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STANDARD RESPONSE FUNCTIONS FOR PROTANOPIC AND DEUTERANOPIC VISION

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ABSTRACT

The color matches set up by the normal observer can be predicted satisfactorily by three functions of wavelength defining the ICI standard observer. It has been found possible by a transformation of coordinate system to express these three functions in a form such that two of the three pairs also represent the color matches of the two recognized types of red-green-blind observer, the protanope and the deuteranope, within the rather small uncertainties to which they are known. The remaining pair of functions represents, within the comparatively large uncertainties to which they are known, the color matches of the tritanope, a more rare type of observer who confuses reddish blue with greenish yellow. These three functions, therefore, serve to relate the color matches made by dichromats to those made by normal trichromats, and so make conveniently accessible the color confusions of average dichromatic observers. The use of these three functions in the solution of problems arising in the design of tests for colorblindness is illustrated by solution of three such problems, and their connection to theories of color vision is discussed.

CONTENTS

I. INTRODUCTION

When the two halves of a photometric field are illuminated to the same degree with light of the same spectral composition, they cannot be distinguished; that is, they produce a color match. But it is possible also to produce a color match between lights of different spectral compositions; such lights are called metamers [22, 51][1] and

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[1] Figures in brackets indicate the literature references at the end of this paper.

are said to form metameric pairs. The functions that give the conditions to be met by the spectral composition of any two lights in order that they shall form a metameric pair for a given observer serve to define the characteristics of that observer; and in 1931 the International Commission on Illumination adopted such functions, which define the standard observer [23, 27].

Protanopic and deuteranopic vision are the two most common forms of dichromatic vision, each form occurring in about one percent of otherwise normal human males [31]. They are known as reduction forms because neither a protanope nor a deuteranope ordinarily distinguishes the members of any pair of normal metamers; that is, a color match set up by a normal observer will not be objected to by a dichromat unless the pigmentation of his eye media, including the macula, be grossly different from that of the normal observer [38b]. The standard color specifications are therefore generally valid for dichromats in the sense that if two lights have the same standard color specifications, no average dichromat will be able to distinguish them.

These specifications are, however, unnecessarily complicated for such observers. In addition to normal metamers, dichromats find many other pairs that they cannot distinguish. These additional metamers are often called confusion colors; they are colors that are distinct to the normal although identical to the dichromat. The normal color specification consists of three numbers because the normal visual system is capable of three independent modes of variation and on that account is often called trichromatic. The dichromatic specification need consist of but two numbers. These two numbers may be derived from the standard color specification in a simple way. The connection between trichromatic and dichromatic specifications was indicated in complete detail by Maxwell in 1855 [44] by appeal to the Young theory of vision. The original simple form of this theory has been disproved, the extensions of it vary with the phenomena to be explained, and there is doubt whether the theory, however extended, can yield more than a partial explanation of the complicated facts of vision. However, the connection between dichromatic and trichromatic color specifications first worked out by Maxwell on theoretical ground does not depend upon the portions of Young's hypothesis now given up; on the contrary this connection has been repeatedly proven to be correct. It is the purpose of this paper to derive, by this principle, response functions for protanopic and deuteranopic vision, to show how they may be used to find whether or not two colors will be confused by an average protanope or average deuteranope, and finally to point out a few of the theoretical implications of the functions chosen.

II. THEORY

Maxwell wrote in his letter of January 4, 1855 to George Wilson [44]: "If we find two combinations of colours which appear identical to a Colour-Blind person, and mark their positions on the triangle of colours (now called the Maxwell triangle), then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two.

"We may in the same way find other lines passing through the series of colours which appear alike to the Colour-Blind. All these lines either pass through one point or are parallel, according to the standard colours which we have assumed, ... Knowing this law of Colour-Blind vision, we may predict any number of equations which will be true for eyes having this defect.

"The mathematical expression of the difference between Colour-Blind and ordinary vision is that colour to the former is a function of two independent variables, but to an ordinary eye, of three; and that the relation of the two kinds of vision is not arbitrary, but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement, which forms one-third of the apparatus by which we receive sensations of colour."

The straight lines on the Maxwell triangle serve to indicate the chromaticity confusions of the colorblind observer. The point of intersection of these lines indicates the normal chromaticity of the primary process not possessed by the colorblind. Suppose for the protanope that the chromaticity-confusion lines are copunctal at (x_p, y_p) on the (x, y)-plane of the standard ICI colorimetric coordinate system [23, 27], and suppose for the deuteranope that the corresponding point is (x_d, y_d). Let us inquire how to derive from the functions (of wavelength) (X, Y, Z) defining the standard normal observer, three new functions (W_d, W_p, K) such that all three taken together represent the normal observer; such that (W_d, K) taken together represent average deuteranopic vision, and such that (W_p, K) taken together represent average protanopic vision. This terminology (W_d, W_p, K) is taken from v. Kries [39, p. 164] who followed König [37] closely. The symbol W is intended to suggest "warm"; and K, "cold" (*kalt*), corresponding to whatever warm color (orange, yellow, greenish yellow) and whatever cold color (blue or violet) is sensed by red-green-blind observers.

The derivation consists of making new choices of primary processes such that two of them correspond to (x_p, y_p) and (x_d, y_d), respectively. It was pointed out by König in 1886 [37] for this very purpose, and by many others since [13, 24, 25, 43, 54] for other purposes, that such choices result from defining the new functions (W_d, W_p, K) as weighted sums of the old, thus

$$\left.\begin{aligned} W_d&=k_1X+k_2Y+k_3Z\\ W_p&=k_4X+k_5Y+k_6Z\\ K&=k_7X+k_8Y+k_9Z \end{aligned}\right\}, \tag{1}$$

where k_1 to k_9 are constants that may be given any arbitrary values whose determinant differs from zero:

$$\begin{vmatrix} k_1, & k_2, & k_3\\ k_4, & k_5, & k_6\\ k_7, & k_8, & k_9 \end{vmatrix} \neq 0. \tag{1a}$$

Since the point (x_p, y_p) corresponds to the primary, W_d, not possessed by the protanope, $W_p=K=0$, for $X/(X+Y+Z)=x_p$, and $Y/(X+Y+Z)=y_p$, and we may write from eq 1:

$$\left.\begin{aligned} k_4x_p+k_5y_p+k_6z_p&=0\\ k_7x_p+k_8y_p+k_9z_p&=0 \end{aligned}\right\}, \tag{2a}$$

where z is defined as $Z/(X+Y+Z)$ and is equal to $1-x-y$.

Similarly, since the point (x_d, y_d) corresponds to the primary not possessed by deuteranopes, we may write:

$$\left.\begin{aligned}k_1x_d+k_2y_d+k_3z_d=0\\ k_7x_d+k_8y_d+k_9z_d=0\end{aligned}\right\}. \tag{2b}$$

The four conditions expressed by eq 2a and 2b are the only conditions that have to be met to insure that W_p and W_d represent the primary color processes not possessed by the deuteranope and protanope, respectively. Since nine conditions are required to determine the nine constants of eq 1, it may be seen that many coordinate systems can serve both for normal trichromatic visual systems and for dichromatic systems by neglect of the one or the other of two of the three normal components.

Two of the remaining five degrees of freedom are required to insure, as is convenient, that the equal-energy stimulus be kept as the basic stimulus [14] of the system, that is, that it be represented at the center of the Maxwell triangle of the new coordinate system as well as in the standard ICI system. This requirement is satisfied if the distribution curves of the new primary color processes throughout the equal-energy spectrum be adjusted, like those of the ICI system, to equal areas; that is, if $k_1+k_2+k_3=k_4+k_5+k_6=k_7+k_8+k_9$. Another degree of freedom must be expended to set the arbitrary units in which the distribution curves are expressed. The three conditions together may be expressed conveniently as:

$$\left.\begin{aligned}k_1+k_2+k_3=1,\\ k_4+k_5+k_6=1,\\ k_7+k_8+k_9=1.\end{aligned}\right\}. \tag{3}$$

König [37] used the other two conditions to fix the third primary at an imaginary color of dominant wavelength near to that usually corresponding to unitary blue, that is, a stimulus which is perceived under ordinary observing conditions as a blue which is neither reddish nor greenish. In this way König derived the "fundamental sensation" curves incorporated by Ladd-Franklin [41] into her theory of color vision. Following further studies of dichromatic vision, however, König gave up the attempt to make any of the primaries correspond to a unitary hue [38]. In this similar reduction of present-day data on the vision of dichromats, it has likewise seemed advisable to pay no attention to the color perception ordinarily corresponding to the primaries adopted, but rather to strive for the simplest possible adequate representation of the data. Thus, it is possible to satisfy eq 2 and 3 in such a way as to yield wavelength functions for the primaries, each consisting of a curve possessing the single-peak shape of the luminosity function. To avoid functions for W_d and W_p having two maxima, it is sufficient simply to require

$$\left.\begin{aligned}k_3=-0.22\ k_1\\ k_6=-0.22\ k_4\end{aligned}\right\} \tag{4}$$

These requirements prevent W_d and W_p from being large near 440 mμ by utilizing only enough of the Z function to cancel approximately the secondary maximum of the X function in that region.

III. CHROMATICITY CONFUSIONS OF RED-GREEN-BLIND OBSERVERS

As first pointed out by Maxwell, the chromaticity confusions of either type of dichromat may be represented on the chromaticity diagram, or Maxwell triangle, for normal trichromatic vision by a family of straight lines passing through a single point, (x_p, y_p) for the protanope, (x_d, y_d) for the deuteranope. The first determination of these points was that by König and Dieterici [37] based upon two protanopes and two deuteranopes. From these points were derived the "fundamental-sensation" curves (Grund-Empfindungs-Curven) R', G', and B'; and from the approximate relation [30] between these curves and those of the present standard coordinate system for colorimetry, an estimate of the coordinates of the points may be made. From the previously evaluated transformation equations [30, eq 7b] from standard color specifications (X, Y, Z) to (R', G', B') may be found the reverse transformations [30, eq 2]:

$$\left.\begin{aligned} X&=0.244R'-0.058G'+0.014B' \\ Y&=0.056R'+0.150G'-0.005B' \\ Z&=0.000R'+0.000G'+0.200B' \end{aligned}\right\} \qquad (5)$$

By setting $G'=B'=0$ in eq 5, we find $X=0.244R'$, $Y=0.056R'$, $Z=0$, whence:

$$x_p=0.244R'/(0.244R'+0.056R')=0.81$$
$$y_p=0.056R'/(0.244R'+0.056R')=0.19$$

Similarly, by setting $R'=B'=0$, we find $x_d=-0.63$, $y_d=1.63$.

Of course, no very great dependence can be placed on the results of this pioneer work. The connection with the present standard coordinate system is uncertain not only because it is based upon but two partially dark-adapted normal observers (König and Dieterici, who incidentally differed importantly from each other) but also because the basic stimulus of the system ("sunlight reaching the earth's surface through atmosphere of highest transmission") is essentially undefined. If we assume $x=0.33$ and $y=0.34$ for this basic stimulus [46] the protanopic and deuteranopic "neutral" points in the spectrum are found graphically on the (x, y)-plot of the standard ICI system to be 496 and 511 mμ, respectively. This agrees only approximately with the wavelengths (495 and 504 mμ, respectively) read from the König-Dieterici triangle [37, fig. 7] and indicates that the possibilities for considerable error have been realized.

In 1935, however, Pitt [52] published data giving the average chromaticity confusions of eight deuteranopes and seven protanopes in terms of the WDW coordinate system proposed by Wright [59] for visual research because it takes account in a simple way of variations in ocular pigmentation (macula lutea, crystalline lens, humours). From the primaries of this system (spectrum lights at 460, 530, and 650 mμ, respectively) and from the fact that ICI standard illuminant *B* [23, 27] has in this system for the standard observer the chromaticity coordinates $r=0.249$, $g=0.399$, we may, for the ICI standard observer, derive equations to connect these chromaticity coordinates (r, g) with those (x, y) of the ICI standard coordinate system:

$$\left.\begin{aligned} x&=\frac{0.874r-0.023g+0.144}{0.402r-0.222g+1.000} \\ y&=\frac{0.354r+0.597g+0.030}{0.402r-0.222g+1.000} \end{aligned}\right\} \qquad (6)$$

By means of these equations the chromaticity-confusion lines, shown on Pitt's figures 15 and 16, have been transferred to the ICI system and are shown as dotted lines in figures 1 (a) and 1 (b). It will be noted that the protanope chromaticity confusion lines plotted in figures 1 (a) all run very closely through a single point ($x_p=0.747$, $y_p=0.253$) in conformity to the principle enunciated by Maxwell from the Young theory. The deuteranope chromaticity-confusion lines (figure 1(b)) are not so perfectly copunctal as this, but most of them come fairly close to the point $x_d=1.08$, $y_d=-0.08$.

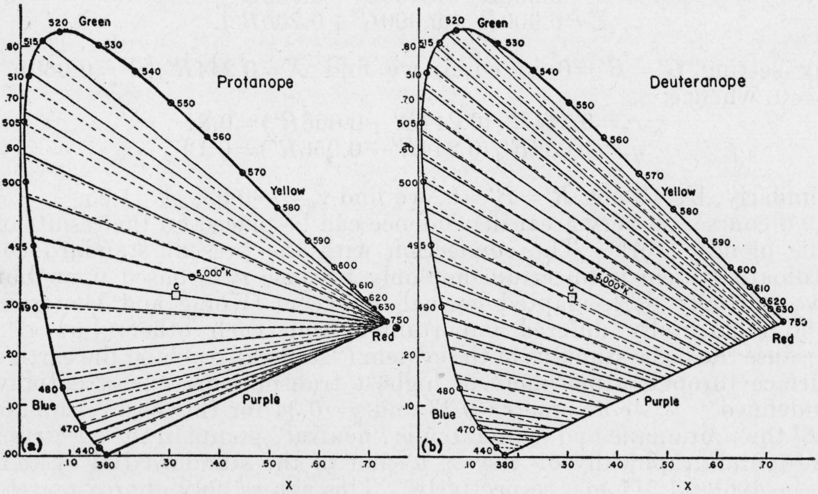

FIGURE 1.—*Chromaticity confusions of the protanope and deuteranope (after Pitt).*

Note how closely the chromaticity-confusion lines (dotted) intersect at a single point for each of the two recognized types of red-green blindness. The solid lines included for comparison are copunctal (protanope at $x=0.747$, $y=0.253$; deuteranope at $x=1.000$, $y=0.000$).

In 1936, Hecht and Shlaer [15] published complete data for two dichromats, one protanope and one deuteranope. These data include luminosity functions, wavelength discrimination, and color-mixture data. The color-mixture data take the form of the energy ratio of two spectral primaries (458.7 and 570.0 mμ) required to produce a chromaticity match for each part of the spectrum. This energy ratio can be accurately defined by the dichromatic observer for the middle portion of the spectrum, but near both 460 and 570 mμ a given energy ratio is found to apply to a considerable spectral range, which continues to widen as the spectral extremes are approached. Table 1 gives average values of these energy ratios found from the Hecht-Shlaer data taking due account of the rapid decline in the ability of these observers to discriminate wavelength change near 460 and 570mμ. These data may be made independent of the degree of ocular-media pigmentation by transforming them to the red and blue primaries of the *WDW* [59] system. In this system spectral primaries (460 and 650mμ) are used, but, instead of energy units, arbitrary units such as to make the amounts of the primaries equal for 494 mμ are adopted. By the usual methods [26], equations of transformation have been found connecting trichromatic coefficients (r, b) in the *WDW* system to the energy ratios, f_d and f_p, for the protanope *HJ* and the deuteranope *AWG*:

$$\left.\begin{aligned} r &= (0.492-0.003f_p)/(0.492+0.997f_p) \\ b &= f_p/(0.492+0.997f_p) \end{aligned}\right\} \qquad (7a)$$

$$\left.\begin{aligned} r &= (31.85-0.021f_d)/(31.85+0.979f_d) \\ b &= f_d/(31.85+0.979f_d) \end{aligned}\right\}. \qquad (7b)$$

TABLE 1.—*Comparison of the Hecht-Shlaer color-mixture data for 1 protanope and 1 deuteranope with Pitt's data for 8 protanopes and seven deuteranopes*

Wave-length	Hecht-Shlaer protanope *HJ*		Pitt average protanope, $b=1-r$	Hecht-Shlaer deuter-anope *AWG*		Pitt average deuter-anope, $b=1-r$
	f_p	$b=1-r$		f_d	$b=1-r$	
mμ						
450	--------	--------	1.03	--------	--------	1.03
460	150.	1.00	1.00	150.	1.00	1.00
470	6.4	0.93	0.93	28.	0.92	0.93
480	2.1	.81	.80	11.2	.79	.80
490	0.77	.61	.57	5.0	.62	.57
500	.294	.37	.34	1.5	.32	.34
510	.100	.17	.17	0.526	.14	.14
520	.033	.06	.09	.182	.05	.06
530	.0113	.02	.06	.062	.02	.03
540	.0040	.01	.04	.025	.01	.02
550	.0005	.001	.02	.010	.003	.01
560	--------	--------	.01	.0025	.001	.00
570	--------	--------	.00	--------	--------	.00
580	--------	--------	.00	--------	--------	.00
590	--------	--------	.00	--------	--------	.00

Table 1 shows the color-mixture data for the protanope *HJ* and the deuteranope *AWG* transformed by eq 7a and 7b to the red and blue primaries of the *WDW* system, and for comparison it also shows the average values found by Pitt [52] for eight protanopes and seven deuteranopes, respectively. It is seen that the Hecht-Shlaer color-mixture data resemble those of Pitt considerably, and comparison of the discrepancies with the known wavelength discrimination of dichromats [15, 52] and with the individual differences among the dichromats studied by Pitt [52, p. 15] shows that the corroboration is wholly satisfactory. It is to be concluded, therefore, that the Hecht-Shlaer data, when referred to the same degree of ocular pigmentation as is included in the ICI standard observer, indicate the same values of (x_p, y_p) and (x_d, y_d) as the Pitt data.[2]

From figure 1 it may be seen that a source of color temperature 5,000° K ($x=0.344$, $y=0.351$) would possess for an average protanope the same chromaticity as the spectrum at 496 mμ; and for an average deuteranope, the neutral point would be at 500 mμ. As pointed out by v. Kries [38b], a dichromat having ocular-pigmentation heavier than that of the standard observer would have his neutral point shifted toward the long-wave end of the spectrum; for a lightly pigmented dichromat a displacement toward the short-wave end is expected. Wright [59] found that, among the 35 normal observers studied, variations in pigmentation made the point representing 5,000° K vary from ($x=0.29$, $y=0.27$) for no macular pigmentation to ($x=0.39$, $y=0.43$) for the most heavily pigmented observer studied. Table 2 shows the wavelengths of the neutral points to be expected from protanopes and deuteranopes having these extremes of pigmentation, and it also shows for comparison the average and extreme wavelengths of neutral points found by König [35], Pitt [52], and Hecht and Shlaer [15]. The König wavelengths have been increased by 3 mμ for protanopes and 4 mμ for deuteranopes to take account of the chromaticity difference between the average daylight used by him and color temperature 5,000° K. Pitt used illuminant *B* for comparison in determining neutral points; the corrections for the chromaticity difference between this source and one at 5,000° K are negligible and have not been applied. It will be noted from table 2 that the König and Pitt data not only agree in average values with the neutral points read from figure 1, but also fall generally within the limits corresponding to those expected from the pigmentation range of Wright's 35 normal observers. The single deuteranope studied by König who falls slightly outside the pigmentation limits of Wright's 35 normal observers probably corresponds to nothing more than the error of random sampling.

One of the Hecht-Shlaer protanopes and seven of their deuteranopes, however, have neutral points considerably higher than the maximum values corresponding to the most heavily pigmented of Wright's 35 normal observers. That 7 out of 12 deuteranopes are found to have heavier ocular pigmentation than any of 35 normal observers is too much to explain by the error of random sampling. This fact suggests strongly that the populations from which the observers were drawn are significantly different in ocular pigmentation. By this view we must be prepared to accept rather wide individual

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[2] Although the color-mixture data for these two dichromats were not put by Hecht and Shlaer into a form suited to prediction of dichromat color matches between heterogeneous stimuli and spectrum stimuli, such a reduction has been carried out recently by Fry [10]. From this reduction it would be possible to determine independently the chromaticity coordinates (x_p, y_p) and (x_d, y_d) of the primary processes not possessed by the two forms of dichromat, instead of relying for this purpose upon Pitt's transformation from the dichromatic to trichromatic diagram of the *WDW* system by determination of the dichromatic luminosities of the primaries. This reduction indicates, however, that the complete radiator at 5,000° K should have had the chromaticity of the spectrum at 498 mμ for protanope *HJ* and 508.5 mμ for deuteranope *AWG*, but by direct observation these wavelengths were found to be 491.5 and 495 mμ, respectively. These discrepancies amount to 5 or 10 times the just noticeable difference for these observers. Although the major part of the discrepancy for protanope *HJ* can be eliminated by more exact methods of reducing the data and by filling in the wavelength region 380 to 470 mμ from Pitt's data, the Hecht-Shlaer data being uncertain in that region, still it would seem that no great reliance is to be placed upon the predictions of dichromat color matches from these data. These discrepancies suggest that the experimental conditions under which the luminosity functions of these observers were obtained may have brought into play a smaller retinal region than that for the color-mixture data, although ostensibly the conditions were identical. Perhaps the mere fact that the luminosity and color-mixture determinations were made separately is a sufficient explanation of the discrepancy. It is doubtful therefore whether separate determination of the copunctal points (x_p, y_p) and (x_d, y_d) from these data would add appreciably to the information obtained by way of the *WDW* coordinate system.

variations in pigmentation. For example, the deuteranope, verified by Hecht and Shlaer as having a neutral point at 525 mμ must have an ocular pigmentation eight or nine times as heavy as the average found by Ludvigh and McCarthy [42] for ocular pigmentation of 62-year-old eyes, excluding the macular pigment. Table 2 also suggests the possibility that deuteranopes tend to have heavier pigmentation than protanopes or normals. It is concluded that individual variations in neutral point are ascribable to variations in ocular pigmentation (macula, lens, humours).

TABLE 2.—*Individual variation in wavelength of neutral point compared to the expected influence of ocular pigmentation based upon 35 normal observers studied by Wright* [59]

Source of data	Number and type of observers	Wavelength in mμ of the spectral region having the dichromatic chromaticity of a source of color temperature 5,000° K					
		Protanope			Deuteranope		
		Minimum	Maximum	Average	Minimum	Maximum	Average
Figure 1	35 normals	489	504	496	494	508	500
König (1884)	7 protanopes 7 deuteranopes	495	499	497	500	509	503
Pitt (1935)	5 protanopes 6 deuteranopes	494	497	496	495	507	500
Hecht-Shlaer (1936)	10 protanopes 12 deuteranopes	492	508	498	491	525	510

IV. DERIVATION OF RESPONSE FUNCTIONS

It is worth noticing that the point ($x_p=0.747$, $y_p=0.253$) found from Pitt's data as the common crossing point of the protanopic chromaticity-confusion lines is very close to the long-wave extreme of the spectrum ($x=0.735$, $y=0.265$) used as a primary in the OSA coordinate system [57]. Reference to figure 1(a) shows immediately that in spite of this close approach, the long-wave extreme of the spectrum is not eligible to represent (x_p, y_p). For example, the line connecting the extremes of the spectrum cuts all of the protanopic chromaticity-confusion lines (dotted); that is, the protanopic chromaticity of the long-wave extreme differs maximally from that of the short-wave extreme. But if the long-wave extreme of the spectrum locus were itself taken as the point (x_p, y_p), one interpretation would be that the two extremes of the spectrum have the same protanopic chromaticity. On this account the solid lines of figure 1(a) drawn in for comparison have been made copunctal at ($x_p=0.747$, $y_p=0.253$). Although the choice of the long-wave extreme of the spectrum to represent (x_p, y_p) flatly contradicts the facts, it is of interest to inquire what response functions are generated by this choice, because it bears on a question raised in 1798 by Dalton [3] and argued many times since, namely: Are protanopes red-blind?

Similarly, it is worth noting that the point ($x_d=1.08$, $y_d=-0.08$) estimated from Pitt's data as the most probable common crossing point of the deuteranopic chromaticity-confusion lines is fairly close to the X primary ($x=1.00$, $y=0.00$) of the standard ICI coordinate system. The solid lines of figure 1 (b) have been drawn through this point, and it is seen that with one exception the solid lines are either closely parallel to the nearest dotted line or cross a single dotted line. Since the separation of the dotted lines indicates the smallest chromaticity difference perceptible to Pitt's observers, it may be concluded that to set $x_d=1$ and $y_d=0$ is not inconsistent with these data; and, indeed, from a set of lines drawn through the point ($x_d=1.08$, $y_d=-0.08$) it may be concluded that the former choice is scarcely less apt than the latter, which was chosen as the best estimate possible. It will be of interest, therefore, to study both choices.

It is convenient to start this study by investigating the implications of setting $x_p+y_p=x_d+y_d=1$, or, stated another way, since $z=1-x-y$, the implications of setting $z_p=z_d=0$. All four points under consideration meet this requirement.

If $z_p=z_d=0$, from eq 2a and 2b, we find $k_7=k_8=0$, provided $x_p/y_p\neq x_d/y_d$, that is, provided (x_p, y_p) and (x_d, y_d) are different points; and from eq 3, we find that $k_9=1$. From eq 1 we may therefore write $K=Z$. This conclusion is important. It says that since both the protanopic chromaticity-confusion lines and the deuteranopic-confusion lines are copunctal at points lying on the tangent to the spectrum locus at the long-wave extreme, the Z function of the standard observer is adequate to represent the K function used in describing the chromaticity confusions of both deuteranope and protanope. König [37], from somewhat incomplete data [6], drew this conclusion, and v. Kries [39, p. 164] deduced it from the identity of the K function of protanope and deuteranope on the assumption that both are reduction forms of normal vision.

A further conclusion is possible from setting $z_p=z_d=0$. From eq 2b, 3, and 4, we may write three simultaneous equations with only three unknowns, thus

$$\begin{gathered} k_1x_d+k_2y_d=0 \\ k_1+k_2+k_3=1 \\ k_3=-0.22k_1, \end{gathered}$$

from which the coefficients, k_1 to k_3, defining the W_d function by eq 1, are found to be

$$k_1=-y_d/(x_d-0.78y_d),\ k_2=x_d/(x_d-0.78y_d),\ k_3=0.22y_d/(x_d-0.78). \qquad (8a)$$

Thus it is seen that the W_d function (warm curve of deuteranopes) is determined wholly by the coordinates of the point (x_d,y_d) provided $z_d=z_p=0$, as they do for the four choices of interest.

Similarly, from eq 2a, 3, and 4, we find that

$$k_4=-y_p/(x_p-0.78y_p), k_5=x_p/(x_p-0.78y_p), k_6=0.22y_p/(x_p-0.78y_p). \qquad (8b)$$

Thus it is seen that the W_p function (warm curve of protanopes) is determined wholly by the coordinates of the point (x_p, y_p), again provided $z_d=z_p=0$.

The next step is to find the response functions resulting from setting $x_d=1$, $y_d=z_d=0$, as is justified from available data on chromaticity confusions of deuteranopes. From eq 8a, we find immediately that $k_1=k_3=0$, and $k_2=1$. Hence from eq 1, $W_d=Y$. This conclusion is important. It says that since the deuteranopic chromaticity-confusion lines may be taken as copunctal at $x_d=1$, $y_d=z_d=0$, then the Y function of the standard observer is adequate to represent the W_d function (warm curve of the deuteranope) for describing the chromaticity confusions of the deuteranope. The great convenience of being able to use two (Z and Y) of the three functions representing the standard observer directly in deuteranopic chromaticity specifications should be emphasized. It means that the chromaticity of any color stimulus already specified in terms of the standard observer may be specified for an average deuteranope merely by leaving the X-specification out of account. That is, any two stimuli for which the Y-specification and the Z-specification bear the same ratio will be chromaticity matches for an average deuteranope.

The W_p function is found from the values of $x_p=0.747$, $y_p=0.253$, through eq 8b and 1, to be

$$W_p=-0.460X+1.359Y+0.101Z. \tag{9}$$

The W_p function generated from the theoretically interesting but factually inadmissible values of $x_p=0.735$, $y_p=0.265$ is found in a similar way to be

$$W'_p=-0.503X+1.392Y+0.111Z. \tag{9a}$$

The W_d function agreeing as closely to the Pitt average data as is possible ($x_d=1.08$, $y_d=-0.08$, $z_d=0.00$) is found from eq 8a and 1 to be

$$W'_d=0.079X+1.061Y-0.017Z. \tag{10}$$

V. LUMINOSITY FUNCTIONS OF RED-GREEN-BLIND OBSERVERS COMPARED TO THOSE OF NORMAL AND ANOMALOUS TRICHROMATS

We may now inquire into the relationship between the functions, W_d, W_p, W'_d, and W'_p, and the luminosity functions of dichromatic and trichromatic observers. Figure 2 shows, by dotted lines, the luminosity functions of eleven protanomalous observers studied by McKeon and Wright [45] and one studied by Nelson [49]. It also shows by solid lines the luminosity functions of six deuteranomalous observers studied by Nelson [49]. Although at the short-wave end of the spectrum these groups of functions overlap and are not significantly different, the two groups divide quite sharply at the long-wave end. In this respect they are like the groups of functions representing the two corresponding types of dichromatic observer, the deuteranope and the protanope. The circles represent the average of six protanopic luminosity functions evaluated by Pitt [52], and the crosses represent his average of six deuteranopic luminosity functions. The arrows indicate the maximum and minimum values of relative luminosity found by Gibson and Tyndall [11] among their 37 completely studied normal observers. Although because of differences in experimental conditions no highly precise intercomparison is possible between the data shown for anomalous and dichromatic observers and these normal limits, nevertheless figure 2 indicates that the deuteranopic and deuteranomalous observers alike possess luminosity functions generally well within normal limits; but both protanomalous observers and protanopic observers possess luminosity functions that are anbormally low at the long-wave end.

Figure 2 tends to indicate also that protanomalous observers possess abnormally narrow luminosity functions even compared to protanopes, and that deuteranopes possess luminosity functions abnormally high between 570 and 610 mμ with deuteranomalous observers intermediate in this respect; but because of the small number of abnormal observers tested, these indications may fail to be significant. The experimental conditions (field size, field luminance, surrounding-field luminance) and technics (adaptation, control of fixation, length of observing time) govern the participation of the rod mechanism in determining these functions. In any one study the luminosity curves may be thrown more or less to the short-wave side because of partial participation of the rods, either intended or inadvertent. The intertwining of the curves at the short-wave end is probably to be ascribed chiefly to individual variation in the amount of yellow or brown pigmentation of the eye media, such as degree of macular pigmentation.

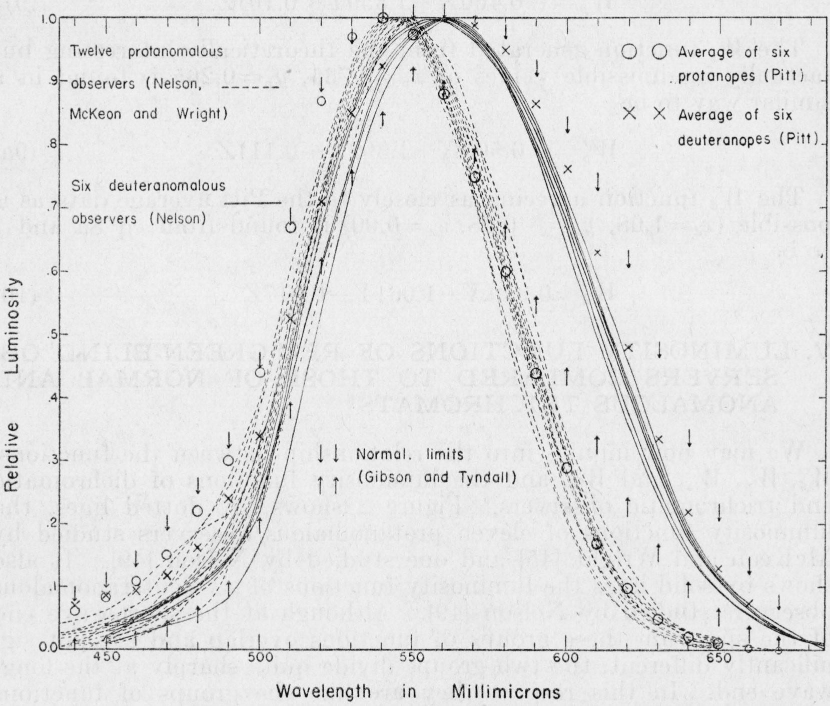

FIGURE. 2.—*Relative luminosity functions of anomalous trichromats (6 deuteranomalous, 12 protanomalous) compared to averages for dichromats and to limits for normal trichromats.*

Deuteranopic and deuteranomalous luminosity functions fall generally within normal limits, but protanopic and protanomalous luminosity functions fall outside of normal limits.

Since from figure 2 both deuteranopic and deuteranomalous luminosity functions fall generally within the limits of 37 normal observers, it is just as valid to predict the brightness judgments of these abnormal observers from the standard luminosity function as it is those of an observer chosen at random from the large population group to be classed as of normal color vision. Figure 3 compares the standard luminosity function, $Y=W_d$, (dotted line) with the upper and lower limits (arrows) for the deuteranopic and deuteranomalous luminosity functions referred to in figure 2. The standard luminosity function is seen to fall satisfactorily within these limits except for the wavelength region between 570 and 610mμ. Studies by Sloan [56], corroborated by the recent work of Walters and Wright [58], show that the shape of the luminosity function within this wavelength region is particularly subject to variation with experimental conditions. It is therefore perhaps reasonable to ascribe the whole of this small discrepancy to variation in experimental conditions.

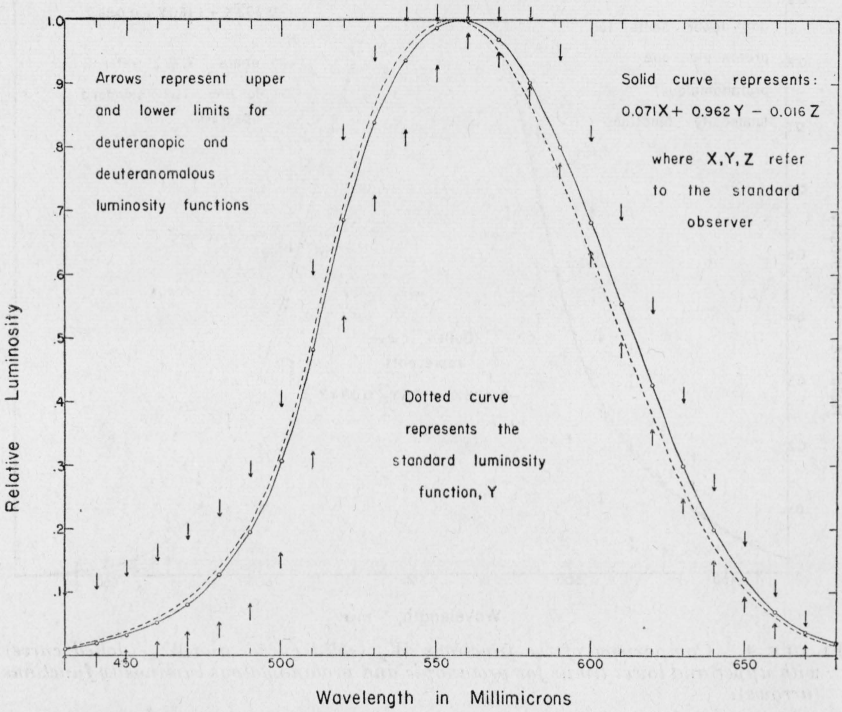

FIGURE. 3.—*Comparison of the functions* W'_d *(standard luminosity function, Y) and* W'_d *(solid curve) with upper and lower limits for deuteranopic and deuteranomalous luminosity functions (arrows).*

The standard luminosity function is seen to fall satisfactorily within these limits except for the wavelength region between 570 mμ and 610 mμ. The function W'_d yields slightly better agreement.

Figure 3 also shows the function W'_d (eq 10) adjusted to unit maximum as a solid line. This function represents available data on deuteranopic luminosity slightly better than the standard luminosity function $Y=W_d$, just as, combined with the K function, it represents deuteranopic chromaticity data slightly better. There is therefore some basis for arguing, as in the most common form of Young-Helmholtz theory, that the normal luminosity function should be thought of as consisting of three components, a red, a green, and a violet.[3] Whether this possible justification of the usual Young-

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[3] An account of dichromatic vision following this view has recently been worked out by Pitt [53].

Helmholtz view would be borne out by a study of an adequate sample of normal and deuteranopic observers remains to be decided by further extensive experiment. The two functions, W_d and W'_d, differ so little, however, that it would rarely be of practical importance to use one rather than the other. Because of its greater simplicity it seems better to take $W_d=Y$ as the function representing deuteranopic and deuteranomalous luminosity.

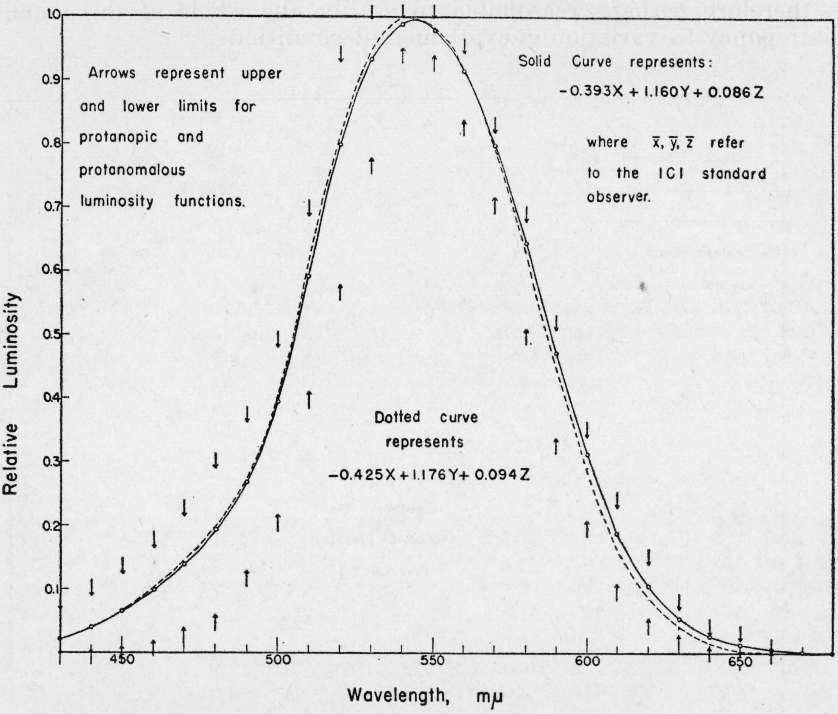

FIGURE 4.—*Comparison of the functions W_p (solid curve) and W'_p (dotted curve) with upper and lower limits for protanopic and protanomalous luminosity functions (arrows).*

Both functions are seen to fall satisfactorily within these limits, except that W'_p is perhaps too low for wavelengths greater than 650 mμ.

Figure 4 compares the functions 0.854 W_p and 0.845 W'_p with the upper and lower limits (arrows) for the protanopic and protanomalous luminosity functions referred to in Figure 2. The constants 0.854 and 0.845 serve to adjust the functions approximately to unit maximum, and from eq 9 and 9a, respectively yield

$$0.854\,W_p=-0.393X+1.160Y+0.086Z \text{ (solid curve)},$$
$$0.845\,W'_p=-0.425X+1.176Y+0.094Z \text{ (dotted curve)}.$$

It is seen that both functions fall satisfactorily between the limits at least for wavelengths shorter than 650 mμ. The function W_p, therefore, not only represents adequately, when combined with the K function, the chromaticity confusions of the average protanope, but also, taken by itself, his brightness judgments. The function W'_p, although it grossly fails, when combined with the K function, to represent the chromaticity confusions of the protanope, nevertheless succeeds in representing adequately his brightness judgments except for wavelengths greater than 650 mμ. This rather minor difference in luminosity function is all that differentiates red-blind vision (luminosity zero for wavelengths greater than 700 mμ) from the experimentally determined facts of protanopia. The old name, red blindness, is therefore nearly but not quite accurate.

By way of contrast it may be pointed out that the corresponding old name, green blindness, for the type of defect known since v. Kries [46] as deuteranopia is almost wholly wrong. Since the standard luminosity function represents the brightness judgments of deuteranopic observers just as closely as those of a considerable fraction of normal observers, and is far from being zero anywhere within the spectrum region that ordinarily appears green (490 to 550 mμ), deuteranopes are far from being insensitive to the green part of the spectrum. It is true that protanopia and deuteranopia can logically be described as red blindness and green blindness, respectively, in terms of the Young-Helmholtz theory, but in a purely descriptive nontheoretical sense, no observer has ever been found to be quite red-blind, and none even remotely deserving the name green-blind.

The following functions therefore may be taken to be representative of normal, deuteranopic and protanopic vision:

$$\left.\begin{aligned} K&=Z\\ W_p&=-0.460X+1.359Y+0.101Z\\ W_d&=Y \end{aligned}\right\} \qquad (11)$$

K and W_d for deuteranopic vision, K and W_p for protanopic vision, and all three for normal vision. It is also true, although no adjustments other than eq 4 have been made for it nor any detailed quantitative comparisons presented, that W_p and W_d taken together closely represent tritanopic vision [31, 33, 38, 47]. Table 3 gives these three functions for 10-mμ intervals of the visible spectrum. The function W_p is taken as the luminosity function of a typical protanope, the function W_d is taken as the luminosity function of a typical deuteranope as well as the normal observer (and perhaps also that of a typical tritanope [7, 8, 34]), and the function K is unassociated with luminosity.

The representation of normal vision by K, W_d, and W_p is just as adequate as that afforded by the functions Z, Y, and X of the standard ICI coordinate system, and indeed functions closely like these were proposed by the author in 1931 for international adoption through the American representative, Irwin G. Priest, but were rejected because the corresponding chromaticity diagram gave considerably less uniform chromaticity scales than the coordinate system (X, Y, Z) finally adopted. There is therefore nothing to be gained by supplanting the present X-function by W_p to represent normal vision, and this substitution is not recommended.

TABLE 3.—*Response functions for protanopic* (W_p, K) *and deuteranopic* (W_d, K) *vision*

Wave-length	W_d	W_p	K
$m\mu$			
380	0.00004	0.00006	0.0065
390	.00012	.00024	.0201
400	.0004	.00082	.0679
410	.0012	.00257	.2074
420	.0040	.00883	.6456
430	.0116	.0251	1.3856
440	.0230	.0476	1.7471
450	.0380	.0759	1.7721
460	.0600	.1163	1.6692
470	.0910	.1638	1.2876
480	.1390	.2270	0.8130
490	.2080	.3150	.4652
500	.3230	.4642	.2720
510	.5030	.6953	.1582
520	.7100	.9437	.0782
530	.8620	1.0997	.0422
540	.9540	1.1650	.0203
550	.9950	1.1537	.0087
560	.9950	1.0791	.0039
570	.9520	0.9434	.0021
580	.8700	.7610	.0017
590	.7570	.5568	.0011
$m\mu$			
600	0.6310	0.3690	0.0008
610	.5030	.2224	.0003
620	.3810	.1248	.0002
630	.2650	.0646	.0000
640	.1750	.0318	.0000
650	.1070	.0150	.0000
660	.0610	.0070	.0000
670	.0320	.0033	.0000
680	.0170	.00157	.0000
690	.0082	.00070	.0000
700	.0041	.00035	.0000
710	.0021	.00018	.0000
720	.00105	.000089	.0000
730	.00052	.000044	.0000
740	.00025	.000021	.0000
750	.00012	.000010	.0000
760	.00006	.000005	.0000
770	.00003	.000003	.0000

The representation of deuteranopic vision by K and W_d, and protanopic vision by K and W_p, is valid in the same way as representation of normal vision by X, Y, and Z. No single normal observer, or protanope, or deuteranope, can be found, except by accident, who conforms to the standard representation of the respective group, but the departures of each single individual from the standard may be expected to be smaller than some of the individual differences within the group.

VI. USE OF THE RESPONSE FUNCTIONS

The response functions K, W_p, and W_d have been used by the ISCC Committee on Colorblindness [4] to find whether or not two colors will be confused by an average protanope or an average deuteranope. Such use will be illustrated by solution of three problems: First, what is the influence of the choice of standard neutral source on the dichromatic neutral points in the spectrum; second, how much darker to an average protanope are normally reddish colors; and third, what arrangements of the colors of the Farnsworth dichotomous test B–20 [9] are characteristic of average deuteranopic and protanopic vision.

1. WAVELENGTH OF NEUTRAL POINT

The determination of the neutral point in the spectrum of a dichromatic observer involves setting up a chromaticity match between some standard neutral source (ICI illuminant A, ICI illuminant C, black-body at 5,000 ° K, and so forth) and some part of the spectrum (see table 2). Deuteranopic and protanopic chromaticity is conveniently specified by chromaticity coordinates w_d, k_d and w_p, k_p, the

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[4] Committee on Colorblindness Tests, Inter-Society Color Council, Co-Chairmen LeGrand H. Hardy and the author.

first pair for deuteranopic vision, the second for protanopic. The definitions of these coordinates and their equivalents in terms of X, Y, Z, from eq 11 are as follows:

$$\left.\begin{aligned} w_d &\equiv W_d/(W_d+K)=Y/(Y+Z) \\ k_d &\equiv K/(W_d+K)=Z/(Y+Z) \end{aligned}\right\} \quad (12a)$$

$$\left.\begin{aligned} w_p &\equiv W_p/(W_p+K)= \frac{-0.460X+1.359Y+0.101Z}{-0.460X+1.359Y+1.101Z} \\ k_p &\equiv K/(W_p+K)=Z/(-0.460X+1.359Y+1.101Z) \end{aligned}\right\} \quad (12b)$$

Table 4 shows these coordinates for the various parts of the spectrum, particularly the portions near 495 mμ where chromaticity for these types of dichromatic vision varies most rapidly with wavelength. Since from their definitions the two coordinates sum to unity, only one of them is required to specify the dichromatic chromaticity of a stimulus.

TABLE 4.—*Chromaticity coordinates for deuteranopic and protanopic vision*

Wave-length	Chromaticity coordinates			
	Protanopia		Deuteranopia	
	w_p	k_p	w_d	k_d
mμ				
380	0.012	0.988	0.006	0.994
430	.018	.982	.008	.992
450	.041	.959	.021	.979
460	.065	.935	.035	.965
470	.113	.887	.066	.934
475	.156	.844	.098	.902
480	.218	.782	.146	.854
485	.301	.699	.216	.784
490	.404	.596	.309	.691
495	.518	.482	.423	.577
500	.631	.369	.543	.457
505	.730	.270	.657	.343
510	.815	.185	.761	.239
515	.881	.119	.845	.155
520	.923	.077	.901	.099
530	.963	.037	.953	.047
540	.982	.018	.979	.021
550	.992	.007	.991	.009
600	.998	.002	.999	.001
650	1.000	.000	1.000	.000
750	1.000	.000	1.000	.000

In order to find the neutral point in the spectrum of, say, an average protanope for a standard neutral source specified in the standard ICI system by X_o, Y_o, Z_o, it is necessary only to compute w_p from eq 12b and find by interpolation among the values of w_p for the spectrum given in table 4, the wavelength for the corresponding value. This wavelength corresponds to the same chromaticity as the standard neutral source and is therefore the desired neutral point. This method is the analytic equivalent of a graphical method using the chromaticity diagram of the standard system (see fig. 1). The graphical method is to draw a straight line from the copunctal point through the point representing the chromaticity of the standard neutral source and extend it until it cuts the spectrum locus. The wavelength corresponding to this intersection is the desired neutral point in the dichromatic spectrum.

Table 5 gives the standard chromaticity specifications, x and y, for several sources that might be taken as standard neutrals, the values of w_p and w_d for these sources, and the corresponding neutral points found by the analytical method from table 4. It will be noted that the change in neutral point depending upon choice of standard neutral source is considerably greater than the difference between the neutral points of the typical protanope and typical deuteranope for any one source.

TABLE 5.—*Protanopic and deuteranopic neutral points in the spectrum derived from the dichromatic chromaticity coordinates, w_p and w_d*

Standard source	ICI chromaticity coordinates		Dichromatic chromaticity coordinates		Wavelength of the neutral point	
	x	y	w_p	w_d	λ_p	λd
					mμ	mμ
ICI illuminant *A*	0.4476	0.4075	0.7144	0.7377	504.0	508.9
ICI illuminant *B*	.3484	.3516	.5369	.5396	495.8	499.8
Complete radiator at 5,000° K	.3445	.3512	.5346	.5358	495.7	499.7
ICI illuminant *C*	.3101	.3163	.4652	.4585	492.7	496.4
Illuminant *D* [1]	.2999	.3120	.4559	.4457	492.3	495.8
Illuminant *S* [2]	.2319	.2318	.3286	.3018	486.4	489.6

[1] Illuminant *D* is produced by an incandescent lamp and Corning Daylite glass filter [32, p. 60] and has a nearest color temperature [28] of about 7,500° K.

[2] Illuminant *S* was found [32, p. 58] by weighting Abbot's "sun-outside-atmosphere" energy data by the inverse λ^4 scattering relation, and has been designated as "limit blue sky."

2. PROTANOPIC LUMINOUS REFLECTANCE

The second problem is to find how much darker to an average protanope are normally reddish colors. For spectrum colors the solution is given simply by comparing W_p with $Y=W_d$ in table 3. It is seen, for example, that relative to an equal-energy source, an average protanope finds the spectrum from 700 to 770 mμ to have less than 10 percent of the normal luminosity. It could be said therefore in a sense that he is more than 90-percent blind to the long-wave end of the spectrum; but we would also have to say in the same sense that he is supersensitive by a factor of 2 for the spectral region near 450 mμ. It is more convenient to say that the luminosity function of the average protanope is shifted about 10 mμ toward the short-wave end of the spectrum relative to normal.

If the color is that of a surface, it is customary to estimate its lightness or darkness from the luminous reflectance of the surface for ICI illuminant *C* relative to magnesium oxide. For magnesium oxide, $X=0.9804$, $Y=1.0000$, and $Z=1.1812$, and from eq 9, $W_p=1.0274$. Protanopic luminous reflectance relative to magnesium oxide is therefore equal to $W_p/1.0274$ or $0.9733\ W_p$. It is convenient to represent on the (x, y) chromaticity diagram the dependence of protanopic luminous reflectance of a surface on the normal chromaticity of its color.

If Q be the ratio of the protanopic luminous reflectance of a surface to its normal luminous reflectance, Y, we may write

$$Q=0.9733\ W_p/Y=0.9733\ (-0.460X+1.359\ Y+0.101\ Z)/Y$$
$$=0.9733\ (-0.460\ x+1.359\ y+0.101\ z)/\ y$$

whence it is found

$$x=y(Q-1.224)/(-0.546)+0.180. \qquad (13)$$

Examination of eq 13 shows that it represents a family of straight lines on the $(x,\ y)$ chromaticity diagram passing through the point $(x=0.180,\ y=0.000)$. For $Q=1$, the line passes through the point representing illuminant C $(x=0.3101,\ y=0.3163)$. All surfaces

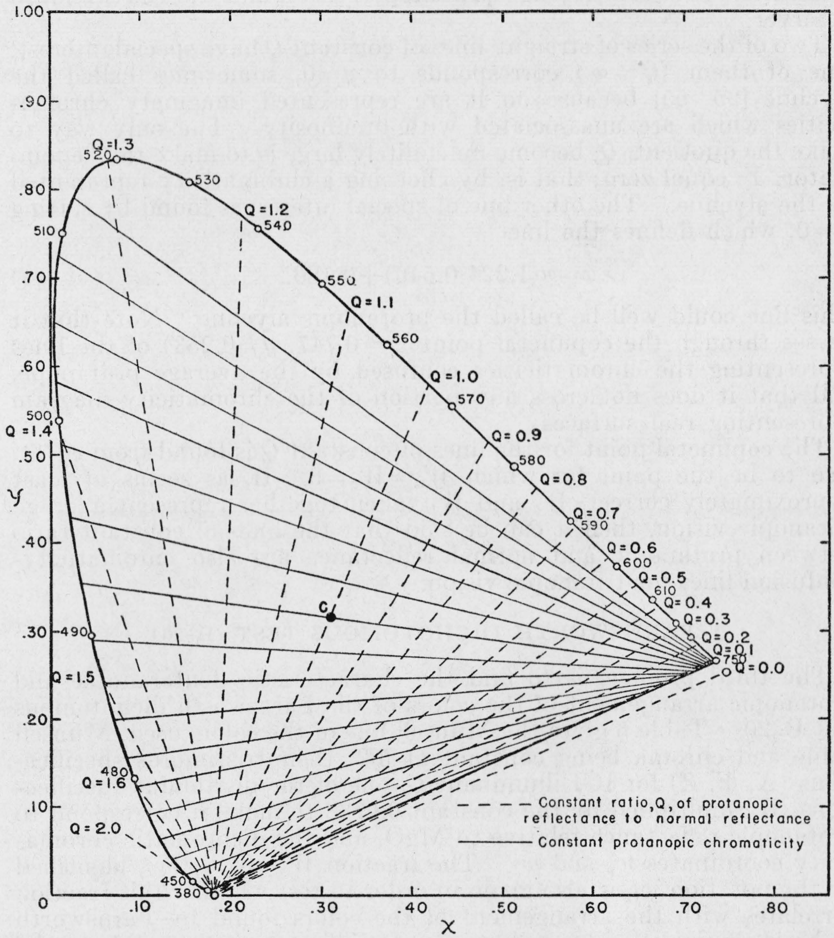

FIGURE 5.—*Relation of protanopic to normal vision.*

The solid lines (as in fig. 1) represent constant protanopic chromaticity; the dotted lines represent constant values of the ratio, Q, of protanopic luminous reflectance to normal luminous reflectance. Both reflectances are taken relative to that of magnesium oxide and refer to ICI standard illuminant C. Note that the line (dotted) for $Q=1$ passes through the point $x=0.310$, $y=0.316$, representing illuminant C. Note also that the line for $Q=0$ passes through the copunctal point $x=0.747$, $y=0.253$; this line could be called the protanopic alychne (lightless line), just as the line for $Q=\infty$ ($y=0$, see eq 13) has sometimes been called the normal alychne.

whose chromaticities under illuminant C are represented on this line have identical protanopic and normal luminous reflectances. For $Q=2.0$, the line passes close to 450 mμ; all surfaces represented on this line have protanopic luminous reflectance twice the normal luminous reflectance. For $Q=0.10$, the line passes close to 650 mμ; all surfaces represented on this line have protanopic luminous reflectance equal to 10 percent of the normal luminous reflectance. Figure 5 shows a number of these lines of constant Q (dotted) on the (x, y) chromaticity diagram; it indicates completely and quantitatively how much darker to an average protanope are normally reddish colors. It also shows the lines of protanopic chromaticity confusion; it therefore gives a convenient, concise and quantitative summary of the connection between average protanopic vision and the ICI standard observer.

Two of the series of straight lines of constant Q have special interest. One of them ($Q=\infty$) corresponds to $y=0$, sometimes called the alychne [25, 55] because on it are represented imaginary chromaticities which are unassociated with luminosity. The only way to make the quotient, Q, become indefinitely large is to make the denominator, Y, equal zero; that is, by choosing a chromaticity represented on the alychne. The other line of special interest is found by setting $Q=0$, which defines the line

$$x=y(1.224/0.546)+0.180. \tag{14}$$

This line could well be called the protanopic alychne. Note that it passes through the copunctal point ($x=0.747$, $y=0.253$) of the lines representing the chromaticities confused by the average protanope, and that it does not cross any portion of the chromaticity diagram representing real surfaces.

The copunctal point for the lines of constant Q is found from eq 11, also to be the point for which $W_a=W_b=0$. If, as seems at least approximately correct, W_a and W_b taken together represent average tritanopic vision, then it can be said that the lines of constant ratio between protanopic and normal reflectance are also chromaticity-confusion lines for tritanopic vision.

3. FARNSWORTH DICHOTOMOUS TEST (B-20)

The third problem is to find the characteristic deuteranopic and protanopic arrangement of the colors of the Farnsworth dichotomous test B-20. Table 6 gives the Munsell hue of the colors used, Munsell value and chroma being constant at 5/2, their tristimulus specifications (X, Y, Z) for ICI illuminant C, one of the distimulus specifications, W_p multiplied by the constant 0.9733 to make it correspond to protanopic reflectance relative to MgO, and the dichromatic chromaticity coordinates w_p and w_d. The fraction $W_d/(W_p+W_d)$, identified by the notation w_t, is also given in order to test whether this fraction correlates with the arrangement of the colors found by Farnsworth to be characteristic of a tritanope. It is first to be noted from the relative constancy of the daylight reflectance, $w_d=Y$, and the protanopic reflectance, $0.9733W_p$, for these colors that the papers used in the Farnsworth dichotomous test have nearly the same reflectances for illuminant C not only for the standard observer and typical deuteranope ($W_d=Y$), but also for the typical protanope ($0.9733W_p$)

the variation being between 0.2321 and 0.1818 which correspond to only 0.55 of a Munsell value step [50]. Since these papers are exhibited with a dark-gray border that interferes with perception of lightness differences more than it does chromaticity differences [29, p. 425] the basis for arranging the colors in order must be chiefly according to chromaticity. The characteristic protanopic arrangement is therefore to be found in accord with w_p, and the characteristic deuteranopic arrangement according to w_d.

TABLE 6.—*Tristimulus and distimulus specifications of the colors of the Farnsworth dichotomous test for color vision*

Serial number	Munsell hue (at 5/2)	Tristimulus specifications			Protanopic reflectance	Distimulus chromaticity coordinates		
		X	Y	Z	$0.9733 W_p$	w_p	w_d	w_t
1	10B	0.2121	0.2229	0.3292	0.2321	0.4201	0.4037	0.4831
2	5B	.1749	.1917	.2831	.2030	.4242	.4037	.4789
3	10BG	.1820	.2043	.2790	.2162	.4432	.4227	.4791
4	5BG	.1782	.2077	.2538	.2199	.4709	.4501	.4790
5	5G	.1618	.1888	.2017	.1972	.5011	.4835	.4824
6	10GY	.1766	.2017	.1873	.2061	.5307	.5185	.4878
7	5GY	.1908	.2097	.1804	.2096	.5442	.5376	.4933
8	10Y	.2135	.2292	.1798	.2253	.5628	.5604	.4975
9	5Y	.2057	.2117	1795	.2056	5406	.5412	.5006
10	10YR	.2170	.2143	.1780	.2038	.5405	.5463	.5058
11	5YR	.2029	.1927	.1755	.1813	.5149	.5234	.5084
12	10R	.2364	.2185	.2055	.2034	.5042	.5153	.5111
13	5R	.2158	.1954	.2041	.1818	.4779	.4891	.5113
14	10RP	.2426	.2229	.2453	.2103	.4684	.4761	.5077
15	5RP	.2304	.2138	2559	.2048	.4512	.4552	.5040
16	10P	.2256	.2092	.2751	.2027	.4309	.4320	.5011
17	5P	.2070	.1959	.2818	.1942	.4145	.4101	.4954
18	10PB	.2199	.2118	.3301	.2140	.3998	.3908	.4906
19	7.5PB	.2055	.2035	.3036	.2071	.4121	.4013	.4888
20	5PB	.2052	.2087	.3277	.2164	.4042	.3891	.484

Table 7 shows the serial order according to w_p compared to that for a protanope reported by Farnsworth. That is, color number 8 was picked by the Farnsworth protanope as the warmest of the 20 colors, and it also has the highest value of w_p (0.5628 from table 5); therefore number 8 heads both the second and third column of table 6. A similar comparison is given between the arrangements according to w_d and a deuteranope reported by Farnsworth, and between w_t and the tritanope reported by Farnsworth. It is evident from table 7 that the correlations indicated by these comparisons are high; that is, the results chosen by Farnsworth to be typical of protanopia, deuteranopia, and tritanopia agree well, though not perfectly, with the chromaticity arrangements according to w_p, w_d, and w_t, respectively.

Failure of the agreement to be perfect might be due to failure of the tristimulus specifications reported for certain samples of the Munsell papers to apply exactly to the particular samples used in the Farnsworth test B–20, to careless arrangement of the colors by the dichromatic observer or to arrangement on a basis other than chromaticity, or to significant difference between the visual system of the dichromat and that represented by the pairs of functions, K, W_p, and W_d taken here to represent the average protanope and deuteranope. By plotting the orders given in table 7 according to w_p, w_d, and w_t on the Farnsworth chart for analysing results on the dichotomous test (see his fig. 15), it is discovered that most of the minor differences are ascribable to a real difference between the typical Farnsworth dichromats and the corresponding hypothetical observers defined by the functions K, W_p, and W_d. The axes of confusion for the hypothetical observers fall, however, within the limits indicated by Farnsworth (see his fig. 15) as embracing the three types of dichromat delineated by the dichotomous test.

TABLE 7.—*Comparison of serial orders of the colors of the Farnsworth dichotomous test according to w_p, w_d, and w_t with the orders reported by Farnsworth for a protanope (red-bluegreen confuser), and deuteranope (green-redpurple confuser), and a tritanope (violet-greenishyellow confuser)*

Normal order (based on hue)	Serial numbers in order from warmest to coldest dichromatic color according to—					
	w_p	Farnsworth protanope	w_d	Farnsworth deuteranope	w_t	Farnsworth tritanope
1	8	8	8	10	13	13
2	7	10	10	9	12	14
3	9	7	9	8	11	15
4	10	6	7	7	14	12
5	6	9	11	11	10	11
6	11	11	6	12	15	16
7	12	5	12	6	16	10
8	5	12	13	13	9	9
9	13	13	5	14	8	17
10	4	4	14	5	17	18
11	14	3	15	15	7	8
12	15	15	4	16	18	19
13	3	14	16	4	19	7
14	16	2	3	17	6	20
15	2	16	17	3	20	1
16	1	17	2	2	1	6
17	17	1	1	18	5	2
18	19	19	19	20	3	5
19	20	20	18	19	4	4
20	18	18	20	1	2	3

Although this corroboration was expected for protanopic and deuteranopic vision because of the firm basis for choosing the functions K, W_p, and W_d, it is interesting to note that substantially the same agreement is found for tritanopic vision. The discrepancies are of the sort to be ascribed to individual differences. It is evident that the Farnsworth test is well adapted for surveys of the characteristics of dichromats; a statistical treatment of an extended body of data found by this test would serve to determine whether the functions proposed here for dichromatic vision describe, as was intended, visual systems intermediate to actual observers of the corresponding type; also whether the functions W_p and W_d, taken together, describe a type of tritanopic vision intermediate to actual tritanopes.

VII. THEORETICAL IMPLICATIONS OF THE RESPONSE FUNCTIONS

The functions W_d, W_p, and K defined in eq 11 have theoretical implications, the validity of which in no way interferes with the practical applications just discussed. It is convenient to bring out these implications by indicating the relation of these functions to the later theoretical views of Arthur König, to the recent dominator-modulator theory of Granit, and to the zone-theory of G. E. Müller.

1. KÖNIG THEORIES

In 1886 and in 1893 Arthur König published accounts of his monumental work on the color systems of normal and abnormal observers [36, 37]. The coordinate system chosen by him to represent his experimental results satisfies eq 1 and 2 but does not satisfy eq 4, since one of the "fundamental sensations" was taken as unitary blue.

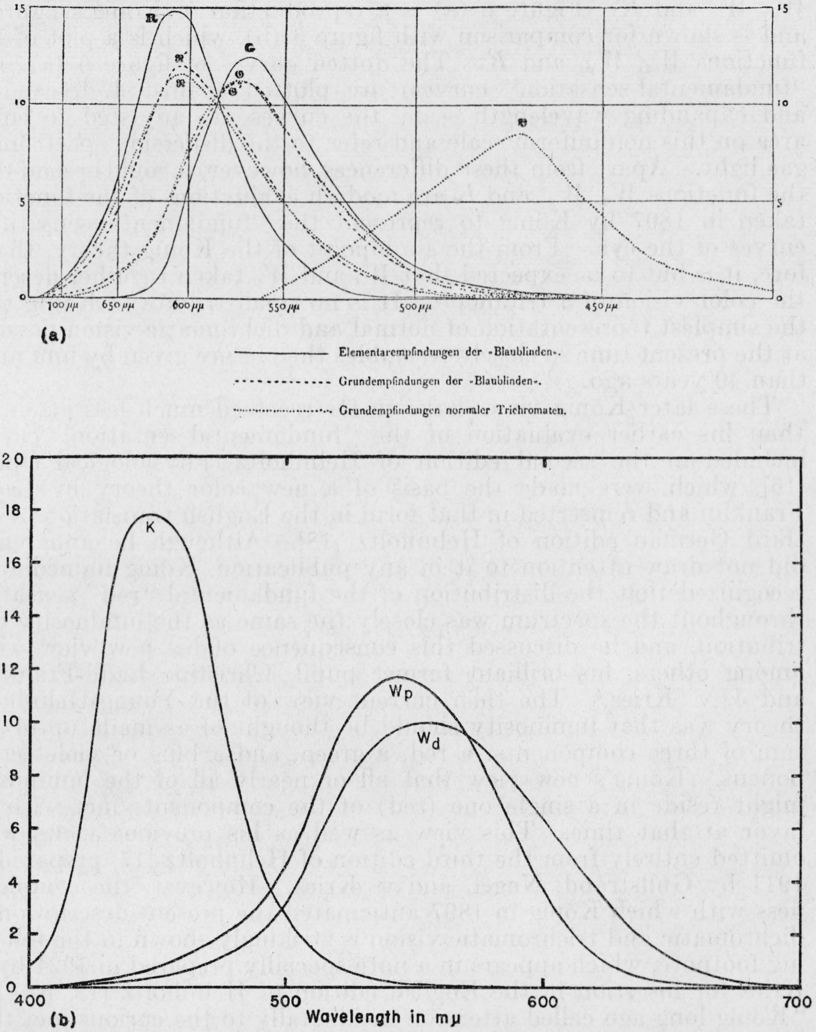

FIGURE 6.—*Comparison of the functions W_d, W_p, and K with the "fundamental-sensation" curves (dotted) derived by König in 1897.*

If allowance is made for the descending and expanding wavelength scale and for the energy distribution of gas light used by König, it will be noted that there is no essential difference between the two sets of functions.

In 1897, however, König published an important paper on tritanopia or "blueblindness", in which the choice of unitary blue as a "fundamental sensation" was given up. He says in a footnote [38, p. 716; 38a, p. 396], "I will immediately remark here that regarding the quality of this third fundamental sensation of the Young-Helmholtz color theory—whether it be blue or violet—I can still pass no definite judgment. In any case, however, I am now inclined to move it far closer to violet than was the case in the year 1886 (compare Sitz. Akad. Wiss. Berlin, July 29, 1886)."

As a result of this change in choice of third primary the "fundamental-sensation" curves bear a close resemblance to the functions W_d, W_p and K. Figure 6 (a) is a reproduction of König's figure 2 and is shown for comparison with figure 6 (b), which is a plot of the functions W_d, W_p, and K. The dotted curves of figure 6 (a) (the "fundamental-sensation" curves) are plotted against a descending and expanding wavelength scale, the curves are adjusted to equal area on this nonuniform scale and refer to the dispersion spectrum of gas light. Apart from these differences, however, it could be said that the functions W_d, W_p, and K are modern evaluations of the functions taken in 1897 by König to represent the "fundamental-sensation" curves of the eye. From the standpoint of the König theory, therefore, it is but to be expected that W_d and W_p taken together describe the color vision of a tritanope. It is no small tribute to König that the simplest representation of normal and dichromatic vision possible at the present time so closely resembles the picture given by him more than 40 years ago.

These later König views have so far received much less attention than his earlier evaluation of the "fundamental-sensation" curves included in the second edition of Helmholtz' Physiological Optics [16], which were made the basis of a new color theory by Ladd-Franklin and reinserted in that form in the English translation of the third German edition of Helmholtz [18]. Although he apparently did not draw attention to it in any publication, König immediately recognized that the distribution of the fundamental "red" sensation throughout the spectrum was closely the same as the luminosity distribution, and he discussed this consequence of his new view with, among others, his brilliant former pupil, Christine Ladd-Franklin, and J. v. Kries.[5] The then current view of the Young-Helmholtz theory was that luminosity should be thought of as made up of the sum of three components: a red, a green, and a blue or violet component. König's new view that all or nearly all of the luminosity might reside in a single one (red) of the components met with no favor at that time. This view as well as his previous views were omitted entirely from the third edition of Helmholtz [17] prepared in 1911 by Gullstrand, Nagel, and v. Kries. However, the completeness with which König in 1897 anticipated the present description of dichromatic and trichromatic vision is strikingly shown in the following footnote, which appears in a note specially prepared in 1924 by v. Kries for insertion in the English edition of Helmholtz [18, p. 412], "König long ago called attention incidentally to the curious fact that the luminosity values of the colours . . . . turn out to be very nearly the same function of the wave-length for deuteranopes and

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[5] Communicated privately to the author by Ladd-Franklin in 1928.

persons with normal vision. Since in the case of deuteranopes light of long wave-lengths has no effect except on the red component, we must suppose also that this action depends on the wave-length in the same way. But then the further result is that the distribution of luminosity is not affected, or at least almost inappreciably affected, by the addition of the green component by which the deuteranopic visual organ is converted into a normal organ; in other words, that in the case of normal vision also the luminosity goes practically hand in hand with the action of the red component."

2. DOMINATOR-MODULATOR THEORY OF GRANIT

In 1943 Granit [12] gave a brief outline of a physiological theory of color perception based upon his oscillographic studies of the responses of isolated fibers from the optic nerves of animals (cat, guinea pig, frog, snake, rat). In the light-adapted eyes of these animals the simple spectral sensitivity curves recorded with his microelectrode technique were found to be of two types: (a) Broad absorption bands, called *dominators;* and (b) narrow bands, called *modulators*. The dominator of the light-adapted animal eye is found to have its maximum in the spectral region around 560 mμ. The form and spectral location of the dominator were found to be practically identical with the average curve obtained from massed receptors in the light-adapted eyes of the same species. This and its good correspondence with respect to form and location with the luminosity curve of the light-adapted human eye led Granit to the conclusion that the dominator is responsible for the sensation of brightness, which thus is taken as our dominant impression. Modulation of the dominant impression of brightness to color would seem to be the task of the much rarer modulators which occupy narrow bands of sensitivity in three preferential regions around 580 to 600 mμ, 520 to 540 mμ, and 450 to 470 mμ.

It will be noted from figure 6 (b) that the functions W_d, W_p, and K proposed here to represent dichromatic and normal trichromatic vision conform fairly well with the dominator-modulator theory. The K function is the narrowest of the three, and since it is unassociated with luminosity, could be thought of as a modulator for normal, protanopic, and deuteranopic vision. The W_p function is somewhat narrower than the W_d function and could be thought of as a modulator in the case of normal and tritanopic vision. The spectral location of these functions does not, however, conform perfectly with any of the three preferential regions found by Granit in his study of the eyes of animals.

Granit's view of deuteranopia differs somewhat from that suggested by the functions W_d and K. He says [12, p. 13], "Colour-blindness need not, but *can* be possible without parallel change of the photopic luminosity curve. A colour-blindness of this type would be the common form of red-green blindness known as deuteranopia, to be interpreted as absence of the 'red' and 'green' modulators, with the remaining dominator alone giving the normal luminosity curve." The view suggested by the functions W_d and K is that deuteranopia consists of the absence of only the green modulator, W_p.

Granit attacks the classical trichromatic theory of color blindness as follows [12, p. 14]: "The trichromatic theory regards white as due to the summed effects of, chiefly, the 'red' and the 'green' sensitivity curves. This forces the theory to accept the consequence that removal of 'red' and/or 'green' should cause removal of the perception of luminosity in the same region of the spectrum. Hence there can be no colour-blindness without profound changes in the form and locus of the luminosity curve. It is an admission of failure to have to explain so important a phenomenon as deuteranopia by pushing it aside to be taken care of by the 'higher centres'." It is interesting to note that this criticism of classic three-components theory does not apply to the later König form of this theory.

It may be concluded that the functions W_d, W_p, and K are nearly, if not perfectly, consistent with Granit's dominator-modulator theory, and also that this theory was foreshadowed by König's later views. König, himself, however, began to be interested in the theoretical possibilities of an hypothetical post-receptor center of the visual mechanism apparently first suggested by Donders [5]. He states [38, p. 730; 38a, p. 414], "This is not the place to go into any further discussion of the hypothesis of the existence of a second more centrally located structure of the color system. I wish only to call special attention to the fact that with its assumption many difficulties still existing in the explanation of color vision in the extrafoveal and peripheral parts of the retina, for what I have called the blueblindness of the fovea, for the apparent achromaticity of sensations in the lowest brightness levels, and so forth, would be removed. Through assumption of pathological processes in this more centrally located color apparatus there could be explained further many cases of erythropsia, chloropia and so forth and finally perhaps color sense disturbances due to hysteria and so forth."

This view had been favored for some years by v. Kries [39, p. 269] and was adopted later by Adams [1] and Müller [47, 48]. Lest it appear that the concise account of dichromatic and normal trichromatic vision afforded by the functions W_d, W_p, and K establish the correctness of the three-components theory of vision according to König and Granit, and render further theorizing useless, two of the shortcomings of this form of three-components theory may be mentioned, and the way the zone theory of Müller overcomes these deficiencies may be indicated.

In the long-wave end stretch of the spectrum from 700 to 770 mμ the chromaticity is constant; that is, any part of this region of the spectrum can be matched perfectly for a normal observer by any other part, provided only that the luminances of the two be equalized. This constancy of chromaticity is indicated in the standard ICI coordinate system by the fact that X and Y bear the same ratio over this wavelength range, and the same thing is indicated from the functions W_d, W_p, and K from the fact that W_d and W_p also bear a constant ratio over the same range. If, as in König's later view, we regard W_d and W_p as the spectral distributions of the fundamental red and green sensations, the question may be asked, "What could account for the fact that while varying by a factor of 100 these two distributions still manage to bear exactly the same ratio?" There is only one answer at all plausible; the only way we would expect two processes to bear a constant ratio is that they are both functions of some other single process. Perhaps, for example, W_d and W_p are proportional to excitation of the fibers of the optic nerve in the red and green sense, resulting from receptors (retinal cones, say) containing preponderantly a red-sensitive and a green-sensitive substance, respectively. If the green-sensitive substance were entirely insensitive for wavelengths greater than 700mμ, that is, suppose its sensitivity curve corresponded to W'_p (see fig. 4), the explanation for constant chromaticity for wavelengths longer than 700mμ would be very simple and believable; that is, only one photosensitive substance, the red sensitive, is affected by radiant energy of this wavelength range. The dependence of the fundamental red sensation upon this kind of radiant energy is customarily ascribed to leakage of a small fairly constant fraction of the green-sensitive substance into the predominantly red-sensitive cones. Thus it is seen that the functions W_d, W_p, and K, though they give a concise account of the connection between normal and dichromatic vision at the level of, perhaps, the fibers of the optic nerve, do not suffice to explain plausibly the constant chromaticity of the long-wave end stretch of the spectrum for the normal observer. To provide a satisfactory explanation, these functions have to be supplemented by another three-component system referring to the photosensitive substances themselves.

Although formulation of normal and dichromatic vision in terms of the functions W_d, W_p, and K gives a clear and satisfactory definition of the samples of radiant energy confused both by an average normal observer and average dichromatic observers, it does not provide any mechanism to explain the usual sensations. That is, these functions describe what the observers confuse but not what they see. Thus for the normal we have the fundamental sensations of red, green, and violet in equal amount summing to neutral. But when the red sensation is missing, as in protanopia, instead of having green and violet left, we find that blue and yellow are seen. Similarly when the green sensation is missing, as in deuteranopia or as in certain extra-foveal portions of the normal retina, blue and yellow are seen rather than red and violet, as might seem reasonable from the formulation of deuteranopia from W_d and K taken without W_p. And finally, when both green and violet are missing, neutral colors only are seen as in acquired total colorblindness, rather than the red which might be expected. This failure of the three-components theory to indicate correctly the sensations derivable from the various presumed combinations of the fundamental sensations has been pointed out innumerable times, chiefly by psychologists who could scarcely be expected to be satisfied with a formulation giving false implications of visual experience, even though it did predict correctly which samples of radiant energy are found to be identical in appearance. This failure led to the Hering and Ladd-Franklin theories of vision, and set König, Donders [5], and v. Kries to thinking about higher zones in the color apparatus. This lack is also felt by Granit, who remarks [12, p. 13], "The experiments with the cone-eye of the snake suggested that the dominator itself is composed of modulators joined together in such a fashion—either photochemically or by connections in the retinal synapses—as to operate as a *functional* unit. This assumption would explain why stimulation of all modulators together also causes an impression of white, and not of all colours confused. . . . Alternatively, the modulators could be coupled in antagonistic pairs which simultaneously neutralized each other at the retinal or some higher level." The first suggestion accords closely with the Adams theory [1, 2], which is probably the first to follow explicitly the dominator-modulator form. The latter suggestion corresponds somewhat to the zone theory of Müller [47], which by 1924 had been elaborated so as to deal satisfactorily in a qualitative way with all known forms of dichromatic and monochromatic vision.

3. ZONE THEORY OF G. E. MÜLLER

The Müller theory includes three separate color systems, one in the zone of the initial photosensitive substances, a second in the zone of sensory retinal processes aroused by action of the initial photosensitive substances, and a third in the zone of excitations of optic-nerve fibers. The first color system has red, green, and violet primaries and accords with the classical Young-Helmholtz theory. The second system includes two pairs of antagonistic chromatic processes: Yellowishred-greenishblue (yR-gB), and greenishyellow-reddishblue (gY-rB). The first named of each pair has a lightening effect; the last named, a darkening effect. And, finally, the third system includes two pairs of antagonistic chromatic excitations and one nonantagonistic pair of achromatic excitations: Red-green (r-g), yellow-blue (y-b) and white-black (w-s). White does not cancel black, but combines with it to give gray.

Failure of the yR-gB process corresponds to protanopia. Yellowish red and greenish blue are the hues of the colors confused by the protanope with gray. Since the normal lightening effect of the yR-process is missing, the theory indicates that the luminosity function of the protanope is shifted toward the short-wave end, which corresponds to the facts.

Failure of the gY-rB process corresponds to tritanopia. Greenish yellow and reddish blue are the hues of the colors confused by the tritanope with gray. Since the normal lightening effect of the gY-process is missing, the theory indicates that the luminosity function of the tritanope is higher than normal at the short-wave end. The few data available [7, 8, 33, 34] are inconclusive concerning this indicated deviation from the normal luminosity function.

Failure of the r-g excitation corresponds to deuteranopia. Purplish red and green are the hues of the colors confused by the deuteranope with gray. The theory indicates that the deuteranopic luminosity function corresponds to the normal, which is the same indication given by the functions W_d, W_p, and K (see eq 11).

If, as seems likely, it is possible to quantify the Müller theory so as to make it embody the same dichromatic confusion colors as those indicated by the functions W_d, W_p, and K, a much more complete account of visual phenomena would be so achieved. The shortcomings of the König-Granit three-components theory outlined above would be overcome. One could then take his choice between a single color system yielding conveniently all of the dichromatic color confusions, or a three-system theory yielding the same confusions together with a prediction of dichromatic sensations.

The major theoretical point of difference between these two views lies in the account of deuteranopia. By the König-Granit theory, deuteranopia and protanopia are defects of similar character. By the Müller theory, deuteranopia is a defect of the optic-nerve fibers; protanopia and tritanopia, defects of the cones. Though there is no conclusive evidence pointing to which, if either, view is correct, it may be significant, first, that diseases of the nerve-fiber layer of the retinal optic nerve and tract always produce the deuteranopic form of red-green blindness [33], never the protanopic form, and second, there have been several observers found who have tritanopia or protanopia in one eye [4, 19, 20, 21] and normal vision in the other, but never a case of congenital monocular deuteranopia [4, p. 78].

VIII. SUMMARY

There have been derived, chiefly from the work of Pitt, three functions of wavelength which permit easy definition of the samples of radiant energy found to be identical (metameric) not only by the ICI standard observer but also by the two most common types of red-green-blind observers, protanopes and deuteranopes. These three functions W_d, W_p, and K may be taken together to indicate the metamers of the normal observer; W_d and K, taken together indicate the metamers of the average deuteranopic observer; and W_p and K, those of the average protanopic observer. The functions W_d and W_p taken together also yield a close approximation to the less well determined metamers of the average tritanopic observer. Methods of using these functions to derive the variation in wavelength of neutral point for dichromatic observer with choice of neutral source are given, together with a convenient graphical solution of the ratio of protanopic reflectance to normal reflectance as a function of normal chromaticity. A derivation of the average protanopic, deuteranopic and tritanopic responses to the Farnsworth dichotomous test for color blindness is also given by way of these three functions. The functions W_d, W_p, and K, therefore, serve to make conveniently accessible the color confusions of dichromatic observers. These three functions also conform to the type of three-components color theory outlined in 1897 by Arthur König, and they are consistent with the dominator-modulator theory proposed by Granit in 1943. They do not, however, form the basis of a complete theory of vision such as that of Müller.

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