2. Measurement of Reactor Period and Reactivity

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Inhour Equation

Point Kinetics Equation

- The reactor kinetics equations based on one-point reactor approximation with one-energy-group theory are as follows:

$$\frac{dn(t)}{dt} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t), \quad \text{(1a)}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{i,eff}}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \cdots, 6), \quad \text{(1b)}$$

where $n(t)$ = neutron density or total neutron population,

$C_i(t)$ = i-th group delayed neutron precursor density,

λ_i = decay constant of the i-th group delayed neutron precursor,

Λ = prompt neutron generation time, which is the prompt neutron lifetime l divided by k_{eff},

β_{eff} = effective delayed neutron fraction

$\beta_{i,eff}$ = effective delayed neutron fraction of i-th delayed neutron precursor group

Inhour Equation

- All of the coefficients in Eq. (1) are physical constants, in practice, except the reactivity, which can be changed by variation of a operation parameter.
- In the case where reactivity does not vary, the system is a "constant coefficient" differential equation system, and its solution can be found by merely seeking exponential solutions of the form

$$n(t) = a \cdot \exp(\omega t), \qquad C_i(t) = b_i \cdot \exp(\omega t) \ (i = 1, 2, \cdots, 6), \quad \cdots\cdots (2)$$

where ω, a, and b_i are constants.

- Insertion of Eq. (2) into Eq. (1) gives

$$\omega a \cancel{\exp(\omega t)} = \frac{\rho - \beta_{eff}}{\Lambda} a \cancel{\exp(\omega t)} + \sum_{i=1}^{6} \lambda_i b_i \cancel{\exp(\omega t)}, \quad \cdots\cdots (3a)$$

$$\omega b_i \cancel{\exp(\omega t)} = \frac{\beta_{i,eff}}{\Lambda} a \cancel{\exp(\omega t)} - \lambda_i b_i \cancel{\exp(\omega t)} \quad (i = 1, 2, \cdots, 6). \quad \cdots\cdots (3b)$$

Inhour Equation (Contd.)

- By substituting b_i derived from Eq. (3b) into Eq. (3a), we can obtain a characteristic equation as

$$\rho = \omega\left[\Lambda_{eff} + \sum_{i=1}^{6}\frac{\beta_{i,eff}}{\lambda_i + \omega}\right] \quad \cdots\cdots \quad (4a)$$

- Because $\Lambda_{eff} = l/k_{eff}$ where l denote the neutron lifetime is Eq. (4a) can be expressed as

$$\rho = \omega\left[-l\left(1 - \frac{1}{k_{eff}}\right) + l + \sum_{i=1}^{6}\frac{\beta_{i,eff}}{\lambda_i + \omega}\right]$$

$$\Rightarrow \rho = \frac{\omega l}{\omega l + 1} + \frac{\omega}{\omega l + 1}\sum_{i=1}^{6}\frac{\beta_{i,eff}}{\lambda_i + \omega} \quad \cdots\cdots \quad (4b)$$

- Eq. (4) is called the "inhour equation" because it gives a quantity that can be expressed in hour^{-1} (inverse hour).

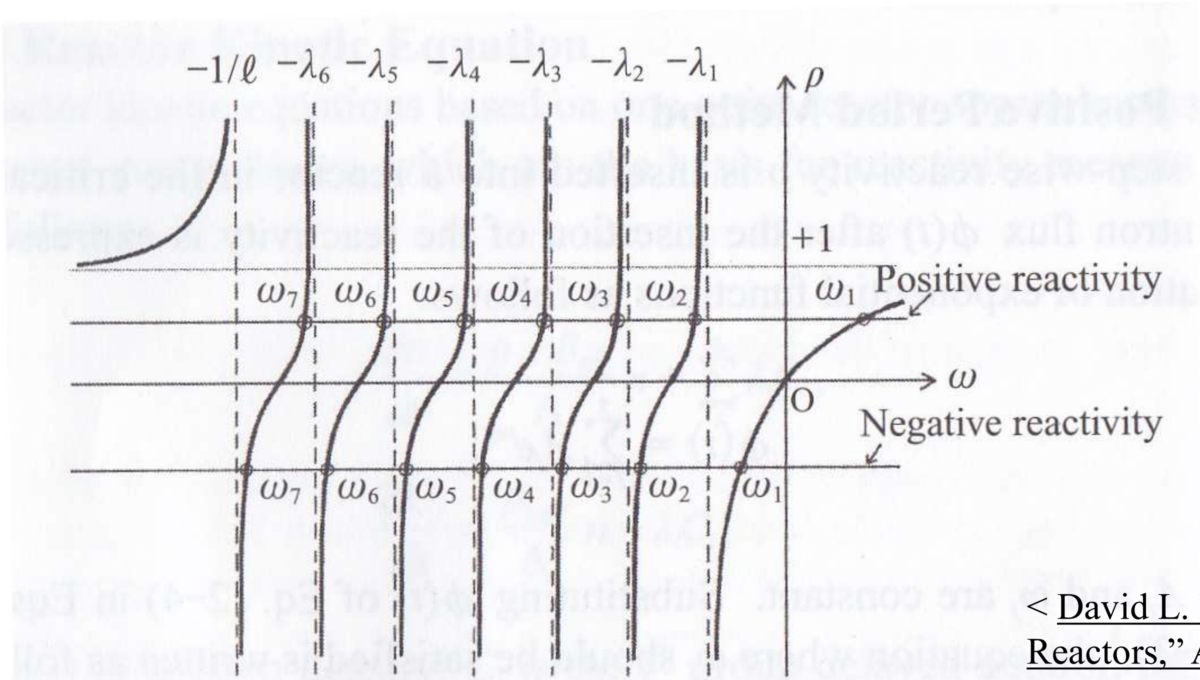
$\cdots\cdots$ (7)

Relation betw. Reactivity and Reactor Period

- When a step-wise reactivity change from critical state happen as ρ, the neutron level $n(t)$ after the change of reactivity can be expressed as a summation of exponential functions as follows:

$$n(t) = \sum_{j=1}^{7} A_j \exp(\omega_j t) \qquad \text{······} \quad (5)$$

where ω_j should satisfy Eq. (4).



< Relation between reactivity ρ and ω >

< David L. Hetrick, "Dynamics of Nuclear Reactors," American Nuclear Society, Inc., IL, USA (1993) >

Relation betw. Reactivity and Reactor Period (Contd.)

- When ω_1 is the largest value among all the seven ω_j, from the figure of <Relation between reactivity ρ and ω>, one can see that only ω_1 is positive when ρ>0 and every ω_j is negative when ρ<0.
- Then, one can imagine that the time-dependent behavior of neutron population due to the reactivity change from the critical state will follow the function of $\exp(\omega_1 t)$ after contributions of the other components decay out as

$$n(t) \cong A_1 \exp(\omega_1 t) \quad (t >> 0) \qquad \text{.......... (6)}$$

- Here, the inverse of ω_1, T, is defined as the stable reactor period (or, merely, the period):

$$T \equiv \frac{1}{\omega_1} \qquad \text{.......... (7)}$$

- Because ω_1 should satisfy Eq. (4b), replacing of ω by 1/T in Eq. (4b) gives

$$\rho = \frac{(1/T)l}{(1/T)l+1} + \frac{(1/T)}{(1/T)l+1}\sum_{i=1}^{6}\frac{\beta_{i,eff}}{\lambda_i + (1/T)}$$

$$\Rightarrow \rho = \frac{l}{l+T} + \frac{T}{l+T}\sum_{i=1}^{6}\frac{\beta_{i,eff}}{\lambda_i T + 1} \qquad \text{.......... (8)}$$

Period Measurement

Experimental Procedure

1) Make a reactor critical at a low power level and stay more than 2 minutes.
2) Prepare stop watches & record sheets.
3) Read and write an initial counts from your detector choice.
4) Move up coarse CR by 1cm in one push action and read and write detector counts at every 10 seconds during at least 200 seconds.
5) After reading, make the reactor critical at the new power level and stay more than 2 minutes.

Exchange the roles of each person and do the same procedures with different control rod (fine CR) or different reactivity insertions (rod move down) mode.

Experiment Worksheet

Experiment #2 - Reactor Period Measurement

Group #: _____ Name: ___________ Time: _________ Date: _________

Experiment Condition			Temp.	Analog Console	
Source Position				Digital Reactor	
Gamma	In/Out	/		Digital Water	
neutron	In/Out	/		Digital Room	

Time	#	Rod Position		Count Rate		
		CR	FR	Channel #	Channel #	Channel #
Initial S.S.						
0	0					
10 sec	1					
20 sec	2					
30 sec	3					
40 sec	4					
50 sec	5					
60 sec	6					
70 sec	7					
80 sec	8					
90 sec	9					
100 sec	10					
110 sec	11					
120 sec	12					
130 sec	13					
140 sec	14					
150 sec	15					
160 sec	16					
170 sec	17					
180 sec	18					
190 sec	19					
200 sec	20					

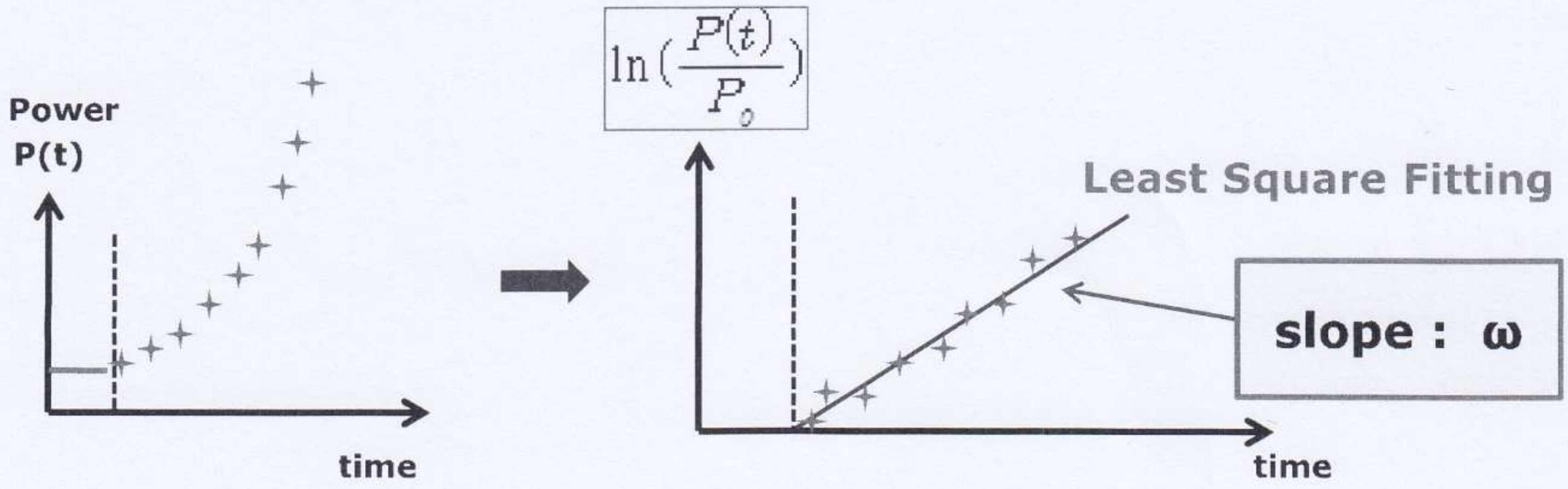
Reference Data: (from Digital Console) Period: ___________ Reactivity ___________

(from DDRCS) Reactivity ___________

Video for Exp. #2

Calculation of Period – (1) Least Square Fitting

(1) The Use of Least Square Fitting



$$P(t) = P_0 e^{\omega t} = P_0 e^{t/T}$$

$$\ln \frac{P(t)}{P_0} = \omega t \rightarrow f(t) = \omega t$$

$$T = \frac{1}{\omega}$$

Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

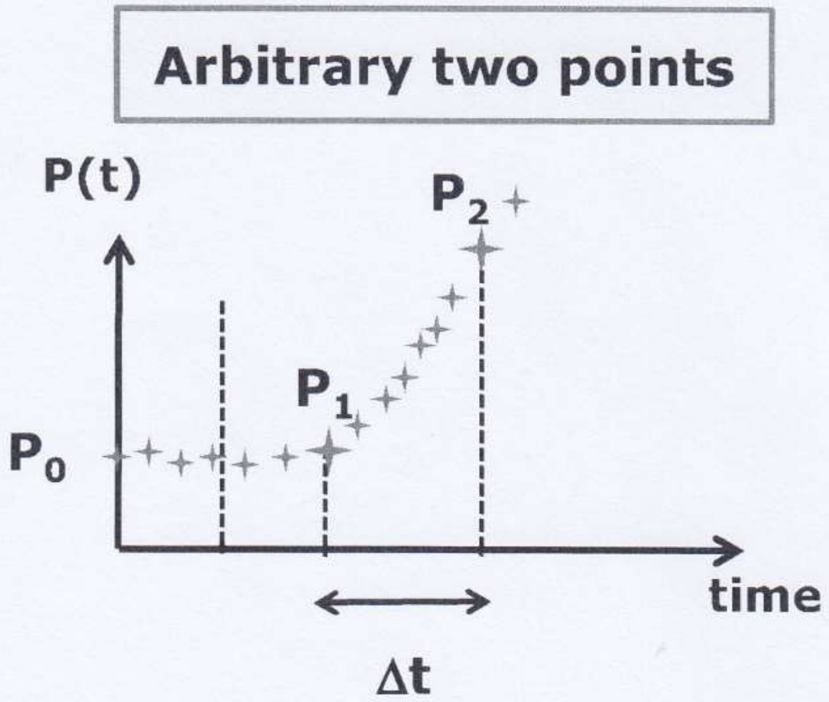
Calculation of Period – (2) Two Point Calculation

(2) The Use of the Simple Math

$$P_2 = P_1 e^{\omega \Delta t} = P_1 e^{\Delta t / T}$$

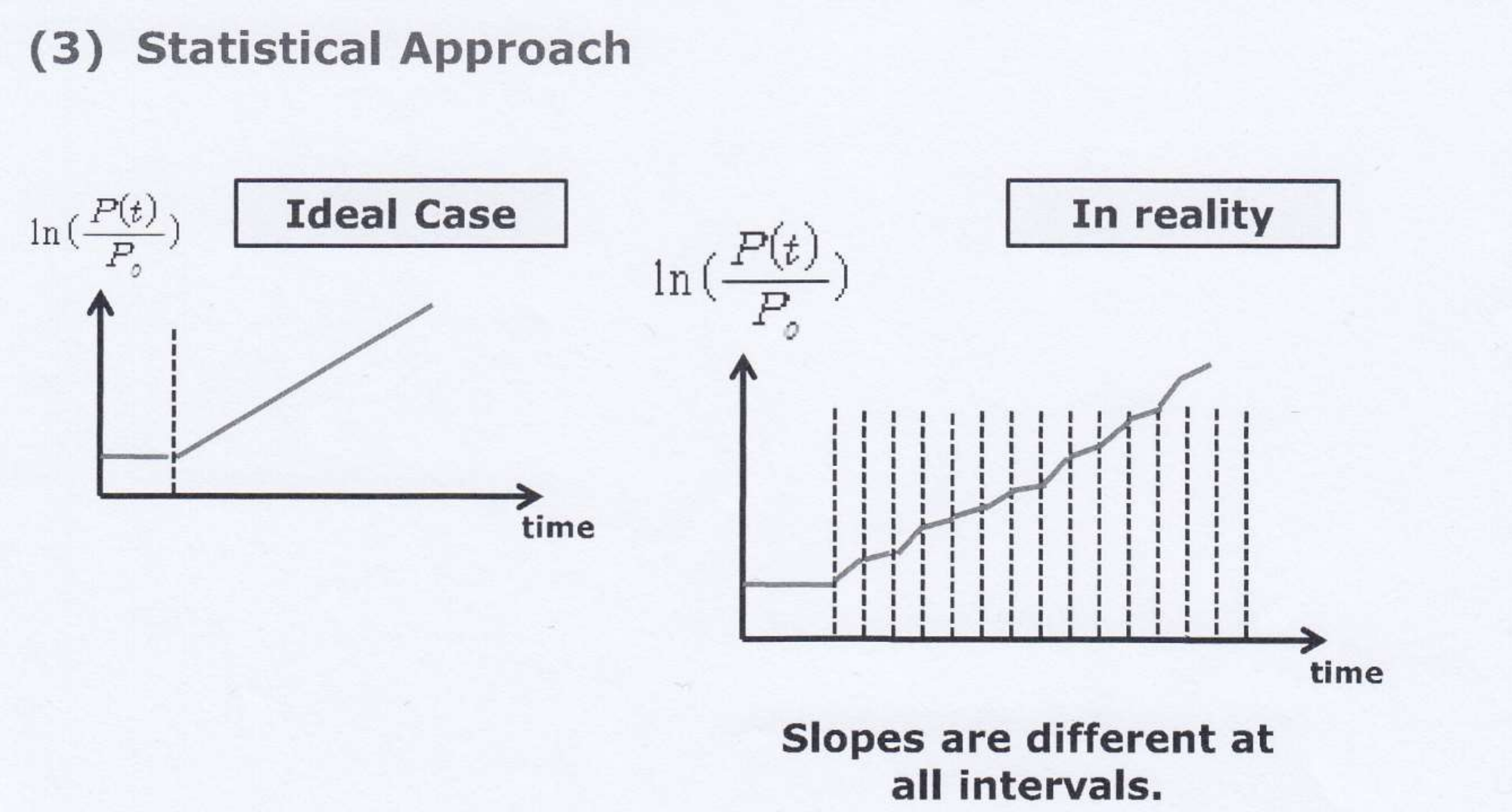
$$\omega = \frac{\ln(P_2 / P_1)}{\Delta t}$$

$$T = \frac{1}{\omega}$$



Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

Calculation of Period – (3) Statistical Approach



Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

(3) Statistical Approach (Contd.)

t	P_i (Amp.)	$\ln P_i$	$\ln \frac{P_i}{P_{i-1}}$	Δt_i	$\left(\frac{\Delta \ln P}{\Delta t}\right)_i$	$\Delta \chi_i$	$(\Delta \chi_i)^2$
0	2.8×10^{-11}	−24.29882	0	0			
10	3.7×10^{-11}	−24.02010	0.27872	10	0.027872	0.014936	2.231×10^{-4}
20	4.3×10^{-11}	−23.86982	0.15028	10	0.015028	0.002092	4.376×10^{-6}
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
150							
		$\Delta \ln P = 0.12936$			Average	$(\Delta Xi)^2 = 2.89\times10^{-4}$	

$$\overline{\omega} = \left(\overline{\frac{\Delta \ln P}{\Delta t}}\right)$$

$$\Delta \chi_i = \omega_i - \overline{\omega}$$

$$\sigma^2 = \frac{1}{N-1}\sum_i (\Delta \chi_i)^2$$

$$T = \frac{1}{\overline{\omega}} \pm \frac{\sigma}{\overline{\omega}}$$

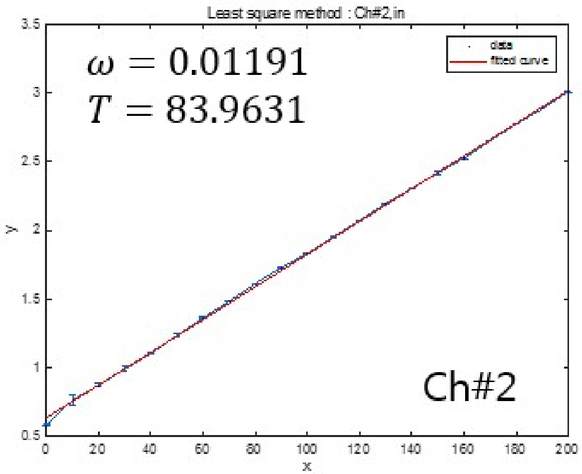
Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

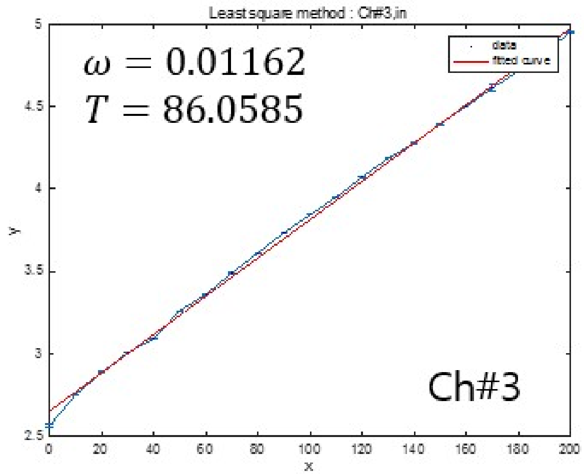
Example of Measurements

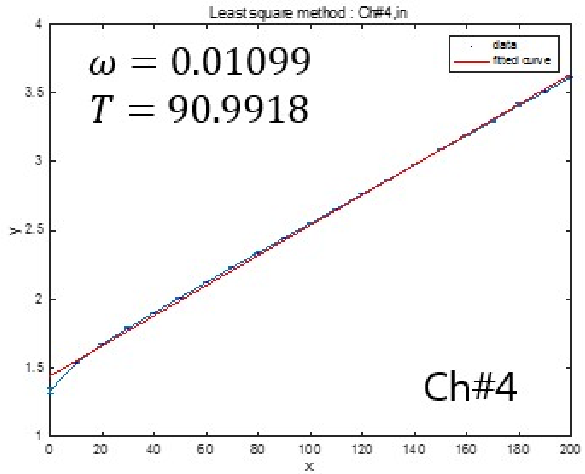
1. CR 1cm 상승				이재국	김진수	박태준	평균	재욱	구자룡	김경민	평균	박종현	박상진	박경찬	평균
Time	#	CR	FR	Ch#2	Ch#2	Ch#2		Ch#3 [%]	Ch#3	Ch#3		Ch#4	Ch#4	Ch#4	
Initial S.S.		19.57	15.64	1.4	1.34	1.38	1.373333	9.5	9.75	9.4	9.55	3.46	3.45	3.475	3.461667
0	0	20.57	15.64	1.78	1.78	1.8	1.786667	12.9	13.2	12.9	13	3.86	3.74	3.76	3.786667
10 sec	1			2.2	2.2	2.04	2.146667	15.7	15.7	15.7	15.7	4.64	4.64	4.64	4.64
20 sec	2			2.4	2.35	2.41	2.386667	18.05	17.85	17.85	17.91667	5.3	5.3	5.3	5.3
30 sec	3			2.66	2.66	2.75	2.69	20.25	20.25	20.25	20.25	5.97	6	5.97	5.98
40 sec	4			3.03	2.968	3.03	3.009333	22	22	22	22	6.69	6.69	6.69	6.69
50 sec	5			3.4	3.408	3.5	3.436	26.1	26.1	26.1	26.1	7.49	7.46	7.46	7.47
60 sec	6			3.91	3.86	3.93	3.9	28.8	28.8	28.8	28.8	8.33	8.33	8.33	8.33
70 sec	7			4.41	4.415	4.41	4.411667	32.65	32.95	32.95	32.85	9.31	9.31	9.31	9.31
80 sec	8			5.01	5.01	5.01	5.01	37.1	37.1	37.1	37.1	10.37	10.3	10.37	10.34667
90 sec	9			5.65	5.65	5.59	5.63	41.85	42.05	41.85	41.91667	11.51	11.57	11.51	11.53
100 sec	10			6.23	6.23	6.27	6.243333	46.75	46.75	46.75	46.75	12.82	12.82	12.78	12.80667
110 sec	11			7.03	7.07	7.03	7.043333	52	52	52	52	14.24	14.24	14.24	14.24
120 sec	12			7.94	7.94	7.94	7.94	58.65	58.65	58.85	58.71667	15.87	15.87	15.87	15.87
130 sec	13			8.96	8.96	9.01	8.976667	65.8	65.8	65.8	65.8	17.6	17.6	17.6	17.6
140 sec	14			10	10.007	10.1	10.03567	73.05	71.75	71.75	72.18333	19.63	19.63	19.63	19.63
150 sec	15			11.1	11.15	11.4	11.21667	81.7	80.9	80.9	81.16667	22	21.9	21.91	21.93667
160 sec	16			12.5	12.44	12.5	12.48	90.25	90.25	90.25	90.25	24.23	24.27	24.2	24.23333
170 sec	17			14.2	14.2	14.2	14.2	99.14	99.15	104.5	100.93	27	27	26.9	26.96667
180 sec	18			16	16	16	16	111.79	111.8	111.79	111.7933	30	30.07	30.77	30.28
190 sec	19			17.9	17.88	17.9	17.89333	125.07	125.1	125.07	125.08	33.34	33.43	33.34	33.37
200 sec	20			20.3	20.18	20.3	20.26	141.35	141.35	141.83	141.51	37.13	37.13	37.13	37.13
			Period	Reactivity											
Reference Data	from Digital Console		78 sec	0.105 $											
	from DDRCS			66.2 pcm											

Example of Experimental Results

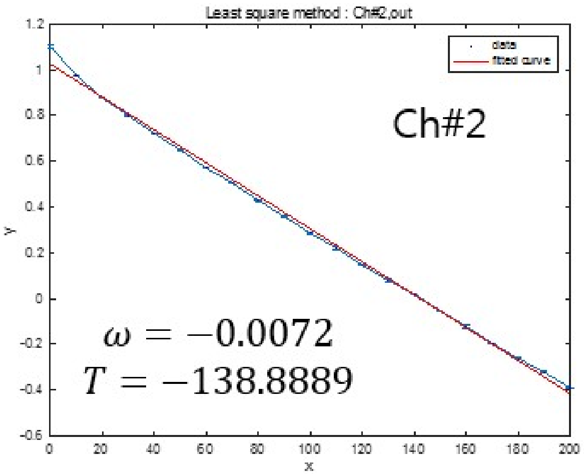
- Control rod 1cm in :

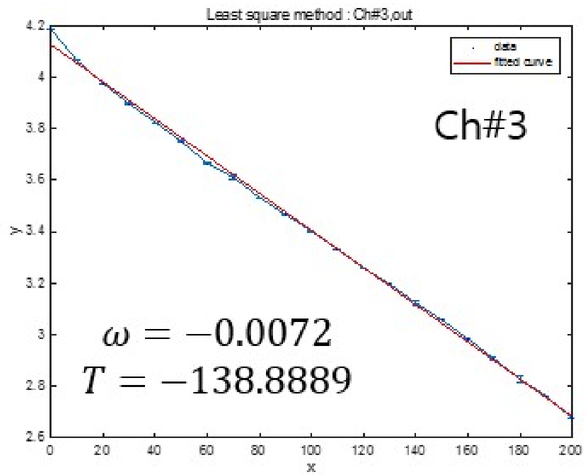


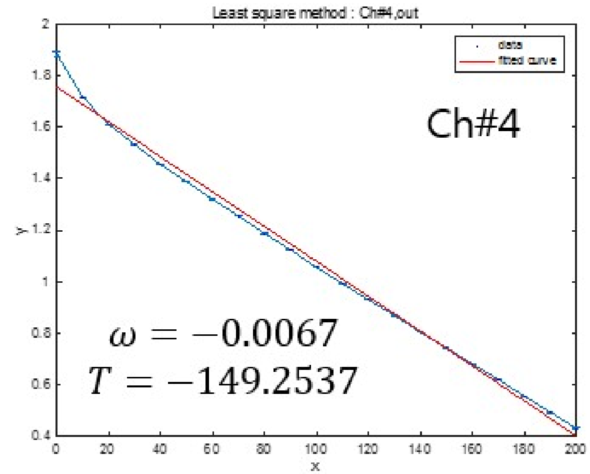




- Control rod 1cm out :







Discussion Points

1. What is the most reliable method for period measurement among three based on your experience?

2. If you want to apply for the nuclear power plant, which method will be adaptable to a real world?

3. Did you find any differences from different detectors in use?

4. Did you find the expected results from different experimental runs done by other groups?