derived by any of several means, all depending on some variation of the relation between observed energy flux ( $f$ ), emitted flux ( $F$ ), and angular diameter ( $\theta$ ):  $f = F(\theta^2/4)$ , where the flux is either monochromatic or integrated. That is, the effective temperature can be inferred for any star for which the angular diameter and total integrated (bolometric) flux (outside the Earth's atmosphere) can be measured. This latter quantity is often difficult to obtain for late-type stars since the bolometric correction is so large. In fact, the desired information—complete spectrophotometry—is seldom available for the M, S, and C stars (Strecker et al., 1979, for M stars; Goebel et al., 1980, for a carbon star; and Augason et al., 1986, for S stars), and fluxes obtained from broadband colors are used instead (e.g., Ridgway et al., 1980a).

Angular diameters are measured or inferred by several means:

1. They can be directly measured by lunar occultation for stars near the ecliptic, and this is the principal source of our current data (e.g., Ridgway et al., 1980; Beavers et al., 1982).
2. For a few stars, values of angular diameters are available from Michelson interferometry or from speckle interferometry (e.g., Balega et al., 1982; Bonneau et al., 1982; see also Chapter 2).
3. Angular diameters can be indirectly inferred from the relation between visual surface brightness and unreddened ( $V-R$ ) color (Barnes and Evans, 1976; Barnes et al., 1978; Eaton and Poe, 1984) although the validity of this relation for the coolest giant stars has not been established beyond doubt.
4. Substitution of the monochromatic flux from an appropriate model for the flux at the stellar surface allows one to infer the angular diameter of any selected star for which the monochromatic flux can be observed (see above formula).

If the integrated flux can be observed, the effective temperature can also be calculated. Since the infrared region is often least influenced by molecular bands, it offers the best choice for application, and the method is generally called the method of infrared photometry (Blackwell and Shallis, 1977; Blackwell et al., 1980). For the coolest red giants generally, this method provides the best present information and has been used to define a temperature scale for M giants (Tsuji, 1981a), for S stars (Augason et al., 1986), and for N-type carbon stars (Tsuji, 1981b).

A final method of determination of effective temperature, that of model fitting, does not depend on the angular diameter. Instead, one directly compares computed and observed flux curves and chooses the effective temperature of the best-fitting model. Widely used in hot and intermediate stars, the method has not been much used in cool stars (e.g., Querci et al., 1974; Bouchet et al., 1983; Steiman-Cameron and Johnson, 1986) because of the lack of complete spectrophotometry and the deficiencies of the models, although it has been applied to calibrate a scale of effective temperatures for M giants (Tsuji, 1978). As these deficiencies are corrected, this method will assume greater importance. In this scope, the comparison between observed and synthetic spectra should give an effective temperature (e.g., Querci and Querci, 1976). However, it remains the problem of uniqueness of the solution.

**Table 1-4**  
**Effective Temperatures ( $T_{\text{eff}}$ ) for M Giant Stars\***

Spectral Type	$T_c$ (K)	V-K Color	$T_{\text{eff}}$ (K) <sup>1</sup>	$T_{\text{eff}}$ (K) <sup>2</sup>	$T_{\text{eff}}$ (K) <sup>3</sup>
M0	3750	3.78	3895	3900	3800
M1	3640	4.02	3810	3800	3865
M2	3530	4.30	3730	3700	3800
M3	3400	4.64	3640	3600	3640
M4	3250	5.10	3560	3500	3460
M5	3000	5.96	3420	3300	3310
M6	2600	6.84	3250	3200	3280
M8	---	---	---	2300	---

1. Effective temperature from broadband colors and lunar occultation angular diameters (Ridgway et al., 1980a,b).
2. Effective temperature from model fitting (Tsuji, 1978).
3. Effective temperature by method of infrared photometry (Mira variables are not considered) (Tsuji, 1981a).

\*Color temperature is on the Wing system (Ridgway et al., 1980).

In the past, a rough method was to use a spectroscopic temperature or excitation temperature for the effective temperature.

Results for M giant stars are collected in Table 1-4. Although these are the best current values, there is still considerable uncertainty, and much more work is needed. A graphic comparison of the results obtained for M stars by lunar occultation (Ridgway et al., 1980a, b), by infrared photometry (Tsuji, 1981a), and by the method of model fitting (Steiman-Cameron and Johnson, 1986) is shown in Chapter 7 (Figure 7-14). As yet, there is no reliable temperature scale for M giants or supergiants later than M6, where most stars are Mira variables, the very red ones being OH-IR objects.

Several obstacles hinder the decisive determination of effective temperatures for the R

stars. No angular diameters are available, molecules distort all broadband colors, the method of infrared photometry fails because of an excess of flux in the L band compared to G-K giants, blackbody colors are inconsistent, and insufficient spectrophotometry is available for model fitting. These and other problems have been exhaustively discussed by Dominy (1982, 1984), who finds that effective temperatures for three R0 stars lie in the range  $4500 \leq T_{\text{eff}} \leq 4850$  K. However, some R0 stars may be even hotter. Dominy finds that R5 stars are nearly as hot as R0 stars. The temperature scale for the later R stars is presently quite uncertain, and these stars may not be closely related to the early R stars.

Effective temperatures for N-type carbon stars have been notoriously difficult to obtain,



and even now they are uncertain except for a few stars. Angular diameters have been measured for six non-Mira N-type carbon stars, of which five (TX Psc, X Cnc, AQ Sgr, SZ Sgr, and TW Oph) are irregular variables and one (Y Tau) is an SRa variable (Ridgway et al., 1977; Walker et al., 1979; Ridgway et al., 1980b; Ridgway et al., 1982). Although one star (TW Oph) is so heavily reddened that the effective temperature is unreliable, values for the other stars form the cornerstone of any further effort.

Tsuji (1981b) has obtained values of effective temperatures of 31 N-type carbon stars (SRb and Lb variables) by the method of infrared photometry (Blackwell et al., 1980). Angular diameters were deduced from previously published models (Querci et al., 1974; Querci and Querci, 1975) and L band photometry (Noguchi et al., 1977), corrected for the effects of molecular bands (however, see discussion by Bouchet, 1984a). Combined with complete broadband photometry, these yield effective temperatures. For the stars in common, the results of Tsuji agree with those obtained from measured angular diameters, and we believe that these are the best values currently available. The effective temperatures are all confined within the range  $2400 \leq T_{\text{eff}} \leq 3200$  K, and most fall between 2600 and 3100 K. It is well known that the "temperature classes" of the C star classification are not well correlated with effective temperatures (e.g., Tsuji, 1981b); Tsuji (1985a) presents arguments to explain this situation.

A scale of effective temperature for non-Mira S stars has been deduced (Augason et al., 1986) both by model fitting and by the method of infrared photometry, and the complicating effect of the unknown C/O ratio is revealed. There is also no temperature scale for S or C Mira variables.

Once fundamental effective temperatures have been determined for a sufficient number of calibrating stars, it will be useful to calibrate a color temperature derived from a carefully chosen set of filters (Wing, 1985). When this is possible for all types of peculiar red giants,

values of effective temperature can be routinely obtained. Many effective temperature indicators are presently taken into account among the cool stars. Since the bolometric flux is very sensitive to temperature variations and the infrared monochromatic flux is not, the color index,  $m_{\text{bol}}-L$ , can be used as a temperature indicator (proportional to  $T_{\text{eff}}^3$ ) (Tsuji, 1981a, 1981b, 1985a). Another is the I(104)-L(400) index by Wing and Rinsland (1981) because the two spectral regions, 104 (1.04  $\mu\text{m}$ ) and 400 (4.00  $\mu\text{m}$ ), are free from molecular absorptions in the cooler stars.

For cool carbon stars, Tsuji (1981c) shows that all broadband photometric colors correlate fairly well with his effective temperature scale. Interestingly, although the intrinsic ( $R-I$ ) color is nearly the same for the N-type irregular stars, these have a range of temperature, and ( $R-I$ ) is therefore an indicator, not of temperature, but of interstellar reddening. The ( $I-L$ ) color is suggested by Tsuji as the best temperature discriminant for both carbon stars and M giant stars. Wing (1985) also suggests the use of the  $I-K$  index.

Although the intrinsic infrared colors, ( $J-K$ ) and ( $H-K$ ), for cool carbon stars correlate with effective temperature and have been used to infer effective temperature, the range in both colors is relatively small ( $\sim 0.6$ ), and the scatter in ( $J-K$ ) is large. These colors do not appear to be reliable temperature indicators. One reason for the scatter might be the imprecision of the effective temperatures. On the other hand, the claim has been made, on the basis of comparative studies of carbon stars in the Magellanic Clouds and the Galaxy, that the infrared colors of carbon stars are primarily determined by strengths of molecular bands and only secondarily by effective temperatures (Cohen et al., 1981). Perhaps the answer lies somewhere between; infrared colors may well be influenced both by effective temperature and by chemical composition.

The temperature range covered by the RCB stars as a group is very large. For example, RS Tel has a spectral class as late as R8 with a surface temperature of 2500 K. Two very hot

stars—the B star, MV Sgr, and the 09 star, V348 Sgr—are also considered as RCB stars because of their variability behavior.

**Radii.** The giants, and chiefly the supergiants, are enormously distended. Most of them have sizes approaching that of our solar system. Some examples follow. If we adopt an effective temperature of 3000 K for  $\mu$  Cep, one of the most luminous supergiant M stars, we find a radius of some 1590 times the solar value; this means that the photospheric layers of the star fill in our planetary system out to the orbit of Saturn! The extent of the  $\mu$  Cep outer envelope layers are not yet measured nowadays. The photosphere of the supergiant Betelgeuse ( $\alpha$  Ori: M2 Iab) is roughly the size of Jupiter's orbit ( $R \approx 650 R_{\odot}$ ). Polarimetric observations give a radius of the  $\alpha$  Ori outer envelope of roughly  $3700 R_{\odot}$ . (Our planetary system would be filled out to the Uranus orbit!) Observed from the Earth, the outer envelope of  $\alpha$  Ori has an apparent diameter equal to 1/10 of that of the Sun. When we consider the HR diagram, we have to bear in mind that two supergiants of the same luminosity but with different temperatures do not have the same diameter. For example, the M, S, and C stars have radii 100 times that of a main-sequence B star. The cool hypergiants are still more extended stars. Finally, the S Mira,  $\chi$  Cyg, has a photospheric radius  $R \sim 240 R_{\odot}$  (Hinkle et al., 1982).

The direct measurement of angular diameter of single stars uses the lunar occultation technique (e.g., Ridgway et al., 1980a, 1980b; Beavers et al., 1982) and numerous interferometric techniques such as speckle interferometry (e.g., see the review of Dainty, 1981, and references therein), as noted in the previous section. A photometric approach may also furnish the stellar radius; if one knows the absolute bolometric magnitude,  $M_{bol}$  (or the luminosity), and the effective temperature,  $T_{eff}$ , the radius of the star comes from:  $L = 4 \pi R^2 \sigma T_{eff}^4$ .

Description of the geometrical shell extension is given by M. Querci (this volume). As an example, let us add that, using the very long

baseline interferometer (VLBI), Bowers et al. (1980) found some masing regions in OH/IR stars from  $> 0.5''$  to  $> 0.04''$ . The extent of the 1612-MHz region can be quite large, from 1000 to 10000 AU. The apparent diameter is consistent with the general predictions of the maser models (Goldreich and Scoville, 1976).

From the knowledge of the stellar radius and mass, the acceleration of gravity in a stellar atmosphere may be determined. In these stars, the  $g_{eff}$  values are rather badly known since the masses are linked to an uncertain evolution theory (Wood, 1985); they are considered to be  $0.1 < g_{eff} < 10$ . Determination of the surface gravity for these peculiar red-giant stars remains a pressing unsolved problem. Much insight can be gained by examining the exhaustively discussed parallel problem for the well-observed K2 giant, Arcturus (Trimble and Bell, 1981). (Examples of indirect methods of estimating the surface gravity of late-type giants are given in Chapter 7.)

#### Aspects of Evolution of Long-Period Variables (LPV's) in the Magellanic Clouds

Studying long-period variables (i.e., the most extreme—in luminosity—red giants and supergiants) in the Magellanic Clouds releases us from the problem of distance determinations (hence, absolute magnitude determinations) in comparison to LPV's in the galactic disc for which existing distance determinations are generally based on statistical parallaxes (since they are not known to be related to open clusters). Also, the LPV's in the Magellanic Clouds cover a wide range in mass and luminosity and are generally free from significant interstellar reddening (Wood, 1982).

Wood et al. (1983) obtain IR photometry (*JHK*) and low-dispersion red spectra of 90 LPV's in the Small Magellanic Cloud (SMC) and the Large Magellanic Cloud (LMC), completing the IR photometry data of Glass and Feast (1982 and references herein) on Miras in the LMC. Their results, which are summarized below, largely contribute to progress in the knowledge of the red-giant or supergiant phase

of evolution that ends by drastic events such as planetary nebula ejection or supernova explosions. One must also recall that these observations include the brightest and, hence, most massive stars, some considerably more massive than the intermediate-mass stars that form the bulk of the M, S, and C stars.

Evolutionary calculations show that the LPV's fall into two distinct groups: (1) non-degenerate core helium (or carbon) burning supergiants with mass  $M \geq 9 M_{\odot}$  on their first appearance as red giants; (2) asymptotic giant branch (AGB) stars (among them Miras) with degenerate carbon/oxygen cores and hydrogen- and helium-burning shells, climbing up the giant branch for the second time. Relevant properties of the AGB stars must be noted: (1) theoretical maximum possible luminosity for an AGB star is  $M_{bol} \sim -7.1$ ; (2) their main-sequence masses are  $\leq 9 M_{\odot}$ ; and (3) they are able to dredge up carbon and s-process elements to the surface through undergoing helium shell flashes, so that they participate in the enrichment of the interstellar medium in these elements at the onset of current loss of their envelopes by stellar winds.

Observational data of LPV's in the Magellanic Clouds confirm the division of these stars in supergiants and AGB's: (1) An absolute K-magnitude ( $M_K$ ) against period ( $P$ ) plot (see Figure 2 in Wood et al., 1983) shows that the stars fall into two sequences separated by the theoretical AGB limit in  $M_K$ : below this limit, stars are AGB stars, whereas above the limit, they are supergiants. (2) In this plot, carbon (N) stars and S stars (rich in s-process element Zr) are located in AGB sequence only, as expected from theoretical evolution calculations for which C and s-process elements are dredged up to the surface during helium shell flashes suffered by AGB stars. (3) K light curves indicate that the AGB stars have pulsation amplitudes of 0.5 to 1.0 mag, whereas the supergiants have smaller K amplitudes (<0.25 mag). (4) Finally, the mean  $J-K$  colors are rather similar for the supergiants whatever the period, while they become redder with period for the AGB stars.

Wood et al. (1983) discuss the evolution and physical properties of the LPV's in the Magellanic Clouds through the ( $\langle M_{bol} \rangle, P$ )-diagram (Figure 1-3) on which the two regions occupied by the supergiants and the AGB stars are again clearly delimited (by dotted lines on the figure). For each bolometric luminosity, the core mass,  $M_c$ , of the AGB stars is indicated (theoretically, a linear relation exists between these quantities), as well as the main-sequence mass,  $M_{MS}$ , for the supergiants. (These quantities are related regardless of subsequent mass loss.) Theoretical lines of constant mass (the pulsation mass) are also plotted assuming the LPV's are first-overtone pulsators (see details in the section *Modes of Pulsation of the Long-Period Variables*). From Figure 1-3, Wood et al. note that the pulsation masses: (1) for the supergiants, range from  $\sim 7$  to  $\sim 30 M_{\odot}$ , with a strong concentration from  $\sim 10$  to  $\sim 25 M_{\odot}$ ; (2) for the AGB oxygen-rich stars, tend to range from  $\sim 0.7$  to  $7 M_{\odot}$ ; and (3) for the carbon stars in their sample (mainly in the field of NGC 371 and, hence, perhaps coeval), tend to concentrate from  $1.0$  to  $1.5 M_{\odot}$ . The evolutionary implications of Figure 1-3 are as follows.

The supergiants (including some galactic supergiants) describe a continuous band from ( $P, M_{bol}$ )  $\sim$  (100 days,  $-6.5$ ) to (850 days,  $-8.2$ ), with all but three of the Magellanic Cloud objects having  $P > 400$  days. On the other hand, the initial main-sequence masses in this range of luminosities and  $P > 400$  days go from  $\sim 16$  to  $23 M_{\odot}$ . The pulsation masses, which are present-day masses, indicate that the supergiants have lost up to half their mass since the main-sequence phase: the more luminous and the initially more massive the supergiants, the more mass they have lost in previous evolutionary phases. The supergiant evolution results from: (1) the decrease in mass, causing  $P$  to increase ( $P \propto M^{-1/2}$ ), and (2) the change in radius with evolution ( $P \propto R^{3/2}$ ).

As AGB stars evolve, they increase their luminosity with time, and correspondingly, they increase their periods,  $P$ . It is worth noting that, for the first time, stars on the upper AGB (i.e., with luminosity right up to the AGB limit

( $M_{bol} \sim 7.1$ ) have been identified. The arrows on the constant mass lines for AGB's in Figure 1-3 indicate that these lines are approximate evolutionary tracks. (Mass-loss effects are shown to be qualitatively unimportant.) To each luminosity corresponds an observed maximum period, the approximate position of which is given on the dotted line. As discussed by Wood et al. (1983), the normal evolution of an AGB variable evolving on an evolutionary track with  $M < 3.5 M_{\odot}$  stops when its track crosses the dotted line (maximum period reached); at this moment, the total mass of the star is still greater than the core mass, showing that the envelope hydrogen and helium has not yet been fully destroyed by nuclear-burning shells. The authors suggest that the evolution is terminated by the loss of the stellar envelope, followed by the formation of a planetary nebula, with nebula mass they estimate to be from  $\sim 0.1 M_{\odot}$  (for the oldest evolved stars) to  $2.1 M_{\odot}$  (for stars of initial mass  $5 M_{\odot}$ ). The mechanism they suggest for ejection of the envelope is the switch in mode of pulsation from first overtone to fundamental. This suggestion is supported by theoretical pulsation models, indicating that, as an AGB star increases its luminosity, it pulsates in modes of lower and lower order until the fundamental one—thought we have to keep in mind that the mode of pulsation remains a disputed question (see the section *Modes of Pulsation of Long-Period Variables*).

The galactic OH/IR stars (see the section *Classification*) with  $P \sim 1000$  days are a good example of these stars pulsating in the fundamental mode and being in a transition phase between LPV and planetary nebula. When the evolution track of a more massive AGB star with  $M > 3.5 M_{\odot}$  reaches the AGB maximum luminosity limit, such a massive star will produce a supernova by the ignition of carbon in the degenerate core.

Finally, Wood et al. (1983) discuss further important points related to nucleosynthesis on the AGB. During helium shell flashes,  $^{12}\text{C}$  and s-process elements are dredged up to the stellar surface. Carbon stars have dredged up suffi-

cient carbon to be characterized by  $\text{C/O} > 1$ . The S stars, located on the AGB between M and C stars, have not yet dredged up sufficient carbon to have  $\text{C/O} > 1$ , but they have dredged up sufficient Zr, an s-process element, so that ZrO bands are observed to be enhanced in these stars. Carbon stars in the Magellanic Clouds were already known to have their higher luminosity at  $M_{bol} \sim -6$ . This is confirmed by Figure 1-3, in which all the more luminous AGB stars are noncarbon stars, among them S stars. It is clear that s-process elements are dredged up in these upper AGB stars, but  $\text{C/O}$  remains less than unity. An explanation favored by Wood et al. to the fact that some AGB stars of type S are found above the most luminous C stars is that the  $^{12}\text{C}$  dredged up is converted to  $^{14}\text{N}$  during quiescent evolution between shell flashes by CNO cycling at the base of the envelope convection zone; therefore, the AGB stars are the source of primary nitrogen (since  $^{14}\text{N}$  is synthesized from the hydrogen and helium in the star at its birth), as well as s-process and carbon elements.

#### **VARIABILITY TYPES OF GIANTS AND SUPERGIANTS: A SURVEY OF OBSERVATIONS AND INTERPRETATIONS**

The brightness fluctuations of these intrinsic or physical variable stars are caused by geometrical and physical factors: fluctuations in diameter, temperature, pressure, molecular and dust opacity strengths, etc. Apparently, complex phenomena make up their entire unstable atmosphere: shock waves, strong coupling of convection and pulsation, and certainly motions of clouds of matter from the photosphere to the circumstellar layers (if any) and infalling of this matter at other phases of the variation, perturbation in the layers due to companion tidal effect.

The *visual* light curves tend to be regular, with large amplitudes for the Miras; they tend to be less regular and/or erratic, with much smaller amplitudes for the semiregular and irregular variables. Among all these variables, we

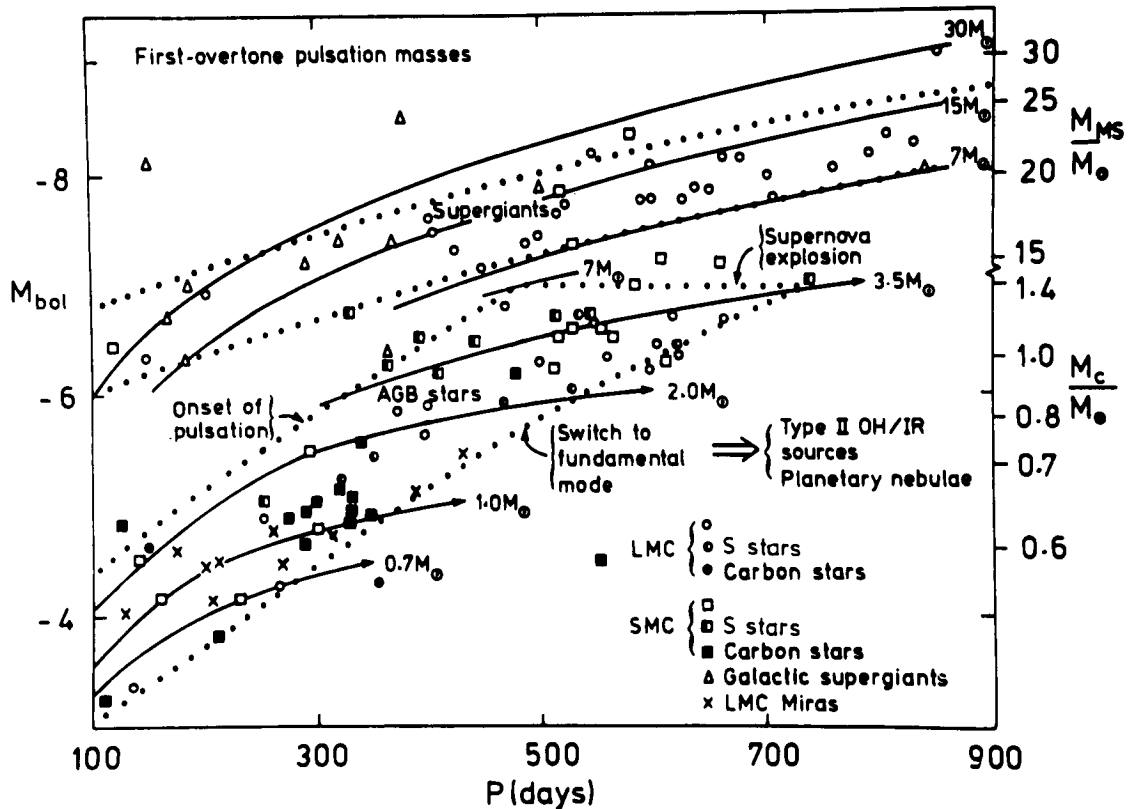


Figure 1-3. Bolometric luminosity ( $M_{bol}$ ) plotted against period ( $P$ ) for long-period variables (LPV's) in the Magellanic Clouds (SMC and LMC). Also included are some galactic supergiant variables. Stars labeled "LMC Miras" are LPV's studied by Glass and Feast (1982). Dotted lines delineate the regions occupied by supergiants and AGB stars. Continuous lines are lines of constant mass assuming that LPV's are first-overtone pulsators. Also indicated are theoretical values of the core masses ( $M_c$ ) of AGB stars and the initial main-sequence masses ( $M_{MS}$ ) of supergiants. See text for comments (from Wood et al., 1983).

find stars with all different values of the ratio C/O ( $<1$  for M-type stars,  $\leq 1$  for S-type stars, and  $>1$  for C stars). They may be either giants or supergiants (Kukarkin et al., 1958). All these stars can also be identified as pulsating or eruptive variable stars (Strohmeier, 1972).

In the pulsating group, we recognize three types—Miras, semiregulars (SR), and irregulars (L)—as follows:

1. The Mira Ceti-type stars are long-period stars with visible amplitude variation over 2 magnitudes (and for some cases up to 5 and larger), with well-expressed periodicity, good regularity, and periods from 2 months to 1 year or more. (They belong to the AGB long-period variables (LPV); see Figure 1-3.) Around the max-

imum, characteristic emission spectra are observed, mainly the hydrogen Balmer series. They occupy the tip of the AGB. Representative stars are  $\circ$  Ceti, R Lep,  $\chi$  Cyg, etc.

2. The semiregular variables are further classified SRa, SRb, and SRc. Their visible amplitude variations are smaller than those of the Miras. The SRa light curves are not regular and have strong variations from one period to another. Different durations of cycles with irregularities and some constancy of brightness are observed in the SRb light curves. The SRa and SRb variables are giants; the SRc's are supergiants with SRb behavior. Some of the giants are

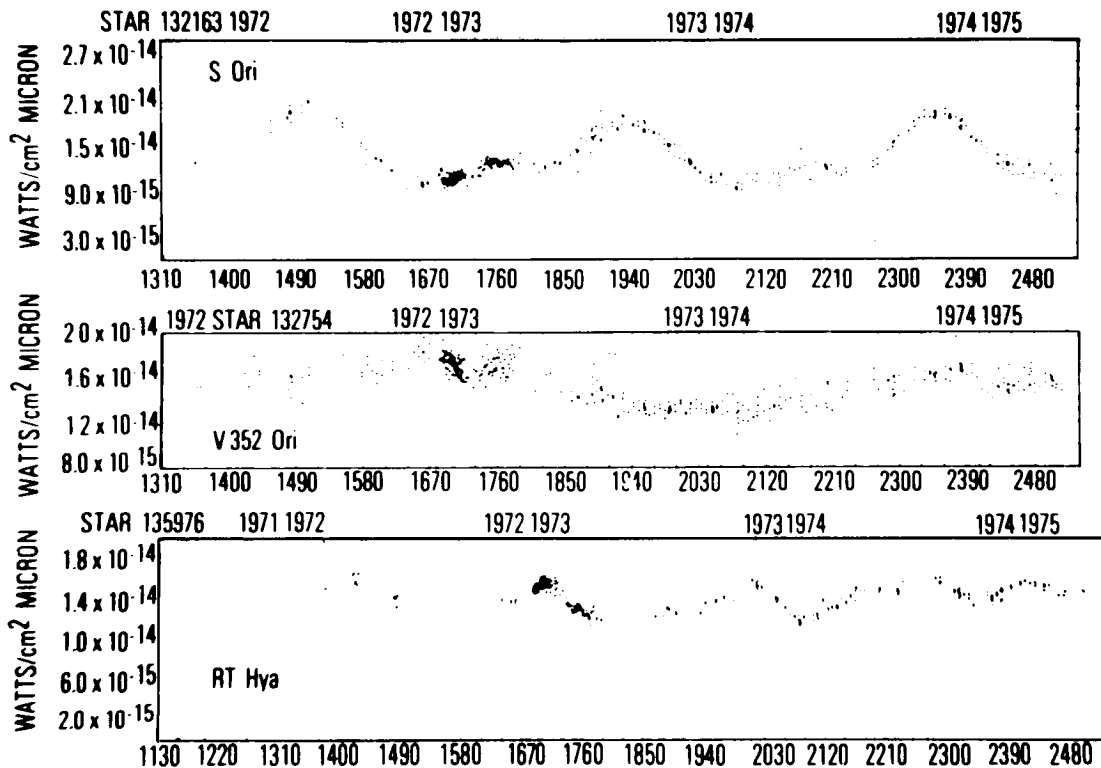


Figure 1-4.  $2.7\text{-}\mu\text{m}$  light curves: Top: S Ori is a Mira-type variable with a period of 419 days; bottom: RT Hya is an SRa-type variable; center: the irregular (Lb) variable V352 Ori shows a small flux variation on a time of 2 years (from Maran et al., 1977).

probably on the AGB, others on the first ascent of the red-giant branch (RGB). Typical representatives are WZ Cas,  $\rho$  Per, S Per, etc.

3. The true irregular variables are called Lb or Lc. The former are giants; among them,  $\beta$  Peg, CO Cyg, and V Aql are the typical representatives. The latter are supergiants; the best known is TZ Cas.

Figure 1-4 shows the three characteristic curves of Mira, SRa, and Lb stars (Maran et al., 1977).

R CrB is the typical representative of the RCB stars, *the eruptive group*. These stars are of spectral classes, F, G, K, and R (hot carbon stars) and have a very high luminosity, as described in the previous section. They are characterized by slow nonperiodic drops in brightness of different amplitudes (1 to 9 magnitudes) and of different duration (up to several

hundred days for some). The increase in brightness is generally rapid (a few days). Besides R CrB, other representative stars are XX Cam, RY Sgr, and SU Tau. The star,  $\rho$  Cas, is one of these stars which appear in the literature as pulsating (SRc) or eruptive (RCB) stars.

Let us remark that we do not intend to study here certain peculiar but rare types of variables, mainly among the carbon spectral-type stars such as Isb (V374 Aql, UX Cas, and DY Aur), RVa (AC Her, etc.), or the cepheids with carbon molecular bands (RU Cam, etc.). These types of variables are described by various authors (e.g., Payne-Gaposchkin, 1954a).

In the late-type star family, the carbon stars are not only different by the C/O ratio, which is greater than one, but also by the behavior of their light curves (i.e., perhaps by specific reactions of their atmospheres to mechanical energy transport). First of all, the distribution

**Table 1-5**  
**Percentage of Variability Types**  
**in M, S, and C Stars\***

Spectral Type	Type of Variability(%)		
	Mira	SR	Lb
M	46	29	25
S	58	23	19
C	20	38	42

\*From Alksne and Ikaunieks (1971).

of the different types of variability is very different for the three spectral classes, M, S, and C. Table 1-5 (Alksne and Ikaunieks, 1971) shows that the number of semiregular and irregular stars increases sharply as one goes to carbon stars. This might be partially due to a selection effect, since these stars are faint and their light variations are not so easily followed as in the M stars. Half of the variable stars of M and S classes are Mira variables. For carbon stars, only 20 percent are Mira variables, 40 percent are SR, and 30 percent are Lb. The Mira and semiregular carbon stars have the largest mean periods; carbon Miras have the smallest photographic amplitude variations among all the Mira variables, whereas the irregular carbon stars have the largest ones among all the Lb variables.

*The study of the cool star variability is at present based on visible and IR photometric data:* Far-UV and far-IR data are too rare (see the section on *Photometric Observations*) to study the photometric variability. The present data are borrowed from a large number of papers, and consequently, they suffer from too many inhomogeneities like observations at different phases, use of different photoelectric systems (*UBV*, *DDO*, etc.), and data reduction method not always being adapted to cool stars. To achieve homogeneous data covering several periods, Bouchet (1984b) carried out a photometric analysis (from the blue to far-infrared ranges) of a sample of galactic carbon

stars at ESO-La Silla during 4 years. This work is an example of a long set of data for Mira, SR, and L carbon stars made under the same observational conditions. It will be a good sample for testing the evolution and/or dynamical pulsation theories. Another extensive example concerning infrared photometry on M, S, and C stars comes from Catchpole et al. (1979).

Before describing the general characteristics of each group of variables, we focus on two important remarks:

1. The Mira variables, as well as the SRb, Lb, or HdC stars, have microvariations with a duration much smaller than the length of the period or the pseudoperiod generally known (some hours or days compared to several months or years).
2. In the HR diagram, some stages of the evolution of the variables can be followed during a human life. For example, some of these stars become apparently planetary nebulae in less than 10 years (e.g., HD 59643, HM Sge, or V1016 Cyg). (See the section *Irreversible Changes in SR and L Stars*.)

### The Miras

By "Miras," we distinguish a type of light curve, not a spectral type of stars. Many papers have confused the term and sometimes speak about Miras without any definition of the spectral type of the stars; this can occasionally be confusing for a nonspecialist. Therefore, we have to be clear: we point out that the Mira variable-type stars exist among the three spectral types: M (R Aqr, R Aql, R Cae, R Cen, o Cet, R Hya, U Ori, etc.), S (R And, R Cyg, χ Cyg, etc.), and C (U Cyg, V Cyg, T Dra, R Lep, SS Vir, etc.). Moreover, the chemical equilibrium of the elements inside the atmosphere of these later stars shows that it is impossible to find OH in carbon stars. Consequently, the papers on OH masers usually refer to the spectral-type M stars. This is meaningful when looking for eventual correlations among the observed features.

The Miras are the most observed and best known among the intrinsic variable stars. The earlier curves of light were recorded during the 18th century:

During the autumn of 1596, the shepherd David Fabricius discovered a variable star in the constellation of the Whale, a star of the 3rd magnitude. In 1603, the astronomer Bayer added this star, which was of the 4th magnitude, onto his famous atlas and labeled it with the letter  $\sigma$ . In 1635, Dutch astronomers Holwarda and Fullenius likewise showed that this star is variable, and in 1638, Holwarda gave the amplitude of variation to be from the 2nd to the 10th magnitude. Hevelius observed this star during 50 consecutive days and determined its period: 331 days. At that time, this star was the sole variable known in the sky. It was therefore named "Mira Ceti" or the "magic star of the Baleen constellation" (Jacchia, 1933). In 1839, Argelander, the German astronomer who is often called the father of variable star astronomy, observed a maximum of 2.2.

$\chi$  Cyg was discovered by G. Kirch in 1681, R Hya by G.P. Maraldi in 1704, and R Leo by Koch in 1782. In the following century, Baxendell discovered, among many stars, S Aql (1863) and U Boo (1880); Pogson discovered R Cyg in 1852 and R Cas, R UMa, and S UMa in the following year. R Car was discovered at Cordoba in 1871.

Let us quickly describe the famous work of Padre Angelo Secchi. In 1868, he observed nearly 100 red stars with the 9-inch Merz refractor of the Collegio Romano and a small direct-vision spectroscope. Among them, he included 15 long-period variables from Schjellerup's catalog (1866). The first slit spectrogram of LPV stars could indeed be this of  $\sigma$  Cet taken by J. Wilsing with the Potsdam refractor in February 1896. Vogel (1896) remarked on the great intensity of the bright hydrogen lines and the absence of H $\epsilon$ . In 1897, the Reverend Walter Sidgreaves (1897) pointed out the rapid variation of  $\sigma$

Cet during a week, "but a marked change in the relative intensities of the yellow-green and the blue radiation appears to have taken place during the cloudy week between December 2 and December 11." He confirmed the absence of H $\epsilon$  noted by Vogel the previous year: "Of the hydrogen lines H $\epsilon$  is still absent, lost, or much weakened in the calcium absorption." The notion of stellar envelope is already contained here, 100 years ago....

Nearly all photoelectric V magnitudes of  $\sigma$  Cet and T Cas at bright phases lie above the curves given by the American Association of Variable Star Observers (AAVSO) and may indicate a systematic difference between the AAVSO and the V magnitude (Lockwood and Wing, 1971). Stanton (1983) gives an experimental relation between the visual magnitude ( $m_v$ ) and the photometric measurements  $V$  and  $B-V$ .

The Mira light variations with phase are sometimes very large in the visible. For example, the Mira,  $\chi$  Cyg (spectral type S), has a visual amplitude variation that is about 9 magnitudes (4000 times brighter at maximum light than at minimum). Consequently, UV and visual spectrograms can hardly be obtained, especially near the light minimum of the Mira stars (Maehara, 1971). The Mira stars were analyzed mainly by the Japanese group (Maehara, 1968; Yamashita and Maehara, 1977, 1978; Yamashita et al., 1978; and others).

Most of the Mira changes are cyclic, with a period equal to that of the visual light curve. Some obvious inconsistencies appear in the observational data, as well as phase lags between light curves in different colors, which are caused by the actions of the cyclical temperature variations on the various layers of their very extended atmospheres. Joshi et al. (1980) give the evolution of the effective temperature during 12 days for  $\sigma$  Cet: on October 6, 1979, Mira was an M5.5 III star with 2470 K, and on October 18, 1979, it reached M4.5 III with 2760 K. Because the period of individual light cycles sometimes differs from the star's mean period by several percent, the observable properties are



not repeated exactly from cycle-to-cycle (Barnes, 1973; Wing, 1980).

Fluxes of some M Miras show a significant deviation from a stellar blackbody beyond  $8\ \mu\text{m}$  due to the presence of a cold thin dust shell ( $10\text{-}\mu\text{m}$  excess radiation), whereas some have a great excess of infrared (see M. Querci, this volume) due to the presence of a thick circumstellar dust envelope (Epchtein et al., 1980). Among the M Miras with the largest infrared excess at  $10\ \mu\text{m}$ , we know: IRC + 10011, IRC + 50137, and IRC -10529. Some of them have masers; the OH satellite line at 1612 MHz is seen in many sources, representatives of which are IRC + 10011, WX Sgr, R Aql, NML Tau, IRC + 50137, VY CMa, and NML Cyg. U Her and U Ori are known to be 1665-MHz emitters. The M star, RV Hya, has no OH microwave emission (Zuckerman, 1980). For the first time in a carbon-star envelope, Henkel et al. (1983) found a maser line in IRC + 10216. Like OH, the observed SiS maser has two horn features.

Some changes from OH type I to OH type II Miras were observed during recent years, such as on R Leo, for which Lesqueren (1983) reports an unusual intensity variation: the flux at 1667 MHz has steadily decreased from  $1.5\text{E-}22\ \text{W/m}^2$ , reached during the 1980 cycle, to a smaller value  $0.1\text{E-}22\ \text{W/m}^2$  at the maximum of the 1982 cycle. Another striking example is U Ori. Pataki and Kolena (1974a, 1974b) found a bright 1612-MHz flash up to 22 Jy and a disappearance of the 1667-MHz line below the 0.4-Jy detection level on May 28, 1974 (the 1612-MHz line is characteristic of the OH type II maser star), whereas before 1974, particularly on July 28 and November 26, 1973, only the main lines (at 1667 and 1665 MHz, characteristic lines of OH type I maser stars) are present in emission. Moreover, on May 28, 1974, no radical change is registered in the output of the 1665-MHz main line. Coincidentally, the 43.122-GHz SiO maser line was not detected on June 2, 1974, by Snyder and Buhl (1975), but it was detected exactly two U Ori periods later by Balister et al. (1977) with an energy level of 89 Jy. On the basis of these observations, Cimerman (1979) develops a

monitoring that shows no variation on the 1667-MHz main line until May 1977. On January 25, 1978, this line appears with 0.8 Jy and  $-42\ \text{km/s}$ . The 1665-MHz line was not observed as often as the 1612-MHz line because of its weakness. On March 1, 1978, the 1612-MHz line has a flux peak of 2.2 Jy, which is about one-half of that observed by Pataki and Kolena (1974b) in May 1974. On July 15, 1975, the spectrum of 1612 MHz consists of a sharp feature at  $-42\ \text{km/s}$  and a group of blended lines peaking at  $-46\ \text{km/s}$ . This large blend evolves to a sharp feature at  $-47\ \text{km/s}$ , and the  $-42\text{-km/s}$  peak remains constant (spectrum of January 25, 1978). Besides the variation of the line profile, a variation on the intensity of the 1612-MHz line occurs: the average level of  $45\text{E-}22\ \text{W/m}^2$  detected by Pataki and Kolena (1974a, 1974b) remains constant until the beginning of 1976, when it drops to one-half its previous level in 100 days and continues to decline (see correlations with visible variations below). This decline is confirmed by Jewell et al. (1979; see their Figure 15). The main-line emission has exhibited highly circular polarized components and large semiperiodic variations (Fix, 1979). Cimerman (1979) attributes the origin of these events to the central star; this assumption is corroborated by the coincidence in velocity between one of the 1612-MHz lines and the optical emission lines. All these lines are interpreted as the outcome of a shock or magnetohydrodynamic (MHD) wave originating under the photosphere of U Ori (Merrill, 1960; Wallerstein, 1973). The model of Elitzur et al. (1976), which explains the 1612-MHz maser in sources with a thick shell like IRC + 10011, seems to be inadequate for U Ori, where the dust shell is very thin. In this case, another pumping process for the 1612-MHz line may exist in addition to the one that uses the dust grains (Cimerman, 1979). A mapping of the U Ori neighborhood was made by the VLBI on the principal and satellite frequencies. Unfortunately, the variations on 1665-MHz lines (Fix et al., 1980) were made 3 years after the 1612-MHz ones (Reid et al., 1977), and comparisons are therefore hazardous.

**Phase lag.** Since their discovery, the Miras have attracted generations of astronomers until the present day; the pioneers are Herschel, Secchi, Pettit and Nicholson, Hetzler, Hoffleit, Campbell, Stern, Joy, Merrill, and others. Occasionally, astronomers of more recent times have made the same discoveries as those of their forerunners for want of looking at their old publications. The study of the phase delay between the visible, photographic, and near-infrared light curves illustrates this fact to perfection.

Traditionally, the epoch of zero phase refers to the visual maximum. The phase lag (or shift or delay) of the maximum of an infrared or radio curve is the epoch of this maximum, minus the epoch of the visible maximum coming immediately before it. The first correlations between the variations at different wavelengths were made by Pettit and Nicholson (1933). As an example, they have shown that  $\alpha$  Cet, R LMi, R Leo, R Hya, R Aql, and  $\chi$  Cyg have a real maximum of energy shifted by a phase lag of about 0.14 after the visual maximum. Hetzler (1936) has monitored 30 Mira variables with photographic plates at an effective wavelength of 8500 Å ( $\Delta\lambda = 100$  Å). His light curves have a much smaller amplitude than the corresponding visual light curves because of the longer wavelength effect and the decreased blanketing by TiO bands. Hetzler showed that the maxima of the infrared and the visual curves are reached at about the same time, but the infrared maximum persists longer. R Lyn and R Cam are two characteristic examples of simultaneity. R Hya is an exception; the infrared maximum precedes the visual one. In this case, the IR phase lag is negative. This phenomenon is to be confirmed by careful monitoring of many Mira-type stars at several wavelengths. Generally speaking, the infrared light maximum typically lags the visual by 0.1 to 0.2 period, agreeing with all the previous quoted observations and with those of Mendoza (1967), Lockwood and Wing (1971), and Frogel (1971).

An analysis of the spectrum of the spectral M class Miras allowed the astronomers of the

late 1960's to distinguish the behavior of the continuum from that of the absorption lines. With the I104 filter, Wing (1967a) showed that the I(1.04- $\mu$ m) magnitude of  $\chi$  Cyg was brighter at phase +49 days than at 0 day by 0.1 magnitude; this is very similar to the phase lag noted in radiometric curves obtained with the vacuum thermocouple by Pettit and Nicholson (1933). He explains this phase lag by a two-layer model. The infrared magnitude measured at a continuum point refers to the photosphere, while the heavily blanketed visual magnitudes presumably refer to the upper layers of the atmosphere. He concluded that the temperature variations of the two regions appear to be out of phase. At present, such an explanation must be viewed in the context of shock-wave interpretation (see M. Querci, this volume).

Fillit et al. (1977) have analyzed the time variation of some type I OH Mira sources and the correlation with their visual magnitude. More than 2-year monitoring of seven OH/IR stars was done on six Miras (R LMi, R Cas, Y Cas, RS Vir, U Her, and S CrB) and on an SRC variable (S Per). Even allowing for experimental uncertainties, a systematic phase lag of about 20 days between OH lines and visible light is observed on S CrB (Figure 1-5). These authors implicitly assume that the shape of the radio and visible curves are the same. This assumption must be proved.

The phase delay of the [V = 1, J = 2-1] SiO curve on R Leo, is greater than that of the 2.7- $\mu$ m curve, whereas the 2.7- $\mu$ m and SiO curves have the same lag as on  $\alpha$  Cet. The SiO masers observed by Spencer et al. (1981) are interpreted as collisional pumping by Bujarabab and Nguyen-Q-Rieu (1981). This requires high temperatures and densities in the SiO layers ( $T_K \sim 2000$  K,  $n_H \sim 10^{10}$  cm<sup>-3</sup>), and consequently these layers are close to the star ( $R \sim 1.5 R_*$ ). Hinkle et al. (1976) indicate that the SiO strength in Miras varies markedly with phase (see also Nguyen-Q-Rieu, this volume).

The behavior of another maser line, H<sub>2</sub>O at 1.35 cm, and of lines at 2.2  $\mu$ m and in the visible were compared (Schwartz et al., 1974) for some Mira stars (U Her, S CrB, U Ori, R Aql, and

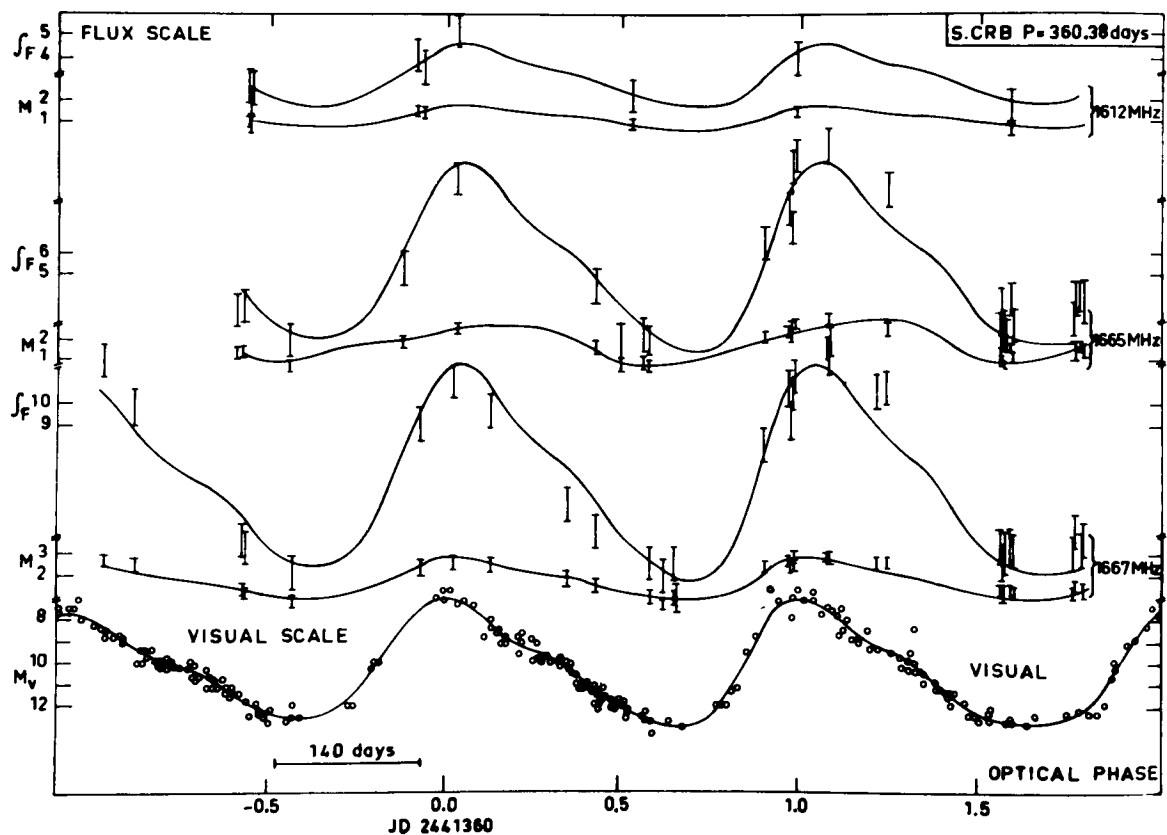


Figure 1-5. The flux and the magnitude for each of the OH maser lines: 1612 MHz, 1665 MHz, and 1667 MHz lines of S CrB were fitted by least squares using the visual light curve (from Fillit et al., 1977).

W Hya; see their Figures 1 through 5). The mean curves show the phase lags of radio and infrared curves relative to visual ones. These lags are characteristic on U Her and S CrB. Some stars do not have the usual phase lags (e.g., the H<sub>2</sub>O maser curve of U Ori has a large lag compared to the infrared curve at 2.2  $\mu$ m). Although the IR curve of R Aql has a phase lag of 0.2 compared to the visual curve, the 1.35-cm curve has the same phase as the visual one. W Hya shows an analogous behavior. This nonconventional behavior requires comment. Is it real? Yes, if the observed values at the different wavelengths are obtained simultaneously; no, if there is a time lag between them: in this case, we find the presence of large amplitude variations during small intervals of time (a small percentage of the period). If the quoted values are from different cycles, another

interpretation is possible: the behavior of the maser lines with the phase is not exactly the same in all the cycles. This was already demonstrated by Wing (1967b). Cox and Parker (1979) demonstrate that the H<sub>2</sub>O was often not stable for more than a few cycles of stellar brightening. (A more comprehensive description of the nature of the different maser pumps is made by Nguyen-Q-Rieu in this volume.)

From 1972 onward, observations of some M Miras were obtained by using the satellites of the U.S. Air Force (Maran et al., 1977). Because the spectral interval on the water-vapor bands around 2.7  $\mu$ m was chosen, correlations with the H<sub>2</sub>O maser should be possible without the perturbations introduced by the variable telluric water vapor. Figure 1-6 shows the 2.7- $\mu$ m infrared and the visual curves of two Miras, S CMi and o Cet.

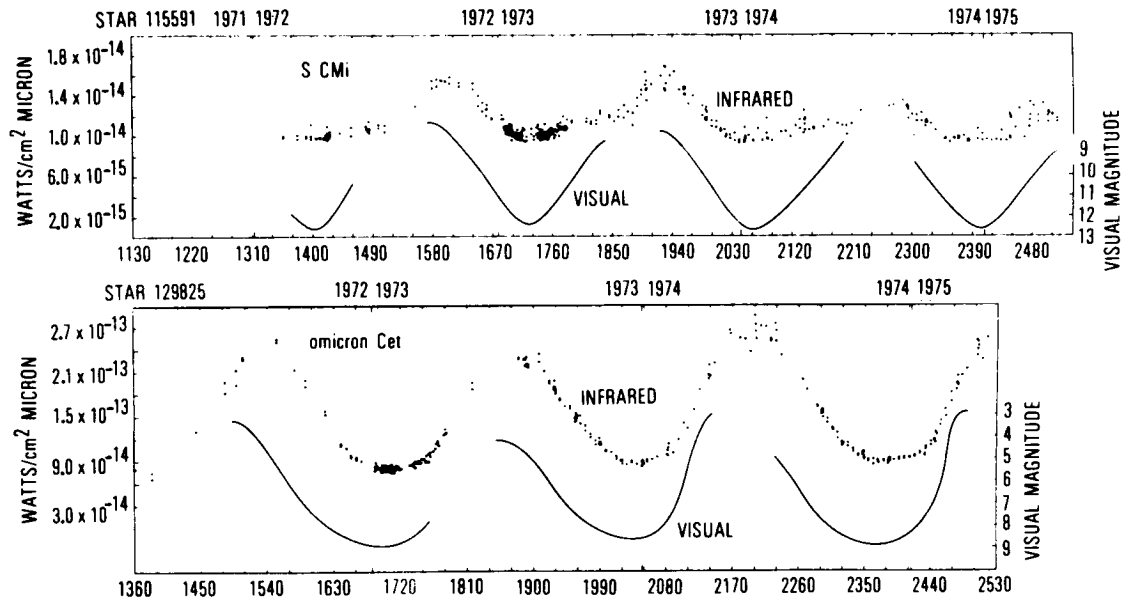


Figure 1-6. 2.7- $\mu$  infrared curve (top) and AAVSO visual curve (bottom) for two Mira variables, S CMi and o Ceti. The left ordinates are the infrared fluxes and the right ones are for the visual magnitude (from Maran et al., 1977).

**Amplitude of Light Variations at Different Wavelengths.** The S Mira,  $\chi$  Cyg, is a very good example for this purpose. Maehara (1971) observed a visual amplitude variation of about 9 magnitudes, whereas the photographic infrared variation was only about 2 magnitudes. Although similar data for the ultraviolet region is incomplete, some information on the variability of the energy distribution of  $\chi$  Cyg from 2500 to 3350  $\text{\AA}$  at two different phases is shown on Figure 1-7. The flux longward of about 3050  $\text{\AA}$  becomes considerably fainter at phase 0.18 than at the light maximum (phase 0.04). The total energy emitted in the range from 2000 to 3240  $\text{\AA}$  is  $4.5 \text{ E-}11 \text{ erg/cm}^2 \text{ s}$  at phase 0.04 and is lower by a factor of 3.53 at phase 0.18. The stellar continuum is even more depressed at phase 0.22 on a high-resolution spectrum. This remarkable change is mainly due to a change in the temperature of the region of continuum-formation (Cassatella et al., 1980).

Lockwood and Wing (1971) show examples of some M Miras for which the curves of the filters, V, 78, 87, and 88, are depressed by TiO,

and the filter-105 curve by VO. These curves are strongly correlated with the differential temperature effect, the local continuum, and the variation of diameter of the forming layers, so that these curves are the best reflection of the physical parameter variations of the atmosphere along the period. Figure 1-8 describes the curves of two Miras, o Ceti and T Cas, in which the minima appear to occur at the same phase. The behavior of R Aql is different from that of o Ceti and T Cas because the visual minima of these two stars have a phase lag relative to the infrared minima. This is shown on the 2.7- $\mu$  curve obtained with the U.S. Air Force satellite (Maran et al., 1977, 1980). When one considers the infrared spectral range, one sees that the amplitudes of variation  $[F_{\text{max}}/F_{\text{min}}]$  decrease with increasing wavelengths from 1.2 to 3.5  $\mu\text{m}$  and are roughly constant between 3.5 and 10  $\mu\text{m}$ . The amplitudes of the 1612-MHz OH masers which are linked with the IR fluxes are significantly less than, or at most equal to, the infrared amplitudes (Harvey et al., 1974). Herman (1983) followed OH/IR Miras during a period

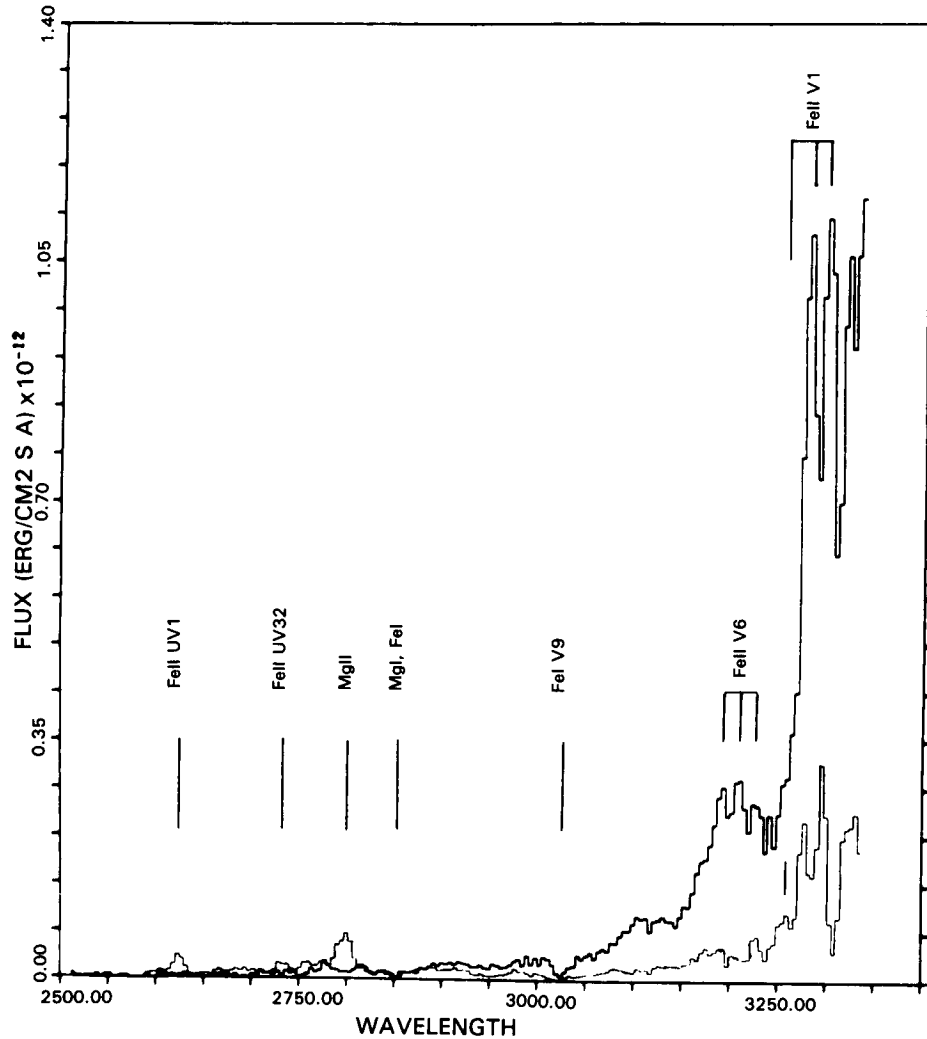


Figure 1-7. Ultraviolet energy distribution of the S Mira variable  $\chi$  Cyg from 2500 to 3350 Å at phases 0.04 (thick line) and 0.18 (thin line). The total energy flux in this spectral region is less at the latter phase by a factor 3.53 (see text) (from Cassatella et al., 1980).

of 3 to 5 years with the 25-m Dwingeloo radiotelescope in order to study their variability.

After considering the amplitude variations in visible, infrared, radio, and ultraviolet wavelengths, we now follow the amplitude variation at one wavelength and point out the characteristic behavior of the Miras. Maran et al. (1980) show five well-observed minima of S CMi (with the U.S. Air Force satellite), where we note the agreement of flux density level at all five minima. The flux of S CMi is remarkably repre-

sentative of all minima. The authors emphasize that the minimum is the "normal state of a long-variable star." However, there has been a good deal of controversy about the reproduction of Mira minima. Glasby (1968) said that  $\alpha$  Cet has variable minima: "on occasion [the minimum] has only been as faint as magnitude 8.0 while at other times it has sunk as low as 10.2 magnitudes." Campbell and Payne (1930) show variation of magnitude of the minima of S Pav. On the other hand, we sometimes observe a strong contrast between consecutive

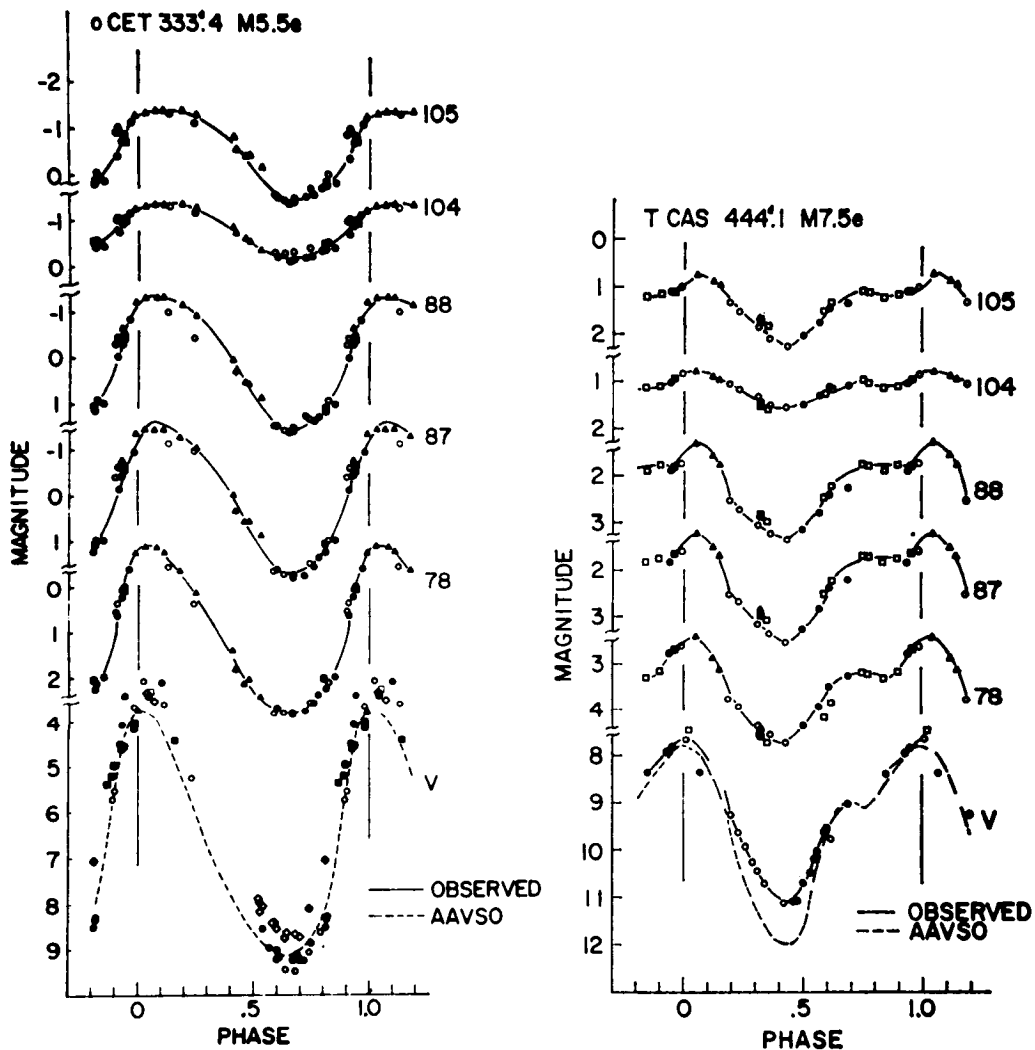


Figure 1-8. Light curves of *o* Cet (*Mira*) at left and *T* Cas at right, in *V* and at five infrared wavelengths. The wavelength designations to the right of the curves are defined in Lockwood and Wing's (1971) Table 1. The observed cycles of the stars are numbered serially; the IR observations start with cycle 1 in 1965 and are coded as follows: -1 =  $\blacklozenge$ , 0 =  $\diamond$ , 1 =  $\bullet$ , 2 =  $\circ$ , 3 =  $\blacksquare$ , 4 =  $\square$ , 5 =  $\blacktriangle$ , 6 =  $\triangle$ , 7 =  $\blacktriangledown$ , 8 =  $\triangledown$ , 9 =  $\blacklozenge$ , and 10 =  $\diamond$  (from Lockwood and Wing, 1971).

maxima of *Mira* stars, such as the two maxima of *R Aql* in 1972-1973 and those of *S CMi* in 1973 and 1974 (Maran et al., 1977). In the late 18th century, the irregularity of the maximum light was already noticed by William Herschel on *o* Cet, where it is extremely well pronounced in the visual curve. In 1779, it attained the first magnitude, being equal to Aldebaran in brightness. For many *Miras*, the cycle-to-cycle differences are not erratic; bright and faint max-

ima tend to alternate, and bright maxima tend to occur before the predicted date (Harrington, 1965; Keenan, 1966). This could be correlated with the intensity of the pulse (i.e., the size of the energy reservoirs in the cycle). *RS Cen*, *R Cet*, *V Cen*, *T Gru*, *R Vir*, *RT Cen*, and *SS Her* are *Miras* with alternative bright and faint maxima.

**Different Profiles of Light Curves.** The AAVSO report 38 (1983) gives many visual light

curves of Mira variables. We warmly congratulate all the AAVSO members for their arduous and comprehensive work; we are convinced of its usefulness to professional astronomers.

The light curves of carbon stars are more gradual and more symmetrical than those of M class stars (Figure 37 in Alksne and Ikaunieks, 1971). Campbell (1955) shows that the S star light curves are very similar to those of M stars. As a rule in M and S spectral types, the rise to maximum is somewhat more rapid than the decline to minimum. In general,  $D \sim 0.44$ , with  $D = (\text{epoch of maximum} - \text{epoch of minimum}) / \text{period}$ . However, some exceptions exist, such as:  $\alpha$  Cet ( $D = 0.62$ ) and R Cyg ( $D = 0.7$ ). (Because  $D$  is always larger than zero, it represents the fraction of the period needed to go from the minimum to the maximum.)

All the Mira light curves do not have the same shape (Figure 1-9). They can present:

1. A rapid rise to the maximum and a slow decline to the minimum: the majority of the Mira light curves have this shape, with  $D \sim 0.45$  (R Gem, U Her, and U Ori).
2. A slow and long rise to the maximum with a rapid decline with  $D \sim 0.7$  (S CMi and U CMi).
3. A narrow and sharp maximum with a large minimum (R Aqr, R Cas, Z Cas, U Cet, T Col, R Crv, R For, R Pav, T Sgr, RU Sgr, and R UMa).
4. A large and round maximum and a narrow and deep minimum (Z Cap, S Car, RT Cen, X Cet, T Hor, and S UMa). These stars have a period  $P \leq 225$  days (Celis, 1977) so that the magnitude varies rapidly around the minimum.
5. A sawtooth shape with sharp maxima and minima with  $D \sim 0.5$  (RT Cyg and V Oph). The variations between their extremes are rectilinear segments. These stars have periods shorter than 150 days (SS Her, W Pup, and R Vir; Celis, 1977).
6. A smoothed sinusoidal shape (SS Vir, R Dra, W Cas, and U UMi).
7. Two maxima of the same intensity and two minima with different depths per cycle (R Cen and R Nor, both discovered by Gould at Cordoba in 1871). The main maximum is followed by the main minimum. This curious light curve was reproduced theoretically by Wood (1979) in an attempt to produce a larger post-shock velocity maximum in an isothermal pulsation model. It appears that this situation is apparently stable (over 75 cycles) where shocks with different velocities alternate. This visual curve also shows variation from cycle to cycle in the rate of rise and fall, in the amplitude and in the height of the maxima, and in the depth of the minima (Marino et al., 1979). CR Mus shows a scatter of approximately one magnitude in the level of the V light curve; the shape of this maximum varies extensively from cycle to cycle.
8. A hump on the rising branch, around the phase 0.7, so that the minimum is moved at earlier phase (R Lep, T Tuc, and S Vol). Such a hump is also observed by Hetzler (1936) on R Tri, R Vir, R Hya, T Cep, and  $\chi$  Cyg. For R Tri (Figure 1-10a) and R Vir (Figure 1-10b), the shoulder (or hump) is much more important in the photographic infrared ( $\lambda = 8500 \text{ \AA}$  and  $\Delta\lambda = 100 \text{ \AA}$ ) than in the visible, whereas for T Cep (Figure 1-10c), the shoulder rises in proportion to the brightness variation in the  $8500 \text{ \AA}$  region and in the visible. To explain the  $8500 \text{ \AA}$  behavior of R Tri and R Vir, Wing (1967b) suggests that the effective wavelength of Hetzler's photographic infrared agrees well with his curves 87 and 88, which are likewise affected by moderately strong TiO absorption. For these stars, it therefore seems that the visible region is less affected by the TiO opacity. The humps on the rising branch of the light curve are generally centered at  $\phi = 0.7$

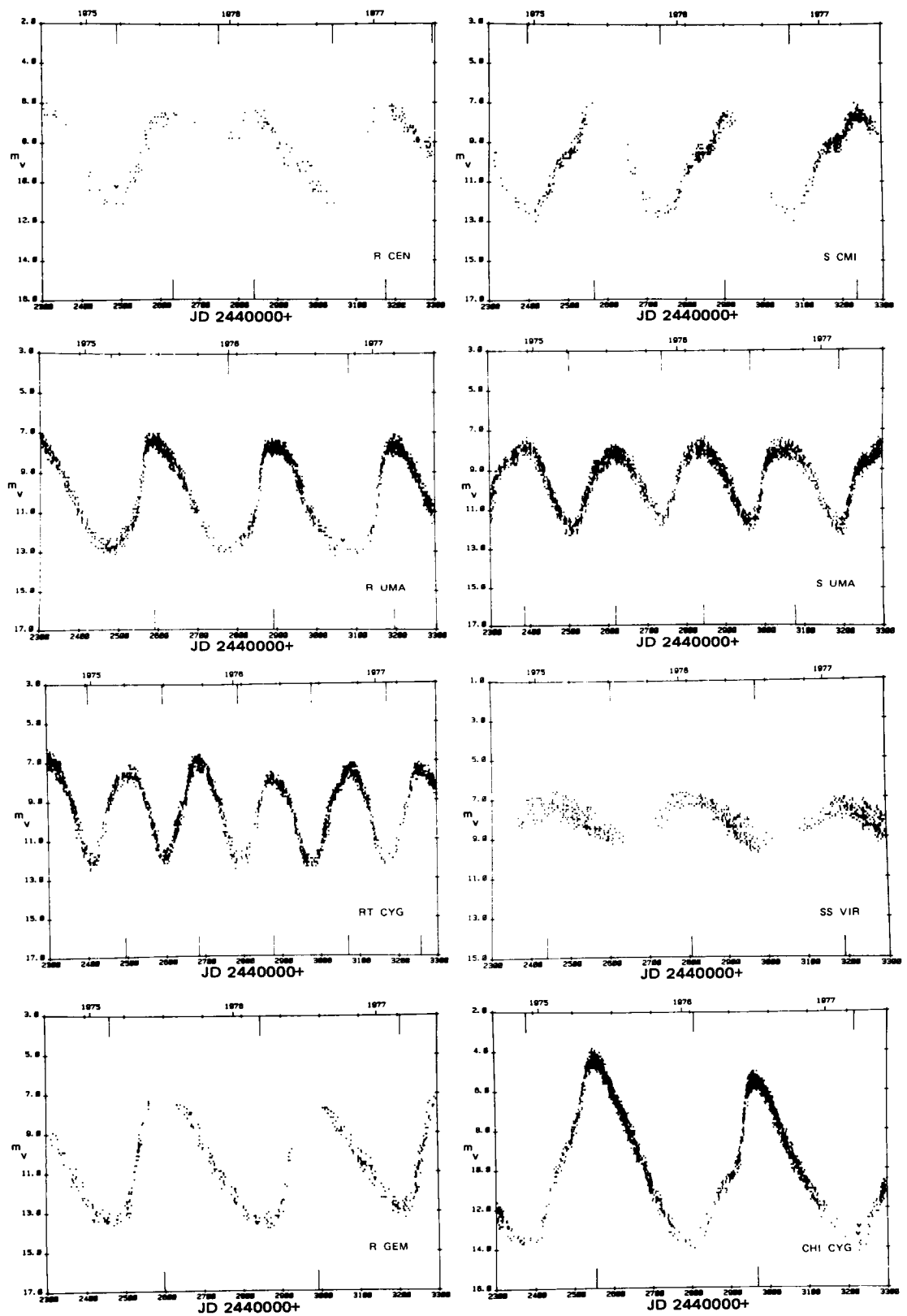


Figure 1-9. Various shapes in the Mira light curves (from AAVSO, 1983).



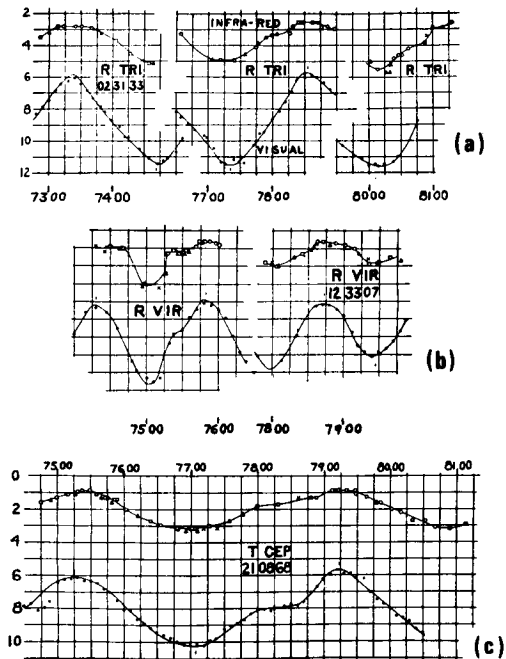


Figure 1-10. Mira light curves illustrating humps on the rising branch (from Hetzler, 1936).

(e.g., T Cas in Figure 1-8). The majority of the stars studied by Lockwood and Wing (1971) show evidence of humps in at least a few cycles. In some of these stars, the humps have reoccurred (R Cam and T Cep), but in some stars, the humps are spread out during the phase interval 0.6 to 0.8. Glasby (1968) indicates the presence of humps in the descending branch of the light curves. Vernon-Robinson (1929) observed secondary oscillation on the descending branch of XY Cas. The hump phenomenon is present in some Miras and not in others, and it comes and goes in some of them. The visual curve of  $\chi$  Cyg is a good example of a hump on the ascending branch (Figure 1-9). T Cas has a hump on the ascending branch of the light curve; its maxima and minima and the size of the hump vary from one cycle to the following one (Leung, 1971). The humps are considered as a rapid drop of the radius

of the star (Lockwood and Wing, 1971).

9. Inflection points in the increasing and decreasing parts of the light curves (R Cha, T Hya, R Ind, V Mon, T Nor, and RT Sgr); their periods range from 200 to 350 days (Celis, 1977).

Nearly all the short-period stars (less than 150 days) do not have very constant elements  $p$  and  $q$  with respect to the length of a given period  $P$  (Celis, 1977). As an example, T Gru ( $P = 136.6$  days, M1 Iae–M2 Ibe) has  $P$  periods between 121 and 138 days,  $Q$  periods from 124 to 145 days, and partial period  $p$  from 54 to 83 days. The amplitudes go from 1.80 to 3.05, and the light minima differ up to 1.5 mag.

All these light curves might reflect the interaction of the opacities and density variations of the different atmospheric layers and the propagation of shock waves (see M. Querci, this volume).

Are light curves at other wavelengths similar to the visual ones? The two stars in Figure 1-8 have V light variations of a strikingly different nature, which is preserved at the different observed wavelengths. The hump on the rising branch of the AAVSO curve for T Cas also appears on the infrared curves, including the continuum curve, I104.

**Variation in Colors During Mira Light Cycles.** All the observed Miras (for M, S, and C spectral types) show variations in color during their cycles of light, which are first indicated by the analysis of color indices obtained from visual observations (generally from AAVSO or Association Française des Observateurs d'Etoiles Variables (AFOEV)) and broadband photometry  $U, B, V, I, K$ , etc. Narrowband photometry with adapted filters for characteristic atomic (He I,  $H\alpha$ , and Na I) and molecular (TiO, ZrO, VO, etc.) features is proposed, at first by Wing (1967b). Spectrophotometry is also a good technique for investigating the variation in colors during the Mira phases. We will not elaborate here on the variations of line intensity or profile; they are dealt with by M. Querci, this volume.

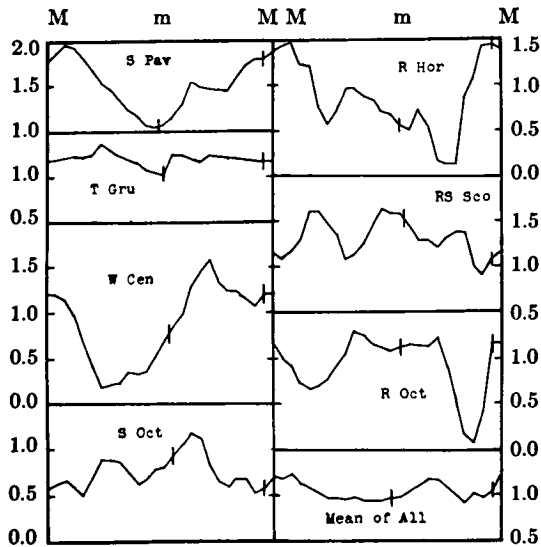


Figure 1-11. Mean color curves of seven Mira variables: *S Pav*, *R Hor*, *T Gru*, *RS Sco*, *W Cen*, *R Oct*, and *S Oct*. Ordinates and abscissas are color index and phase, respectively, all periods being reduced to the same horizontal scale. Maxima and minima are marked with short vertical lines and labeled "M" and "m" (from Campbell and Payne, 1930).

As early as 1930, Cannon (1930) showed that the color index (difference between the photographic and photometric magnitudes) is at its greatest about 40 days before maximum and at its minimum for 100 to 150 days after maximum. Campbell and Payne (1930) obtained photographic light curves and used the AAVSO visual ones to give the color indices from observations assembled into 10-day means; these are obtained by subtracting the visual magnitude from the photographic one (Figure 1-11). Two opposite effects contribute to the photographic minus visual index: (1) the great strengthening of TiO absorption, which cuts down the red part of the spectrum and diminishes the visual brightness more than it reduces the photographic light; and (2) during the fall, the decrease of the temperature of the photographic and visual forming layers which should strengthen the red end of the spectrum.

During the Mira light variations, the  $B-V$  values vary and the spectral classes usually

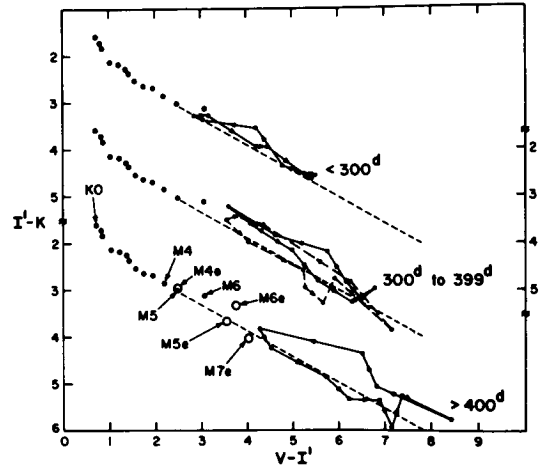


Figure 1-12. Color-color curves for luminosity class III nonvariable stars and Mira stars of spectral type M and S. Ordinates, in  $(I'-K)$  magnitudes; abscissas, in  $(V-I')$  magnitudes. Heavy dots (plotted three times on shifted scales of ordinates) = nonvariable stars from K0 to M6; smaller dots = loops described during the cycle of variation for stars of average period 239, 350, and 456 days, class Me; broken lines = Se stars, mean period 380 days; dotted lines extend the relation for nonvariable stars through points corresponding to the fall from maximum. On the lowest curve, points corresponding to K0 III, M4 III, M5 III, and M6 III are identified. Circles show the relation between maximal colors for Mira stars of classes M4e, M6e, and M7e (from Payne-Gaposchkin, 1975).

change, the earliest spectral class appearing near the visual maximum light.

It is interesting to compare the color indices of Miras during their cycles with the indices of the nonvariable stars of the same spectral type and luminosity class. Payne-Gaposchkin (1975; Figure 1-12) shows color/color curves for Mira stars and nonvariable stars of luminosity class III. During the decline from the visual maximum and a little later, the representative points of the Miras follow the relation shown by the nonvariable stars of the same spectral type. The  $I'$  filter used in Figure 1-12 is related to the Johnson system by  $(I-K) = 0.745(I'-K) - 0.13$ . Two remarks will be made about this figure: the length and the maximum width of the loop

(I'-K), and consequently, its area versus (V-I') seem to be correlated with the period/amplitude relation of the M Mira stars; secondly, during the maximum phase, the stars are bluer than during the minimum. This was also obtained by Mendoza (1967), using his multicolor photometry. The analyzed set of stars indicates that, with the light variation in V, the  $B-V$  index and the other color indices change. Typically, as the star becomes fainter in V (it is running to the minimum), it becomes redder; how-

ever, the C star, LW Cyg, is an exception.

Spinrad and Wing (1969) demonstrate that the variation of the molecular (TiO + VO) index around  $1 \mu\text{m}$  is correlated to the observed temperature (Figure 1-13). On the other hand, the nonvariables and small amplitude variables are represented by their mean line running from near the origin to the box labeled RX Boo in Figure 1-13. (This box contains all five scans of the semiregular variable, RX Boo.) Each Mira moves on the color index/temperature

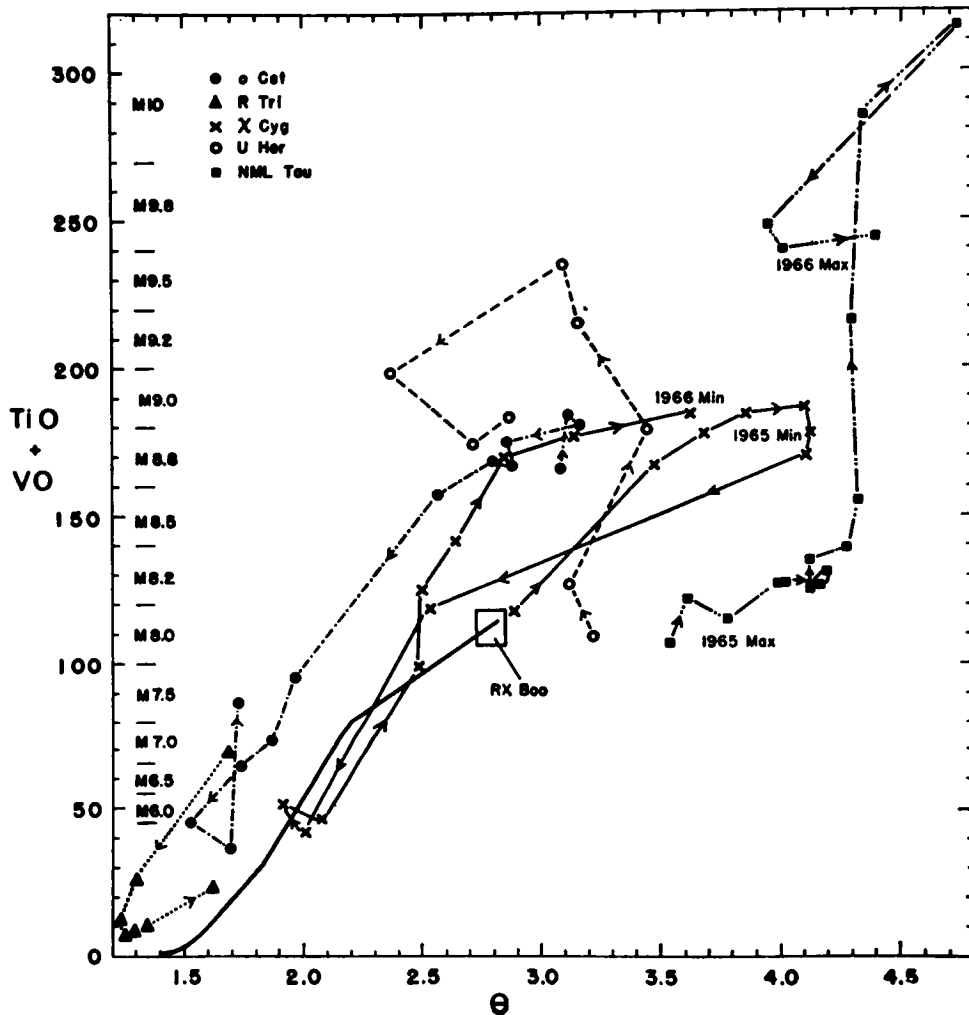


Figure 1-13. The relations between band strength and temperature are obtained from observations in the  $1\text{-}\mu\text{m}$  region. The ordinate is the sum of TiO and VO indices (unit = 0.01 mag) and is calibrated in terms of spectral type. The abscissa is  $\theta = 5040/T$ , where temperature  $T$  is obtained from a blackbody fit to the continuum. Observations are connected in chronological order (from Spinrad and Wing, 1969).

plane along the period:

1. R Tri ( $\blacktriangle$ ) is a relatively early-type Mira; its earliest spectral type was M3, obtained at its 1965 maximum, but its corresponding temperature was that of a normal K4 giant.
2.  $\chi$  Cyg ( $\times$ ) was observed during a large part of two successive cycles, and its minimum approach paths were seen to be different in the two cycles.
3.  $\circ$  Cet ( $\bullet$ ) is shown from minimum to the post-maximum in the 1965 cycle. Its molecular index is much stronger than in non-Miras of the same temperature; consequently, this star is much hotter than the temperature obtained from its spectral type.

Another extensive project is from Maehara and Yamashita (1978), who use a photoelectric scanning spectrophotometer. The wavelength region covered is 3700 to 5500 Å with 13 Å resolution. They give the energy distribution measured at flux peaks for a suitable sample of stars of luminosity class Iab to III and spectral range from M0 to M8 stars. Three S stars (V Cnc, R And, and W And) and one carbon star (SS Vir) are also included. The comparison of these curves with those of nonvariable M stars gives the spectral type variation of each Mira star during the period:  $\circ$  Cet ranges from M5 to M8, R Aur from M6 to M8, R Cnc from M6 to M8, etc. The S stars have an M-star behavior in the spectral region of this investigation because they suffer from TiO absorptions. The C-type Mira, SS Vir (C6,3e), shows an entirely different distribution. The flux is too small to be measured shortward of  $\lambda$ 4250. The variations of the color indices in the spectral range 4250 to 5000 Å are very small compared to those of M and S stars; the variation of the C<sub>2</sub> band head at  $\lambda$ 4737 is also relatively small.

Krempec (1975) shows changes of the central depth in spectral features, such as in C<sub>2</sub> and CN molecular bands and in atomic D-lines, with the light phase.

A comparison of the color changes observed on two prototype stars with two maxima per period (R Cen and BH Cru) shows differences: (1) the two  $U-B$  curves have reversed shapes, and (2) on the R Cen curve, the color index decreases at the second maxima, while in BH Cru, a sharp increase is observed. The  $B-V$  curves of these two stars show only small differences.

To conclude this study on the variations of color indices, we point out that there is a large variation in the color index for stars of the same spectral subclass; the strength of the hypothetical shock wave that causes the variability should not be correlated with the spectral subclass. The same remark applies to the SRa variables according to Alksne and Ikaunieks (1971). In their review of carbon stars, these authors emphasize that the mean values of  $B-V$  and  $V-I$  indices decrease somewhat in the sequence Mira, SR, L.

**Period Changes.** Little is known about the frequency of period changes in Mira-type stars because a long-time base of observations is required and the different shape and amplitude of successive cycles make the inferences more doubtful. Nevertheless, three kinds of period changes are listed:

1. A secular evolution by smooth increasing or decreasing of the length of the period or by a sine or cosine variation of the latter.
2. Some shape changes of the light curve between two or several consecutive periods.
3. An abrupt change of period and an abrupt shift of epoch.

From a 20-year survey, Hoffleit (1976) shows that  $\log(\Delta P/P)$  is spread around a linear function of  $\log P$ , where  $\Delta P$  is the maximum difference between the various periods of the same star and  $P$  is the mean period. This observed relationship holds for Miras, SR variables, W Vir stars, classical cepheids, and RR Lyr stars.

Hoffleit (1979) has drawn a catalog of 356 stars that have, at one time or another, been recorded as having changing periods; these stars

were selected in the *General Catalogue of Variable Stars* (Kukarkin, 1976), in the *Geschicht und Literatur des Lichtwechsels* (Prager and Schneller, 1934, 1963), and in some *Information Bulletins of Variable Stars* (IBVS). Among 332 Miras, 273 individual increases in period (e.g., for V Cas and U Her) and 290 decreases (e.g., for T Cep) were noted. This catalog is available on request to its author. To have some idea as to the number of variable stars with different kinds of changing period, we must refer to the 108 Sagittarius stars examined in sufficient detail by Hoffleit (1979). The observations could not be suitably represented by a constant period over 50 years, but the O-C plots for a constant period are best represented by one *abrupt change*. Three stars have the O-C curve represented by a parabola, indicating that the period is progressively increasing or decreasing. One star has its O-C curve represented by a sine-term, which suggests that the period is alternatively increasing and decreasing.

The four stars, V462 Cyg, V734 Cyg, HO Lyr, and MV Sgr, have cyclical changes in period. The O-C curves are represented by sine or cosine terms (Hoffleit, 1979). U Boo and S Ser have sinusoidally changing periods (Sterne and Campbell, 1937), as well as six others (R And, RS Cen, Z Cyg, TU Cyg, W Her, and W Hya) given by Prager and Schneller (1934). However, this kind of ephemeris is a very simple mathematical representation of very complex physics, and also it breaks down during 50 years. Thus, several later catalogs dropped these representations in favor of listing discrete periods for successive time intervals.

The conclusions of Nudjenko (1974) about the period changes are noticeably different from those of Hoffleit (1979). Forty-three (O-C) curves of the former demonstrate that the changes appear to be smoothly continuous rather than abrupt like these observed by Hoffleit. Investigations of all of the data are required. Table 1-6 summarizes the number of stars that have changed their period  $n$  times (Hoffleit, 1979).

Sterne and Campbell (1937) and Hoffleit (1979) come to the same conclusion that the

Table 1-6  
Number of Stars Having Changed Period\*  $n$  Times

$n$	No. Stars	$n$	No. Stars
2	146	7	1
3	110	8	0
4	46	9	1
5	15	10	0
6	4	$\geq 10$	2

\*From Hoffleit (1979).

stars, R Cnc, R Cen, and X Lib, have changed periods.

Some examples of stars that have a progressively decreasing period length are:

1. HS Aql, with its period ranging from 267 days in the 1930's, through 263.8 days in the late 1950's, to approximately 260 days in 1970's (Thompson, 1981). This star might be climbing the asymptotic branch.
2. W Tau, with its period ranging from 273 days in 1887 to 253 days in 1956 (Schneller, 1965).
3. R Aql, with its regularly decreasing period (Hoffleit, 1979; Schneller, 1965) in a sinusoidal steady manner (Payne and Campbell, 1930).
4. R Hya, with its period of about 500 days in the 18th century to about 390 days as noted by Merrill (1946). These period changes are used by Wood and Zarro (1981) to provide direct observational confirmation of the theory of helium flash shell (see their Figure 3).

A list of such stars may be found in Hoffleit's catalog (1979).

On the other hand, some stars, like V Cas and U Her, have increasing period length (Lockwood and Wing, 1971). Again, Hoffleit's

catalog (1979) lists stars with likely smooth varying periods.

It is useful to know that the prototype star,  $\alpha$  Cet, was noted in Prager and Schneller (1934) to have 12 discrete periods ranging from 322.5 to 335.4 days. For a total of 335 epochs of maxima, the best constant period is 331.7 days, giving a *nonsystematic* spread in O-C of 82 days and  $\Delta(O-C)/P = 0.247$ . Consequently, Hofleit (1979) concludes that "this star is no longer considered as having a significantly changing period."

In 1936, Hetzler (1936) shows some small variations on consecutive light curves of Miras. The perturbation appears in the visible AAVSO light curves and in the infrared ones at  $\lambda 8500$ . The latter wavelength is affected moderately by strong TiO absorption (Wing, 1971). Figure 1-10b shows two light curves (IR on the top and visual on the bottom) for two periods of R Vir separated by a period which is not indicated. The hump shown in the first period is not present in the second one (or is much less noticeable).

R Car is an astonishing star; it shows a wide maximum cycle followed by a narrow maximum one (Celis, 1977).

Abrupt changes of period (rapid fluctuations of the length of period) appear in some Miras; they are visible from the differences (O-C) between the observed epoch of maximum and the computed dates derived from formulas ( $JD_{Max} = P \times E$ ). This phenomenon was observed on T Phe and U Tuc by Campbell (1926) and on RT Eri (Payne and Campbell, 1930); it is not correlated with the length of the next cycle, such as the shift of maximum described in the next paragraph.

Abrupt shifts of epoch (fluctuations of phase) are also pointed out by Ludendorff (1928) on R Lup and by Shapley (1929) on Z Aqr. For the latter, the (O-C) curve demonstrates that the period remains constant over the interval covered by observations (1896 to 1929), although there appears to be a jump in epoch of about -55 days near JD 2420900. This change seems to take place within about two periods, but the observations are not

spaced closely enough to determine whether the change is due to an abnormal lengthening of a maximum or a minimum. New photometric and spectroscopic observations are needed on such stars.

**Multiple Periods.** The light curves can also be analyzed by other means, such as the superposition of two or more distinct periodic light curves. Payne-Gaposchkin (1954b) notes that the ratio  $P_2/P_1$  is approximately the same for the stars of a given spectral type, but it differs from one spectral type to another. She gives  $P_2/P_1 \approx 9.4$  for the M stars and 12.2 for N stars. The frequency distribution of  $P_2/P_1$  for M-type variables has a rather flat maximum between 7.5 and 10, with a slight peak between 9 and 10 (Houck, 1963). With a small sample of N-type stars, Houck shows a maximum frequency of  $P_2/P_1$  between 12 and 13. These values agree well with Gaposchkin's earlier results. Houck (1963) gives six Mira variables with secondary periods: SV And, U Per, V Hya, V545 Cen, Y Per, and V1280 Sgr. Among the variables with M-type spectra, there are five with  $P_2/P_1$  ratio in the range 22.5 to 51.5 and one N star with this ratio about 45.4. In these quoted papers,  $P_2$  should be interpreted in terms of a beat frequency phenomenon resulting from interference between two slightly different periods within the atmosphere of the stars.

Leung (1971) describes a method of analysis of superposition of two or more nonsinusoidal period components. Leung (1980) applies this method to 50-year observations on six selected M and S Miras:  $\alpha$  Cet, R And, T Cep, T Cas, R Hya, and S CrB. Two components were determined for the majority of the observed Miras (Figure 1-14). For five variables, the period ratios,  $P_2/P_1$ , of the two individual nonsinusoidal period components are around the value,  $1.04 \pm 0.01$ . In general, the shorter period  $P_1$  is associated with the larger amplitude of variation, and the amplitude ratios,  $A_2/A_1$ , range from 0.15 to 0.28. The amplitude and period ratios of period components of S CrB do not fit into the general behavior pattern described

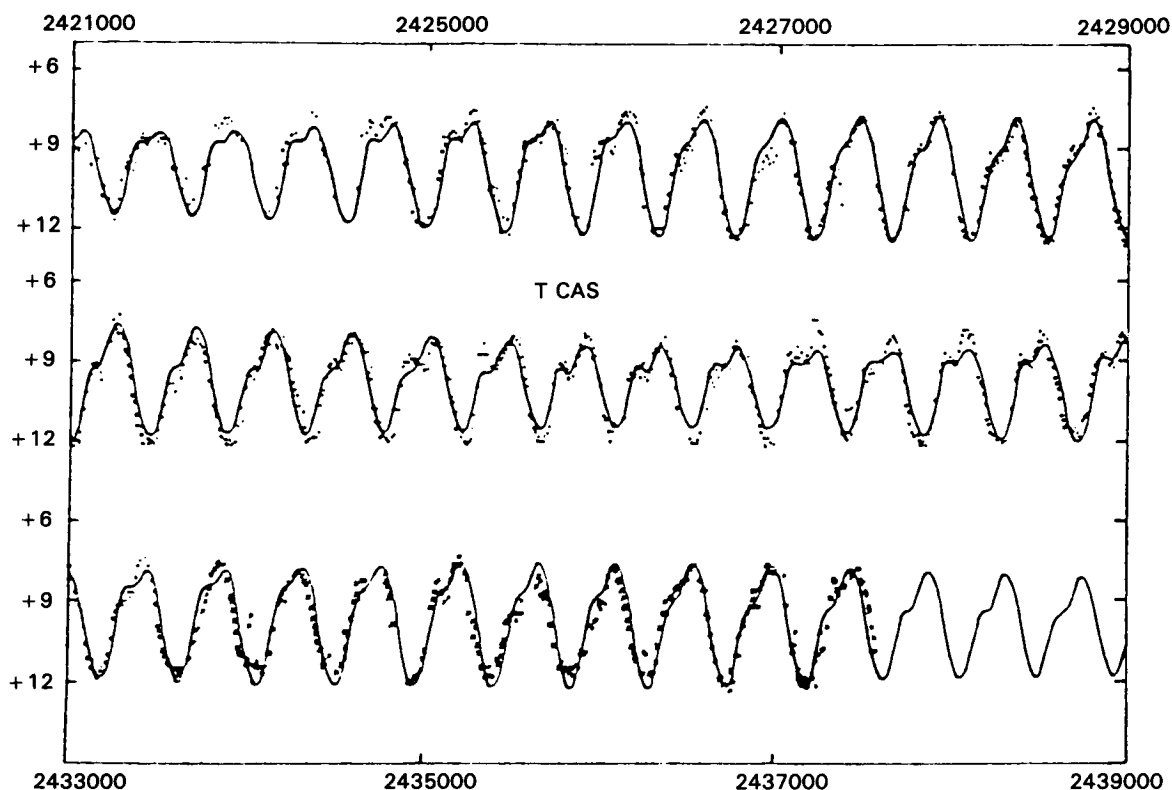


Figure 1-14. AAVSO observations of *T Cas* (10-day mean). The computed light curve (smooth curve) is based on two period components. The observations not included in the determination of period components are denoted as crosses. The abscissa is Julian Day (from Leung, 1980).

here. The  $\alpha$  Ceti light curve needs the superposition of three periodic components.

Houck (1963) gives two periods for the C star,  $V$  Hya:  $P_1 = 533$  days and  $P_2 = 6500$  days (beat frequency). These values are of the same order as those of Campbell (1943), who finds  $P_1 = 539$  days and  $P_2 = 18$  years. The latter period gives extra-deep minima reached in 1889, 1908, 1925, and 1943. Analyzing the data gathered by AAVSO from 1860 to 1968 and using the method of analysis of Leung (1971), Leung (1973) points out one period of 6670 days with an amplitude of about 3.5 magnitude. The other component has a period of 530.7 days, an amplitude variation of 1.1 magnitude, and a shape close to sinusoidal, so that the larger period component has the larger amplitude. Note that the opposite holds for the red supergiant variables (Leung, 1973). He concludes that  $V$  Hya may be a late Mira of Ib

type, for which the long-period component is not due to a normal mode of radial pulsation, and that the dip preceding the minimum is probably due to nonlinear effects or cold spots on the star surface. Reanalyzing the data of  $V$  Hya, Leung (1980) concludes that this carbon star has an extremely peculiar light variation and proposes four distinct period components.

To conclude, we recall Leung (1980), who suggests that the primary period,  $P_1$ , is the pulsational period and that the modulating period,  $P_2$ , may be derived from a nonradial mode. The multiple periodicity may reveal information on the modes of oscillation of these stars.

**Short-Term Fluctuations.** In addition to the rapid evolution in the  $\alpha$  Ceti effective temperature on 12 days reported by Joshi et al. (1980), we want to detail some scarce observations

which seem to indicate short-term fluctuations in the Mira variables.

The  $J = 1-0$ ,  $v = 1$ , and  $v = 2$  SiO maser emission lines of the Mira variable, R Leo, show phase-dependence effects in both line shape and intensity. At the premaximum phases, they are strong narrow features of decreasing intensity. Around the visual maximum ( $\varphi = 0.98, 0.02, 0.04$ ), they appear to be respectively about 30 and 15 percent broader without any trace of narrow features (Clark et al., 1982). The effective duration of these changes continues up to  $\varphi \approx 0.40$ .

With the Griffin-type radial-velocity spectrometer and masks especially designed for Mira spectra, Pierce et al. (1979) find nightly variations of radial velocity of emission and absorption lines in the spectra of Miras. More precisely, the range in the measurements for each night is larger than the average uncertainty for a set of observations covering 7 nights between  $\varphi = 0.9$  and  $\varphi = 0.2$ . For example: on T Cep, the nights at  $\varphi = 0.02$  and  $\varphi = 0.12$  show a variable behavior of the absorption lines (the emission lines are too faint to be measured); on  $\chi$  Cyg, the nights at  $\varphi = 0.07$ ,  $\varphi = 0.11$ , and  $\varphi = 0.13$  show a variable behavior of the emission lines; and on  $\circ$  Cet, the radial velocity is variable in the absorption lines for the nights around  $\varphi = 0.76$  and  $\varphi = 0.21$  and in the emission lines for the nights at  $\varphi = 0.12$ . The range of these nightly radial-velocity variations is between  $-2$  and  $-13$  km/s. These short-term variations should be interpreted by the *interaction of shock waves with nonuniform atmospheric layers*. Also, they should be linked to eventual flares such as those observed in R Aql (Woodworth and Hughes, 1973, 1977) or V Cyg (Querci et al., 1979); however, simultaneous observations are not available. Finally, it might be that these variations in radial velocity are a consequence of matter swept along magnetic tubes (see M. Querci, this volume).

**Correlations Between Different Observed Quantities.** Numerous correlations between the observed quantities such as period length, light-curve shape, IR excess, visible and IR color diagram, and OH radio-line separation exist in

the literature. Apparently, some of them are surprising, because we do not understand for the moment why they happen. However, we shall note in detail here those of the greatest importance and shall endeavor to draw conclusions on the relationships between the derived physical parameters.

The first correlations between the observed quantities were made at the Harvard Observatory. Campbell (1925) first demonstrated that a statistical relationship exists between the shape of the light curve and the period. The asymmetry becomes gradually more pronounced as the period increases. In all three spectral classes (M, S, or C), Mira light amplitudes are smaller for shorter periods and greater for longer periods. This was again explained by J.G. Garcia (1980).

Using the data of Kukarkin et al. (1958), Merrill (1960) found that the period of M, S, and C stars is distributed in a broad maximum around 300 days for M stars, 360 days for S stars, and 400 days for C stars. Consequently, the C Miras have, on the average, the longest periods.

Using two groups of Miras, one in which emission lines have low radial velocities ( $< 20$  km/s) and another in which they have high radial velocities ( $> 80$  km/s; from Merrill's catalog, 1941), Ahnert (1969) finds that the larger the scattering of cycles in percentage of periods, the earlier the subgroup of Me stars. A corresponding relation exists between the scattering and the period length, a relation which seems trivial because it is a well-known fact that the longer the period, the later the Me subgroup (Glasby, 1968).

Harrington (1965) investigated 165 Miras from among the data of Campbell (1955) and found a correlation between the difference in magnitude of two successive maxima and the time interval separating the two maxima. When the interval is under the average value, the second maximum is brighter (45 percent of the considered stars show this correlation with a significance of at least 95 percent). These light curves may be interpreted by the outward propagating disturbances taking more or less time



to reach the star surface, this time being a function of the energy carried.

The kinematic studies of Feast (1962) imply that the shorter period Mira variables are Population II stars, while those of longer periods belong to more intermediate populations. These conclusions suggest that the period amplitude of the Mira stars may be a function of their metal abundance.

To determine the difference in metal abundances between short and long-period Mira variables, DeGioia-Eastwood et al. (1981) used the relationship derived between  $|W|$ , the velocity of the star perpendicular to the galactic plane, and  $\delta(U-B)$ , the ultraviolet excess, given by Eggen et al. (1962). To calibrate the relationship between  $[Fe/H]$  and the kinematic properties, they use Carney's (1979) relation between  $\delta(U-B)$  and  $[Fe/H]$ . They have examined two groups of Mira variables: the stars with periods  $149 < P \leq 200$  days were found to be deficient in  $[Fe/H]$  by more than 1 dex, compared to the group with  $350 < P \leq 410$  days. The metal deficiencies in the short-period Mira variables are substantial, and consequently, the mass lost is proportional not only to the outflow velocity and the mass of the shell, but also to the abundance of heavy elements contained in the ejected matter.

Eggen (1973) shows that the periods and colors of small-amplitude red variables are not related. Eggen (1975) deduces a linear relation in the plane  $(\log P, (R-I)_0)$  for the Miras, the index  $(R-I)_0$  being measured at phase 0.25 (approximately medium light). This linear relation is given by:

$$(R-I)_0 = -0.45 \text{ mag} + 0.90 \log P$$

It is probable that this relation becomes non-linear for stars with periods longer than 500 days that are redder than  $R-I = 1.8$  mag (Eggen, 1975); although the number of such stars is not important, observations are needed to determine the  $((R-I)_0, \log P)$  relationship. The halo Mira variables have no correlation between the period and the color  $R-I$  at phase 0.25 (medium light). Feast (1980) points out that Miras show a slow rise in brightness with

period, from  $M_{bol} \sim -4.1$  at 200 days to  $M_{bol} \sim -4.6$  at 400 days. The short-period ( $\sim 135$  days) Miras do not follow this law and are quite fainter.

The extension of the visible to the red and radio spectral ranges, and consequently, the detection of the IR excess and of the maser lines, leads to many correlations for an improved knowledge of the Mira behavior. Moreover, the analysis of M, S, and C stellar spectra using model atmosphere synthetic spectra is very useful because it shows that the stellar continuum could be reached in small intervals around 1.04, 3.6, and 8  $\mu\text{m}$  (see H.R. Johnson, this volume).

Lockwood and Wing (1971) and Wing and Lockwood (1973) find a direct correlation between the period length and the amplitude variation at 1.04  $\mu\text{m}$  for a large sample of M stars, whereas Keenan et al. (1974) find a direct correlation between the mean spectral type at maximum light and the period for the Me stars. Ukita (1982) shows (Figure 1-15) this relationship for the Mira stars with strong OH emission, weak OH emission, non-OH Miras, and Miras which have not been observed in OH. Frogel (1971) established a correlation between the variation of the  $[1.2 \mu\text{m}]-[3.5 \mu\text{m}]$  color index and the amplitude variation at 2.2  $\mu\text{m}$  for stars with little or large excess 10- $\mu\text{m}$  radiation. A tendency for larger period stars to have longer amplitudes and larger  $[0.8 \mu\text{m}]-[2.2 \mu\text{m}]$  or  $[I-K]$  color indices was noted by Hyland et al. (1972). The continuum amplitude variations can be explained by variations in the effective temperature, whereas both the effective temperature and molecular opacities act together on other spectral ranges. Using the 2.2- $\mu\text{m}$  spectral range, Harvey et al. (1974) again found the period-amplitude relation previously obtained in the visible and at 1.04  $\mu\text{m}$ ; they also established a relation between the variation of the  $[3.5 \mu\text{m}]-[10.0 \mu\text{m}]$  color index and the period amplitude. DeGioia-Eastwood et al. (1981) found that the  $[8.7 \mu\text{m}]-[11.4 \mu\text{m}]$  color augments monotonically with increasing period for a sample of 41 Mira variables of M spectral type. However, since the photometric phase at the time of each measurement had not been

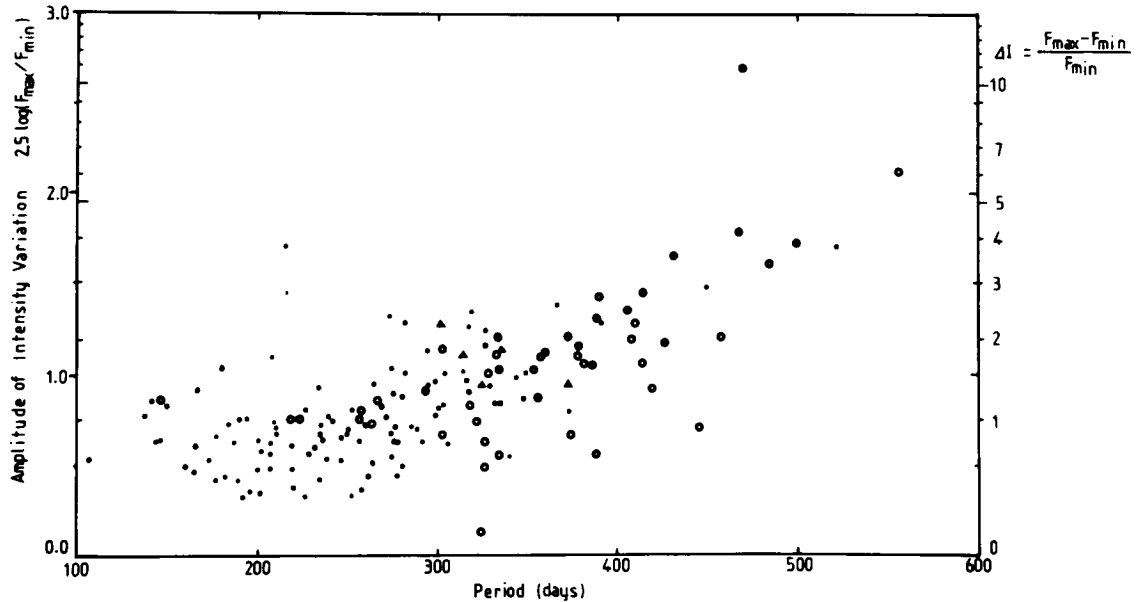


Figure 1-15. Plot of the  $1.04\text{-}\mu\text{m}$  intensity variations against stellar period for oxygen-rich Mira variables: filled circles represent OH Miras in which OH emission was detected [ $\text{OH luminosity} \equiv \text{SD}^2 \geq 0.1 \text{ Jy}(\text{kpc})^2$ ]; filled triangles are OH Miras with weak OH emission [ $\text{SD}^2 < 0.1 \text{ Jy}(\text{kpc})^2$ ]; open circles are non-OH Miras; dots are Miras not having been observed in OH or for which upper limits are poorer than the above criterion (from Ukita, 1982).

taken into account (of course, how could it?), some of the scatter may be due to intrinsic variability. We conclude that the height of the silicate feature increases with the period amplitude, and consequently, the ratio of mass loss and dust formation, as well as the temperature of the silicate emitting layers, are directly linked to the stellar parameters which dictate the period amplitude. The height of the silicate feature does not seem to be due to a stochastic process in any Mira variable.

Like DeGioia-Eastwood et al. (1981), we point out that the IR excess is not indicative of the total amount of mass in the shell, which is governed by the mass loss, but only of the mass of dust (and not gas) contained in the *thermosphere*, which extends to that radius in the shell where the dust is sufficiently warm to radiate significantly at  $10 \mu\text{m}$ . Therefore, the  $[8.7 \mu\text{m}]-[11.4 \mu\text{m}]$  versus period relation can be interpreted as an indication that the observed mass of dust in a given thermosphere is an almost linear function of the period.

Bearing in mind that the Population II stars

have a short period and that they are metal-deficient, we assume that two stars are equal in mass and luminosity and differ only in period; the previous correlation permits us to conclude that the Population II stars are also infrared-excess-deficient, and consequently, that dust formation may be a function of the metal abundance.

In some stars, Wilson (1970) noted that the OH 1612-MHz masers and the IR variations are clearly correlated. Harvey et al. (1974) developed a monitoring program to measure, at monthly intervals, the maser at 1612 MHz and the broadband infrared fluxes at six wavelengths for OH/IR sources. They essentially confirm the observed correlation between the infrared and the 1612-MHz variability of the OH/IR stars and the OH clouds. This coupling mechanism is consistent with a radiative pumping of the masers, possibly at 2.8, 35, 53, 80, and  $120 \mu\text{m}$  (Litvak, 1969; Litvak and Dickinson, 1972; Elitzur, 1978; Elitzur et al., 1976; Bujarrabal et al., 1980; Epchtein et al., 1980).

A monitoring of OH maser emission from the M-type stars, IRC + 10011, NML Tau, U Ori, S CrB, WX Ser, U Her, R Aql, and R Cas is made by Jewell et al. (1979). The maser emission is observed in the 1612-, 1665-, and 1667-MHz lines of OH and 22-GHz line of H<sub>2</sub>O. They conclude that, in most cases (their Figures 7 through 13), for all main and satellite lines, the maser pumping mechanism must be radiative in nature. For the satellite line, this has already been established with some certainty by Harvey et al. (1974). The data presented by Fillit et al. (1977) also suggest a radiative pumping for the satellite line and, with less certainty, for the main lines. For these latter lines, the work of Jewell et al. (1979) establishes conclusively that the main lines are also related to the optical flux of the star (see later for controversy). Schwartz et al. (1974) suggest two possible correlations between the 1.35-cm H<sub>2</sub>O maser and the 2.2- $\mu$ m infrared flux, either an exponential rate which fits the intensity of the maser versus the IR flux at 2.2  $\mu$ m, or a linear relation between them after a threshold of the infrared flux value (which is probably correlated with some processes quenching the maser at low level of infrared flux or a possible collisional deexcitation of H<sub>2</sub>O molecules). The pump may be linearly proportional to the infrared flux above the threshold if the maser is saturated.

Werner et al. (1980) simultaneously monitored far-IR, near-IR, and radio observations on five very red objects with 1612-MHz emissions. The direct comparison of these data strongly supports the hypothesis that the maser emission is pumped by 35- $\mu$ m photons (see also Nguyen-Q-Rieu, this volume).

In the case of U Ori, the comparison of different line velocities produces three velocity groups (Cimerman, 1979). The most important is the group at -42 km/s, which includes the zero-volt optical absorption lines, one line of the SiO transitions, and an OH line of each frequency seen in all observations. The second group includes the optical emission lines and one line of the 1612-MHz features at -47 km/s. Lines scattered around the velocity of -36 km/s

form the last group. Garrigue (1980) attempts to correlate the OH maser emission with the observed visible light curve. In his U Ori monitoring, he finds an entire period of light shifted by 2 mag above its usual mean value for each phase, whereas the light curve of this Mira is usually very regular. Garrigue and Mennessier (1980) point out that the beginning of this epoch of higher brightness began approximately 650 days before the 1612-MHz flare discovered by Pataki and Kolena (1974b) in November 1974 and that the lifetimes of both visual brightness and 1612-MHz satellite increases have 550-day durations. This confirms that the OH emission in the satellite line is due to a perturbation which propagates throughout the stellar atmosphere. The proposed scenario is as follows:

1. An overbrightening perturbation occurs in the photosphere, disturbing the visual light curve during 550 days.
2. It propagates throughout the stellar atmosphere with a mean velocity of 20 km/s (Slutz, 1976).
3. After about 650 days, the perturbation reaches the dust shell and disturbs the IR radiation emitted by it (at about 10<sup>14</sup> cm; from Gehrz and Woolf, 1971).

The shell IR radiation reaches the U Ori OH-emitting region (10<sup>15</sup> cm; from Reid et al., 1977) in 1 day, and "by OH radiation coupling, is responsible to the maser effect during 550 days" (Garrigue and Mennessier, 1980). This interpretation is a little different from the one proposed by Cimerman (1979), who does not take account of the 650-day delay between the beginning of the visible and the radio variations.

Four papers (Dickinson et al., 1973; Harvey et al., 1974; Dickinson et al., 1975; Morris et al., 1979) argue in favor of a direct correlation between the expansion velocity given by the OH maser circumstellar lines and the period. Lépine et al. (1976) show that this correlation is doubtful for their Mira sample. Using the relationship of Figure 1-15 and the hypothetical correlation expansion velocity versus period

amplitude, one might conclude that a direct correlation exists between the OH expansion velocity and the amplitude variation on the I104 filter. Research in this way was attempted by Ukita (1982), who confirms the uncertainty stressed by Lépine et al. (1976). With data of type I and II OH Miras (Figure 1-16), he finds that the expansion velocity has a direct correlation with the amplitude variation at  $1.04 \mu\text{m}$  and an inverse correlation with the period length. Knowing that the type II OH/IR Mira stars have larger color indices than the OH Miras and, consequently, have thicker envelopes (Harvey et al., 1974; Dickinson et al., 1975; Olton, 1977), Ukita (1982) assumes that the mass-loss rates increase in the order: non-OH Miras, OH Miras, and type II OH/IR stars. Ukita interprets his result in terms of a dust-driven wind enhanced by pulsation, a hybrid model of mass loss (Wood, 1979; Deguchi, 1980).

Looking for the SiO masers in a large sample of Miras, SRB, and supergiants, Spencer et

al. (1981) conclude that no global properties of these stars correlate with the SiO luminosity; this indicates that the SiO masers are located very close to the stars, where the local conditions affect the maser intensity much more than the global properties of the photosphere, such as the stellar temperature, affect it. The local conditions could be governed by clumps of gas, convective cells, or turbulent eddies. The variation of the SiO maser intensity and profile pointed out by Clark et al. (1982a, 1982b) in the Mira, R Cas, is a good example (see their Figures 1 through 5). Moreover, in Miras, the SiO maser flux is correlated with the bolometric flux (Cahn and Elitzur, 1979; Cahn, 1981) and with the infrared flux (Hjalmarsen and Olofsson, 1979). (Let us remark that, when the lines are weak, broad, and roughly parabolic, they are due to nonmaser emission formed in a thick expanding envelope (Robinson and Van Blerkom, 1981).)

Using VLBI observations, Bowers et al. (1980) correlate the mass-loss rates with the size

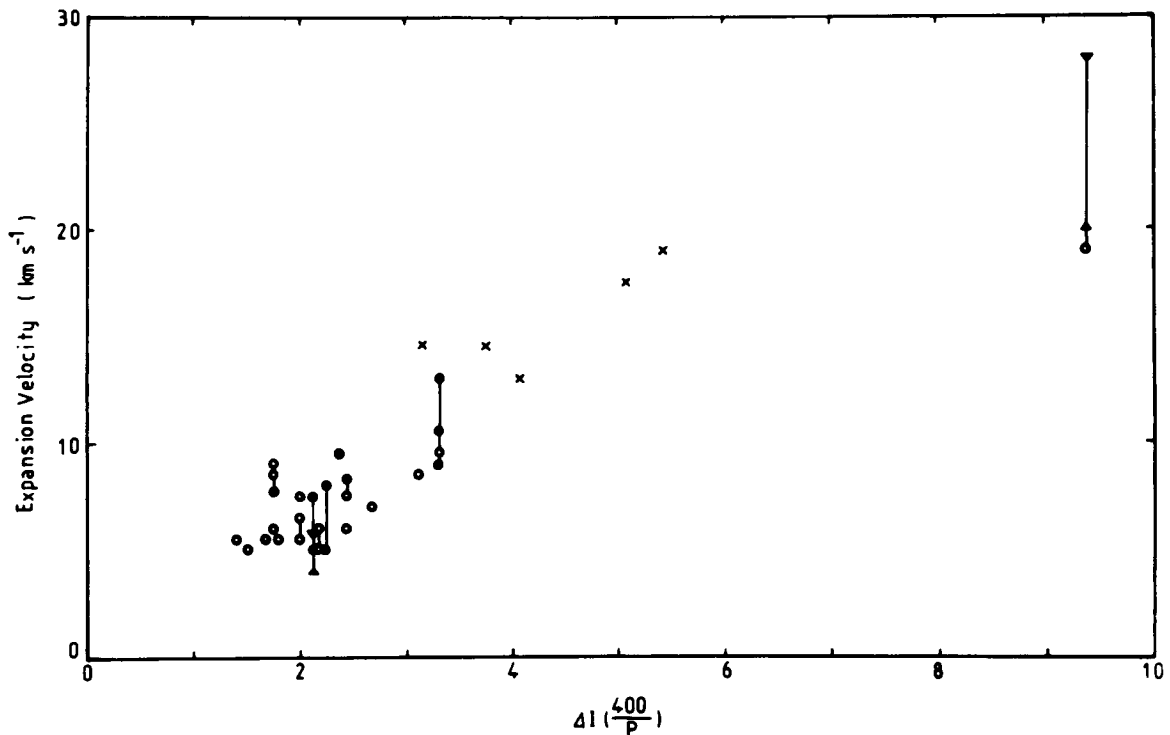


Figure 1-16. Plot of expansion velocity versus the quantity  $\Delta I \times 400/P$ , for oxygen-rich Mira variables. ( $\Delta I$  and marks are defined in Figure 1-15.) The crosses represent type II OH/IR stars (from Ukita, 1982).

of the masing regions; stars with mass-loss rates less than  $10^{-5} M_{\odot} \text{ yr}^{-1}$  have masing regions  $<1000$  AU in extent, while stars with stronger mass-loss rates have larger masing regions.

Wolff and Carlson (1982) point out a "general tendency of both line width and integrated intensity of SiO thermal emission lines to increase with infrared color excess."

Scharlach and Woolf (1979) look for a correlation between the maser activity and the amount of ejected matter, which is correlated with the subphotospheric activity (Thomas, 1973) as indicated by the hydrogen emission. They therefore searched for a correlation between the maser activity and the H $\delta$  emission line (which is the only Balmer line not disturbed by TiO absorptions); no correlation was found.

#### Modes of Pulsation of Long-Period Variables.

Miras are radial pulsators and are driven mainly by hydrogen ionization effects (e.g., Cox, 1984). The real germane problem is to know which radial mode is the primary mode of pulsation, the fundamental or the first overtone. Both modes have been proposed by different groups through comparisons between observational values of the pulsation constant  $Q$  and theoretical  $Q$ -values.

The pulsation constant  $Q$  (in days) relates the period  $P$  (in days) and the density  $\rho$  by:  $Q = P(\rho/\rho_{\odot})^{1/2}$ , otherwise:

$$Q = P (M/M_{\odot})^{1/2} (R/R_{\odot})^{-3/2},$$

with  $M$ , the mass, and  $R$ , the radius (i.e., a period-mass-radius (PMR) relation exists).

A primordial observational constraint to the  $Q$  determination concerns the knowledge of radii. Their evaluation by direct methods (giving angular diameters) such as occultation technics or speckle interferometry, or by photometry is debated by Wood (1981) and Willson (1982). (See also the section on *Radii*.) In the direct method, it is shown that radii vary from one spectral region to another; also distances to individual objects must be known. The photometric approach involves the luminosity definition:

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

The radius may be expressed in the function of  $L$ , hence the absolute bolometric magnitude  $M_{\text{bol}}$ , and of  $T_{\text{eff}}$ . Therefore, knowing observationally the  $M_{\text{bol}}/\log P$  relation and the  $\log T_{\text{eff}}/P$  relation for LPV's, we get observational  $Q M^{-1/2}/P$  relations. However, the uncertainty in the effective temperature determination (hence, in the radius determination) for the Miras remains the major problem in estimating  $Q$ -values from observations. (See the extensive discussions in Willson, 1982, and in Wood, 1981, 1982.) This is shown in the summary of observed  $Q$ -values given by Fox and Wood (1982, and references herein): (1) for the Population II small-amplitude variables (e.g., in globular clusters 47 Tuc, M4, M22, and  $\omega$  Cen), using the Ridgway et al. (1980) temperature scale (an extension to Miras of a scale based on nonvariable M giants), the pulsation mode is the first overtone for stars in the two first-quoted clusters, and the fundamental for stars in the two latter, without clear explanation; (2) for Miras in the solar neighborhood, using a blackbody temperature scale based on lunar occultation angular diameters, the mode is the first overtone, whereas the use of the Ridgway et al.  $T_{\text{eff}}$  scale might lead to the fundamental mode; (3) in the Magellanic Clouds, for the two groups of long-period variables (see the section *Aspects of Evolution of Long-Period Variables in the Magellanic Clouds*), the pulsation mode appears to be first overtone if blackbody temperatures are used, or fundamental mode if the Ridgway et al.  $T_{\text{eff}}$  are favored. In fact, one can have confidence in the blackbody color temperatures for LPV's of large amplitude that are linked to infrared lunar occultation diameters of Mira variables (as described in Glass and Feast, 1982); therefore, first-overtone pulsations are favored.

Pulsation models, as well as linear nonadiabatic models (Fox and Wood, 1982), have been calculated as nonlinear nonadiabatic models (Wood, 1974; Tuchman et al., 1979; Ostlie et al., 1982). In the extensive grid of linear nonadiabatic pulsation models of Fox and Wood (1982), theoretical values of the

pulsation constant are derived for the first three radial pulsation modes in red giants (galactic disc and Population II) and supergiants. Main results are as follows:

- In the standard models, for Miras in the galactic disc,  $Q$  is not constant for either mode.  $Q$ -values for the first overtone asymptotically approaches  $Q_1 \sim 0.04$  at low luminosity or high mass, and short  $P$ , but  $Q_1$  may take values up to about 0.075 (see Figure 3 in Fox and Wood, 1982); no simple period-mass-radius relation exists. For the fundamental mode,  $Q$ -values increase with  $P$ , and the relation  $P M^a R^{-b} = Q$  with  $a \sim 1.0$  and  $b \sim 2.0$  holds.
- The ratio  $P_0/P_1$  can take values from  $\sim 2$  to 3 for values of  $P_1 \leq 630$  days;  $P_0/P_1 \sim 7$  occurs when  $P_0 \sim 2000$ –5000 days in some massive luminous stars. Beat periods exist between  $P_1$  and  $P_2$  of  $\sim 3 P_1$  to  $\sim 20 P_1$  (Wood, 1982); some observed beat periods are given in the section *Multiple Periods*.

Note that, for the LPV's in Magellanic Clouds (models with slightly decreased metal abundances), the  $Q$ -values for the first overtone tend to cluster around  $Q$  (days) = 0.38.

Therefore, although the ratio of  $P_0/P_1$  seems to be well determined theoretically, determination of the mode of pulsation itself of the Miras requires more than the  $Q$ -value determination. Wood (1981) suggests that the strongest evidence in favor of the *first-overtone pulsation mode* is the presence of secondary periodicities caused by resonant coupling between the first-overtone (corresponding to the primary mode of pulsation) and the fundamental mode (the secondary mode of pulsation) when the latter has a period *almost twice* that of the first overtone. In fact, Wood (1982), among various examples, quotes massive supergiants (see Figure 1-3) which present secondary periods with  $P \sim 2000$ –7000 days (i.e., about seven times the length of the primary pulsation period) in agreement with the

$P_0/P_1 \sim 7$  found in supergiant models of Fox and Wood (1982). Also, carbon stars among AGB variables are known to have such an observed  $P_0/P_1$  ratio. Another support to the first-overtone primary mode of pulsation comes from Figure 1-3. As commented in the section *Aspects of Evolution of Long-Period Variables in Magellanic Clouds*, an AGB LPV switches from the first-overtone mode to the fundamental mode when crossing the dotted line corresponding to the long-period edge of the AGB region, to subsequently form a planetary nebula. If the primary mode is the fundamental one, the event that could suddenly stop the AGB evolution of an LPV still having substantial envelope mass is not clear. Evidence for the fundamental mode of pulsation in Miras has been given by Willson (1981, 1982, and references herein). Again, the controversy comes mainly from the used radii. Also, Willson analyzes shock propagation in Mira atmospheres to determine  $Q$ -values indicating the fundamental mode of pulsation. Wood (1981, 1982) extensively discussed her arguments and did not find them convincing.

### The SRa-SRb-SRc Variables

Sir Williams Herschel discovered the first two variable stars of this type:  $\mu$  Cep in 1782 and  $\alpha$  Her in 1796, with their magnitude varying in a "random" manner between 4–6 and 3–4, respectively. John Herschel attributed these irregularities to some periodic veiling effects caused by interposing dark nebulous matter.

The amplitudes of SR variables are smaller than those of Miras, and the lengths of their cycles are generally smaller than the period of the Miras, except for certain supergiants in which periods of many years are observed (e.g.,  $\alpha$  Ori).

It is very difficult to define a period for stars in which the random brightness fluctuations are more marked than the regular period. The duration of the observed cycle often varies by more than 1 month, and the individual maxima by more than a magnitude. However, the classification into the three following groups

(more or less arbitrary) is generally accepted:

1. The semiregulars of late spectral class (M, S, and C), denoted SRa, are giants. Many of these stars differ from the Mira types only in the smaller amplitude of light variations. Frequently, their curves have strong variations from one cycle to another. Typical representative stars are RU And, S Aql, Z Aqr, T Cnc, WZ Cas, T Cen, RS Cyg, RS Gem, and R UMi.
2. The SRb-type semiregulars have a poorly expressed periodicity. Different periods of the individual cycles prevent the prediction of the epochs of minimum and maximum brightness. These stars sometimes temporarily replace periodic changes by slow irregular variations or even by the constancy of the brightness. Typical stars are V Aqr, V Boo, RX Boo, UU Aur, X Cnc, Y CVn, TT Cyg, R Dor, RY Dra, UX Dra, T Ind, R Lyr, VY Leo, W Ori, L<sub>2</sub> Pup, ρ Per, SW Vir, and S Sct. The SRb stars are giants.
3. The SRC-type semiregulars are supergiants with an SRb behavior. The visual light changes are generally of the order of a single magnitude or less (Maeder, 1980). Typical representatives of the SRC type are VY CMa, μ Cep, RS Cnc, RW Cyg, TV Gem, Y Lyn, α Ori, S Per, and VX Sgr.

It appears that we find a larger fraction of semiregular variables among the carbon stars (C spectral class) than among the M and S stars. Almost half of the known semiregulars are N stars, the remainder being of M class; very few of S and R classes are known. Moreover, it is of interest to note that the N-type stars have larger primary and secondary cycles than the M class stars (for example, see Table 9 in Glasby, 1968).

The frequency-distribution curve of class C Mira and SR stars are very different. The former has a large maximum between 350 and

450 days, while the latter has two maxima around 150 and 400 days, clearly separated by a minimum around 270 days (Alksne and Ikaunieks, 1971). Analyzing this minimum in the distribution, Guzeva Yakimova (1960) noted the absence of carbon stars with periods (Miras) or cycles (SRa) between 250 and 285 days. With observations based on 316 stars (mainly spectral class M), Glasby (1968) uses the shortest cycle for SR stars (cycle intrinsically related to a possible pulsation of the star itself) and constructs a frequency-distribution curve for the cycle length. Two maxima are detected, around 85 and 135 days, on a period range from about 30 to 1000 days or more.

**Different Time-Scale Light Variations of Semiregular Variables.** The first approach to these stars could be light curves obtained by a visual estimate of the brightness; such tasks are well suited to amateur observers. Visual curves of semiregular stars do not have the strong similarity among them as do the Miras, but are quite individual. Moreover, instead of the mean periods of the Miras, the SR variables may be characterized by a form of periodicity which is hidden by irregular variations in brightness (Glasby, 1968).

Among the semiregular variable stars, the SRa are most similar to the Miras, with which we find many common characteristics such as Balmer emission around some light maxima as in WZ Cas, changing periods (Hoffleit, 1979), and other characteristics.

As for the SRb stars, there is often an erroneous classification between them and the irregular variables, Lb, because the observations are not continued for a sufficient duration. Maran et al. (1980) give a good example with VY Leo (56 Leo), which clearly shows the slow irregular variations that temporarily replace the periodic changes. Two groups of Brazilian astronomers have separately detected variations with a time scale of about 1 hour on the M8 II star, R Crt: Gomes Balboa et al. (1982) in the 22-GHz H<sub>2</sub>O line and Livi and Bergmann (1982) in the DDO magnitudes and colors (over TiO bands and Ca I lines). R Crt is the first SRb

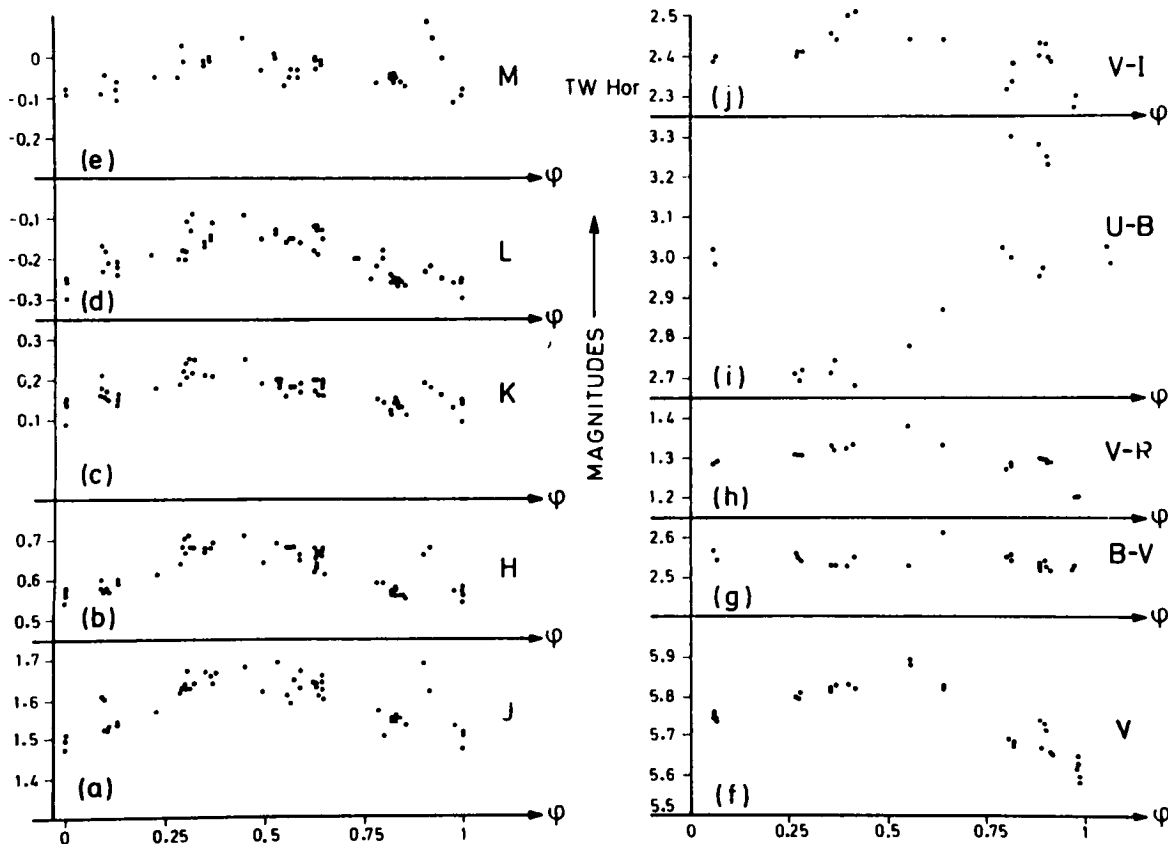


Figure 1-17. Photometric light curves of the SRb variable, TW Hor (from Bouchet, 1984a).

variable to show such rapid variations.

Bouchet (1984a) analyzes the C star, TW Hor, by broadband photometry from the U band to  $30 \mu\text{m}$  (i.e., using U, B, V, R, I, J, K, L, M, N, Q, and P filters; Figure 1-17) carried out over a 4-year period and by spectrophotometry from 1 to  $13 \mu\text{m}$ . The variations of brightness are similar in the J, H, L, and V bands. They are less pronounced in the K and M bands. The  $V-R$  and  $V-I$  indices vary slightly in phase with the V-magnitude. The  $B-V$  index remains nearly constant over the period, but the  $U-B$  varies oppositely to the other colors and magnitudes. Strong and rapid oscillations on  $U-B$  are pointed out between  $\phi = 0.80$  and  $\phi = 0.90$  (see his Figure 2); they are probably connected with the  $\lambda 3280 \text{ Fe II V1}$  emission-line variations (Bouchet et al., 1983; see also M. Querci, this volume). These oscillations should duplicate rapid temperature or opacity variations due to *nonthermal chromospheric excita-*

*tion* (Querci and Querci, 1983, 1985a). Surely, important scientific information is contained in these variations, but many opacity contributors such as  $\text{C}_3$ , violet CN, SiC, or graphite are included in the flux of each filter, and the restoration of each one is impossible. A new photometric system for carbon stars is needed (Querci and Querci, 1985b). A feature noted in the Bouchet's (1984a) paper is the variation on the N flux where SiC particles may form, which is either due to the variation of the amount of emitting SiC or to the fading of the exciting emission. Moreover, the time-scale variations over a week on N and P filters are not quoted, but they are not negligible. At phase 0.25, the N flux varies from  $8.1$  to  $6.3 \times 10^{-25} \text{ Wm}^{-2} \text{ Hz}^{-1}$ , while Q varies from  $2.8$  to  $5.7 \times 10^{-25} \text{ Wm}^{-2} \text{ Hz}^{-1}$ .

Lastly, we describe the supergiants (SRc). Because the time interval between two consecutive maxima for the supergiants is between



several hundred and several thousand days with amplitude variations which are generally small ( $\leq 1.5$  mag), several years of observations are needed to decide whether a supergiant such as  $\rho$  Cas belongs to the SRc class or to the Lc class (Smith, 1976). Maeder (1980) observed on the Geneva photometry file that the cyclical amplitude variations of the SRc stars decrease by several tenths of magnitude over some years; he concludes that "over half a century or more they may exhibit changes up to 2 magnitudes." Moreover, he points out that the limit in the HR diagram where the light variation amplitudes of the G-M supergiants become important coincides very closely with the limit of appearance of very deep convective zones in these stars. Using these two observations, we may conclude that the SRc supergiants stay no more than 100 years near the Hayashi limit because, in this area of the HR diagram, the amplitude variations are at their maximum and the observations show that the light maxima decrease about 2 magnitudes in one century.

The hypergiant,  $\rho$  Cas, shows large variations in luminosity and spectral class. Before 1930, it had a K class spectrum; in 1943, Morgan et al. (1943) classified it as F8Ia,b, and during 1946-1947, the star underwent a deep light minimum ( $\Delta m_v \sim 2.0$ ), probably because of the large amount of ejected matter, and consequently, the spectrum looked like that of an M star. In 1948, Greenstein (1948) pointed out signatures of matter falling back on the star a day after a sudden expansion. Joshi and Rautela (1978) found that the color temperature of  $\rho$  Cas was higher than that of  $\delta$  CMa (a non-variable star of the same spectral class) by about 650 K in 1920 and 100 K in 1974. As quoted by Glasby (1968): "One marked peculiarity of the light curve of  $\rho$  Cas is the apparent existence of three fairly distinct modes with quite abrupt changes occurring from one to the next. Such changes have been observed in 1911, 1922, and 1946 when a very deep minimum occurred which was well followed visually and photoelectrically."

Some other observational proofs of long-term variations have been made by the analysis

of the variable polarization. For example, Tinbergen et al. (1981) interpret the yearly variations of  $\alpha$  Ori and  $\alpha$  Sco by very large eddies (size about  $1.5 \times 10^8$  km) with a lifetime of 1 year. One, or at most two or three, of these large moving elements are seen at the same time; this agrees with Schwarzschild's (1975) estimate that a cell moving up and down in the stellar atmosphere with a sonic speed (supposedly constant and equal to 5 km/s) crosses the whole stellar diameter in 1 year. In  $\alpha$  Ori, Goldberg (1979) finds random fluctuations on the time scales of 1 year or less noted above. He also finds a cycle of about 5 years with both brightness and photospheric radial-velocity variations, which demonstrates the correctness of the period of 5.781 years of Jones (1928), Sanford (1933), and Pettit (1945). (However, see the discussion by M. Querci, this volume.)

In addition to these long-term disturbances, night-to-night variations are observed in some super or hypergiants. In  $\rho$  Cas (Figure 1-18), Joshi and Rautela (1978) show variations in the slope of the Paschen continua and the Balmer jump and suggest that they are brought about by the circumstellar cloud surrounding the star. The daily disturbances are probably due to local interaction between the photosphere and surrounding matter in small puffs or to local motions in the atmosphere itself, leading to irregular variations of brightness and polarization. Sargent (1961) suggested that the weak Balmer absorption lines of  $\rho$  Cas during the late 1950's are probably filled in by emission lines, and he argued that there is no deficiency of hydrogen in the irregular variables. We conclude that a monitoring with high-resolution spectroscopy could clarify the problem.

Looking for the middle-term color changes shown by  $\rho$  Cas (Beardsley, 1961), Landolt (1968) concludes that "such a very long-term project as this color change problem is one to which serious amateur-astronomers using standard UBV filters, a photoelectric cell, and a small reflector could usefully contribute."

The above variabilities remind us of those observed in the hotter supergiants, from A to

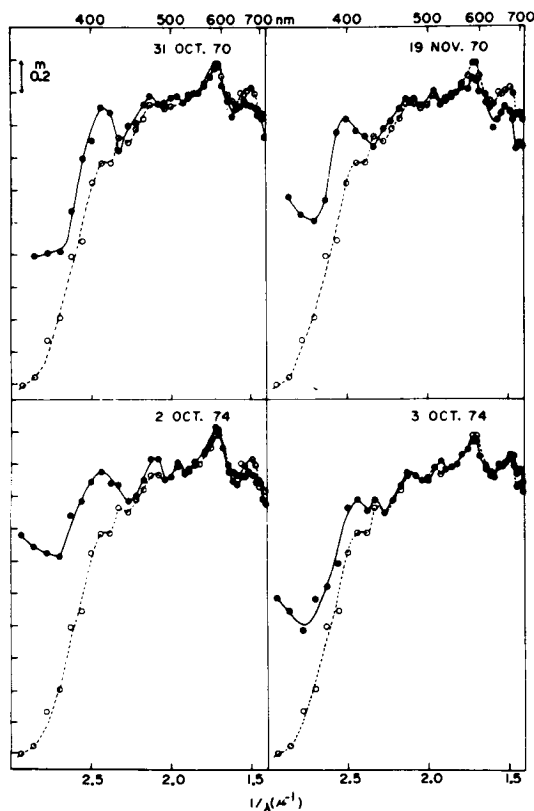


Figure 1-18. Energy distribution curves of  $\rho$  Cas and S CMa: filled circles =  $\rho$  Cas; open circles = S CMa (from Joshi and Rautela, 1978).

G spectral classes, with the three different time scales: (1) the nightly variations, (2) the cyclic variations of several hundred days, and (3) the long-term variations of several years which are sometimes quasi-catastrophic for the star.

#### Correlations Between Observed Quantities.

Definite correlations between periods, absolute magnitudes, spectral classes, and phase lags are less obvious in the SR variables than in Miras. Generally, the clouds of points in the figures (Feast et al., 1980) are scattered around the mathematical relation deduced from these figures. However, efforts have been made to rectify the situation: Glasby (1968) reports studies from Palmer which conclude that, statistically, the amplitude variations may be accepted as constant. Or rather, he points out that the behavior of the maximum and minimum

apparent brightness is virtually the same in the SRa stars, whereas it is very different in the Miras, in which two successive maxima could vary appreciably while the minima show very little variation. Moreover, he notes that the longer the period, the redder the spectrum—an association also noted by Strohmeier (1972).

Although the monitoring of  $\text{H}_2\text{O}$  microwave (Schwartz et al., 1974) on the SR variables, RX Boo (SRa), NML Cyg, and VX Sgr (SRc), was of such low signal-to-noise ratio that no correlation between microwave variations and visual luminosity was possible, Cox and Parker (1979) found a steady increase of the  $\text{H}_2\text{O}$  line with no sign of cyclic variation seen at optical wavelength in RX Boo (SRb). Moreover, sharp maxima appear in the maser strength with no apparent correlation with the visible. The monitoring of the two  $\text{H}_2\text{O}$  peaks of the SRb star, R Crt, gives a different behavior of these two features against the visual brightness. Spencer et al. (1981) conclude that the SiO maser intensities are correlated not with the stellar properties but with local conditions in the stellar atmosphere of the SRa, SRb, and SRc stars (like the Miras). When the visual brightness of VX Sgr (SRc) monotonically increases by about 3 magnitudes (Schwartz et al., 1974), the  $2.2\text{-}\mu\text{m}$  flux increases by a factor 2. Then, and then only, does the  $\text{H}_2\text{O}$  microwave emission appear. It seems that the masering regions and the photosphere of the star are not usually coupled mechanically because the stellar photosphere suffers some periodic variations, but the masers have quite nonvariable radial velocities.

In his investigation on the SRa carbon star, Y CVn, with photoelectric B, V, and R filters and spectroscopic observations, Vetesnik (1982b, 1982c) gives a  $\text{C}_2$  Swan radial-velocity oscillation of  $\pm 2$  km/s. Although a gap exists in the photometric observations, he suggests a likely relationship between the light variations and the radial-velocity oscillations. Moreover, he gives the light elements of the minima by  $\text{JD}_{\text{Min}} = 2436097.3 + 251.6E$ . However, this new period disagrees with the mean period

given in the GCVS ( $P = 158$  days); the changing period is quite evident unless there is a mistake in the GCVS.

Another SRa carbon star, UX Dra, shows a light curve with a marked regularity, alternating one deep and one shallow minimum, which suggests the light curve of a photometric binary (Vetesnik, 1982a). On the other hand, the radial velocity deduced from the C<sub>2</sub> Swan has a half-amplitude of 2.2 km/s and a period of 340 days, which is double the mean period obtained with the light curves. Confirmation by continuous observations is needed.

Referring to 20-year observations in the Geneva photometry, Maeder (1980) gives the V amplitude variations for several SRc supergiants of type Ia, Iab, and II. For a given spectral type, the larger the brightness amplitude, the more luminous is the supergiant; however, Strohmeier (1972) argues that the cycle length has no apparent correlation with the luminosity. This apparent contradiction indicates that a very high quality of measurement is necessary for investigating SR stars. (The Geneva photometry is a good example.) Moreover, Maeder (1980) points out a new observed correlation for the G-M supergiants; for the same amplitude variation, the higher the luminosity, the earlier the spectral type is. Observations of numerous supergiants are necessary to confirm Maeder's conclusions because he has used only three Ia supergiants, nine Iab and three II supergiants of M0-M2 class, and three Iab and two Ib supergiants for spectral classes later than M2. Because the irregular variable stars are generally poorly monitored by ground-based observers, numerous observations are needed to contribute to the understanding of these stars.

Nevertheless, from 24 red supergiant variables in the LMC, Feast et al. (1980) deduce a period-luminosity relation. The measurements of Glass (1979) come close to a theoretical P-L relationship derived from the work of Stothers and Leung (1971).

The satellites hail a new epoch. Maran et al. (1980) demonstrate that VY Leo is clearly an SRa variable with a period of about 1 year,

although it was classified Lb in the GCVS. Thus, this star can no longer be used as a normal giant (Wing, 1980). Consequently, it is urgent to use satellites for long series of uninterrupted observations of selected variable stars; such sustained observations will help to point out the various time-scale phenomena involved in these star atmospheres.

**Superposition of Periods, Chaos, or Randomness in Light Curves.** A primal question is: are the semiregular stars multiperiodic, chaotic, or truly random?

In the last decades, astronomers have mainly developed multiperiodic analysis, and many apparent light-curve irregularities may be explained by a complex mixing of two or more oscillations, each of which is more or less regular and varies independently. Harmonic analysis of the observations must be employed to identify the individual oscillations. RS Gem (SRb, M II) is a representative star, with a secondary variation which is observed as a second maximum progressing along the light curve relative to the primary cycle (Figure 1-19). UZ Per is one of the SRa variables which has been studied in depth over several years; it apparently has three cycles of different length: a short period of 90.8 days, a longer period of 922 days, and a third one which is estimated to be more than 5000 days long (Glasby, 1968).

Variable stars with a double period are more often observed among C stars than among M and S stars. A list of those stars are given in Table 26 of Alksne and Ikaunieks (1971). Values of the ratio of the secondary to the primary periods ( $P_2/P_1$ ) are quoted in the works of Payne-Gaposchkin (1954b), Houck (1963), and Alksne and Ikaunieks (1971), in which the majority of the stars are SRa or SRb of both M and C classes. We will not discuss it again. Leung (1980) found that this period ratio (secondary to primary) is about 10 for the M supergiants (SRc). Stothers and Leung (1971) suggest two possible explanations for these two periods. First, the primary period is connected to the radial fundamental mode of pulsation, and the long secondary period is tentatively interpreted as the convective turnover time of

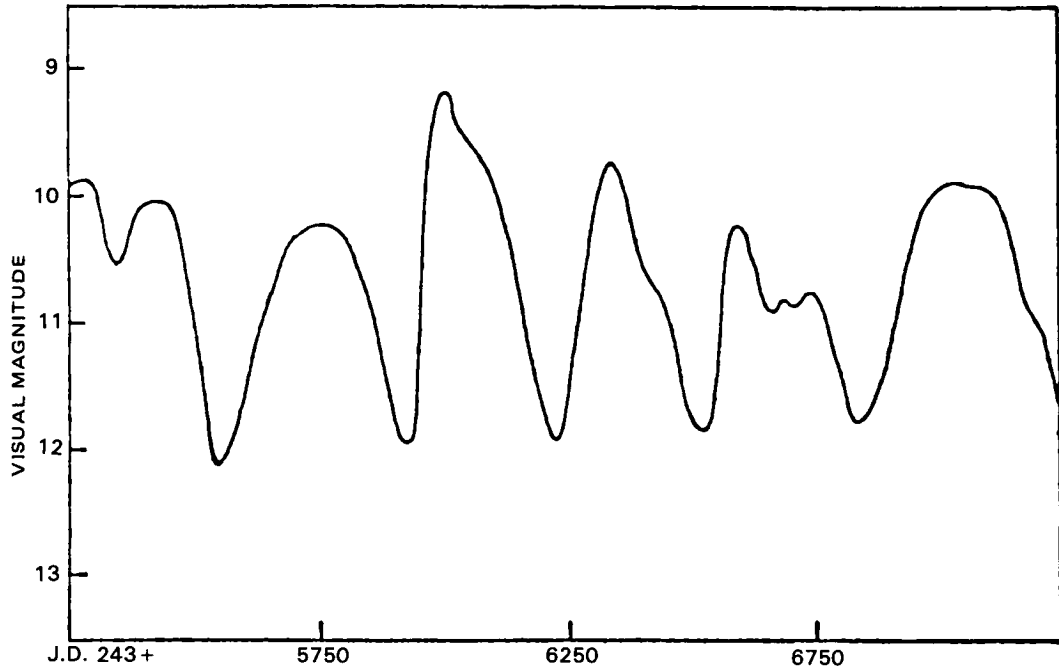


Figure 1-19. Light curve of RS Gem, a typical semiregular variable (from Glasby, 1968).

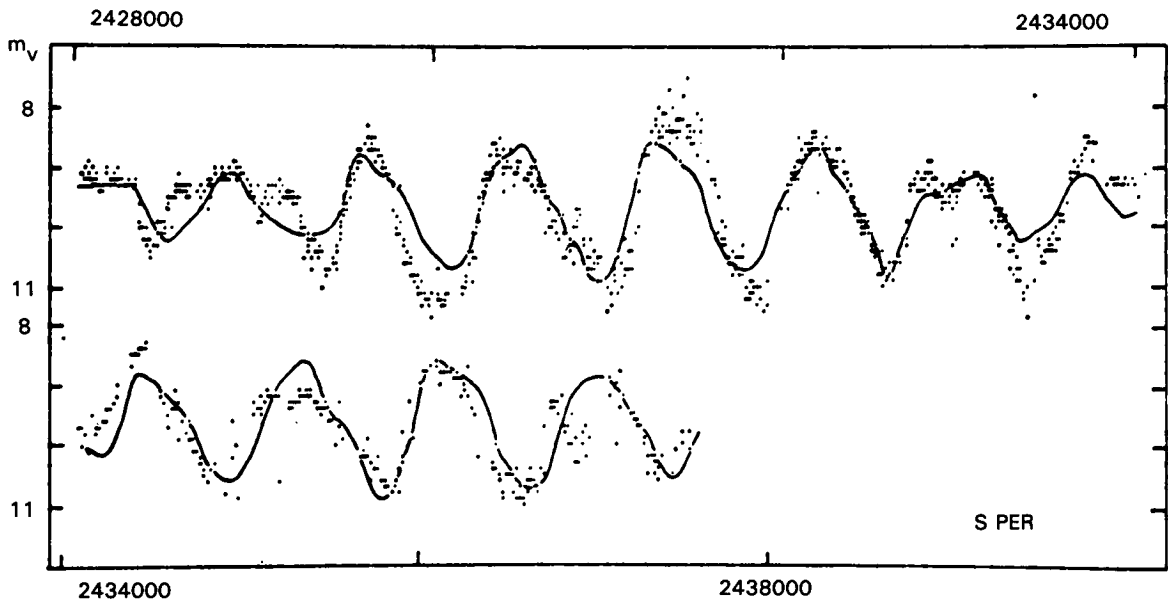


Figure 1-20. AAVSO observations of the supergiant semiregular variable (SRc), S Per. Each point is a mean of 6-day observations. The computed light curve (smooth curve) is based on two period components. The abscissa is in Julian days (from Leung, 1980).

giant cells in the stellar envelope. A second possible interpretation is that the primary period may represent the first radial overtone, while the secondary period may represent the fundamental mode itself. Stothers (1972) prefers the first explanation, while Fox and Wood (1982) argue in favor of the second one and give a period ratio of about 7 (see the section *Modes of Pulsation of Long-Period Variables* and de la Reza, this volume). Figure 1-20 shows the S Per light curve and the computed two-component light curve from Leung (1980).

Nowadays, two other types of physical explanations for the behavior of the semiregular and irregular stars are proposed: the randomness and the chaos. In the randomness process, the events are largely independent of each other, and it is impossible to make accurate predictions of any range of variations with certainty. However, in a chaotic process, which has been mainly analyzed in mathematics and physics, the observed variations reflect the unstable motions which are produced by a collective and cooperative behavior of the matter when some acting forces are largely amplified (Whitney, 1984). Nonlinear equations are used to investigate such behavior.

Here, we make two observational remarks:

1. The analysis of semiregular and irregular variables requires a much longer series of uninterrupted observations than have been done until now; only very few stars (Houck, 1963; Glasby, 1968; Hoffleit, 1980; Leung, 1980) have been monitored to this extent.
2. Although observations of new periods in cepheids or binaries with improved accuracy permit us to obtain more precise quantitative parameters of these stars, the accuracy of the observations on the semiregular stars may determine the nature of the variations amidst the processes described above and may allow us to choose between two competing physical theories (Whitney, 1984). Observations with improved accuracy are crucial for semiregular and irregular stars.

As we know, there are many more irregular variables among the carbon stars than among the M stars. Let us give some examples of semiregular C stars with apparent behavior changes in their light curve, illustrating chaotic variations. First, we quote the SRa star, S Cam. This star was observed from the beginning of the 20th century. Ludendorff (1923) derived a period of 325.5 days and a light amplitude of 2.8 mag from his 1911-1912 observations. Later, Campbell (1941, 1947) and Payne-Gaposchkin (1944) indicate a period of 326.5 days and a light amplitude of 2.5 mag. Nielsen (1952) obtained a period of 324 days. All these results seem homogeneous. However, Mayall (1960a, 1964, 1966) suggests that S Cam light amplitudes decreased to 2.0 mag, and Romano (1950) points out time intervals with constant brightness and other ones *with additional waves* on the light curve. From her 1966-1971 observations, Krempec (1973) deduces a period of 326.4 days and a variable behavior in the light amplitude. During some time intervals, the light amplitude is above 1.3 mag, and during some other ones (presenting very flat minima), the light amplitude is only about 0.8 mag. During the Krempec's observational period (1966-1971), the SRb star, RV Aur, presented regular time variations with a period of 229 days, but the behavior of this star was sometimes quite irregular. UV Cam is another SR star with an apparent period (294 days) and considerable irregularities (Krempec, 1973). U Cam is one of the SRb stars with multiple periods. Payne-Gaposchkin (1944) finds three periods for this star (223, 435, and 3000 days). Kukarkin (1949) confirms the two periods of Jacchia (1933): 419 days (as the period of fundamental oscillations) and a superposed wave of 3000 days. Krempec (1973) shows that the period of 400 days is probable. The light variations were often irregular.

As quoted by Krempec (1973), there is no regularity in the light variations of the SRb star, X Cnc, and the observations give considerably different periods, such as 970 or 365 days. The light amplitude does not exceed a few tenths of magnitude, but Mendoza (1967) and Eggen

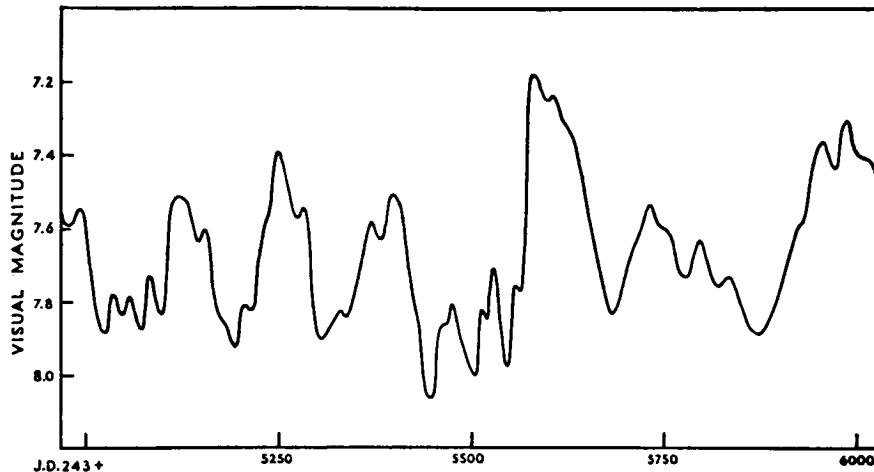


Figure 1-21. Light curve of *V Aql* (from Glasby, 1968).

(1967) stated that the photometric color index,  $B-V$ , ranges from 2.9 to 3.4 and 3.0, respectively. Hagen (1925) notes that *X Cnc* is imbedded in a dark symmetric cloud of size  $0^{\circ}8$ . This cloud is confirmed by a strong continuous ultraviolet absorption (Mendoza, 1967) and a strong continuous infrared emission (Gillett et al., 1971). *TT Cyg*, also an SRb carbon star, is an astonishing object; it shows very rapid irregular light variations during the 1923–1924 and 1928–1929 observations of Parenago (1938). During other time intervals, it has a period of  $118 \pm 18$  days (Payne-Gaposchkin, 1944; Krempec, 1973).

To conclude this analysis of the semiregular stars, we emphasize some occasional striking behavior like the one observed by Garcia et al. (1977) on the SRb star, *R Dor*. On the basis of 455 visual observations, these authors show the occasional appearance of an 8-day harmonic period besides the 332-day fundamental period. At the present time, it is difficult to give a physical explanation to the excitation of the 8-day mode.

#### The Lb and Lc Stars

**Generalities.** The Lb and Lc stars, giants and supergiants, respectively, are slowly varying without any trace of periodicity and with small

amplitude ( $< 1.5$  mag). Deep minima appear at irregular intervals. The light curves seem to be composed by slow waves of varying amplitude. Strohmeier (1972) notes that an extremely weak periodicity occasionally appears.

Some variables are initially classified Lb or Lc and are later shifted to other variability classes as new observations reveal unsuspected periodicity. Therefore, the red irregular stars with unknown spectral class and luminosity are first assigned to the irregular star class while awaiting new observations.

Few such stars are regularly observed by spectroscopy. Even today the presence of Balmer emission lines has not been successfully demonstrated in these variable stars.

Although the Miras of spectral class M are scattered over a large spectral range—M2 to M10—the irregular stars, as well as the semiregulars, are concentrated at M6 and M6.5, with none later than M7 (Cameron and Nassau, 1956); the latest should be *RX Boo* (M7–M8). Joy (1942) showed that the concentration of irregular M stars occurs at M5 and M6. This discrepancy occurs because a small difference appears between the Mt. Wilson (Joy) and Case (Cameron and Nassau) systems of classification. We must keep in mind that these differences are partially due to the variation of

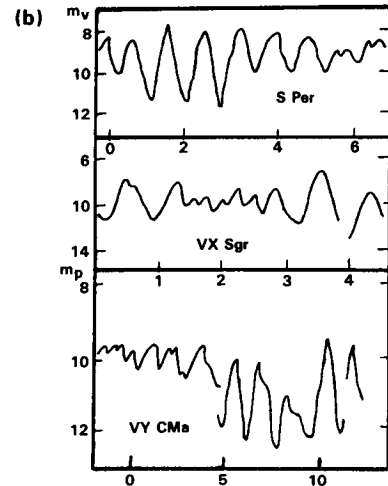
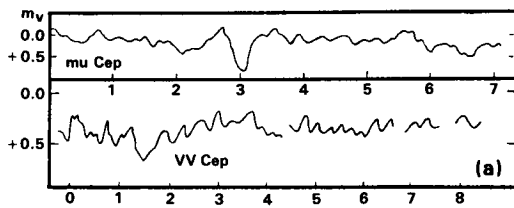


Figure 1-22. (a) Portions of the visual light curves of two supergiant variables of small amplitude. The ordinates are in magnitudes, and the abscissas are in thousands of days from an arbitrary zero-point. Note that the scales for the two variables are not the same. The curve for  $\mu$  Cep is from Hassenstein (1938), and that of VV Cep is from Fredrick (1960). (b) Portions of the light curves of three red supergiant variables of large amplitude. The ordinates of S Persei and VX Sagittarii are in units of visual magnitude, while photographic magnitudes are given for VY Canis Majoris. The abscissas are in thousands of days from an arbitrary zero-point. The visual light curves are based on AAVSO observations, and the photographic curve was determined by Robinson (1970) from Harvard plates (from Smith, 1976).

spectral class of the star during its light variations. Wilson and Merrill (1942) give a long list of irregular variables with M spectral types. They add two S spectral-type stars: SU Mon and AD Cyg. The stars  $\mu$  Cep,  $\alpha$  Ori,  $\alpha$  Her, and  $\alpha$  Sco are also classified irregular by Strohmeier (1972). Many M class irregular variables are connected with young O-associations of Population I (Crawford et al., 1955).

The carbon Mira variables are found in the low-temperature C6-C7 subclasses, while the semiregular and irregular variables are chiefly found in the earlier subclasses (Alksne and Ikaunieks, 1971). Typical representative Lb carbon stars are V Aql (Figure 1-21), W CMa, V460 Cyg, U Hya, V Hya, T Lyr, BL Ori, TX Psc, and VY UMa.

In any case, it is generally difficult to decide whether a supergiant belongs to class SRc or class Lc because the time interval between successive maxima is very long. This time varies from a few hundreds to a few thousands of days, and extensive observations are needed

before a conclusion can be reached. This raises the question that another kind of distinction between the supergiants must be investigated. Smith (1976) suggests a much more physical splitting among the supergiants: those with small amplitude variations and those with large amplitude variations (Figure 1-22a, b). In the first group, we find  $\mu$  Cep and VV Cep, the latter being an eclipsing binary. Their mean maximum amplitude is 1.1 mag with a dispersion of about 0.2 to 0.3 mag. The second group includes stars like S Per, VY CMa, and VX Sgr. Using the published line curves of Robinson (1970), Dinerstein (1973), and Smith (1974), Smith (1976) found a mean maximum amplitude of 4.2 visual magnitudes with a dispersion of 1.1 mag among these three stars. The light curves of  $\mu$  Cep and VV Cep suggest that they have something in common, but they are different from the other three supergiants which are rather inhomogeneously classified in GCVS: S Per as SRc, VY CMa as Lc, and VX Sgr as SRb.

It seems to be necessary to support this division into small and large amplitude variables by studying other stellar properties and observations of other stars. Humphreys (1974) found that the three stars, S Per, VY CMa, and VX Sgr, share a few peculiarities, such as large excess radiation between 0.7 and 1.5  $\mu\text{m}$  and between 1.5 and 8  $\mu\text{m}$ . This last excess has a slope like that of free-free emission in VX Sgr and S Per. Moreover, these stars show absorption-line weakening in their near-infrared spectra, silicate emission features between 10 and 20  $\mu\text{m}$ , and strong OH and H<sub>2</sub>O emissions. S Aur and Y Lyn can be added to this group of large amplitude variables.

Among the observed variable supergiants, the number of large amplitude variables is very limited. Then, if we suppose that the supergiants cross through the small and large amplitude phase, we could conclude that the latter phase is very short. Moreover, no theoretical works give any information on the order and the duration of these two phases; only observations on the cluster,  $\eta$  and  $\chi$  Per, suggest that S Per (large amplitude variable) may lie at the tip of the red supergiant branch in the HR diagram because it appears redder than the small amplitude supergiants of the same cluster (Smith, 1976).

**Different Time-Scale Light Variations of Irregular Variables.** Visual or photoelectric photometry monitoring of a few irregular stars (e.g., VY CMa, and VX Sgr) indicates that their variations result more from chaotic or random processes than from harmonic processes. This could be interpreted as irregular surface features—spots, loops, or cells—that should have an effect on the light variation larger than have the pulsational effects. Spectroscopic observations in support of random variations in these stars appear in literature. As an example, we quote the variability in the emission lines in the IUE low-resolution spectra of the Lb carbon star, TX Psc, over a 3-year period, reported by Baumert and Johnson (1984). These authors show a probable time variation of the Mg II  $\lambda 2800$  lines by a factor of 3 to 25

and the possible variation of the C II  $\lambda 2325$  line by 25 percent in the spectra of this object. On the basis of these data, Avrett and Johnson (1983) attempted to model a chromosphere for cool carbon stars (see de la Reza, this volume). A monitoring during 4 years (1981–1984) was jointly developed by the Bloomington and Toulouse groups on the low-resolution and long-spectral observations of TX Psc (Johnson et al., 1986). The intensity variations of Mg II, C II, and Fe II emission lines were confirmed with a factor of variability of 6 to 15, 5, and 8 to 80, respectively. Moreover, all the lines appear to vary together. The possible mechanisms for the production of these emission lines are reviewed: shock waves such as in Miras, magnetic fields associated with plages, heating by short-period acoustic waves, and changes in absorbing overlying clouds by their own motion. However, the continuous flux in the long-wavelength spectral range (LWR) is essentially constant. A recent high-resolution LWP spectrum of TX Psc confirms unusual profiles and line strengths for h and k lines. These lines are heavily absorbed by overlying Mg II, Mn I, and Fe I absorption features (Eriksson et al., 1985).

As in semiregular variables, changes in light-curve shapes are observed:

1. Some irregular stars, like W CMi, have an irregular light curve, but during some periods of time, their light curve suggests a regular light period and during other periods a very constant brightness (Krempec, 1973). Some other stars have periodic variations during some intervals of time, and shortly thereafter, are not variable at all. BM Gem is a good example pointed out by Krempec (1973); RX Lep is the typical representative star.
2. Some stars have rapid changes of brightness. Krempec (1973) notes a change of 0.80 mag during 30 days on UY And and “very rapid light variations of the order of a few tenths of magnitude during a few days” on the light curve of SV Cyg.



3. Other stars, such as TT CVn, have a period approximately defined at different epochs (Wendel, 1913; Krempec, 1973); however, during some time intervals, the brightness varies rather irregularly (Krempec, 1973).

Ashbrook et al. (1954) describe the light curve of the SRc-Lc star,  $\mu$  Cep, by a second stochastic chain. "The light variation is not explicable by a simple pulsation; rather, it may be interpreted as arising from temporary, random surface disturbances on the star." Sharpless et al. (1966) found that the light curve is the result of the sum of periodic terms, with 4836 days of fundamental period. Its light variations have short-period components and at least four different periods between 700 and 14000 days (Polyakova, 1983). By spectral analysis, Mantegazza (1982) shows that the light variation of  $\mu$  Cep may be explained by the superposition of two periodic terms and their nonlinear couplings, the longer term having a cepheid-like shape. The results of Mantegazza tend to support the second hypothesis of Stothers and Leung (1971; see below) and are in agreement with the theoretical ones obtained by Uus (1976), who demonstrates that some red supergiants are pulsating in the radial fundamental and first-overtone modes.

In general, the Lb and Lc stars have not been sufficiently observed. To have a global point of view of their variations, some of them must be followed individually during many years. However, the analysis of the light curves of many of them should give indications to ascertain if chaotic variations are really possible in the Lb stars. More observations are urgently needed to confirm this suggestion.

Finally, we point out the observations of flares in these stars. In the chaotic point of view of the irregular star variations, flares should represent the shortest time variations. However, they are not really used in the analysis of the star behavior because, at present, a permanent followup is impossible with the conventional telescopes, photoelectric equipment, and adapted filters, which prevent the study of con-

secutive flares and their correlation with other variations. As an example of flares on such variable stars, we describe the recent event recorded on  $\mu$  Cep by Arsenijevic (1985). A sudden fast (20-second duration) brightening of 0.034 magnitude on V was observed on August 3, 1981, at 0h 31.9 mn UT, with a standard deviation in the measurements of  $\pm 0.011$  mag; simultaneously, a shallow minimum in the polarization position angle appeared. The declining phase was 28 mn long. The similarity of this phenomenon with the red-dwarf flares strongly favors a stellar flare in the supergiant,  $\mu$  Cep, although the declining phase was longer than in red dwarfs, maybe being correlated with a larger amount of irradiated energy. The flare appeared during the minimum light, such as in the case of the solar ones which are due to a large number of spots at the stellar surface. The total energy irradiated by the  $\mu$  Cep flare is about  $10^{32}$  joules, fitting flare energies observed in the Sun and in stars.

We also note flare activities in the radio wavelength range, for example, in  $\alpha$  Ori and  $\alpha$  Sco (Hjellming and Gibson, 1980; see also the section on *Photometric Observations*); they infer the presence of magnetic fields (see M. Querci, this volume).

To conclude the discussion on irregular stars, we mention the light variability of the brightest and the most famous irregular variable star, which has been observed more or less regularly over 60 years— $\alpha$  Orionis.

The M2 Iab supergiant,  $\alpha$  Ori (Betelgeuse), was monitored by photoelectric photometry in the visual quite regularly from 1916 to 1931 by Stebbins (1932). Occasionally, Johnson et al. (1966) and Rucinski (as quoted by Goldberg, 1984) have also made some observations. In recent years, Krisciunas (1982) systematically observed  $\alpha$  Ori in B and V filters from 1979 to 1982, and Guinan (1984) began measurements (around  $H\alpha$  and in the blue at  $\lambda 4530$ ) in 1981 that have continued to the present. Besides these data, AAVSO has recorded regular measurements for the last 60 years, and this extensive temporal coverage makes them very useful, especially in the absence of professional works.

However, these data must be viewed with great caution because the scatter is very large in a single measurement. All the above-quoted data are shown on Figures 1 and 3 through 5 in Goldberg (1984) and on Figure 2 (upper panel) in Guinan (1984). The characteristics of the  $\alpha$  Ori light curves, particularly clearly seen on Guinan's Figure 2, are long-term light variations with a period of about 5.78 years, and with a mean amplitude of about 0.4 mag, and short-term irregular variations with rise or decline time scales of a few weeks to some months superposed on the long-term periodic light variations. The following systematic trends are noted:

1. 1916–1931: Stebbins (1932) found a period of 5.4 years after the use of a drastic smoothing process which cancels all the short-term fluctuations (see his Figure 1). From radial velocity data, Jones (1928) and Sanford (1933) obtain a period of 5.78 years.
2. 1934–1938: The photoelectric data of  $\alpha$  Ori show no evidence of periodicity, the points scatter widely, and the radial velocities are not connected to the 6-year period throughout the period of Adams's (1956) observations: 1937–1947.
3. 1939–1965: The brightness variations seem to follow the 5.78-year period and between 1949 and 1961, a phase lag relative to the mean curve previously found by Stebbins (1932) seems to be observed.
4. 1965–1975: Random variations on brightness and radial velocities appear again.
5. 1975–1980(?): The 6-year period was followed surprisingly well by the radial-velocity measurements of Boesgaard (1979) and Goldberg (1979).

Moreover, short-lived abrupt drops in brightness are evident in 1926, 1934, 1942, 1943, 1947, 1951, 1971, 1974, and 1981 and very short time variations appear in the 10- $\mu$ m

region; the intense infrared emission surrounding  $\alpha$  Ori varies in shape, size, and intensity from night to night during September and December 1966. The stars  $\mu$  Cep and  $\alpha$  Tau have the same behavior (Low, 1965).

As noted by M. Querci (this volume), the atomic lines giving the radial velocities are not listed in the text of some of the papers mentioned above. Consequently, we suppose that the lines used are from various excitation potentials (i.e., from atmospheric layers of different behavior). Therefore, many possible correlations between the radial-velocity variations and drops or declines of brightness in large bandwidth filters, like UBV, cannot be fully inferred and exploited because we are unable to locate the layers from which the light variations come. Some examples of such unexploitable data are:

1. Between Julian days 2424538 and 2424598, the radial velocity decreases by nearly 4 km/s, while the brightness increases by 0.2 mag in the same time (visual brightness). After 200 days, the radial velocity was found to be lower by 3.5 km/s and the brightness was also declining during this time.
2. On Julian day 2431400,  $\alpha$  Ori suffers a steep drop in radial velocity, followed some days later by a fall in brightness by about 0.6 mag. Goldberg (1984) argues that this rapid change takes place soon after the minimum of the "mean velocity curve" like the changes seen in 1925, and shows that some other large disturbances follow the two velocity minima of 1972 and 1978.

During the short-term variations, the change on the linear polarization of the visible light variation varies with the same time scale as the brightness. This suggests a high degree of asymmetry and a local origin in the features involved in these variations (Tinbergen et al., 1981). Hayes (1981) suggests that the change of polarization follows the formation and growth of the local features on the surface of the star (lower chromosphere) and progressive changes

in their orientation. Viewing the entire phenomena described by Goldberg (1984), it seems "equally possible that a sharp drop in radial velocity was accompanied by an initial brightening followed by rapid fading," which can be explained by a gaseous matter ejected from the star which diffuses and condensates into grains and becomes optically thick in the visible region (Goldberg, 1984).

To conclude, it appears that the long-term light variations probably arise from the radial pulsation of the star, the short-term variations having been believed to be caused by large-scale photospheric convective cells (Schwarzschild, 1975). Hayes (1982), Goldberg (1984), and Antia et al. (1984) favor such an explanation, the two first authors for explaining polarization changes. However, Roddier and Roddier (1983, 1985), Petrov (1983), Roddier et al. (1984), and Karovska (1984) argue against it (see M. Querci, this volume), and Guinan (1984) notes a lack of correlation between the short-term variations in the TiO band strength and the short-term light changes. This fact suggests that "the mechanism responsible for the short-term light enhancement is not linked to temperature increases as would be expected from ascending giant convective cells... and that the brightness enhancements are produced above the stars' surface."

The problem might be resolved if the recently claimed companion objects to  $\alpha$  Ori are confirmed. That circumstance would also seem to explain some variations in chromospheric emission-line radial velocities, as well as circumstellar dusty clumps (see M. Querci, this volume). (Anecdotally, Bottlinger (1910) mentions  $\alpha$  Ori as a binary star for which he finds a 6-year period. Other authors (Wilson and others) as reported by Karovska (1984), also classified  $\alpha$  Ori as a binary star.)

#### **Eruptive Variables: The RCB Stars**

**Generalities.** The main characteristics of the eruptive RCB stars are the abrupt drops in brightness, followed by a longer climb back to normal light, probably due to the sudden ejection

of highly absorbent matter. They are occasionally thought to be the progenitors of type I supernovae (Wheeler, 1978), novae (McLaughlin, 1935), planetary nebulae (Webster and Glass, 1974), or helium stars (Tutukov and Iben, 1985), and white dwarfs (Schönberner, 1977).

In 1795, E. Piggott discovered the variability of R CrB, the prototype of the well-known RCB stars. In the southern hemisphere, a few random observations of RY Sgr have been made since 1751 (Lacaille, 1847). From 1895 to 1908, the brightness of UV Cas dropped twice by 1 to 1.5 mag (Florya, 1949); its variability was recognized with the 4-mag fall of 1913 (Shenavrin, 1979). Innes (1903, 1907) published the first series of observations of some RCB stars. After monitoring R CrB for several years, Sterne (1935) concluded that this star is a "perfect irregular" with its visual variation from 5.8 at maximum to 14.8 for the deepest minimum, whereas the light of SU Tau falls almost annually.

As in the other variables, the spectral type of the RCB stars changes during their brightness variations, and accordingly, changes in the spectra and colors of the RCB stars are noted. The latter are extremely complex because the star waxes and wanes. A general behavior of the stellar spectrum during one of the abrupt drops in brightness and recovery may be described as follows, mainly on the basis of R CrB and RY Sgr observations.

At maximum, the spectrum is very similar to that of a supergiant of spectral type F or R, with narrow and sharp absorption lines, the main differences being in the weakness or absence of the Balmer lines and in the strong lines due to carbon. The cooler stars have strong molecular absorption bands of CN and C<sub>2</sub>.

As the star dims, we continue to see the absorption spectrum, but it is veiled. However, in general, the molecular spectrum becomes more and more intense as the minimum is reached. There is a *sharp* emission spectrum which appears during the fall and suffers

gradual changes. Some of these narrow emission lines sometimes replace the atomic absorption lines (Herbig, 1949). First, the sharp lines of Fe II appear (Payne-Gaposchkin, 1963). As the fading continues, the spectrum becomes dominated by Ti II lines. Occasional anomalies are detected. For example, Alexander et al. (1972) found the same intensity for the two Sc II lines at 4354.61 and 4320.75 Å as in the laboratory. Spite and Spite (1979) showed that, in the next drop of RY Sgr, the 4354.61 Å line has disappeared and the 4320.75 Å line is approximately 500 mÅ. In general, this sharp emission spectrum is slightly displaced to the blue (3 to 10 km/s) relative to the absorption lines at maximum. Its intensity increases rapidly during the initial part of the drop and then grows weaker as the star continues to fade. It is likely produced by an expanding region (chromosphere or large envelope), with decreasing expansion as the star approaches minimum. In the decline of RY Sgr during 1977, Spite and Spite (1979) note that the narrow emission lines and the absorption lines of the atmosphere itself have the same radial velocity. The behavior of this emission spectrum is apparently not related to the absolute magnitude of the star (Feast, 1975), but seems to depend on the time from initial decline (Payne-Gaposchkin, 1963). During the 1948 minimum of R CrB (Herbig, 1949), only the second part of the narrow emission spectrum was seen (Ti II). Perhaps this is connected to the observed low initial rate of decline.

A very *broad* emission spectrum appears some days after the brightness begins to fade (40 days in the case of R CrB in 1948 and 70 days in the case of RY Sgr). In the spectrum of R CrB, the lines of He I 3889, Ca II H and K, and Na I D appear with different widths (Rao, 1975). Using narrowband filters, Wing et al. (1972) discovered the presence of a strong He I 10830 emission line during the minimum of R CrB. As the minimum progresses, the profile of these lines changes, and the intensity remains constant at first, then increases, and finally decreases. In the spectrum of the cooler RCB stars, these broad emission lines are

sometimes accompanied by absorption components with velocities of -250 to -130 km/s (Querci and Querci, 1978; Rao, 1980b). (See M. Querci, this volume, for details on observations of the He I 10830 Å line in red giants and the implied constraints to future modeling.) Because the UV spectrum fades much less than the visible and red spectrum, additional sources of emission shortward of 4000 Å have to be taken into account. As the star approaches minimum light, the relative intensity of the emission lines of each element changes gradually (implying variations in the conditions of excitation). Also, a variable polarization with variable wavelength dependence is observed in stars like R CrB and RY Sgr (Serkowski and Kruszewski, 1969; Coyne and Shawl, 1973). At the end of minimum phase, P Cygni profiles develop (Payne-Gaposchkin, 1963; Querci and Querci, 1978; Rao, 1980b). The bands of CN are observed in emission at this time (Wing et al., 1972). Seeds and Ignatuk (1973) have shown changes in strength of the C<sub>2</sub> emission bands in the R CrB minimum. These changes are probably due to the filling in of the photospheric absorption bands by chromospheric emission. Taking into account the numerous atomic lines in the violet, the chromospheric contribution to the total observed light increases to shorter wavelengths; consequently, the dependence of polarization with wavelength is not linked to the particle size only.

As the star brightens again, the absorption lines gradually come back, while the emission spectrum fades with a varying rapidity in the various lines. Finally, the maximum spectrum is reached approximately 2 magnitudes before the light maximum itself; in R CrB, Rao et al. (1981) observe h<sub>3</sub> and k<sub>3</sub> Mg II components, which demonstrate that this star also has a continuous mass loss during the maximum light through a permanent chromosphere (Payne-Gaposchkin, 1963; Rao, 1974, 1975).

Can we draw a coherent qualitative modeling of the RCB phenomenon?

Many models have been proposed during the past 20 years. Payne-Gaposchkin (1963) suggested that the particles are formed in the

upper photosphere of the star, because the emission lines observed at minimum light are not themselves obscured. Wing et al. (1972) propose that there is a quasi-permanent blotchy cloud orbiting around the star. Humphreys and Ney (1974) associate the particle cloud with the atmosphere of a probable cool LPV companion. These last two models are attractive, but pose problems in so far as they fail to explain why the decline is rapid and the rise is slower.

Nowadays, it appears that the initial hypothesis of Loreta (1934) and O'Keefe (1939), in which the light minimum of the RCB stars is due to dust obscuration, gives the best account of the observed features. Feast (1979), sustaining this basic model of ejection of thick dust clouds, explains how it might act. The gas is ejected at the top of one or some of the large convective cells, and it crosses the deeper layers of the star's atmosphere. This ejection is made radially through a fairly large area of the stellar surface in a semiangle of  $\sim 20^\circ$  and roughly at 20 km/s. If the material is ejected along our line of sight, a major visual luminosity minimum occurs, whereas, if it is ejected at an angle to our line of sight, it causes a minor minimum (e.g., Forrest et al., 1972). The gas expands and cools, and the graphitic carbon condenses. The resultant dust clouds expand and cause an eclipse which allows the chromospheric sharp lines to be seen. The particles move away by radiation pressure. When the new cloud collides with the circumstellar shells, strong and broad emission lines are produced. Because the speed of the collision is greater than the escape velocity, the matter is: (1) partially ejected in space with a drop in density and in optical depth of the medium causing the emission lines to fade, (2) partially replenishing the circumstellar patchy envelope of dust and gas, giving rise to the infrared excess observed in some RCB stars (e.g., in RY Sgr and R CrB). Recent IUE data confirm the Loreta-O'Keefe cloud ejection model and are consistent with the dust to be composed of carbon (Hecht et al., 1984). In addition, they show that 5 to 60 nm glassy or amorphous carbon rather than graphite is formed around the RCB stars.

It is now worth turning to some observations that argue in favor of this model and complete it.

Patterson et al. (1976) showed that their spectrophotometric measurements made on different nights during the R CrB climb in 1974 were consistent with extinction of particles that they thought to be spherical graphite ones of about  $0.07\text{-}\mu\text{m}$  radius, expanding by radiation pressure. No variations of the particle size were detected.

Orlov and Rodriguez (1974, 1981) find a microturbulent velocity of 8.9 km/s in XX Cam and 11 km/s in UV Cas, which supports the idea suggested by Howarth (1976) that the initial condensation may be due to turbulent motions (seed creation).

Forrest, as quoted by Rao (1980a), points out that the extinction during the decreasing light tends to be neutral without any color changes, but that the colors redden during the climb back to maximum. Using these confirmations and the electron pressure computed from the broad emission lines, Rao (1980a) suggests that the gas that produces these broad emissions also produces the neutral extinction by electron scattering during the decreasing light. The grains appear later, and the circumstellar shell is replenished by ejecta at minimum light. Consequently, the light from the photosphere is obscured, and the chromosphere (Payne-Gaposchkin, 1963; Alexander et al., 1972) or the circumstellar gas shell (Hartmann and Apruzese, 1976) is the obvious source of the narrow emission spectrum. As pointed out by Herbig (1949), the emitting gas has a relatively low level of excitation.

The radial velocity of the chromosphere is the same as that of the star itself (Spite and Spite, 1979). When the emission region is suddenly cut off from its source of excitation, it would be an ideal place for the production of an electron recombination spectrum. Feast (1969) suggests that the CN molecule is reasonably abundant in these stars and that its spectrum has a longward edge at  $4000 \text{ \AA}$ . This could explain the blue and UV continuum anomalies by:  $\text{CN} + e \rightarrow \text{CN}^- + h\nu$ . The

emission excess on the V band observed by Shenavrin (1979) on UV Cas tallies with this explanation. Moreover, this model supports the changes in the wavelength-dependent polarization, showing that different particle sizes exist successively.

As noted above, some RCB stars have an excess of infrared. The first observations are recorded by Stein et al. (1969), who find the infrared emission much brighter than the expected emission from the star itself, and who conclude that 40 percent of the star's total luminosity is in the infrared; Shenavrin (1979) finds 60 percent for SU Tau. An excess of infrared is also found by Lee and Feast (1969) in RY Sgr. However, XX Cam and UV Cas have no infrared excess at all (Shenavrin, 1979; Rao et al., 1980), and sometimes R CrB does not have any IR excess during some minima, as in 1972 (Rao et al., 1980). Either the physical conditions are not favorable for grain formation in XX Cam and UV Cas or these stars are between the RCB and the nonvariable HdC stars. To explain the infrared emission, Herbig (1949) believes that grains have to condense far from the stellar surface, whereas Maron (1974) suggests the following way: the freshly ejected particles could have the form of "Platt" particles of 3 to 30 Å; they would absorb the visible radiation better than graphite particles but would not reradiate in the infrared. Then, these particles grow by accretion up to sizes between  $10^{-6}$  and  $10^{-5}$  cm (the classical grain nuclei size). Therefore, the occurrence of the infrared emission waits for the growth of the particles. This growth period is a function of the physical conditions in the shell, mainly low density and velocity of escape, which vary from minimum to minimum.

The broad emission lines which are observed after the beginning of the decline are due to matter which is ejected at high velocity and collides with the circumstellar envelope material with a phase lag of 30 to 70 days. From the radial velocity of the circumstellar lines and this phase lag, we conclude that the shell of gas and grains is at about 4 to 8 A.U. from the star. This agrees more or less with Lee and Feast

(1969), giving 5 to 10 A.U., and with Pecker (1976), giving a dust-cloud envelope of approximately 350  $R_*$  at the IR maximum brightness to 600  $R_*$  at the minimum. Moreover, Pecker estimates the total mass of the envelope to  $5 \cdot 10^{-7}$  solar mass. The broad emission lines are noted violet-displaced by Payne-Gaposchkin (1963). However, during the end of the decline of RY Sgr observed by Spite and Spite (1979), the broad emission lines seem slightly displaced to the red relative to the star. (They extend from -100 to +240 km/s.) This last observation is explained by an ejection of matter in which only a small amount is ejected toward the observer and a larger one is ejected backward. Wdowiak (1975) assumes that 10 to 100 convection cells are responsible for the formation of dust blobs that are ejected by radiation pressure. Rao (1980a) indicates that the broad emission lines are formed in a gas with  $N_e \approx 10^{11} - 10^{12}$  per  $\text{cm}^3$  and  $T_e \approx 10^4 \text{K}$ .

High-velocity shells and/or clumps are observed in MV Sgr at -200 km/s with a temperature of 800 to 900 K (Krishna Swamy, 1972), although the star has a hotter shell about 1300 K (Feast and Glass, 1973).

The presence of the circumstellar shell in some RCB stars is also pointed out by IRAS. It discovers a bimodal distribution of RCB stars: (1) the majority of the RCB stars have a flux such as  $F_\nu \sim \nu^{1.6}$ ; they have an IRAS spectrum that can be understood as the spectrum of a star with a constant ejection rate of dust where the emission is proportional to the frequency (Schaefer, 1985); (2) a fifth of RCB stars exhibit a qualitatively different IRAS spectrum with a flux such as  $F_\nu \sim \nu^{-1}$  and a peak flux at 100  $\mu\text{m}$  or beyond; they are brighter than the majority of the RCB's. Their IRAS spectrum is undistinguishable from normal planetary nebulae (known to have a large amount of dust at a great distance from the central star (Schaefer, 1985)). This fact is in agreement with one of Iben and Tutukov's (1984) evolutionary scenarios that place the RCB stars in planetary nebulae.

**Light Curves.** Surprisingly, the RCB stars are the only class of eruptive variables for which

maximum brightness is the normal state. The duration of this state is variable (up to several years). In addition, epochs exist in which the full maximum is not reached. For example, from 1898 to 1948, XX Cam had only one minimum in 1939–1940 (Yuin, 1948); R CrB gave flat maxima from 1925 to 1935, 1936 to mid-1938, 1953 to 1956, 1969 to 1972, etc. The latter also shows a series of minima without full maxima being reached in the 1860's (Mayall, 1960b) and from mid-1962 to 1965. Howarth (1977) defines fades and minima, respectively, "as an initial drop of one magnitude from maximum" and "a fall of one magnitude relative to local sub-maxima." The events appear to obey Poisson's statistics. For R CrB and SU Tau, the mean intervals between minima are  $532 \pm 57$  and  $625 \pm 104$  days, respectively, and between fades, they are  $1026 \pm 156$  and  $1140 \pm 220$  days, respectively. The cooler star, S Aps (R3), has a mean time between fades of 1249 days (Howarth, 1976). This should suggest a trend of decreasing activity with later spectral type. However, observations of more stars are needed to confirm this correlation.

The speed of the decline has also been investigated. In R CrB, Oberstatter (1972) points out a drop of 0.5 visual magnitude per day during the 1972 decline; amateurs have observed 0.3 visual magnitude per day during the decline in 1983 (Proust and Verdenet, 1983), while the "normal" fade is about  $dm/dt \approx 0.1$  mag per day (Howarth, 1976).

The amplitude, the frequency, and the duration of minima are unpredictable. The declines are much more abrupt than the rises to the maximum. However, the shape of the decline and the rise is not the same for each minimum of the RCB stars; sometimes, "a star may rise halfway to maximum only, to fall again to an even deeper minimum" (Howarth, 1976). This could be caused by many ejection centers.

**Superposition of Light Curves.** Different time-scale variations are very well defined in the RCB variable stars. They are:

1. The unpredictable decline of many

magnitudes (already described) called "obscurational" minimum.

2. The semiregular oscillations with a visual amplitude of 0.2 to 0.4 magnitudes and with a pseudoperiod between 19 and 120 days according to the star, called "pulsational" oscillations.
3. The very short nonpermanent oscillations with a time scale of 1 or 2 hours detected in some RCB stars and probably originating from the star itself.
4. A long-term periodicity in the infrared excess, perhaps due to a natural pulsation of the circumstellar dust shell.

Semiregular oscillations over a 38.6-day period were detected by Jacchia (1933) on RY Sgr. Many observations of RCB stars reveal that these small fluctuations appear at each phase of the variation cycle (Bateson, 1978). Mendoza (1978) detected them photometrically in H $\alpha$  and OI ( $\lambda 7774$ ) filters. Pugach (1977) concludes from observations of RY Sgr that these pulsational variations do not depend on light fading. First, these oscillations are observed during the brightest, largest observable phase (i.e., the maximum). Fernie et al. (1972) report a visual variation of R CrB with an amplitude of 0.15 mag and a period of 45 days. Totochava (1973a) confirms this amplitude variation, but deduces a variation period of 40 days from her 1971–1973 observations. Alexander et al. (1972) confirm the oscillations of RY Sgr with a 0.5-mag average amplitude and a 38.6-day period observed by Jacchia (1933); the light and color curves (Figure 1-23a, b) show smooth variations:  $\Delta V \approx 0.5 \Delta(B-V) \approx 0.3 \Delta(U-B) \approx 0.5$  mag. Sherwood (1976) finds tentative periods from 19 to 54 days for some RCB stars and a period greater than 90 days for S Aps, whereas Waters (1966) suggests 120 days and 0.3 mag of amplitude variation for this star. Later, Kilkenny and Flanagan (1983) find evidence for a rapid period decrease, and Kilkenny (1983) shows that S Aps changes from the 120-day periodicity (present in 1960) to one near 40 days (around 1971). Bateson and Jones

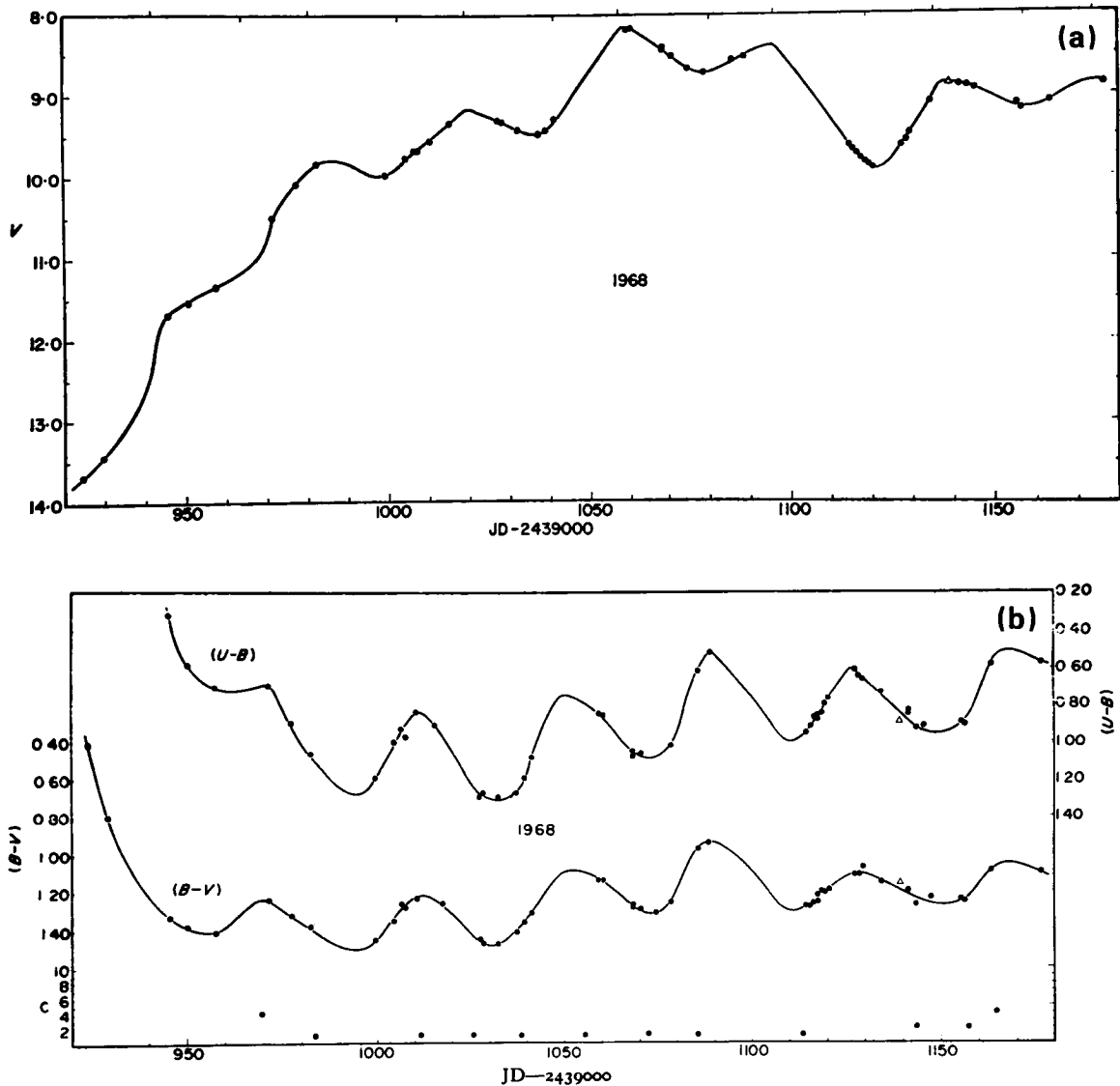


Figure 1-23. RY Sgr (a): V light curve for 1968 plotted against JD. The open triangle is an observation from Lee and Feast (1969). The predicted maxima and minima in V are shown at the top of the diagram. (b): (U-B), (B-V), and spectral classification for 1968 plotted against JD. The same remarks as stated previously apply for the open triangles, maxima and minima. For the spectra, absorption (C) types are shown by filled circles (from Alexander et al., 1972).

(1972) find such semiregular variations in two stars, UW Cen and GU Sgr, with 0.5 mag and 42 days and 0.4 mag and 38 days of amplitude and pseudoperiod, respectively. (See the section *Irreversible Changes in HdC Stars* for details.)

During the deep decline and the rise to maximum, R CrB also has semiregular oscillations

with a time scale of 40 days (Oberstatter, 1972). Isles (1973) confirms that this star occasionally displays oscillations, the amplitude of which decreases as the star brightens. Another RCB star, RY Sgr, shows oscillations during the climb back to maximum, similar to those detected during its maximum; these oscillations



support the idea that no major physical changes took place in the star itself during the minima. The greatest amplitude of the UW Cen variations seems to be when the star is brightening (Bateson and Jones, 1972); it persists along the light curve except during the sudden falls to deep minima (Bateson, 1972). During maximum light, XX Cam varies with the same amplitude as that of R CrB, but the period of the former is more or less shorter than those of the latter (Totochava, 1973b). The light, color, and radial velocity of the RCB's demonstrate that these stars pulsate.

Research on light fluctuations with a smaller time scale requires continuous observations of the light. Herbig (1967) reports that Miskin observed rapid oscillations in R CrB near minimum, but very high frequency pulsations in R CrB were not detected by Horowitz et al. (1971) in 1969. Interesting observations were also made by Totochava (1975) on XX Cam from

1972 to 1974. At first, Totochava conducted her observations with a single-channel photometer, alternatively in U, B, and V. Some variations appear in the yellow filter, with an amplitude of about 0.1 mag over several minutes. Fluctuations alternate with quiet stellar phases (e.g., no light variations are observed during the 1973 autumn). The observations are simultaneously obtained with a three-channel spectrophotometer in the following ranges: 3350 to 3650 Å, 4155 to 4280 Å, and 5120 to 5320 Å. The observed curves obtained in this way show several synchronous fluctuations in the violet and blue regions during certain nights (Figure 1-24a, b). The amplitude of the UV fluctuations are a little larger than in the blue; they amount to approximately 0.1 mag and last for about 2 hours. During a period of several nights, the fluctuations become much less visible in all the colors. At the quiescent phase of XX Cam, Kolotilov et al. (1974) find interstellar polarization characteristics, but a higher value of  $P$  than

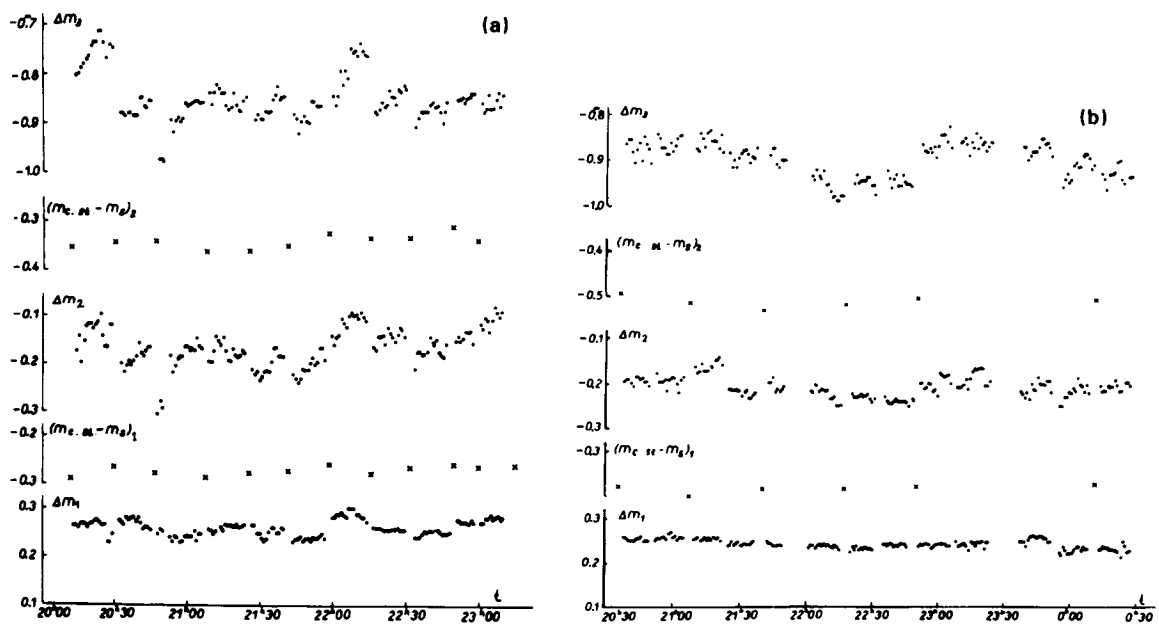


Figure 1-24. Light curves of XX Cam obtained from simultaneous three-color observations (see text): (a) on December 23-24, 1973, where the fluctuations are visible only on the violet (top) and blue (middle) curves; (b) on February 15-16, 1974, where the fluctuations are visible in the three colors. The crosses show the difference between the reference stars (from Totochava, 1975).

for neighboring stars, suggesting that an intrinsic polarization of the star makes some contribution.

Following R CrB itself, Humphreys and Ney (1974) and Strecker (1975) observed a smooth variation between 1.4 and 3.0 mag at  $L$  ( $3.5 \mu\text{m}$ ) with a pseudoperiod of about 1100 days. (The monitoring was done only between JD 2440000 and 2442360.) In RY Sgr, besides the 38-day variation, Feast et al. (1977) also detected a long period variability at  $L$ , which is related to the variation of the star itself. They found a long-term change of 1.5 mag, but the period was not well analyzed for lack of observations. Feast and Glass (1973) and Feast (1975) show how the star can make only a minor contribution at  $L$ , and Feast et al. (1977) testify that 70 to 80 percent of the  $L$  flux is due to the infrared excess (in the case of RY Sgr), with an estimated dust shell radius of 5 to 10 A.U. (Lee and Feast, 1969). "It seems natural to suppose that this is due to pulsation of the circumstellar dust shell itself" (Feast et al., 1977).

It is useful to remember that Bergeat and Lunel (1980) state that "the contribution of shell emissions to the  $J$  values is usually small" for the Miras, SR, and L stars, while Walker (1980) concludes that there is no evidence of dust thermal emission in the near infrared for most of the N-type irregular variables, and Tsuji (1981c) uses the  $L$  flux for his temperature-effective determination of SRb and Lb carbon stars. Consequently, although the C spectral type Mira, SRb, Lb variables, and the RCB stars are all carbon stars, the effectiveness of the circumstellar dust shells is very different at  $L$  wavelength among them.

Additional observations are required to improve our knowledge of the RCB circumstellar dust shells. For example, the phase lag between  $V$ , I104, and  $L$  magnitude curves could help us to define the thickness of the envelope. Some cool variable stars with extensive dust shells have already shown such long-term variability.

**Correlations Between Observed Quantities.** The observations of the RCB stars at different epochs indicate that, in general, the infrared ex-

cess compensates for the loss of light in the visible and UV wavelengths (Alexander et al., 1972); however, this is not true for all the stars.

The oscillations of brightness of RY Sgr during the climb back to maximum, and during the maximum itself, are correlated with the radial-velocity variations of the absorption lines (Figure 1-25) and have 38.6 and 39.0 days of period, respectively (Alexander et al., 1972), confirming that RY Sgr pulsates as a helium star of two solar masses.

The emission lines of RY Sgr do not have the same behavior pattern as the absorption lines (Alexander et al., 1972). We are not able to decide whether the emission lines follow the 38.6-day pulsation or not; large differences on radial velocity between absorption and emission lines are nevertheless observed.

Pugach (1977) found that the beginning of the obscurational decline appears between phases 0.24 and 0.37, and he chose the origin  $\varphi_0$  of the pulsational phase  $\varphi$  at the minimum light of pulsation. If we suppose that these pulsational oscillations are roughly sine waves and if we change the origin  $\varphi_0$  to the maximum, we see that the beginning of the obscurational decline is around the phases 0.74 and 0.87. This reminds us of the famous phase 0.8 of the Mira variables. Can we conclude that the obscurational decline is a result of one pulsational oscillation being stronger than the others? Pugach (1977) concludes that "the serial number of cycles when declines occur, is probably accidental," which seems erroneous today. For the decline of RY Sgr in 1967, the pulsational phase of the drop of light does not confirm the Pugach phase. Howarth (1976, 1977) notes that "the 1967 fade of RY Sgr took place at the time of minimum of the secondary variation."

The occurrence of pulsational minima in RY Sgr can be well represented by a quadratic solution of the cycle number  $n$  ( $\text{JD}_{\text{min}} = T_0 + nP_0 + n^2k$ ). The (O-C) residuals are shown in Figure 1-26a. The value of  $k$  is  $\sim (-51 \pm 2) \times 10^{-5}$  day per cycle. This value is in agreement with theoretical models for the evolution of

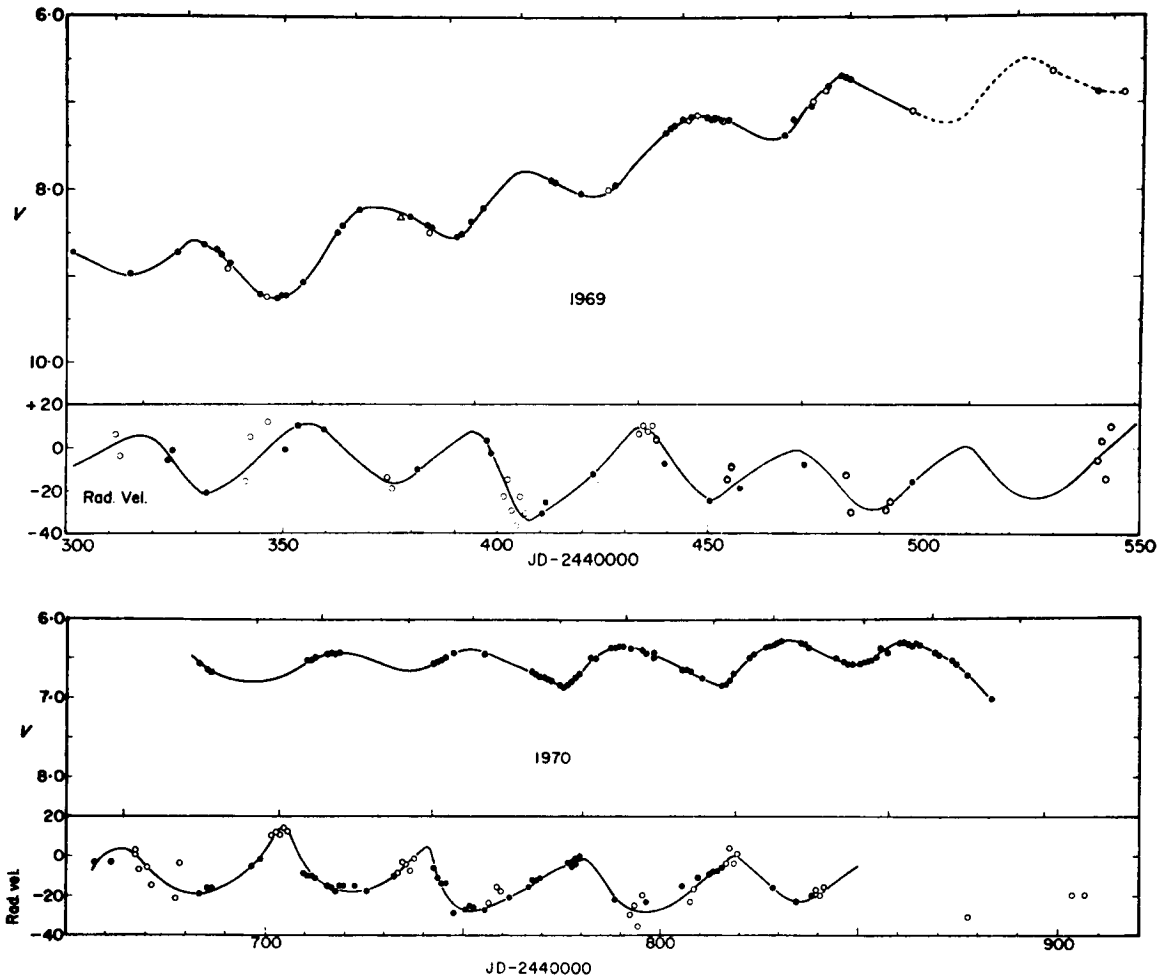


Figure 1-25. RY Sgr (a):  $V$  light curve and radial velocity (km/s) curve during the rising of 1969; (b): during the maximum of 1970. Predicted maxima and minima in  $V$  and predicted maxima in radial velocities are marked above the curves (from Alexander et al., 1972).

HdC stars (Schönberner, 1977). The comparison of the RY Sgr light curve (Figure 1-26b) and the depth estimates of pulsational minima observed on the same time scale (Figure 1-26c), shows that the deeper pulsational minima tend to occur on or near rising branches of the obscurational minima.

Bateson (1972) has noted a similar effect on UW Cen. This behavior remains unexplained (Kilkenny, 1982). Forrest et al. (1972) were the first to stumble on one of the most important clues to the RCB phenomenon: when the star drops into the deep minimum, the infrared ex-

cess is not affected (no change in the emitting flux of the dust shell and no change in the phase of the  $L$  curve; Figure 1-27). The envelope and the atmosphere of these stars seem to be dynamically disconnected. This restricts the model of obscuration, and Hartmann and Apruzese (1976) suggest that the dust is formed above a small region of the stellar surface and is blown away by the radiation pressure. Consequently, comparative studies of the visible and infrared light curves come to the same conclusions as those of the foregoing investigations. Glass (1978) argues in the same vein

about the 1975/1976 visual minimum of S Aps: when  $J$ , mainly due to the star itself, drops to minimum,  $L$  is not so strongly affected.

There is a pressing need to summon observers' help in making observations during light fades of different amplitude and duration to find correlations between observed features and to conclude which parameters are linked together and which are not.

### Nonvariable Stars

After this extensive review of all the types of cool variable giants and supergiants and their probable connections with white dwarfs, planetary nebulae, or supernovae, we have to discuss stars which—though located in the same area of the HR diagram as the red variables—are surprisingly mentioned as nonvariable or normal stars.

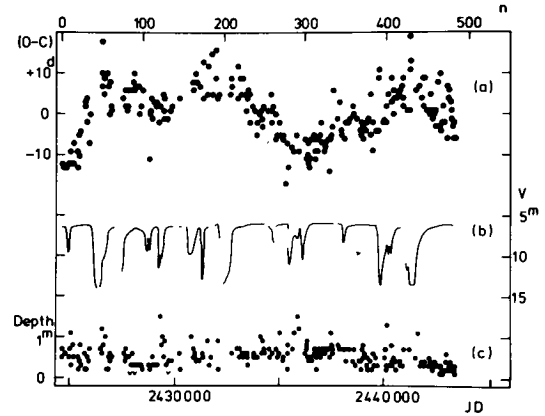


Figure 1-26. *RY Sgr* (a): (O-C) residuals from the quadratic formula of the pulsational minima plotted against cycle  $n$  and Julian date for ephemeris with decreasing period; (b): sketch occurrence of obscuration minima on same time scale as (a); (c): estimates of depths of pulsational minima on same time scale as (a) (from Kilkeny, 1982).

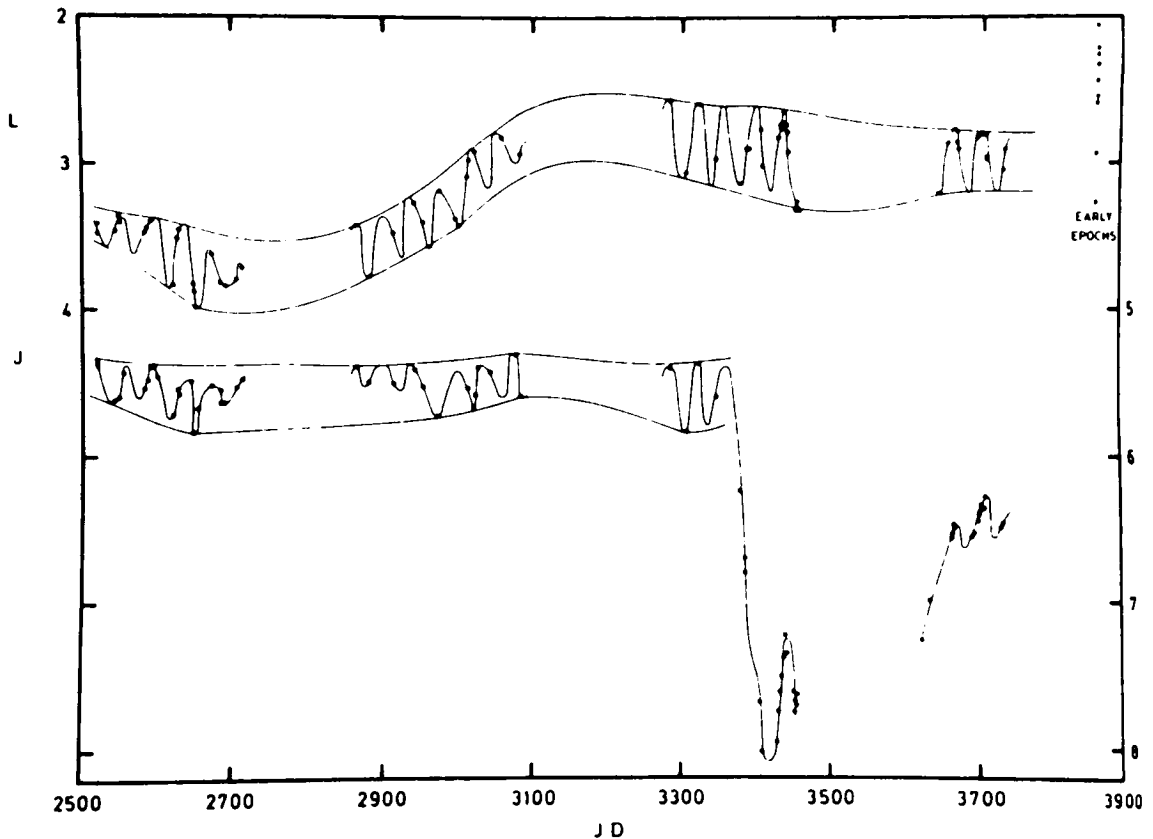


Figure 1-27. SAAO observations of *RY Sgr* at  $L(3.5 \mu\text{m})$  and  $J(1.2 \mu\text{m})$  (from Feast, 1979).

**Nonvariable HdC Stars.** Knowing that the RCB stars have fluctuations of brightness around 0.1 mag or more with periods as large as 40 days (38.9 for RY Sgr and 44 for R CrB), Rao (1980b) searched for variability in the non-variable HdC stars. He observed the cool HdC star, HD 137613, eleven times in *UBV* during 85 days and detected no periodic variations greater than 0.1 mag in *V*. This variation range is larger than the expected RMS error ( $\pm 0.025$  in *V*,  $\pm 0.020$  in *B-V*, and  $\pm 0.025$  in *U-B*), which may indicate a real variability in cool HdC stars. However, we conclude that this number of observations is too low to point out clearly any kind of variability. The total range of 0.1 mag in *V* encourages new observations with a time scale shorter than the week time scale used by Rao. As an example, such a long time scale should prevent us from discovering the short time-scale fluctuations in XX Cam by Totchava (1975), as well as the suspected nonradial-connected phenomena as discussed above.

Feast and Glass (1973) demonstrate that the nonvariable HdC stars do not possess any IR excess, and Rao (1980a) points out that the non-variable star, HD 182040, has no envelope. This should be correlated with the low evidence for large mass loss on the nonvariable hot HdC stars (Schönberner and Hunger, 1978; Herber and Schönberner, 1980). The high-dispersion IUE observations of HD 182040 show no emission lines (i.e., chromospheric indicators; Johnson et al., 1984).

**Helium Stars.** Although all the following helium stars are known to be nonvariable: BD + 37°442, BD + 10°2179, HD 124448, HD 160641, and HD 168476, Herbig (1967) stressed that possible long-period variations exist. Landolt (1968) made intensive observations over periods of a few nights on two or three occasions each year for detecting short-term and middle-term variations; the set of stars are those given above, plus HD 264111 (Stephenson, 1967a) and BD + 13°3224 (Berger and Greenstein, 1963). Only short-term variations are noted on BD + 10°2179: on February 3, 1968,

the star changed by 0.04 mag in *V* between two consecutive nights, a change greater than the probable error of a single measurement. Rao (1980b) also analyzed the variability of this star with 13 observations made on 100 consecutive days. The star does not seem to have any periodic variations in light greater than 0.1 mag in *V*. These variations are in agreement with those observed at the same epoch by Landolt (1973), who gives a total range of 0.09 mag in *V*. Here also, the observations are too few and lead to the same conclusions made for the low-temperature HdC stars.

Observations like these made by Landolt (1973) must be carried out: the large scatter in the data of HD 160641 and the evident brightening of the star by  $\sim 0.1$  mag during 7 hours of observation demonstrate that short time-scale variations exist in helium stars as well as in the cooler HdC stars (Figure 1-28). On the other hand, Herbig (1964) concluded that: "A survey of the bright helium stars on a long-time series of ordinary patrol photographs would appear to be the more promising approach." Today, we endeavor to collaborate with the IAPPP or GEOS members or to use automatic telescopes with automatic photometers or satellites to do such surveys. Correlations between brightness and radial velocity are also seen in the extremely hydrogen-deficient hot star, BD + 13°3224, by Hill et al. (1981). The correlations between observations and evolutionary models (Schönberner, 1977; Hill et al., 1981) are to be analyzed carefully (Tutukov and Iben, 1985).

**Early R Stars.** There are no systematic studies on the relative number of variables and non-variables among carbon (R and N) stars, such that it has been necessary to gather such information from catalogs. For example, among the 122 carbon stars that are brighter than 10.0 mag in the DDO catalog (Lee et al., 1940, 1947; Lee and Bartlett, 1945), 68 percent are variable.

Looking at the early R stars, we discover that only some are variables. (The variability is everywhere!) Among these early R variables, some, like BD + 20°5071 and BD + 69°417

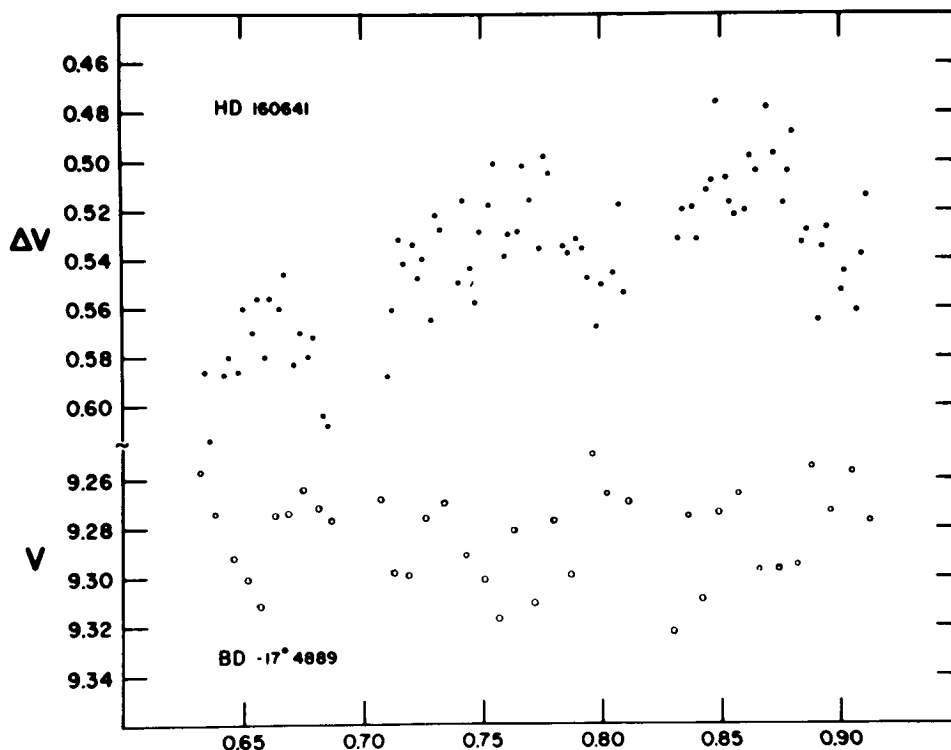


Figure 1-28. Top: plot of the differential  $V$  magnitude for HD 160641 (the gap in data at JD-2441452.70 and 452.83 occurs at times when  $UBV$  standard stars were observed); bottom:  $V$  magnitude plot for the comparison star BD-17°4889. For the 7-hour observing run, the large scatter in comparison star data is due to technical difficulties. The observations are made on May 15, 1972 UT (from Landolt, 1973).

(RU Cam), are bluer than the nonvariables. RU Cam has a period of 22 days (W Virginis type). Its spectrum varies from R0 at minimum to K0 at maximum. BD +57°2161 is also suspected to be variable, but it is redder than the nonvariable R stars. A few further photometric analyses should be undertaken for a better knowledge of these stars. Among the R2 stars, BD +19°3109 and BD +02°3326 are variable and have a higher color index than the normal stars.

**Nonvariable M-Type Stars.** Only a few papers are devoted to normal M stars. Generally, these stars are discussed in papers in which their characteristics are used as reference to the Mira, SR, or L variable stars. One of the most valuable proofs that we observe a normal M

star is the nonvariability of the strength of absorption molecular bands and atomic lines and of the adjacent flux peaks.

The nonvariable M-type stars extend from M0 to M8, with a luminosity class from Iab to III. On the following stars,  $\beta$  And,  $\pi$  Aur,  $\alpha$  Cet,  $\mu$  Gem,  $\psi$  Peg, 119 Tau, 83 UMa, and  $\delta$  Vir, Maehara and Yamashita (1978) obtained repeated observations of their fluxes within about 0.05 mag for most wavelengths. The energy distribution varies gradually from class to class. For stars later than M5, the gradient of the 4000 to 5000 Å region is mainly determined by strong TiO absorption bands rather than by temperature effects on the flux peaks.

We will not elaborate further on the nonvariable M stars which are not especially within

the scope of this volume on variability and non-thermal phenomena. However, a red star can be accepted as nonvariable and, consequently, as a photometric standard, only if it is observed to be constant over a sufficient period of time. It often happens that red stars accepted as UBV standard stars (i.e., nonvariable a priori) prove to have highly variable chromospheric lines (Sherbakov, 1979). As a consequence, they cannot be considered as nonvariable stars in narrowband photometric systems (Wings, 1971; Querci and Querci, 1985b).

## IRREVERSIBLE CHANGES AND RAPID EVOLUTION

### Irreversible Changes in Miras— An Example: R Aqr

Among the Mira variables, one evolved star that illustrates the purpose of this paragraph is the complex object, R Aqr. This binary star—a Mira (M7e) and a blue companion (Ofp, Bep, hot subdwarf, or white dwarf with an accretion disk)—is imbedded in a compact high-excitation nebula (Lampland, 1923a). These three components, which form the spatially unresolved central object of the R Aqr complex, are surrounded by a convex lens-shaped nebula of 2 arc-minutes (Lampland, 1923b) with filamentary and clumpy structures. More recently, an inner jet with an apparent changing structure and discrete variable knots was observed within 10 arc-seconds of the star. A possible counter jet was recently pointed out with radio and optical mappings.

Let us briefly review the main studies looking at the various causes of variability of this object and showing its rapid evolution.

The variability of the classical Mira was discovered by K. L. Harding in 1811, and the first objective-prism spectrograms obtained at Harvard as early as August 1893 show an Me spectrum. One of them, taken on October 17, 1893, shows a faint nebular line at  $\lambda 5007$  and bright hydrogen lines from  $H\beta$  to  $H\xi$ , with no trace of the Me spectrum (Merrill, 1928). The first slit spectrogram, taken at Mount Wilson in October 1919, shows (superposed on the normal

spectrum of the M7e star) several characteristic emission lines of the gaseous nebula, a combination never observed previously.

As pointed out by Merrill (1940), the inner nebulosity is variable in position and brightness. The existence of both inner and outer nebulosities have been confirmed by numerous photographs taken in 1919 by Hubble (1940, 1943) with the Mount Wilson 100-inch refractor. From the measurements of a pair of plates taken at 16-year intervals, Baade (1943, 1944) estimates from the expansion rate that the onset of expansion was about 600 years ago. These observations could confirm the nova-like event noted by the ancient Japanese records in A.D. 930 and which formed the outer extended nebulosity.

During many years, the emission lines of the inner nebula have been photographed repeatedly. Their intensities vary through a large range, apparently without any correlations with the phase of the Mira star. Table 20 in Merrill (1940) describes the behavior of the spectrum of the companion and of the nebular lines from 1919 to 1939, but Merrill neglected the month-to-month variations in his description.

During the 1920–1930 decades, the very high-excitation spectrum marked by bright lines of hydrogen, helium, and iron was ascribed to the companion, which varies in an irregular manner (Merrill, 1935; Campbell, 1938) from the 11th magnitude or fainter to the 8th magnitude in 1933. Slight brightnesses were seen during 1922–1925. In July and September 1925, the nebular lines and the companion spectrum were particularly weak, but by December 1926, a rapid and well-marked increase in brightness occurred; the continuous spectrum of the companion was abnormally strong, and the spectral features were completely different from anything previously seen. The bright hydrogen lines had P Cygni profiles, and numerous ionized iron lines were prominent, whereas the nebular lines,  $\lambda 4959$  and  $\lambda 5007$ , of [O III] were just detectable.

In 1927, the continuous spectrum and the nebular lines were much weaker, but the forbidden iron lines were stronger. Until 1933, no

large variation was noticeable; all the lines were slightly diffuse but not of great width. During 1931–1933, Payne-Gapochkin and Boyd (1946) observed an apparent correlation between the extremely bright active companion and the smeared-out light curve of the Mira in which the minima were bright and the maxima were abnormally low.

In July 1934, the combined flux of the Me and hot companion appeared to be fainter than it had been for several years, and the spectrum was very different. The [Fe II] lines in the blue-violet and the [S II] lines near  $\lambda 4068$  were well marked, while the Ca II H and K, the Fe II, and the nebular lines were very weak. After the Mira maximum of August 1934, the Me spectrum became fainter, and in November, the post-maximum Mg I  $\lambda 4571$  line appeared and became rapidly bright. The active companion was close to the 10th magnitude during the summer of 1935, and its forbidden lines dominated the spectrum during the Mira minimum of July 1935. These forbidden lines remained fainter during 1936–1939, few of them were recorded superposed to the Me spectrum.

Some spectral features must be noted for understanding this complex object. For example:

1. The nebular line,  $\lambda 3967$ , of [Ne III] is not present (Wright, 1919); it could be absorbed by the Ca II H line and could demonstrate the existence of Ca II in the line of sight (i.e., in the outer nebulosity or in a large absorption shell surrounding it).
2. The P Cygni profile of the companion lines has often varied in intensity and structure, perhaps demonstrating important motions inside the inner nebula.

Between 1936 and 1949, the Me component of R Aqr was varying like a “normal” Mira (Merrill, 1950), while the spectrum of the active companion was subject to changes from an Ofp star spectrum with bright lines of N III and Fe II to the spectrum of a Bep star with nu-

merous Fe II lines and a strong continuous spectrum. Its nebular lines, weak in 1936, increased in intensity until 1940 and remained relatively strong during a decade. On 10 Å/mm spectra, their widths appear variable and sometimes slightly unsymmetrical. Variable motions of matter are thus demonstrated in the forming layers of these lines. Merrill (1950) suggests either a probable orbital interpretation (a spectroscopic binary with 27-year period) or large-scale pulsations in the inner nebulosity to interpret the radial-velocity displacement of the [O III] and [Ne III] nebular lines. However, these variations reflect a continuing activity in the inner nebula.

The spectra obtained by Herbig (1965) in the late 1950's show a very strong nebular spectrum that nearly disappears in the observations of Il-ovaisky and Spinrad (1966) made during the 1964 and 1965 minima; present were only the [S II],  $\lambda 4068$ , and the doublet, [O II]  $\lambda\lambda 3726$ – $3729$ , from which an electron density  $N_e \approx 10^3/\text{cm}^3$  was deduced. However, the Mira spectrum at minimum is clearly present, and moreover, there were no traces of the active companion.

The IR emission of R Aqr was detected independently by Woolf (1969) and Stein et al. (1969). The 11- $\mu\text{m}$  emission of R Aqr is not brighter than that of  $\alpha$  Cet or  $\mu$  Cep, but it is much larger (from 8 to 12  $\mu\text{m}$ ). It is not as broad in wavelength as that observed in the planetary nebula, NGC 7027 (Gillett et al., 1968). This peak shape should be correlated with the various sizes of the emission grains located in the nebula. Stein et al. (1969) also note a strong and narrow absorption feature at 7.8  $\mu\text{m}$ . Today, the 7.8- $\mu\text{m}$  feature found in HD 44179 is attributed to coronene ( $\text{C}_{24}\text{H}_{12}$ ) by Léger and Puget (1984).

The IUE spectra show strong emission lines, which are probably formed in the dense compact nebula located around the binary system and close to it ( $d \approx 2 \times 10^{14}$  cm). The spectral type of the active companion is not conclusively resolved; it was identified as a bright white dwarf ( $T \geq 50000$  K) or a subluminous central planetary-nebula star. Whatever it is, it



would have to be less than  $T_{\text{eff}} \approx 65000$  K in order to explain the weakness or absence of the He II 1640 Å line as stressed by Michalitsianos et al. (1980). Its brightness is comparable to that of the solar one. As suggested by Merrill (1950), it can produce enough ionizing photons to excite the continuum and the emission lines, but it is relatively faint for direct observations: "Such a star would photoionize the inner, high density nebula, but lacks a sufficient flux to photoionize the entire extended nebulosity." The observed IUE line fluxes, such as those from He II, C II, C III, C IV, [O II], O III, [O III], O IV, S II, Si III, etc., can be used to obtain the general parameters of the ionized nebula ( $T_e \approx 15000$  K,  $L \approx 2 \times 10^{14}$  cm,  $N_e \approx 10^6$  to  $10^7$  cm<sup>-3</sup>). The observed UV continuum, essentially flat, is attributed to Balmer recombination. Michalitsianos et al. (1980) conclude that the compact nebula (inner nebula) could be entirely due to a mass-loss phenomenon from the primary M7e star with a mass-loss rate larger than  $10^{-7} M_{\odot}$ /year. To explain the 1933 brightness of the companion ( $m_v \approx 8$  mag), they propose that it was provoked by mass transfer from the primary to the secondary. A set of parameters for the companion was deduced in their Table 3. Moreover, from the emission lines such as O I, Mg II, and Si II, they deduce the presence of a warm chromosphere around the M primary star.

A model of such a transfer of mass was developed by Kafatos and Michalitsianos (1982). In this model, the high brightness could be triggered when the companion accretes matter from the Mira component when it crosses over the periastron of its highly elliptical orbit. This hypothesis was one of the two previously suggested by Wallerstein and Greenstein (1980) to explain the apparent correlation observed by Payne-Gaposchkin and Boyd (1946). As suggested by Usher (quoted by Payne-Gaposchkin and Boyd), the companion's outburst might inhibit the pulsation and suppress the maxima by a change in the boundary conditions of the Mira itself. However, Wallerstein and Greenstein (1980) prefer the possibility of a single Mira star with a flaring region made by com-

plex magnetic fields. During such flares, the pulsation of the Mira is inhibited by the magnetic fields and its maxima are smeared out. Jacobsen and Wallerstein (1975) arrive at the same conclusion by radial-velocity measurements of the nebular lines on plates from 1957–1965 and from 1970–1971. They do not find the 26.7-year period of Merrill (1940), and they conclude that a type of activity takes place in the outer layers of the Mira during the interval 1930–1940.

Using the criterion for Roche formation, the value of the primary radius,  $R_1 \approx 2 \times 10^{13}$  cm, and the half-value of the orbit axis,  $a \approx 2.5 \times 10^{14}$  cm, Kafatos and Michalitsianos (1982) found  $0.84 \leq e \leq 0.92$ . With  $e = 0.85$ , they obtain  $M_1 + M_2 \approx 2.5 M_{\odot}$ ,  $P \approx 44$  years, a visible brightness of the disk of the order of  $\approx 1/5$  the Mira brightness, and a disk optically thick with an external temperature of  $\approx 2300$  K and  $R_d \approx 2 R_1$ . If these values are correct, it is probable that the gravitational field of the secondary could be more important than that of the Mira. Kafatos and Michalitsianos (1982) believe that the disk around the unseen companion is not as large as  $2 R_1$ , but has its larger extent at periastron because a moderate dimension of the disk would be more consistent with the expected mass of the secondary ( $1 M_{\odot}$ ).

Viewing the visual light curves of the Mira, Willson et al. (1981) suggest that the 1934 and 1978 low maxima are caused by an eclipse of the Mira itself by the accretion disk or by the gas cloud of the system and conclude that the orbital period is 44 years.

The thermal relaxation time of the envelope of the Mira, as well as the free-fall time scale, is about some years. These times are comparable in range to the 8.5-year eclipse duration observed between 1928 and 1935 and to the appearance of the jet between 1970 and 1977, which occurs about 44 years later (i.e., binary period). Such observations may not represent an eclipse at the periastron ( $\approx 1$  year), but they would in fact infer the characteristic dynamical time scale required by the Mira to recover to its preperiastron quiescent state.

The first hypothesis of Wallerstein and Greenstein (1980), quoted previously, is supported by Bath (1977), who proposes that the mass transfer from the Mira envelope to the companion at its periastron orbital phase builds a spatially thick accretion disk at supercritical accretion rates, accompanied by the formation of a jet, possibly driven by radiation pressure. To confirm this hypothesis, Herbig (1980) found such a spike on the inner nebulosity, ex-

tending 10 arc-seconds from the star, with an angle position of approximately  $22^\circ$  on direct plates obtained with the Lick telescope. Tapia et al. (1982) reveal an  $8''$  jet at position angle (P.A.)  $26^\circ$  formed by discrete variable knots and perhaps a diffuse component and apparently a very small extension at the opposite direction not observed before. They suggest that this violent ejection may have taken place during the current minimum of the variable.

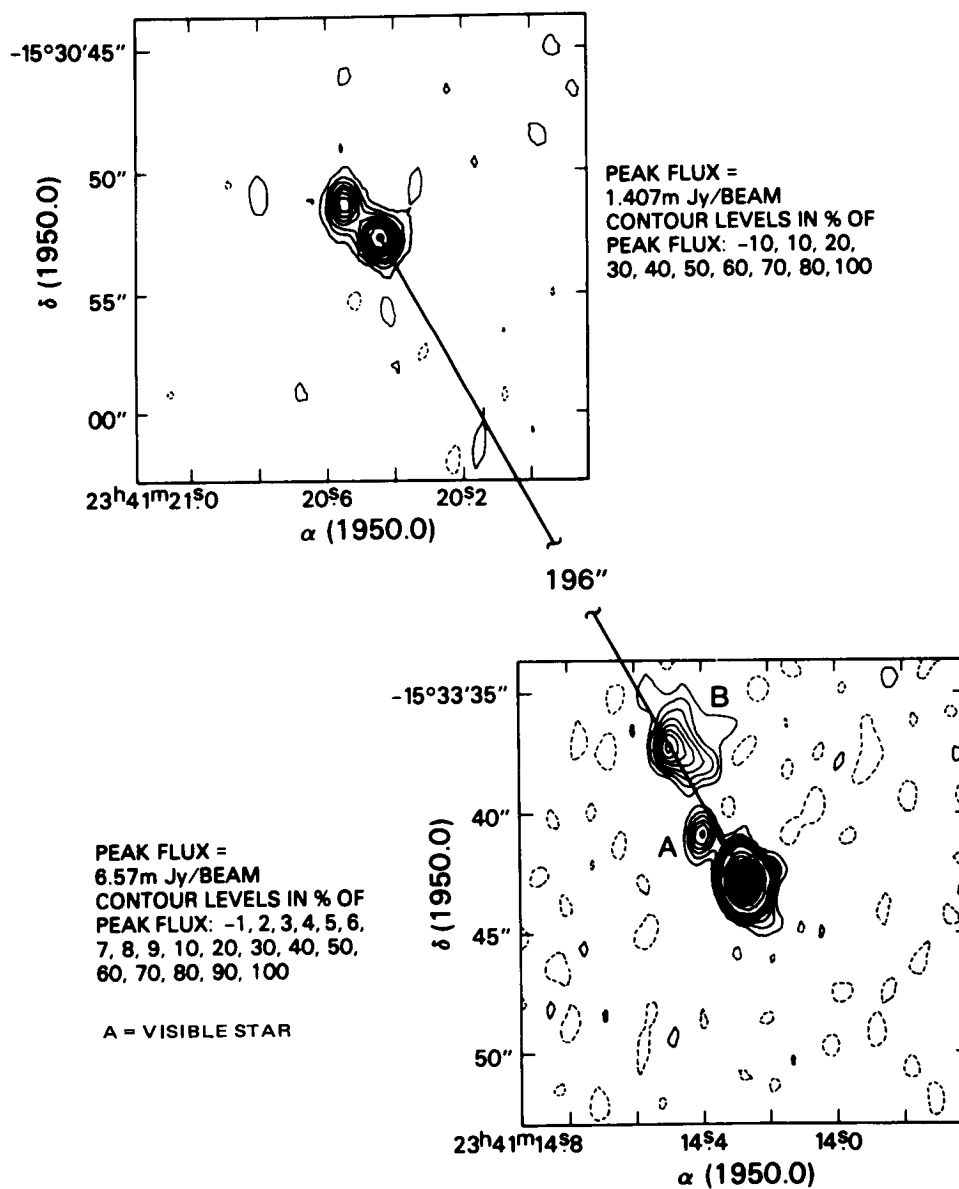


Figure 1-29. A 6-cm map of the R Aqr complex showing the secondary source located at 196 arc-seconds and the neighborhood of the star (from Kafatos et al., 1983).

This optical jet was rapidly confirmed by a radio counterpart. Sopka et al. (1982) describe its observational properties from optical and very large array (VLA) mappings. They found extensions both north and south of the star, at right angles to the outer nebular arcs, and they confirm that the inner nebulosity is variable in both brightness and structure, as already mentioned by Hubble (1940, 1943). These observations must be made again.

The mean radial velocity of the jet is  $-70$  km/s, whereas the mean radial velocity centered on the star is  $-44$  km/s (Sopka et al., 1982). Kafatos et al. (1983) made VLA observations of the complex R Aqr, with a resolution of 1 arc-second. At  $6.4''$  from the radio emission of R Aqr itself, they found a peak radio intensity at  $29^\circ 3$  P.A. (jet B on Figure 1-29). An unresolved radio source is also found at  $\approx 196$  arc-second away on the line defined by the previous jet and the star itself. This feature may represent matter previously ejected by the complex. Besides these two sharp radio features, a new undetached one has been detected at  $\approx 45^\circ$  P.A. and  $\approx 2.7$  arc-second from the central star.

The near-UV map (Figure 1-30) taken by Mauron et al. (1985) duplicates the 6-cm VLA map made by Kafatos et al. (1983), but the northern knots seem a little more resolved; on this near-UV map, the counterjet located in the opposite side of the jet is well identified (C in Figure 1-30).

The optical and the radio structures of knot B are quite similar, suggesting that we see the same emitting region which produces the Balmer continuum in the near-UV and the free-free radiation at 6 cm.

Knot A seems more radially elongated on the near-UV electronic camera photograph than in the Kafatos et al. (1983) 6-cm map. Moreover, near knot A, on the isophotes of the unresolved inner nebula, a bump is visible (E on Figure 2 in Mauron et al., 1985).

Assuming that the R Aqr distance is about 300 pc (Whitelock et al., 1983), Kafatos et al. (1983) deduce from their VLA observations that knot A is located about  $1.2 \times 10^{16}$  cm

and knot B is about  $2.9 \times 10^{16}$  cm from the central cool Mira variable. Since the knot B structure has not changed much in approximately 1 year (from September 1982 to December 1983), if we compare the 6-cm map from Kafatos et al. (1983) to the near-UV direct electronic camera photograph from Mauron et al. (1985), the latter authors deduce that this knot cannot have been ejected later than about 20 years ago. However, the proper motions of the knots obtained by Kafatos et al. (1983) and Mauron et al. (1985) have very poor accuracy; new measurements, chiefly from space, are required.

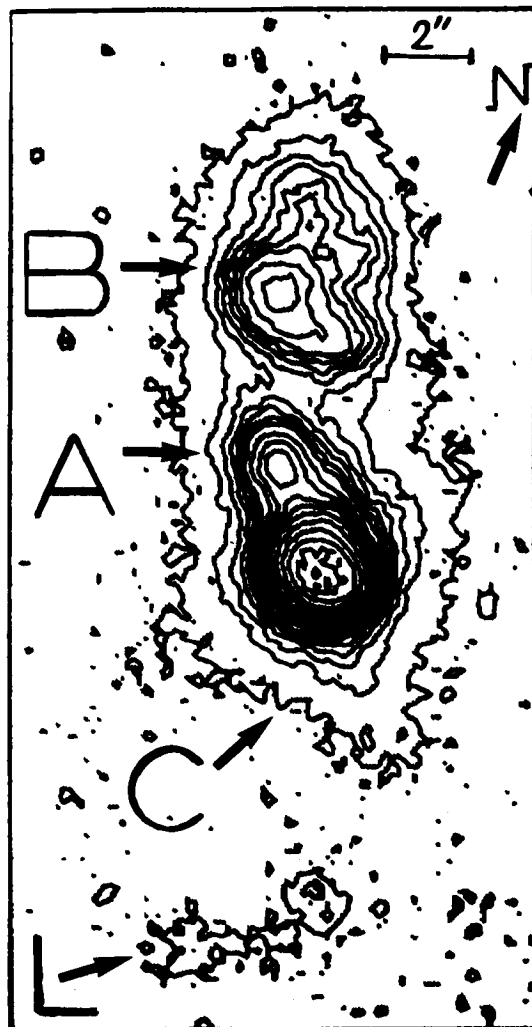


Figure 1-30. The R Aqr inner nebulosity with knots A and B and the possible counter-jet feature (from Mauron et al., 1985).

Taking into account that Si II  $\lambda$ 1815 and Si III  $\lambda$ 1892 are seen in the spectrum of the central region and are absent in the jet spectrum, Michalitsianos and Kafatos (1982) suggest that the material in the jet is comparatively cooler or less dense than in the central region. Quoting the optical spectra of the O II and O III emission lines of R. Fersen, Kafatos et al. (1983) confirm the cooler temperature of the jet.

In conclusion, these knots may be formed in different ways:

1. In the Mira inhomogeneous stellar wind, being perhaps excited by the UV flux from the orbiting companion when they enter its Strömngren sphere (Spergel et al., 1983), with the pressure radiation on dust in the knots making them move outward.
2. During the periastron phase, when the Mira fills in its Roche lobe (accretion at supercritical rates; Ferland et al., 1982).
3. In a probable polar ejection from the disk, giving them a roughly axial orientation relative to the elliptical outer nebula.

Kafatos et al. (1983) write that: "if the jet and the nearby point source (feature A) are ejecta, their difference in position angles could be interpreted as precession of the system while it expels material. The morphology of the outer  $\approx 2'$  nebula could be accounted for in this manner, thus explaining the characteristic lens-shaped filamentary structure." This is to be confirmed by repeated high spatial resolution observations.

The extended nebula has a mass about  $0.2 M_{\odot}$ , a kinetic energy larger than  $2 \times 10^{46}$  ergs. It radiates more than  $5 \times 10^{44}$  ergs/year in Balmer and Lyman emission lines and continua. Its cooling time scale is estimated to be  $\approx 2$  years. It seems unlikely that the photoionization from the companion star is the power source of the extended nebula for the past 600 years because its excitation requires much more energy than the Eddington limit for a  $1 M_{\odot}$  star. However, from purely energetic

arguments, the jet itself could be the source of the excitation of the outer nebula. The jet parameters estimated by Sopka et al. (1982) are used by Kafatos and Michalitsianos (1982) to conclude to a jet kinetic energy of  $\approx 5 \times 10^{45}$  ergs, which the authors estimate in reasonable agreement with the radiation cooling given previously.

The material of the extended nebula is trapped in rings, generally perpendicular to the axis of the jet. Such a structure is favored for the formation of the nebula in a single explosive event hundreds of years ago. This is confirmed by the high content of the nebula in He, and more particularly in N (Wallerstein and Greenstein, 1980), which is also in agreement with models of recurrent novae (Starrfield et al., 1982).

The counterjet feature situated on the opposite side of knot A (C in Figure 1-30) was initially detected by Tapia et al. (1982) on B and V plates taken on September 23–24, 1982. By radio observations, Kafatos et al. (1983) detected a possible counterjet. Using the ratio of the UV C IV to C III] lines at various places on the R Aqr complex, Michalitsianos and Kafatos (1982) found that this ratio at the opposite side of the jet is much different from that elsewhere.

What future projects are now planned for clarifying the R Aqr complex? To study the structure and the evolution of the inner nebula, the knots of the jet, and the counterjet, it is necessary to reach Mira phases and wavelengths when and where the Mira brightness is very low. Observations during the Mira minima and in the UV region could be powerful.

In the aim to diminish or to eliminate the turbulence of the Earth atmosphere usually attached to ground-based observations, observations in good seeing observatories by Speckle interferometry and Space Telescope (ST) investigations are needed. The Faint Object Camera (FOC) was developed especially for high spatial resolution studies. Moreover, the image restoration techniques could help in the analysis of observations which are at the limit of resolution of the ST, such as for the inner

nebula of R Aqr. The proper motions and evolution of the shape of the elements of the inner and outer nebulas could be investigated by this instrumentation. The kinematics of various features of the R Aqr complex could be deduced. High-resolution spectra of the jet and the knots should be made to evaluate the richness of the extended nebula in helium and nitrogen—richness suspected by Wallerstein and Greenstein (1980).

### Irreversible Changes in SR and L Stars

Many M and C class stars showed outstanding and nonreversing variations in one or more decades. We shall elaborate here on some examples of these rapid and definitive changes.

V1016 Cyg (MH $\alpha$  328–116) was classified as a strong H $\alpha$  emission object (Merrill and Burwell, 1950) with an estimated V magnitude around 15 (Fitzgerald et al., 1966). Between July 1963 and August 1965, this object brightened by  $\approx 4$  mag (McCuskey, 1965). Since this outburst, its behavior has been monitored optically by Mammano and Ciatti (1975); in the infrared by Harvey (1974), Puetter et al. (1978), and Aitken et al. (1980); in the radio by Purton et al. (1973) and Altenhoff et al. (1976); and in the UV-IUE by Flower et al. (1979) and Carpenter and Wing (1979). In the visible, many lines indicate a very hot source of radiation although some low-excitation lines are also present, and this, together with the absorption bands of TiO and VO, has been taken as evidence of a binary system with an LPV and a hot star exciting a nebula (Mammano and Ciatti, 1975). From their analysis of the IUE spectrum, Carpenter and Wing (1979) conclude that there has been sudden shell ejection with a very large excitation which developed a rich emission-line spectrum, with lines of O I, Fe II, Mg II, C II, N V, O V, [Ne V], and [Mg V]. As these lines are also seen in the spectra of planetary nebulas such as NGC 7027, something like a planetary nebula was probably formed or excited around the star (see also Flower et al., 1979). Nussbaumer and Schild

(1981) interpret that spectrum with a single star planetary model, the central star having a temperature of  $T = 160000$  K and a radius of  $0.06 R_{\odot}$ . The expansion velocity deduced from the line shape is around 100 km/s. The electron density in the shell is approximately  $3 \times 10^6$  cm $^{-3}$ , and the electron temperature varies from 8000 to 18000 K. From the reddening determined in the IUE observed UV region, they derive a distance of 2.2 kpc.

The carbon star, HD 59643, provides another example of nonreversing and rapid evolution. Shane (1928) and Keenan and Morgan (1941) considered it as a cool, but otherwise rather ordinary, nonvariable carbon star and classified it as R8 or R9. After reduction of its spectrum, Wildt (1941) considered it as a normal carbon star, with normal excess of heavy elements and with Ca II H and K lines in absorption. After more than 25 years, Gordon (1967) and Yamashita (1967) noticed an increase in the equivalent widths of the absorption lines of the heavy elements Ba, La, Sc, and Sr, as well as a deeper molecular G band. Greene and Wing (1971) examined the plates of HD 59643 taken by Keenan in 1949 and found no evidence of hydrogen emission. H $\alpha$  was seen in emission for the first time in November 1966 by Gordon (1967), when it was a little brighter than its nearby continuum. In 1969, Utsumi (1970) saw H $\beta$  quite clearly in absorption. These data suggest that the activity is of recent origin but not easy to understand. Greene and Wing (1971) found an abnormal and variable spectrum below 4000 Å with a filling in of the K line and the (0,0) CN violet band. This activity of February 1970 was also observed near the hydrogen emission lines H $\delta$ –H $\xi$ , but the longward spectrum was not significantly different from that analyzed by Wildt (1941). The main difference is that the Ca II H and K lines were completely invisible in 1970, although they were in absorption in 1941. This absence cannot be interpreted as a calcium deficiency because the Ca I  $\lambda$ 4226 line appeared to be strong. Another difference is the appearance of a very weak H $\alpha$  emission line in 1966, along with the strong H $\delta$ –H $\xi$  and the UV

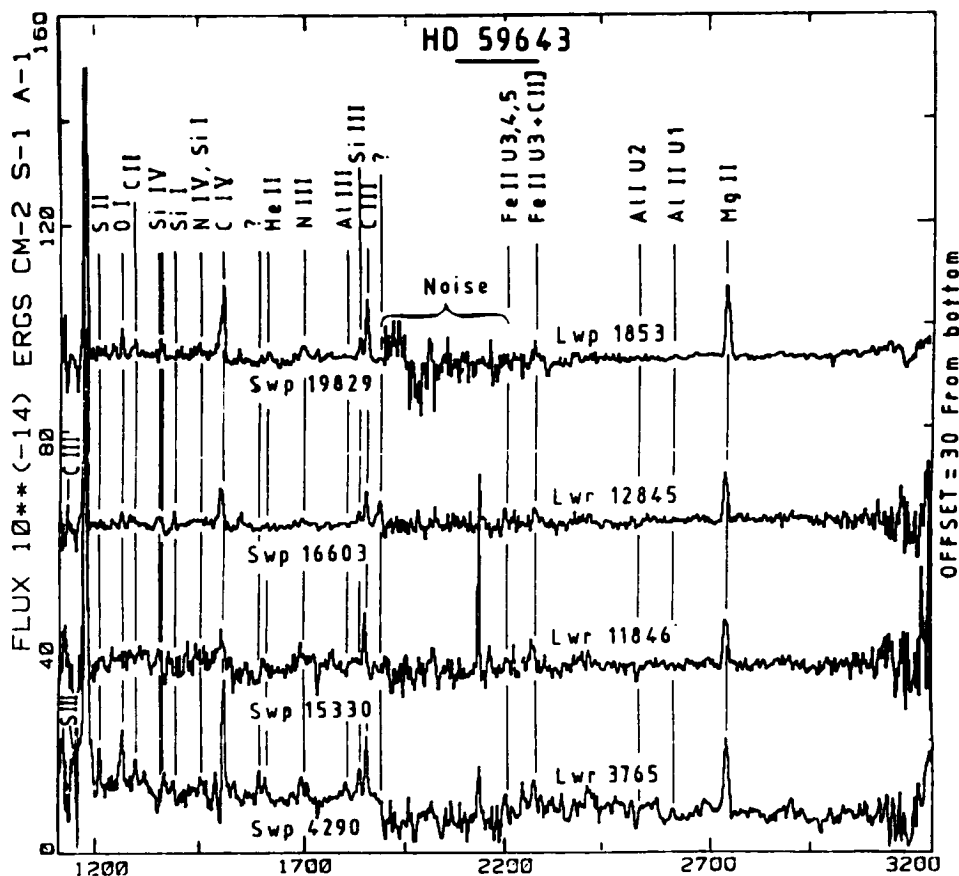


Figure 1-31. Low-resolution IUE spectra of HD 59643. The abscissas are in Angstroms, the ordinates are absolute fluxes. The lower spectra are obtained from the data bank. The observations are made on: Feb. 17, 1979 (SWP 4290); Feb. 14, 1979 (LWR 3765); Oct. 25, 1981 (SWP 15330, LWR 11846); March 23, 1982 (SWP 16603, LWR 12845); April 27, 1983 (SWP 19829, LWP 1853). We note the high level of noise from 2000 to 2200 Å for the LWP spectrum and a spike at 2180 Å in the LWR spectra (from Querci and Querci, 1985b).

continuous emission in 1970. Greene and Wing (1971) conclude that the UV continuous emission does not come from the photosphere; the radiation must emanate either from a hot circumstellar envelope or from the secondary star of the binary system. The latter interpretation disagrees with Wildt's observations, in which no continuum or emission lines are reported. Therefore, the UV emission could be considered to be an activity located above the layers responsible for the violet Ca II H and K lines, giving an IUE spectrum (Carpenter and Wing, 1979) with ions of a great range in ionization:

O I, Mg II, C II, and N IV. Querci and Querci (1985b) demonstrate that the emission lines are variable and that the continuum from 2200 to 1200 Å decreased significantly between 1979 and 1983 (Figure 1-31). The Balmer lines in HD 59643 are formed in the same outer layers because the Balmer decrement is very small, whereas it is very steep in the Mira variables and consequently affected by the violet depression. Through IUE and visible high-resolution spectra, the Bloomington and Toulouse groups are following the evolution of this complex object.

The ultraviolet continuum of the SRb carbon star, UU Aur, decreases with time. Shajn and Struve (1947) measured the Ca II H and K lines to be seven times weaker than those of the gM0 comparison star, HD 80943, while the Ca I 4226 Å line is not weakened and the Sr II 4078 Å line seems to be less affected. They suggest that the weakening of the continuous violet background may be attributed to the effect of many overlapping unknown molecular bands rather than to continuous absorption, in any case not to some temperature effect. They also measured the wavelength of emission lines like  $\lambda\lambda$  4036, 4009, 3959, 3949, 3914, and others, whereas in the regions centered at  $\lambda\lambda$  3983, 4005, and 4052, there is no apparent emission in the N stars. In the 200-inch plates obtained by Bowen (1951) in January 1951, the H and K lines, as well as the Al I 3944 and 3961.5 Å lines, are not visible on a spectrum which has a very low continuous background without absorption features shortward to Mn I 4030 Å multiplet. Only numerous emission lines are observed, especially at 3982.5 Å, which are identified as Ti I, V I, or Zr I low-excitation lines (Gilra, 1976). We therefore suggest an increase of the optical depth of the absorbing layers situated at the temperature minimum, which smears out all the absorption features coming from layers situated beneath. The very extended layers situated above might give rise to emission lines (low-temperature chromosphere) and narrow absorption lines (extended shell) such as those from Mn I multiplets, which eat away the emission lines. It is suspected that the changes in UU Aur are more in the nature of variations than secular changes.

### Irreversible Changes in HdC Stars

The irreversible evolution for the different RCB stars is not exactly the same. In the following, we detail the observed evolutions of only a few chosen RCB stars, all the more so because, unfortunately, the sample of the correctly observed RCB is very poor.

**A Slowly Evolving Star: RY Sgr.** Using data from 1967 to 1970, Alexander et al. (1972) confirm the cepheid-like behavior of RY Sgr by spectroscopic and photoelectric observations. Yet again, they find the cyclic variations of 0.6-mag amplitude and the 38.6-day period found by Jacchia (1933), with data covering the period 1920–1932. They conclude that “neither evolutionary effects nor mass loss by ejection are of enough importance to significantly change the period on the time scale of 50 years.” However, observations with longer time intervals could give slightly different results. Pugach (1977) was the first to point out a shortening of 0.9 days in 40 years, using observations from 1926 to 1977, but with a gap of 168 pulsational cycles. However, he was unable to find a relation between the period shortening and the time. This decrease of the pulsational period length was confirmed by the results of Marraco and Milesi (1980, 1982) and those of Kilkenny (1982). With observations of RY Sgr from 1897 to 1977, Marraco and Milesi (1982) give the instantaneous periods of the pulsational oscillations in 1897, 1926, and 1977 to be 39.3, 39.0, and 38.2 days, respectively. The periods of Jacchia (1933) and Pugach (1977) are again found, and the period decrease of 1.1 day in 80 years is deduced. Kilkenny (1982) reexamines the archive material from 1926 to 1978, material largely due to the efforts of amateur astronomers. He shows that the epochs of the observed minima agree with  $JD_{\min} = T_0 + P_0 n + kn^2$ , where  $T_0, P_0$  are the “zero” epoch and period, respectively, and  $k$  represents the linear rate of change of the pulsational period. The decreasing period thus ties in with the general data rather well, but there appears to be some modulation of  $k$ , with a time scale of 100 or 200 pulsation periods. Therefore, RY Sgr has  $k = -0.0005$  (Kilkenny, 1982), a value which is close to that expected by Schönberner (1977) for a deficient hydrogen star of  $M = 1M_{\odot}$  and  $T_{\text{eff}} = 6900$  K, evolving rapidly from the red-giant to the white-dwarf stage.

**A Star Rapidly Changing Its Period: S Aps.** Waters (1966) analyzed the visual observations

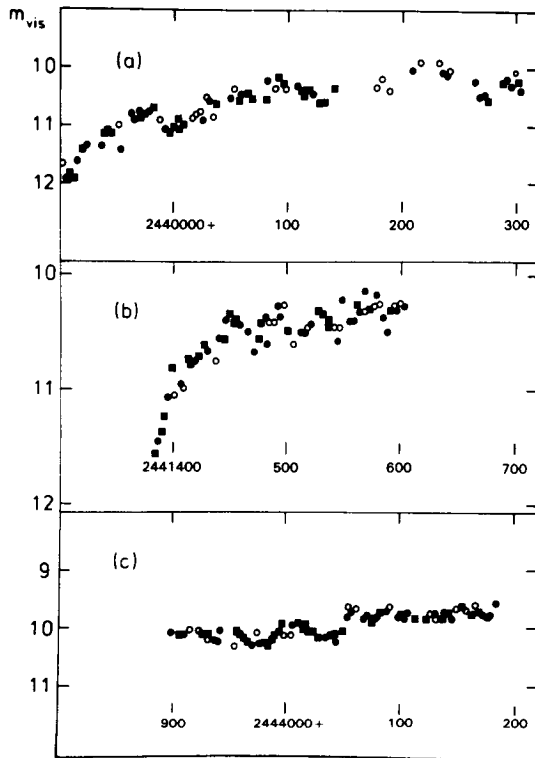


Figure 1-32. Visual observations (3-day means) of S Aps. Abscissas are Julian dates. Symbols indicate the number of observations in the mean: squares have five or more observations; filled circles three or four; and open circles, two; singles observations are omitted (from Kilkenney, 1983).

made during the period 1922–1960 and found a period of about 120 days. Kilkenney (1983) investigates 8000 individual observations covering the interval 1960–1982. After the obscurational minimum of 1967, S Aps shows an instantaneous pulsational period of about 135 days (Figure 1-32a). Surprisingly, after the next obscurational minimum in 1971, S Aps appears to have developed an oscillation of a 37.5-day mean pulsational period (Figure 1-32b). Observations on the 1979 maximum show an instantaneous period of about 40 days (Figure 1-32c). Other observations of such a phenomenon are necessary before we can conclude that the period change is linked to the obscurational minima or if the deep minima facilitate the change of pulsational mode (from fundamen-

tal to first or other overtones). We cannot explain this observation in terms of decreasing pulsational rate, and there is no model of evolution of hydrogen-deficient stars which allows such a rapid change of period (Röser, 1975), although the evolution of a hydrogen-deficient star to a white dwarf is rapid. Therefore, any changes in the observational material should be virtually undetectable on a time scale of a few years (Schönberner, 1977). In addition to these changes on pulsational period, there appears to be a fading of the amplitude of the pulsational variations in some RCB stars (Ferne et al., 1972; Kilkenney, 1983).

As for UW Cen, another example, the situation is not as comprehensible as for S Aps because the reduction of the observations gives many solutions with  $+0.003 \geq k \geq -0.006$ ; consequently, the period could be constant or decreasing/increasing, but at a relatively low rate. We must point out that “if UW Cen has an increasing period, this could mean that it has not yet reached the top of the asymptotic giant branch” (Kilkenney and Flanagan, 1983).

The helium stars could also have irreversible changes. For example, HD 168476 had  $V = 9.37$  mag in 1964 (Hill, 1964); the data of 1969–1971 given by Landolt (1973) and communicated to him by Hill in 1973, as well as the observations of mid-1972 made by Landolt (1973), seem to indicate a slight secular brightening over the decade 1964–1973.

The V mean value and the colors of the HdC star, HD 182040, obtained by Rao (1980b), agree with the data deduced from the one observation of this star made 25 years before by Mendoza and Johnson (1965). Consequently, no middle-term variations seem to appear.

## PROSPECTS AND CONCLUSIONS

At the end of this description of the slow or rapid, reversible or irreversible phenomena presented by the cool giants and supergiants, we should like to emphasize the observations to be performed for having a necessary solid basis of data to model these hydrodynamical



phenomena, and progress in knowledge of the evolutionary status of these stars.

To evaluate the nonthermal energy variations on various time scales involved in the different layers of the entire stellar atmosphere, simultaneous observations in many spectral ranges are required. Consequently, extensive monitoring must be organized from the ground with the present and new well-adapted instruments and with space facilities that will become more and more available in the not too distant future.

Some of the *ground telescopes* and their focal instrumentation could be used for photometric, high-resolution spectroscopic, and polarimetric analysis, as well as in the visible and infrared ranges and in the radio range.

To stimulate long-term variability observations of the carbon-star energy distribution, today partially followed with non-well-adapted means, Querci and Querci (1985b) propose a photometric system covering the main atomic and molecular features over the largest wavelength range. This system is inspired by their previous studies of N-type spectra from the far-UV to the radio frequencies and by synthetic spectra deduced from their model-atmosphere calculations. Also selected are three points on the continuum: at 1.5 mm, 3.8  $\mu\text{m}$ , and 1.04  $\mu\text{m}$  (Wing's I104 filter). Active observations in the visible are presently made in ESO-Chile by the Sterken group with some of the Querci's filters. Variations in color indices such as in  $C_2$ ,  $CN$ ,  $D-Na$ ,  $H\alpha$ ,  $I104$ , etc., can be followed, and various correlations with the known physical parameters can be shown. Monitoring on *other spectral intervals* is urgently required, such as around  $C IV$  (if any, possibly at some phases) and  $CO$  features and at 3.8  $\mu\text{m}$ , 11.3  $\mu\text{m}$ , and 1.5 mm, etc. This should allow a detailed analysis of the behavior of the various stellar layers. In addition, the high spectral resolution presently available, which gives fine details on the structure and dynamics of the red giants (see M. Querci, this volume), should let us envisage the modeling of the entire variable atmosphere.

Collaboration with amateurs on photometric monitoring cannot be neglected. The high

instrumental and scientific levels of some of them permit productive cooperation. The International Amateur-Professional Photometric Photometry (IAPPP) members, mainly in the United States, and the Groupe d'Etudes et d'Observations Stellaires (GEOS), mainly in Europe, are experienced in the photoelectric photometry. In the United States, after the first telescopes and photometers that work on semi-automatic control or remote control, some Automatic Photometric Telescopes (APT's) of 40-cm aperture are already operating, mainly at the Fairborn Observatory in Phoenix, Arizona (Boyd et al., 1985). These telescopes are driven by a computer that has on memory the star program and the comparison stars. The tests on sky quality and the night observations are automatically made on-line (i.e., finding the stars and centering them on the aperture, recording the data, etc.). The computer makes decisions such as when to open the dome, in what order the stars should be observed, and when to shut down. The telescope calls for human assistance if some failures appear. The data are reduced at the end of the night. The accuracy of such measurements made with the  $UBV$  filters of the Johnson system is of  $\approx 0.02$  magnitude. An APT service permits anyone to request differential photometry for his own program stars.

Many efforts are also being made in Europe; among them, the twin-telescope technique is being developed by the Section de Photométrie Photoélectrique of GEOS (Gregory and Querci, 1985). Because the sky transparency is rapidly variable during a night in Western Europe and because the GEOS amateurs aim at an accuracy of 0.002 mag in  $UBV$  filters, human assistance during the night is preferred for various technical and sky-quality controls through the rms of the on-line data on the computer display. In the twin-telescope technique, two telescopes with 28-cm apertures are fixed on the same mounting: one follows the star program, and the other, which is also driven by computer, follows the comparison star. The angle between their optic axes is calculated by the computer, which drives stepper motors for

a differential orientation of the second quoted telescope relative to the first. The two stellar lights are led into the two-channel photometer by liquid optic fibers. This new photometer is the second European Photoelectric Photometer (EPP-2) built by Fontaine (1985). It was derived from the EPP-1 (one-channel photometer) made by Fontaine (1984) and Walker (1985) in a collaboration decided at the IAPPP meeting at Herstmonceux-Castle during the fall of 1984. Full-sky photometry is made by using the standard stars. The *UBV* filters are now being used. Soon, we plan to use narrower filters and larger telescopes.

This photometric monitoring is the basis of research on variability. Narrow-filter photometry with 1-m telescopes and visible and infrared high-resolution spectra with 2.0- or 3.6-m telescopes could be made when interesting phases of variation are detected by photometric monitoring with small telescopes. Larger telescopes, such as VLT's and NNTT's, could be required when the stellar flux is very low.

It is also worth noting that *combined radio and infrared observations* should be developed between radio observatories such as Onsala, IRAM, Eifelsberg, and ESO and infrared telescopes such as UKIRT, CFHT, and ESO to provide constraints on models describing the molecular excitation mechanisms on the emitting layers, since it is well known that the infrared continuum and lines can excite the radio molecular transitions (see Nguyen-Q-Rieu, this volume). New molecules could be detected in the millimetric wavelength. The very far-UV and submillimetric wavelength ranges (i.e., between 200 and 900  $\mu\text{m}$ ) have not been extensively examined. There, photometry, together with spectroscopy, should give insight into the outer layers of the cool stars and special isotopic abundance determinations.

Already, the *space gave a new scope* on the cool giant research through the IUE and IRAS spacecrafts. (See the section *Photometric Observations*.) However, the small aperture of IUE seriously limits the high-resolution analysis to a very small number of cold bright stars.

The first EXOSAT investigations on cool giants and supergiants are presently in progress. (See the section *Photometric Observations*.) It seems to us that X-ray fluxes should be obtained at some phases of Miras (i.e., when the shock waves are the most powerful), and some constraints could therefore be added on the shock-wave interpretation of the Mira variations.

In the near future, new spacecraft will be launched: the Hubble Space Telescope (ST), the largest one with its 2.4-m diameter, by NASA in 1988, the space astrometry satellite, Hipparcos, by ESA in July 1988, and the Infrared Space Observatory (ISO), another ESA spacecraft, foreseen for 1992.

The scientific community has received detailed information on the different focal instruments of the Hubble Telescope via the "Space Telescope Science Institute" in Baltimore, Maryland (The Space Telescope Observatory, 1982). With its planned mission over 15 years, long-term programs on the variability of Miras and semiregular, irregular, and eruptive RCB stars are possible. Such an observational basis could permit great progress in the knowledge of the hydrodynamics of these stars. Many proposals have already been presented on the cool giants and supergiants, some of which are:

- WF/PC observations for the spatial distribution analysis of the clumpy ejected matter around individual stars as a function of star distance (e.g.,  $\alpha$  Ori),
- WF/PC observations of the R Aqr jet in the direction of the secondary radio source seen on the 6-cm map and analysis of the jet formation,
- FOC observations of the R Aqr complex, mainly the high-excitation nebula surrounding the mass-losing Mira variable, and the compact hot companion with its accretion disk for hydrodynamical modeling of the whole object,

- FOC observations of extended chromospheres for an analysis of the spatial properties of the wind such as asymmetric mass loss, radial variations of density and temperature, interaction of cool wind with the cool companion (e.g.,  $\alpha$  Ori), or interaction of cool wind with a hot companion (e.g.,  $\alpha$  Sco and  $\alpha$  Cet),
- FOC observations of the rapidly evolving stars such as V1016 Cyg, HM Sge, and HD 59643 to determine the location of the variable high-excitation region pointed out by IUE spectra,
- FOC exploration of the nearby giants for the structure of their own surface (convective cells, spots, etc.) and for the determination of the spatial and particle-size distributions on their nearby neighborhood (e.g.,  $\alpha$  Ori and  $\alpha$  Cet),
- HRS Echelle spectra of supergiants for dynamical analysis of the various layers by the monitoring of radial velocities and profiles of the spectral lines (e.g.,  $\delta$  Vir,  $\alpha$  Ori, and R CrB).

Hipparcos (High Precision Parallax Collecting Satellite) was named in memory of Hipparcos, who was the first Greek astronomer to measure a parallax (that of the moon) and who cataloged 800 stars. During the 2.5 years of the Hipparcos mission, any particular point of the sky enters in both fields of the satellite several times a year, making accurate photometry of selected stars possible (Turon-Laccarieu, 1978). The absolute coordinates of each star, its parallax, and each component of its annual proper motion will be measured with a maximum mean error of  $\pm 0.002$  arc-seconds (for stars brighter than  $B = 12$ ). Many astrophysical parameters can be deduced from these astronomical data: (a) with the parallax, we get the distance of the star; (b) with its apparent magnitude and distance, we obtain the absolute

magnitude; (c) from already measured apparent angular diameter and distance, we deduce the stellar radius; and (d) with the effective temperature and stellar radius, we have the luminosity and the mass of the stars. During the Hipparcos mission, the period of many Miras could be defined, the calibration of the period-luminosity relation could be improved, and the theory of pulsation of Miras could be checked. Knowing the center-of-mass radial velocity (obtained from the radio thermal lines) and the proper motions, the spatial motion of the various populations of stars could be deduced. Moreover, the satellite will collect a considerable quantity of accurate photometric data on the variability of the Miras and the semiregular and irregular stars; their analysis could provide the various pulsating periods and their temporal changes. Tests on stellar evolution and structure theories could be developed. The variations on various time scales detected on some stars could also be analyzed.

During the 18-month mission of the 60-cm cooled telescope named Infrared Space Observatory (ISO), some interesting programs on late-type giants and supergiants should be developed: (a) temperature distribution on circumstellar material and planetary dust clouds; (b) determination of the shape of various diffuse features (e.g., solid-state spectral features at 11 and 18  $\mu\text{m}$ ) and correlation of these shapes with the physical parameters of the star and its envelopes; (c) mapping of the neighborhood ( $1^\circ$  by  $1^\circ$ ) of some hypergiants and planetary nebulae at 40, 80, 120, and 160  $\mu\text{m}$  and determination of the shell diameters, energy balance, densities, etc.; and (d) determination of mass-loss rates. The problem described by Wesselius (1984) on the origin of the outer envelopes of  $\alpha$  Ori should be solved by the ISO high-resolution spectroscopy (Barlow and Storey, 1984).

Finally, a large organized international collaboration on photometry, spectroscopy, and polarimetry both from the ground in the visible, near-IR, and radio ranges and from space in the X-ray, far-UV, near- and far-IR, and submillimetric frequencies seems unavoidable

if we want to effectively progress in the knowledge of these variable cool stars.

Constructive comments by Hollis R. Johnson have improved the first part of this chapter.

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