Rheological Response of Bituminous Concrete

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This research is part of a continuing effort to develop more rational methods of flexible pavement design. This study has evaluated in the laboratory the temperature-dependent viscoelastic response of five bituminous concrete mixes representing several major categories used in road surfacing. The complicated dependence of the mechanical properties of bitumen-aggregate compositions on the principal variables of loading time and temperature were separated to yield time functions and functions of temperature.

The validity of the time-temperature superposition concept to these materials was verified to a satisfactory degree of accuracy by aggrement between data obtained from dynamic tests and values predicted by superposition of compression creep measurements over a range of temperature from 0 to 120 F at stress levels in the linear viscoelastic range. This made it possible to define the properties of the bituminous concrete mixes by three general functions: the dependence of the magnitude and phase of the complex modulus on reduced frequency and the temperature dependence of the temperature shift factor, a_T. Experimental verification of the phenomenological linear viscoelastic response of bituminous concrete was obtained by unconfined dynamic and creep experiments to a useful degree of approximation.

The experimental results of complex and creep moduli cover approximately twelve decades of reduced frequency and time, respectively. The time-temperature superposition principle used in this study allowed the viscoelastic data to be extended from 10^{-6} to 10^{6} seconds, a range normally inaccessible by conventional experimental methods. This principle also allows the creep and dynamic strength moduli of bituminous concrete mixtures to be evaluated at any intermediate temperature within the tested experimental range by a relatively few tests. Methods are also presented to incorporate the mechanical properties of the material, evaluated by use of the thermodynamic and rheologic concepts advanced in this study, into pavement design procedures.

•EFFECTIVE UTILIZATION of engineering materials must start with a complete understanding of the materials involved. As new materials are developed, and as the use of existing ones are extended, the evaluation of their engineering properties becomes more significant. The study of the mechanical properties of bituminous concrete is by no means an easy task. This is due to the complex composition of the material, and the variety of environmental and loading conditions under which this material must function. Engineering is an applied science and the ability to simplify and idealize is a requirement to achieve workable solutions to practical problems. The evaluation of the temperature-dependent rheological properties of bitumen-mineral aggregate compositions is no exception, and some degree of idealization is inevitable even in the most sophisticated solutions.

Despite the fact that highway pavements have been constructed from the earliest civilizations, all methods of flexible pavement design have the common disadvantage of

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lacking a completely rational basis. The procedure used at this time consists of applying one of several design methods. These design empiricisms rely on past experience, economics, judgment of the design engineer, and correlations of pavement performance with laboratory tests.

In recent years there has been an increase of interest in the development of a theoretical method for the design of flexible pavement structures. This growing interest in a more rational design procedure has evolved from the limitations imposed by the empirical methods in extrapolating beyond the boundaries of previous experience to additional pavement-climate-load problems. If design procedures are to be used beyond their assigned limitations, it seems that theoretical, rather than empirical, methods would provide a better foundation for such extensions.

One of the important requirements for the development of a rational method of flexible pavement design is a procedure which evaluates the mechanical properties of the components of such a layered system under any type of applied load, at any environmental or loading condition. A step in this direction has been taken in this study which has evaluated the complicated dependence of the mechanical properties of five bituminous concrete mixtures representing several major categories used in road surfacing on the important variables of loading time and environmental temperature.

The present study was concerned with defining the rheological response of bituminous concrete over a wide range of loading times or frequencies and with evaluating the temperature dependence of these properties. The linear viscoelastic response of high polymers has been successfully investigated by using the concepts of rheology (1, 2, 3, 4) and a great deal of fundamental information has been developed in this area. Analyses of the viscoelastic behavior of bituminous concrete mixes have been performed using rheologic concepts (5, 6, 7, 8) and this basic research has served as a valuable reference to asphalt technologists in establishing the nature of bituminous concrete. The temperature dependence of the viscoelastic response of bitumens and high polymers has been studied using time-temperature superposition concepts (9, 10, 11). However, the concepts of the kinetic theory and time-temperature superposition techniques or method of reduced variables (1, 12) at the time this study was started have not been extensively applied to the analysis of the mechanical response of bituminous concrete in the time and frequency domains. This paper is concerned with the validity of the application of these useful concepts to bituminous compositions. Therefore, experimental data are presented which compare the rheological moduli of bituminous mixes evaluated by unconfined creep and dynamic tests on the phenomenological level.

Experimental results of two types of test data are presented and discussed: (a) the results of constant load tests used to determine the transient response of the mixture, and (b) the results of direct dynamic and resonant vibration tests of the same material. An essential part of the study is a presentation of the mathematical relations necessary to evaluate the rheologic response of the material by a generalized Voigt linear viscoelastic model for both types of tests.

The objective of this study was to analyze the method of reduced variables concerning the effect of temperature on the mechanical behavior of thermally sensitive materials such as bituminous concrete. The purpose of applying this concept was to determine if time and temperature may have an approximately equivalent effect on the viscoelastic response of bituminous mixes, and if it would be possible to predict the response of the mixture by a relatively few experiments for any temperature within the tested range and at loading times or frequencies both longer and shorter than are normally obtained experimentally. The specific goal of the study was to define the time and temperature dependent mechanical properties of bituminous mixes over a wide range of environmental and loading conditions.

EXPERIMENTAL TECHNIQUES

To evaluate the mechanical properties of the dense-graded asphaltic concrete mixes, two basic types of tests were performed. Creep and dynamic tests allowed the mechanical properties of these pavement materials to be determined independently by each type of test. The applicability of the linear viscoelastic theory and time-temperature superposition concept to represent the response of asphaltic concrete to a useful degree of approximation was also investigated by both types of tests.

Sinusoidal-Stress Dynamic Tests

To supplement the creep tests and provide information about the dynamic response of bituminous concrete mixes at very short loading time intervals, direct dynamic tests were performed in which the stress varied periodically with a sinusoidal alternation at a frequency, ω. In the dynamic tests a periodic stress was applied to the unconfined specimens and the resulting axial and circumferential strains were recorded using the electronic recorder. This sinusoidal stress was applied by a repetitional loading machine (Fig. 1) designed and built at The Ohio State University's Engineering Experiment Station (13). The equipment consists of an electrical constant speed motor which drives a sinusoidal cam of double eccentricity through a set of interchangeable gears, by which it is possible to vary the frequency of applied load. The sinusoidal stress applied to the samples was measured by a load cell connected to the electronic recorder. Two pairs of SR-4 strain gages were attached to the specimens, one pair attached vertically and the other pair attached horizontally to measure the vertical and lateral strains of the test samples, respectively. The two pairs of strain gages were attached to each specimen at mid-height at diametrically opposite points on the periphery of the test cylinders.

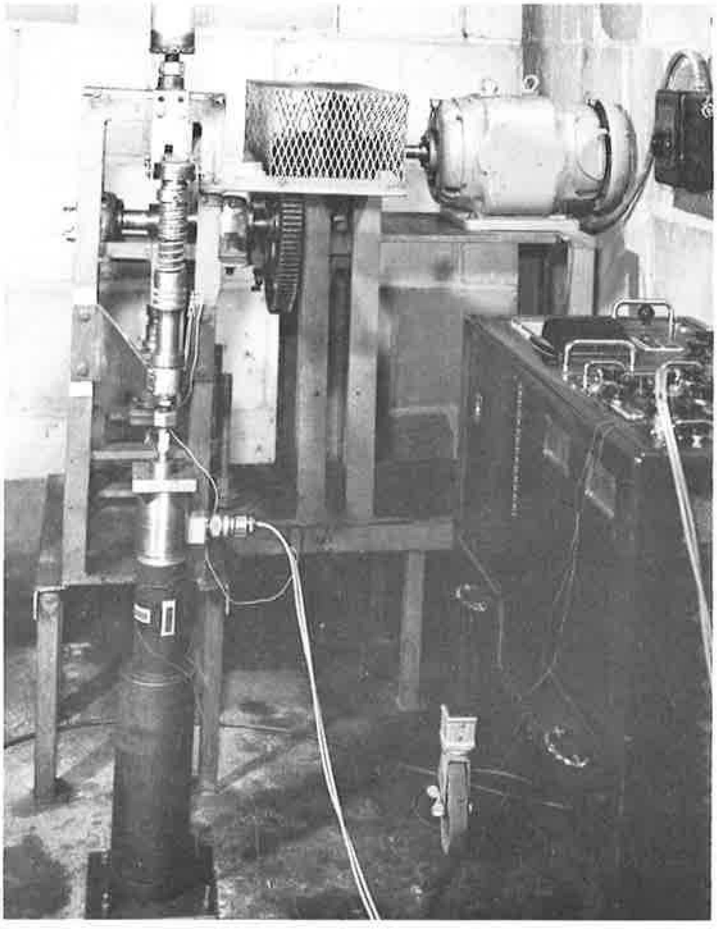

Figure 1. Dynamic test equipment.

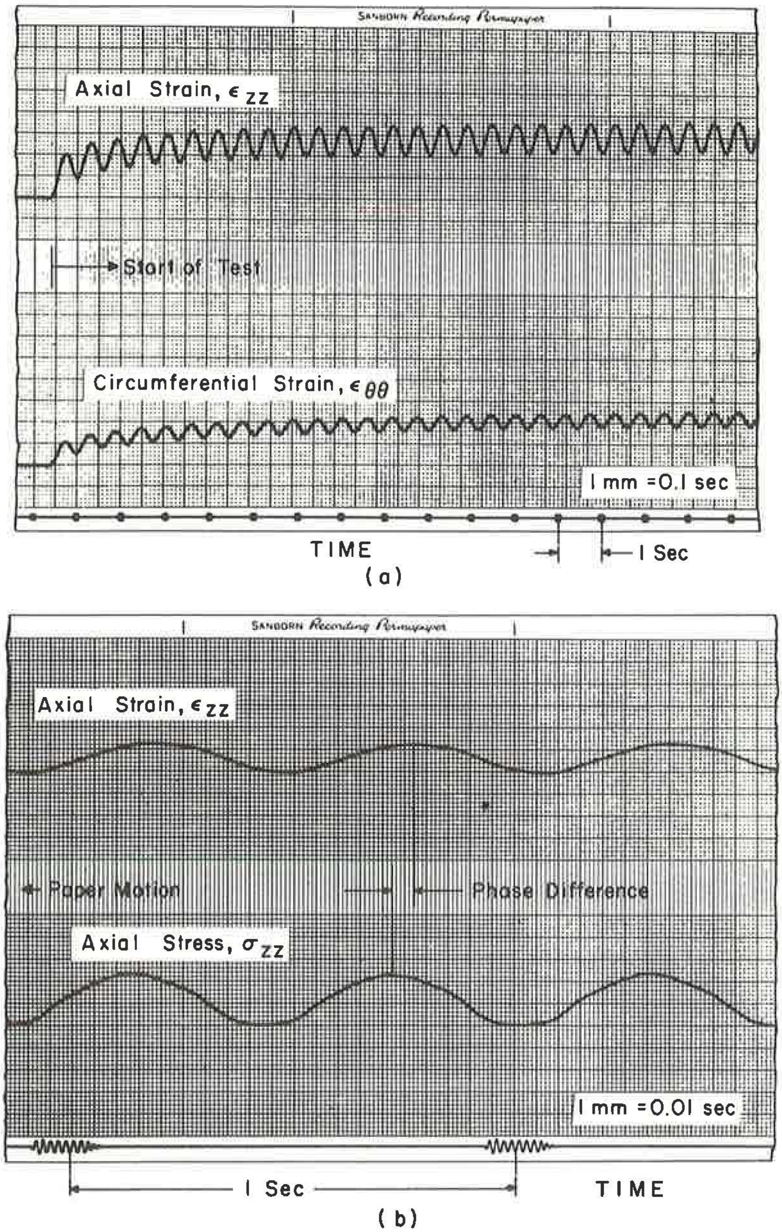

Figure 2. Dynamic test recordings: (a) strain and (b) stress.

Figure 2 shows an actual recording of the stress and strains in a dynamic test. All tests were conducted under isothermal conditions at several frequencies and stress levels. The samples were enclosed in rubber triaxial membranes and placed in a constant temperature water bath for several hours before and during testing. A new sample was used in each test to eliminate any variations due to changes in physical properties as a result of previous testing (14).

Compression Creep Tests

In the creep tests conducted, a constant axial load was quickly applied to the unconfined samples without impact and kept constant throughout the test by a soil consolidation apparatus shown in Figure 3. Continuous recordings of the axial and circumferential strains were obtained by an electronic recorder. A typical test recording of the axial and circumferential strain measured by SR-4 gages, obtained by the electronic recorder, is shown in Figure 4. Vertical deformation of the samples under load was also checked by a linear displacement transducer and by Ames dials. Tests were performed under controlled temperature conditions at several loads at eleven temperatures from 0 to 120 F. The samples were instrumented and strains recorded in a manner similar to the procedure for the identical mixes used in the dynamic tests.

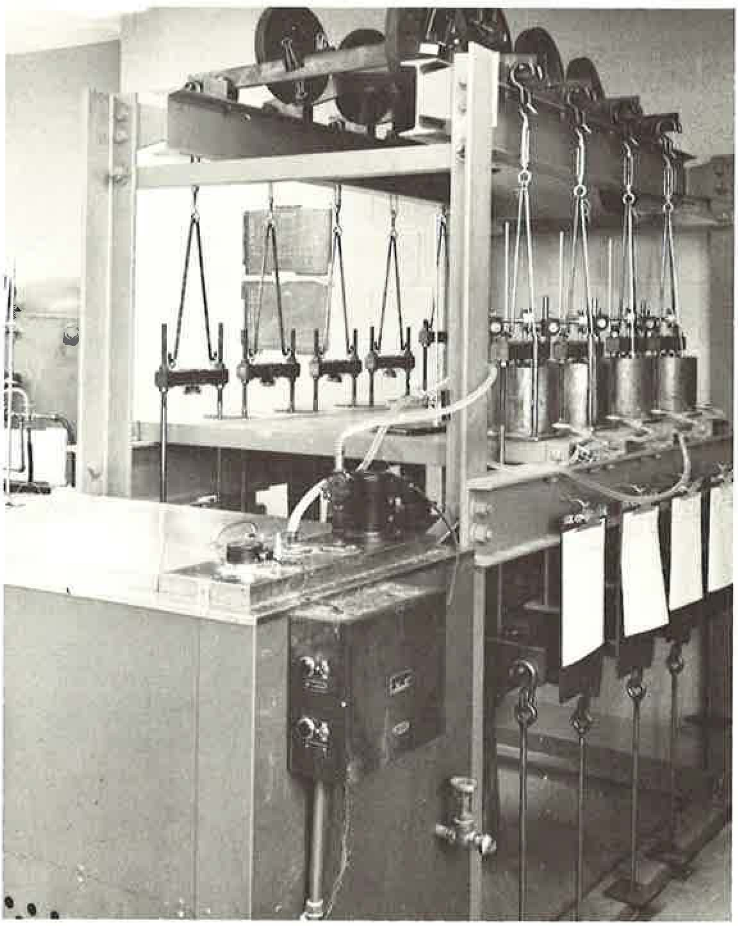

Figure 3. Creep test equipment.

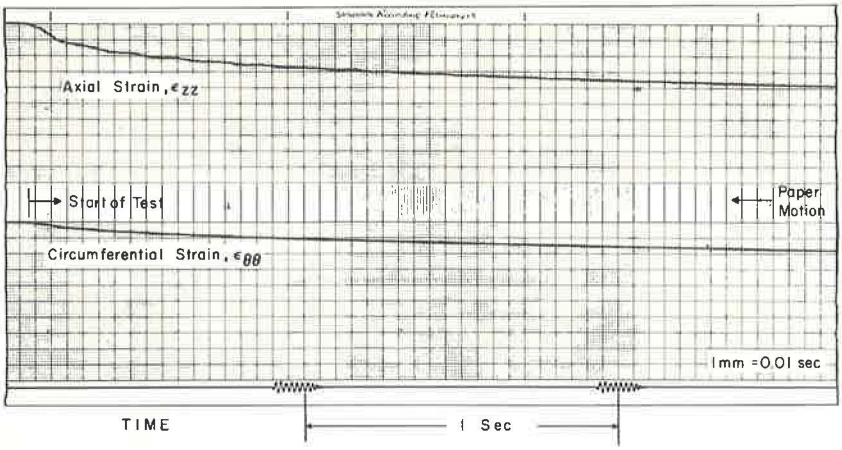

Figure 4. Creep test recordings.

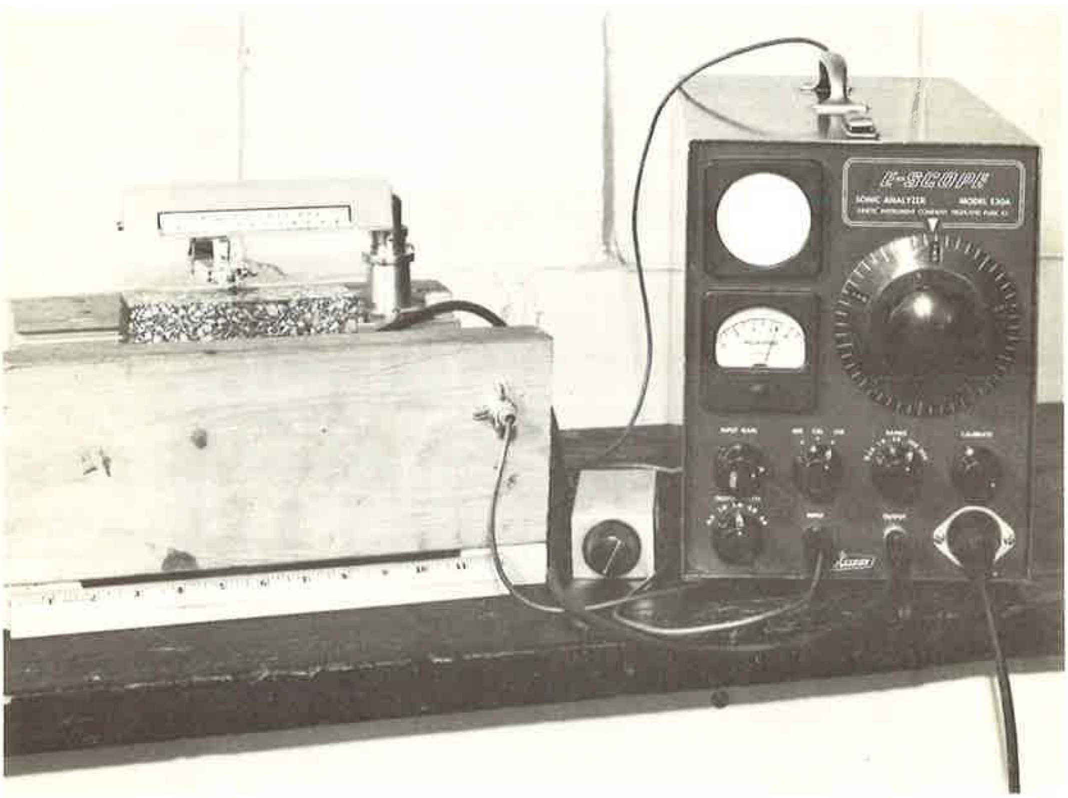

Figure 5. Sonic equipment showing E-scope and testing frame.

Sonic Dynamic Tests

Dynamic resonant frequency tests were also performed on prismatic specimens sawed from the cylindrical samples. The two properties measured in the sonic experiments to characterize the dynamic behavior of the materials were the real component of the complex modulus and the phase angle. The determination of $E_1(\omega)$ and ϕ_E from the resonant frequency vibrations was performed following ASTM Designation C 125-60 and procedures found in other similar references (15, 16). Figure 5 is a photograph of the resonant frequency test in progress showing the sonic apparatus.

PREPARATION OF SAMPLES

To obtain the maximum homogeneity and isotropy in the clyindrical samples, a gyratory compactor was used to mold the specimens. Proportioning of the aggregates and asphalt was performed on the basis of weight. The crushed Columbus and AASHO limestone aggregates were first separated into various sieved sizes, oven-dried and then combined into individual batches with exact proportions. The batches were heated for 8 hr along with the mixing bowls and molds in an oven maintained at 325 F. The exact amount of asphalt which had been heated to 275 F was combined with the heated aggregates, and then thoroughly mixed in a mechanical mixer for a minimum of 2 min or until the aggregate surface was coated with asphalt. The mix was placed in the gyratory compactor mold using a standard procedure which required the mix to be placed in two layers and each layer to be spaded vigorously 25 times. A fixed axial pressure was applied to the samples in the gyratory compactor and maintained constant while the gyration angle of 2° was set and the desired number of gyrations were applied. The samples which measured 2.8 in. in diameter and approximately 6 in. in height were stored before testing at room temperature until age effects (17) had become negligible. Tables 1 through 4 summarize the density and void analyses of the samples, asphalt content and aggregate gradations used, as well as the gyratory compaction details. Included in these tables are the identification tests performed on the bitumens: penetration, specific gravity, and the kinematic viscosity measured by a sliding plate microviscometer. A complete description of the preparation of samples and physical properties of the materials used may be found elsewhere (18).

The degree of homogeneity of the specimens used in this study was analyzed by evaluation of the bulk densities of all cylindrical samples and also small cubes sawed from representative samples. To determine degree of isotropy of the samples, the ultimate compressive strength of the sawed cubes was obtained by performing a constant-rate-of-deformation test on different planes of the cubes. Analyses of bulk densities and ultimate compres-

TABLE 1

DENSITY AND VOID ANALYSIS

Analysis	Mix Designation				
	300	500	700	800	900
Avg. bulk density of samples	2.35	2.28	2.26	2.28	2.34
Max theoretical density	2.40	2.35	2.42	2.39	2.35
Sp gr of total agg.	2.61	2.55	2.58	2.59	2.59
Percent of max theoretical density	97.90	96.90	93.40	95.30	99.50
Vol of voids (% total vol)	2.10	3.10	6.60	4.70	0.50
Vol of total agg. (% total vol)	84.90	84.30	83.50	83.00	84.20
Vol of bitumen cem. (% total vol)	13.00	12.72	10.30	12.60	15.20
Percent of agg. voids	15.1	15.7	16.5	17.0	15.8
Percent of agg. voids filled	86.2	81.0	62.4	74.1	96.3

TABLE 2

SPECIFIC GRAVITY AND PENETRATION GRADE OF BITUMEN USED

Commercial Penetration Limits	Type	Sp Gr	Actual Penetration[a]
85 - 100	AA	1.0209	84
85 - 100	A	1.0309	86
100 - 120	L	1.0295	102

[a]Of 100 g in 5 sec at 77 F.

TABLE 3

COMPACTION DETAILS, BITUMEN CONTENT AND BULK SPECIFIC GRAVITY OF MATERIALS

Analysis	Mix Designation				
	300	500	700	800	900
Mixture type	A	B	C	B	D
Bitumen type	100 - 120 L	85 - 100 AA	85 - 100 A	85 - 100 A	85 - 100 A
Bitumen (% by wt)	5.7	5.7	4.7	5.7	6.7
Gyratory compaction pressure (psi)	244	244	244	244	244
No. gyrations	30	45	45	45	45
Gyratory angle (deg)	2	2	2	2	2
Avg bulk sp gr	2.35	2.28	2.26	2.28	2.34

TABLE 4

BITUMINOUS CONCRETE MIXTURE PROPORTIONS

Type of Material	Passing Sieve	Retained on Sieve	Percent[a]	Percent[b]	Wt per Mix (g)
(a) Type A - Columbus Limestone					
Crushed limestone	½ in.	⅜ in.	10.7	10.1	141
	⅜ in.	No. 4	16.5	15.6	218
	No. 4	No. 10	20.5	19.3	270
	No. 10	No. 20	16.5	15.6	218
	No. 20	No. 40	13.7	12.9	180
	No. 40	No. 80	11.7	11.0	154
	No. 80	No. 200	7.8	7.3	104
Crushed limestone filler	No. 200	—	2.6	2.5	35
Total	—	—	100.0	94.3	—
Bitumen cement	—	—	—	5.7	80
Total	—	—	—	100.0	1,400
(b) Type B - AASHO Materials					
AASHO coarse	⅜ in.	No. 4	18.4	17.4	244
AASHO coarse sand	No. 4	No. 10	22.9	21.6	302
	No. 10	No. 20	18.4	17.4	244
	No. 20	No. 40	15.3	14.4	201
AASHO fine sand	No. 40	No. 80	13.2	12.4	173
	No. 80	No. 200	8.8	8.3	117
Limestone filler	No. 200	—	3.0	2.8	39
Total	—	—	100.0	94.3	—
Bitumen cement	—	—	—	5.7	80
Total	—	—	—	100.0	1,400
(c) Type C - Columbus Limestone and Natural Sand					
Crushed limestone	⅜ in.	No. 4	18.5	17.6	244
	No. 4	No. 10	23.0	21.9	302
	No. 10	No. 20	18.5	17.6	244
Coarse sand	No. 20	No. 40	15.2	14.5	201
Fine Sand + limestone	No. 40	No. 80	13.1	12.5	173
	No. 80	No. 200	8.7	8.4	117
Limestone filler	No. 200	—	3.0	2.8	39
Total	—	—	100.0	95.3	—
Bitumen cement	—	—	—	4.7	65
Total	—	—	—	100.0	1,385
(d) Type D - Columbus Limestone and Natural Sand					
Crushed limestone	⅜ in.	No. 4	18.4	17.2	244
	No. 4	No. 10	23.0	21.4	302
	No. 10	No. 20	18.4	17.2	244
Coarse sand	No. 20	No. 40	15.2	14.2	201
Fine Sand + limestone	No. 40	No. 80	13.1	12.2	173
	No. 80	No. 200	8.9	8.3	117
Limestone filler	No. 200	—	3.0	2.8	39
Total	—	—	100.0	93.3	—
Bitumen cement	—	—	—	6.7	95
Total	—	—	—	100.0	1,415

[a]By wt of aggregate. [b]By wt of total mix.

sive strengths of the experimental mixtures showed the specimens prepared for this study were quite homogeneous and isotropic.

LINEAR VISCOELASTIC THEORY

Phenomenological Treatment of Linear Viscoelastic Behavior

The classical elastic theory deals with the response of purely elastic materials where, in accordance with Hooke's law, stress is directly proportional to strain. However, elastic materials are idealizations. Real materials existing in nature generally show stress, temperature, and time anomalies. A viscoelastic material is one which exhibits both elastic and viscous characteristics, and stress is related to strain by a function of time in the linear viscoelastic range. To describe the response of such a material using the model representation, a mechanical system is used consisting of Hookean springs and Newtonian dashpots connected in series or parallel in various configurations.

The mechanical behavior of materials may be approximated by models composed of a finite number of linear springs and dashpots such as the Kelvin, Maxwell, and Burgers models (19, 20) and have been employed by many engineers. However, if the mechanical behavior of engineering materials is observed closely, it is realized that these models are too simple and cannot adequately depict the response of real materials. Several authors (21, 22) have suggested that it is possible to represent the response of viscoelastic bodies using more refined models consisting of a larger number of elements in the model. The generalized Voigt and Maxwell models consisting of n + 1 and n number of elastic and viscous elements are shown in Figures 6a and 6b, respectively.

The generalized Voigt model used in this study shows instantaneous elasticity, E_o, Newtonian flow with viscosity, η_o, and n different retardation times, τ_i, due to n Kelvin elements connected in series. The Kelvin element with retardation time, τ_i, has a spring with elasticity, E_i, and a dashpot with viscosity, η_i. The retardation time, τ_i, is equal to η_i/E_i and has units of time. Under a constant stress the generalized Voigt model exhibits creep behavior; the creep compliance, $J_c(t)$, used in this study is defined as the constant axial stress, σ, divided by the time-dependent axial strain, $\epsilon_{zz}(t)$:

$$J_c(t) = \frac{1}{E_o} + \frac{t}{\eta_o} + \sum_{i=1}^{n} \frac{1}{E_i}\left(1 - e^{-t/\tau_i}\right) \quad (1)$$

Such a model under given conditions has constant parameters defining the material properties. However, if an outside factor, such as temperature, influences the response of the material, the elements of the model will be functions of temperature.

A continuous model can be constructed starting from the generalized Voigt model which consists of a single spring, a single dashpot, and an infinite number of Kelvin units connected in series. In the continuous model, the Kelvin units with retardation times between τ and $\tau + d\tau$ will contribute to the compliance. The function $L(\tau)$ is called the distribution function of retardation times or retardation spectrum, where $0 < \tau < \infty$. The creep behavior of such a model may be obtained from the creep compliance of the generalized Voigt model. Using the logarithmic

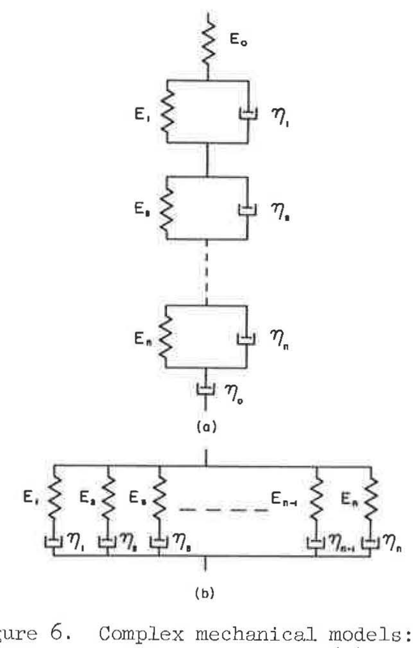

Figure 6. Complex mechanical models: (a) generalized Voigt model and (b) generalized Maxwell model.

distribution function, L (ln τ), the creep response can be represented by the following equation:

$$J_c(t) = \frac{1}{E_o} + \frac{t}{\eta_o} + \int_{-\infty}^{\infty} L(\ln \tau)\left(1 - e^{-t/\tau}\right) d(\ln \tau) \tag{2}$$

Once the retardation spectrum has been evaluated by creep experiments for a linear viscoelastic material, the dynamic response of the material, such as the complex modulus, can be obtained (1, 22).

Similar concepts may be used to evaluate the transverse modulus, T_c (t), which is defined as the constant axial stress divided by the time-dependent circumferential strain, $\epsilon_{\theta\theta}$ (t). The complete theory reviewed for the creep compliance remains valid for all moduli and compliances.

Dynamic Response of Materials

In the previous section, static tests were considered in which the applied stress is essentially a step function and equal to zero up to a given instant and then changes discontinuously to a finite value. The response of materials is also considered in another type of test in which the stress varies sinusoidally with time, $\sigma = \sigma_o \sin \omega t$, and the material undergoes axial and lateral sinusoidal strains, $\epsilon = \epsilon_o \sin(\omega t - \phi)$, at the same frequency as the stress but lagging the stress by a phase angle (6, 23).

The concept of the complex modulus applied in this study to define the viscoelastic response of materials is based on the fact that when a sinusoidal excitation is applied to a material, the response of the material will also be sinusoidal, but out-of-phase with the stress by a phase angle.

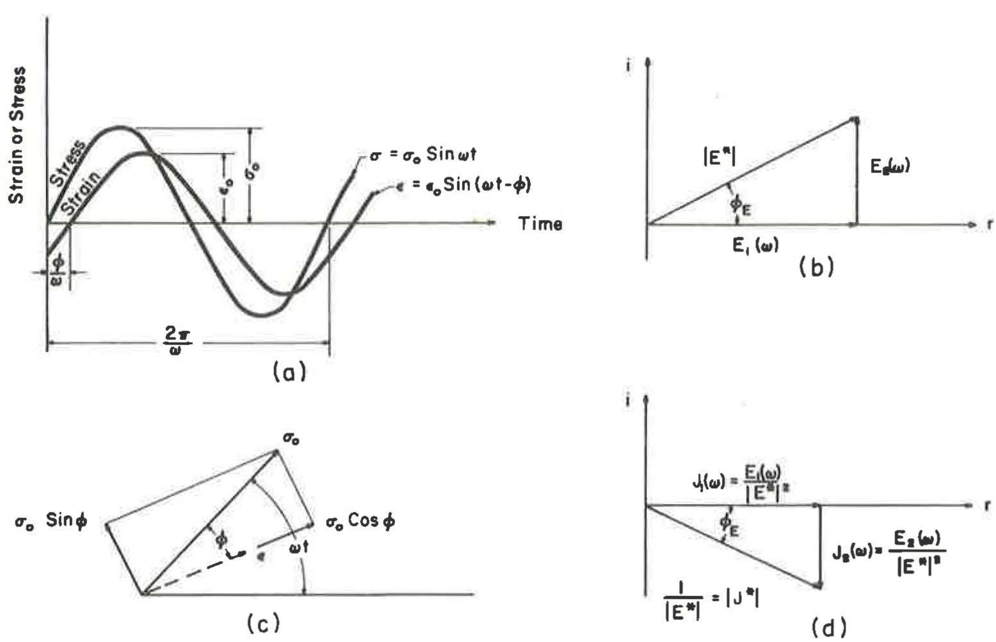

Figure 7. Dynamic viscoelastic response and vectorial resolutions: (a) steady-state response of viscoelastic material to sinusoidal strain or stress, (b) vectorial resolution of modulus components in sinusoidal deformation, (c) rotating vector representation, and (d) vectorial resolution of compliance components in sinusoidal deformation.

A linear viscoelastic material subjected to a sinusoidal stress will reach a steady-state condition after a limited number of cycles, and the amplitude of the stress, σ_0, divided by the amplitude of the strain, ϵ_0, is equal to the absolute value of the complex modulus, $|E^*|$. The phase lag, ϕ, shown in Figure 7, is the angle by which the stress leads the strain.

If a given sinusoidal stress is imposed on a material, the measurement of the amplitude of the stress and strain, as well as the angle by which the strain lags the stress, will define the response of the material at a frequency, ω. Evaluation of ϕ and $|E^*|$ at all frequencies will completely define the viscoelastic response of the material (4). The complex modulus of a material is a complex number and may be resolved into a real and imaginary part or an absolute value and phase angle:

$$E^* = E_1(\omega) + j\,E_2(\omega) \tag{3}$$

and

$$E^* = |E^*|\,e^{j\phi} \tag{4}$$

The dynamic data may also be expressed in terms of a complex compliance which is the reciprocal of the complex modulus as shown in Figure 7:

$$J^* = J_1(\omega) - j\,J_2(\omega) \tag{5}$$

The real and imaginary components of the complex compliance can be represented by the continuous Voigt model (22):

$$J_1(\omega) = \frac{1}{E_0} + \int_{-\infty}^{\infty} L(\ln\tau)\,\frac{1}{1+\omega^2\tau^2}\,d(\ln\tau) \tag{6}$$

$$J_2(\omega) = \frac{1}{\omega\,\eta_0} + \int_{-\infty}^{\infty} L(\ln\tau)\,\frac{\omega\tau}{1+\omega^2\tau^2}\,d(\ln\tau) \tag{7}$$

These equations require a knowledge of the spectrum over a wide range of the frequency scale. In principle, $L(\ln\tau)$ may be obtained from the experimental data of Eqs. (2), (6) or (7). If this is true, the dynamic response of the material can be evaluated at the same temperature using the retardation spectrum as an intermediate value in the calculations.

One important problem is how the $L(\ln\tau)$ functions are calculated from the creep functions, which are presented in graphical form in this study and applied to predict the dynamic response of the material. Because the exact methods are of limited use, the approximate methods suggested by Widder and Leaderman were used in this study (1, 6).

Three-Dimensional Viscoelastic Response

The complete theory developed for the complex elastic modulus, E^*, is also valid for all other complex moduli and compliances. Thus, similar relations may be written for the complex shear modulus, G^*, complex bulk modulus, K^*, and the complex Poisson's ratio, ν^*. The evaluation of two independent complex moduli of an isotropic, homogeneous, linear viscoelastic material allows the formation of general stress-strain equations similar in form to equations for the classical elastic body (1, 7). These equations will differ from the elastic equations in one important respect: all quantities in them are functions of frequency. In these equations E^*, ν^*, G^*, and K^* are the complex moduli of the material, and σ^* and ϵ^* are Fourier transforms of the stress and strain, respectively. A specific complex modulus used in this study is the transverse modulus, T^*, which relates the axial stress to the transverse strain.

TIME-TEMPERATURE SUPERPOSITION CONCEPT

The time-temperature superposition principle whereby phenomenological viscoelastic data at one temperature can be transformed to another temperature by a multiplicative transform of the experimental time scale was independently suggested by several authors (1, 4, 6, 9). This concept shows it may be possible to extend creep viscoelastic moduli such as E_c (t), T_c (t), or the complex moduli obtained at a given temperature to loading times both longer and shorter than can normally be obtained by experimentation. The superposition principle was used in this study to evaluate the extreme portions of the loading time scale and also to determine the creep and dynamic moduli at any intermediate temperature in the experimental range.

By use of the previously discussed concept, master curves of E_c (t), T_c (t), ϕ_E, ϕ_T, $|E^*|$, and $|T^*|$ were developed from the experimental data covering a relatively small portion of the time scale but over a wide range of the temperature scale. Refinements of this procedure are treated in the literature (9).

The empirical criterion for fulfilling the preceding conditions required that the shape of individual creep or dynamic tests data evaluated at different temperatures should coincide, within experimental error, after a horizontal shift along the logarithmic time axis. The horizontal temperature shift factor, a_T, must be evaluated empirically for each temperature, but the requirement of superposition does not permit an arbitrary selection in this choice because in both the dynamic and creep tests, the same value of a_T must be used to bring E_c (t), T_c (t), and the magnitude and phase of the complex moduli into superposition for a given temperature change.

When the criterion is not met, the applicability of reduced variables in this convenient form must be rejected. A material which meets the criterion has been called thermorheologically linear (1, 9) and is defined as one in which a change in experimental temperature alters only the position of the viscoelastic functions on the time or frequency scales and not the general shape of the curves. Even when the response of the material is only approximately thermorheologically linear, it allows the viscoelastic response of the material to be represented by two functions instead of a complex three-dimensional representation in time and temperature space.

The reduction scheme must be slightly altered to make it theoretically more satisfactory. A factor $T_0 P_0/TP$ enters into the coordinate scheme because of the entropy-spring nature of the stored elastic energy in the material as explained by the kinetic theory of rubberlike elasticity and theory of Rouse (1, 9, 24). T_0 is the standard reference temperature, P_0 is the density of the material at this temperature, and P is the density at the absolute experimental temperature, T. The kinetic theory suggests that the equilibrium modulus is proportional to the absolute temperature, and the quantity $T_0 P_0/TP\ E_c$ (t) should be governed by the superposition principle rather E_c (t). The factors of density and absolute temperature result in small vertical shifts of the viscoelastic moduli.

In regions of the time scale where the rheological moduli are changing rapidly with time, it is possible to match adjacent curves empirically without first applying a vertical shift as suggested by theory. Slightly different values of a_T will, of course, be obtained in each case. In sections where the viscoelastic function is flat, the influence of the vertical shift is more significant. However, the vertical factor is clearly indicated by the theoretical considerations.

Experimental moduli of thermorheologically linear materials may now be reduced to a standard reference temperature, T_0, of the master curve. The effect of a temperature increase from T_0 to T on a double logarithmic plot consists of a shift of the creep modulus vertically by the factor, ln $(T_0 P_0/TP)$, and horizontally by the temperature shift factor, ln a_T.

It is suggested that if one of the viscoelastic functions of bituminous concrete obeys the time-temperature relations stated previously, all other viscoelastic functions such as E_c (t), J_c (t), and the complex moduli should obey similar time-temperature relations. The results of the experimental investigation of this concept are included in the subsequent sections.

EXPERIMENTAL RESULTS

The experimental results on the macroanalytical level are presented in graphical and tabular form in an attempt to verify that to a useful degree of approximation, the bituminous mixes used in this study may be represented by thermorheologically linear materials under the experimental conditions investigated. The use of the linear viscoelastic theory to depict the response of bituminous concrete under load was also studied by both dynamic and transient tests.

Creep Tests

The objective of the creep tests was to obtain continuous recordings of the axial and transverse strain over a wide range of the time and temperature scales. For the response of a material to be described by the linear viscoelastic theory to a useful degree of approximation, the creep moduli in a constant stress test must be approximately independent of the stress level in the linear viscoelastic range.

In Figure 8 typical results of several unconfined creep tests are plotted for the 500 series mix. Only the axial stress has been varied in the experiments by simple multiples. Each individual strain curve was obtained by testing three samples under identical conditions and averaging the experimental data. Inspection of the data reveals that the vertical strain data, ϵ_{zz}, are almost proportional to the stress levels at all times, showing that the linear viscoelastic theory is a satisfactory approximation to the response of this material. The creep modulus, E_c (t), at any given time, has approximately the same value for all four stress levels tested. It is suggested that the linear viscoelastic theory will provide a higher level of approximation than the elastic theory to represent the creep response of the material over a range of temperature

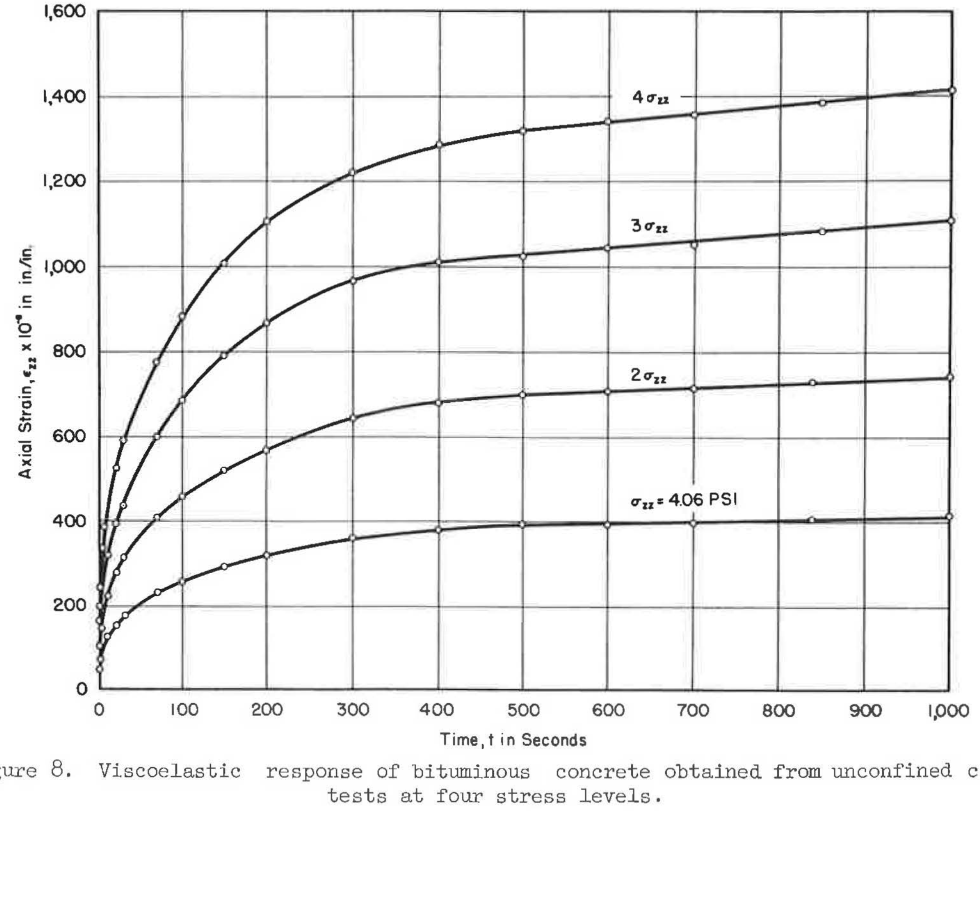

Figure 8. Viscoelastic response of bituminous concrete obtained from unconfined creep tests at four stress levels.

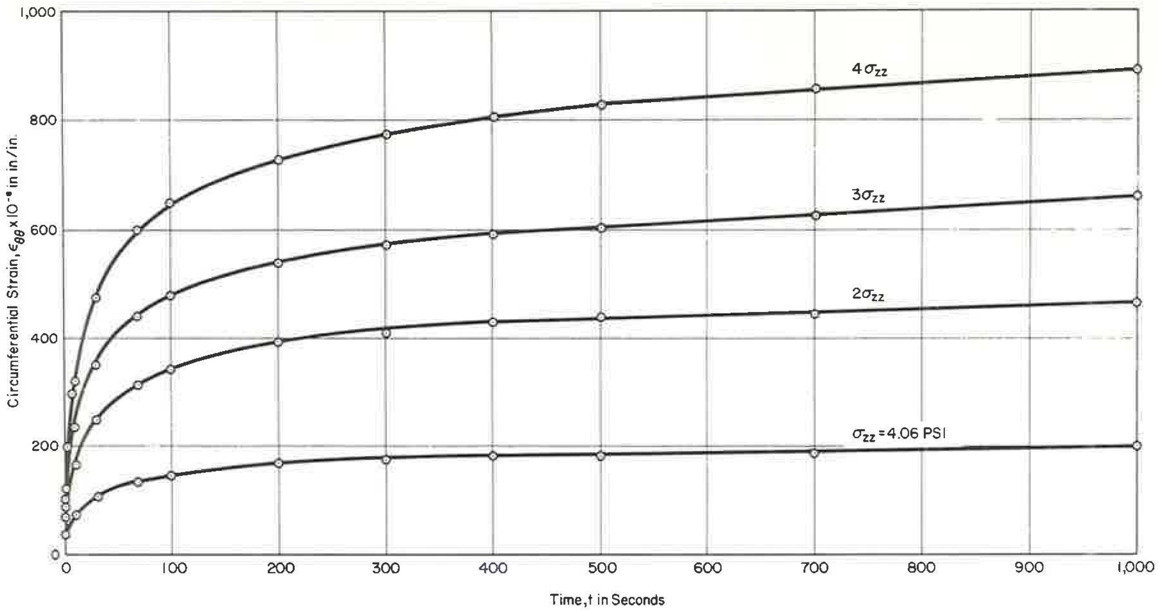

Figure 9. Circumferential strain obtained from creep tests at four stress levels.

TABLE 5

DYNAMIC EXPERIMENTAL RESULTS[a]

Variables			$\|E^*\| \times 10^6$ (psi)					ϕ_E (deg)					$\|T^*\| \times 10^6$ (psi)					ϕ_T (deg)				
Freq. (rad/sec)	Temp. (°C)	Axial Stress (psi)	300	500	700	800	900	300	500	700	800	900	300	500	700	800	900	300	500	700	800	900
10.8	36.1	4.06	0.163	0.140	0.318	—	—	41.4	36.1	26.4	—	—	0.290	0.253	0.330	—	—	49.2	39.5	39.1	—	—
		8.12	0.147	0.129	0.253	—	—	34.8	34.3	24.7	—	—	0.259	0.346	0.333	—	—	46.6	46.3	34.1	—	—
	25.0	4.06	0.412	0.256	0.567	0.329	0.386	29.2	27.2	21.2	28.9	28.1	0.807	0.812	1.136	0.878	0.812	39.5	28.4	28.2	30.3	37.1
		8.12	0.398	0.223	0.441	0.405	0.439	27.3	30.1	25.7	33.4	27.9	0.876	0.888	1.102	1.201	0.903	37.5	36.6	36.6	37.1	31.4
		16.24	0.364	0.206	0.610	0.357	0.465	26.3	27.0	26.3	33.0	32.0	0.874	0.874	1.036	0.822	1.208	35.9	36.5	40.6	34.7	31.1
	5.4	16.24	1.330	1.315	1.333	—	—	19.3	18.8	18.2	—	—	5.461	4.343	3.812	—	—	22.1	20.3	22.8	—	—
		32.48	1.632	1.120	1.290	—	—	20.7	17.0	20.7	—	—	4.636	5.912	3.60	—	—	20.7	24.2	24.2	—	—
21.6	36.1	4.06	0.256	0.152	0.221	—	—	38.6	38.3	21.4	—	—	0.460	0.512	0.301	—	—	41.1	42.4	32.1	—	—
		8.12	0.162	0.103	0.274	—	—	37.4	30.7	23.6	—	—	0.402	0.344	0.351	—	—	44.9	40.2	38.5	—	—
	25.0	4.06	0.380	0.232	0.563	0.486	0.460	24.2	28.0	25.7	31.7	28.9	0.821	0.812	1.791	1.043	1.156	34.2	32.0	29.0	39.0	33.6
		8.12	0.440	0.247	0.551	0.552	0.518	24.9	27.8	25.6	25.4	28.2	0.820	0.839	1.540	1.296	1.174	34.9	29.6	28.1	36.9	32.6
		16.24	0.464	0.351	0.418	0.504	0.502	26.6	29.1	20.5	27.2	26.5	0.990	0.991	0.975	1.165	0.963	32.9	34.8	32.7	36.5	36.1
	5.4	16.24	1.755	1.380	2.011	—	—	15.8	14.1	14.3	—	—	6.018	4.803	3.263	—	—	18.6	24.0	23.5	—	—
		32.48	1.390	1.092	1.911	—	—	18.5	16.2	16.4	—	—	5.149	4.210	3.561	—	—	25.3	17.5	20.2	—	—
86.4	25.0	4.06	0.648	0.397	0.477	0.624	0.698	15.6	26.0	19.6	19.8	23.9	0.972	0.772	2.360	1.622	1.540	24.5	15.5	21.6	17.4	21.6
		8.12	0.570	0.338	0.574	0.616	0.610	22.5	27.6	15.0	18.0	19.6	1.362	0.859	2.295	1.807	1.642	20.1	25.1	20.1	28.2	24.8

[a]Absolute values of complex moduli $|E^*|$ and $|T^*|$ and respective phase angles ϕ_E and ϕ_T for 300, 500, 700, 800, and 900 mixes as designated.

and loading time conditions. Using the linear viscoelastic assumption, the maximum deviation from the average value of ϵ_{zz} by the four vertical strain curves is 10 percent. An identical procedure was used to evaluate the circumferential strains, $\epsilon_{\theta\theta}$, from the same constant stress tests performed on the 500 series mixes. The circumferential strains at four stress levels shown in Figure 9 also illustrated that the linear viscoelastic theory is a good approximation to depict the response of bituminous concrete at low stress levels and for the conditions investigated.

Direct Dynamic Tests

The results of the direct dynamic tests are summarized in Table 5 in terms of the absolute values of the complex moduli, $|E^*|$ and $|T^*|$, and their respective phase angles, ϕ_E and ϕ_T. Besides the 500 mix, four other bituminous concrete mixtures, namely the 300, 700, 800, and 900 mix designations, were investigated in the dynamic experiments. The principal variables studied in this phase of the experimentation were frequency, temperature, stress level, and type of bituminous mixture.

Several important facts are brought out by the inspection of the data, the most important being that the magnitude and phase of the complex modulus $|E^*|$ seem to be independent of the stress level and depend only on mix type, temperature and frequency. If the amplitude of the applied stress is double, the resulting amplitude of axial strain is also approximately doubled so that $|E^*|$ is unchanged. Similar results are noted for $|T^*|$ to demonstrate the linear viscoelastic nature of bituminous concrete.

Resonant Vibration Experiments

The results of the sonic dynamic experiments are summarized in Table 6. The real component and phase angle of the complex elastic and shear moduli were obtained from the experimental results and used to evaluate $|E^*|$ and ϕ_E. The magnitude and phase of E* and T* evaluated by both methods of dynamic tests will be used to study interrelations among the viscoelastic functions.

Evalutation of Thermorheologically Linear Response of Bituminous Concrete

The experimental data of the constant stress tests of the 500 series mix are presented in Figure 10. The creep modulus, E_C (t), has been evaluated at eleven experimental temperatures varying from 0 to 120 F as indicated and plotted on logarithmic plots vs time. The experimental stress levels used were chosen to be sufficiently low (as compared to the ultimate strength of the material) to be within the range for which the linear viscoelastic theory might produce a good approximation for the material at the temperature of test. The stress levels varied from 2 to 90 psi at 120 and 0 F, respectively. A summary of the stress levels used at each temperature in the creep and dynamic tests is presented in Table 7. It was necessary to vary the stress levels at each temperature as it was noted that the upper limit of the linear viscoelastic range is a function of temperature.

TABLE 6

SONIC EXPERIMENTAL DATA[a]

Specimen No.	Length (in.)	Base (in.)	Thickness (in.)	Weight (lb)	Resonant Frequency Flexural N(cps)	Resonant Frequency Torsional N'(cps)	C	B	$E_1(\omega) \times 10^6$ (psi)	ϕ_E (deg)	$E_2(\omega) \times 10^6$ (psi)	$\|E^*\| \times 10^6$ (psi)	$G_1(\omega) \times 10^6$ (psi)	ϕ_G (deg)	$G_2(\omega) \times 10^6$ (psi)	$\|G^*\| \times 10^6$ (psi)
317	5.88	2.37	0.97	1.159	2,185	2,505	0.274	0.0363	1.54	10.1	0.275	1.57	0.264	15.1	0.072	0.274
325	5.87	2.24	1.12	1.261	2,720	2,890	0.196	0.0441	1.84	11.9	0.389	1.88	0.464	7.0	0.055	0.467
337	5.88	2.21	1.10	1.271	2,479	2,180	0.208	0.0523	1.56	14.3	0.398	1.61	0.316	10.3	0.057	0.321
562	6.10	2.20	0.90	0.994	2,281	1,650	0.382	0.0752	1.91	9.1	0.305	1.93	0.204	4.9	0.015	0.204
537	5.94	1.93	0.94	0.885	2,259	1,762	0.388	0.0635	1.76	9.2	0.284	1.78	0.174	7.5	0.023	0.174
570	5.08	2.31	1.02	0.984	3,220	2,205	0.160	0.0474	1.61	11.4	0.324	1.64	0.236	6.0	0.025	0.237
710	5.92	2.33	0.98	1.058	2,453	2,910	0.270	0.0601	1.70	6.3	0.188	1.71	0.538	18.6	0.182	0.568
713	6.03	2.15	0.81	0.885	2,165	2,241	0.512	0.0930	2.12	7.9	0.294	2.14	0.414	15.8	0.117	0.431
706	5.95	2.26	1.01	1.070	2,392	2,627	0.265	0.0564	1.62	9.1	0.258	1.63	0.415	16.2	0.124	0.457

[a] Magnitude and phase of complex moduli obtained from torsional and resonant frequencies using bituminous concrete mixtures 300, 500, and 700 measured at 25 C.

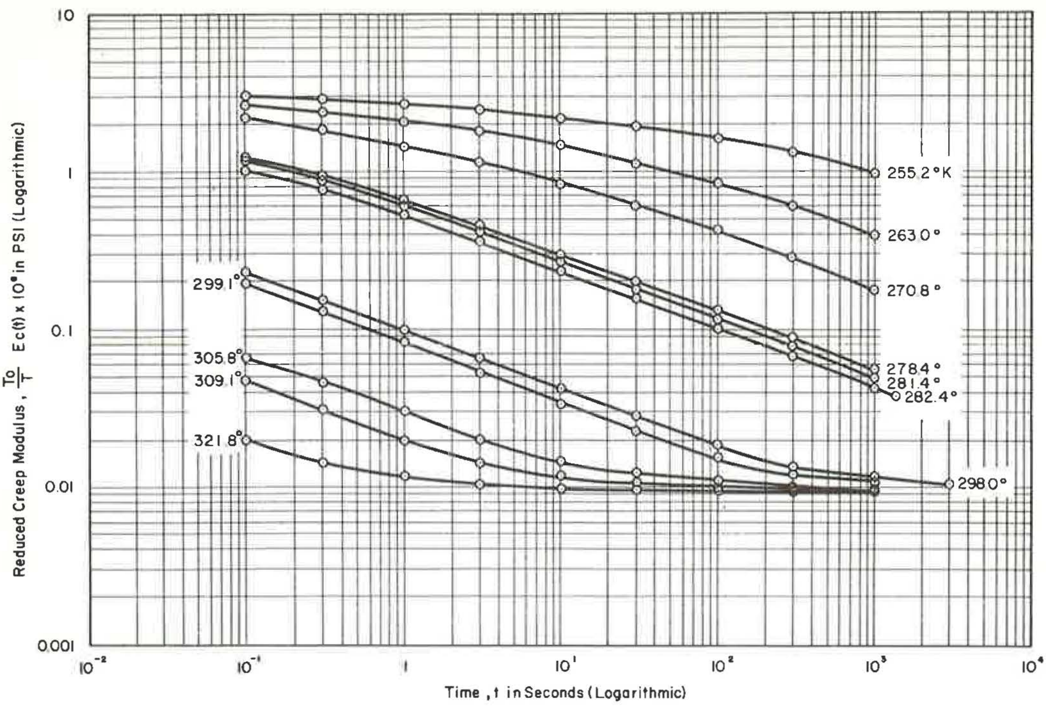

Figure 10. Axial creep modulus reduced to 298 K vs time at indicated temperatures.

TABLE 7

STRESS LEVELS IN LINEAR VISCOELASTIC RANGE AT EXPERIMENTAL TEMPERATURES

Temperature (°K)	Temperature (°F)	Axial Load (lb)	Axial Stress (psi)
255.2	0	550	89.29
263.0	14	500	81.17
270.8	28	450	73.05
278.4	41.7	400	64.94
281.4	47.1	312.5	50.73
282.4	49	312.5	50.73
298.0	77	100	16.24
299.1	79	100	16.24
305.8	91	37.5	6.09
309.1	97	25	4.06
321.8	120	12.5	2.03

A reference temperature, T_0, equal to 77 F or 298 K, was arbitrarily selected within the experimental temperature range. The creep functions were first reduced by the absolute temperature factor, T_0/T, which enters into the coordinate scheme due to entropy-spring nature of the stored elastic energy. The density factor, P_0/P, has been omitted from the calculations as it approaches unity and is within the experimental error of the tests. Each creep function at a given temperature was obtained by testing three samples under identical conditions and averaging the axial and circumferential strain data.

Investigation of the eleven creep functions in Figure 10 shows that a change in the experimental temperature shifts only the position of the reduced creep moduli with respect to the logarithmic time scale, and the general shapes of the curves are not altered when the experimental temperature is changed. The horizontal distance between each pair of adjacent curves was measured and found to be the same within experimental error. Thus, portions of the adjacent viscoelastic functions are essentially parallel lines and have the same numerical values of E_c (t) at different portions of the logarithmic time scale. A change in temperature has shifted only the position of the viscoelastic modulus function on the logarithmic time scale as all retardation times have the same temperature dependence to provide experimental verification that the bitumen-aggregate mixes used in this study may be represented by thermorheologically linear materials.

The viscoelastic data plotted in Figure 10 may now be used to extend the experimentally accessible time scale of the creep modulus obtained at 298 K by developing single

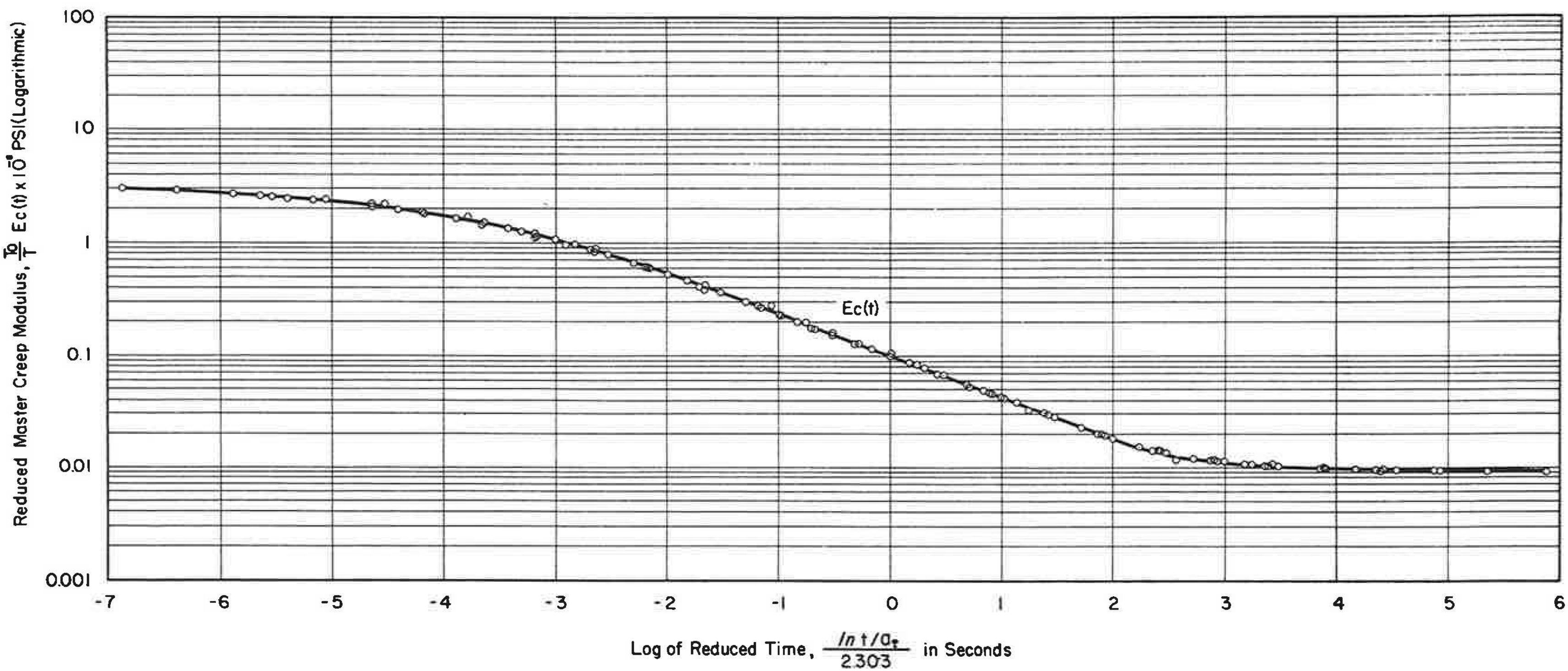

Figure 11. Composite master creep modulus obtained by time-temperature superposition principle, representing viscoelastic behavior over extended time scale at 298 K.

or master creep function at this temperature. Since time and temperature have been shown to have an equivalent influence on the creep moduli in Figure 10, it was possible to shift the remaining ten reduced creep modulus curves to the chosen reference temperature by transforming them horizontally along the time axis until the experimental curves superimpose on the reference curve at 298 K to give a fairly smooth plot as shown in Figure 11. In this figure the time scale of the experimental curve was extended beyond the practical experimental range, and now extends from 10^{-6} to 10^{6} seconds or approximately 12 days. The composite curve represents the behavior of the bituminous mix at a single temperature, T_0; the master curve is actually composed of segments of the eleven creep functions which were evaluated at different temperatures.

The experimental transverse creep functions, T_c (t), determined at the same eleven temperatures are plotted in Figure 12. The equivalence of time and temperature effects on the viscoelastic properties of bituminous concrete can again be noted from the experimental plots. The transverse creep functions were shifted vertically by the T_0/T factor. The density factor was again omitted from the reduction scheme. The master transverse creep function at 298 K shown in Figure 13 was obtained by superposition of the eleven individual creep functions and now covers an extensive range of the time scale.

The experimental creep moduli, E_c (t) and T_c (t), have been reduced vertically by the factor T_0/T before application of the superposition concept to develop the master curves. This procedure was used in this study as it seemed to produce the smoothest master viscoelastic functions. Although master curves of E_c (t) and T_c (t) were also

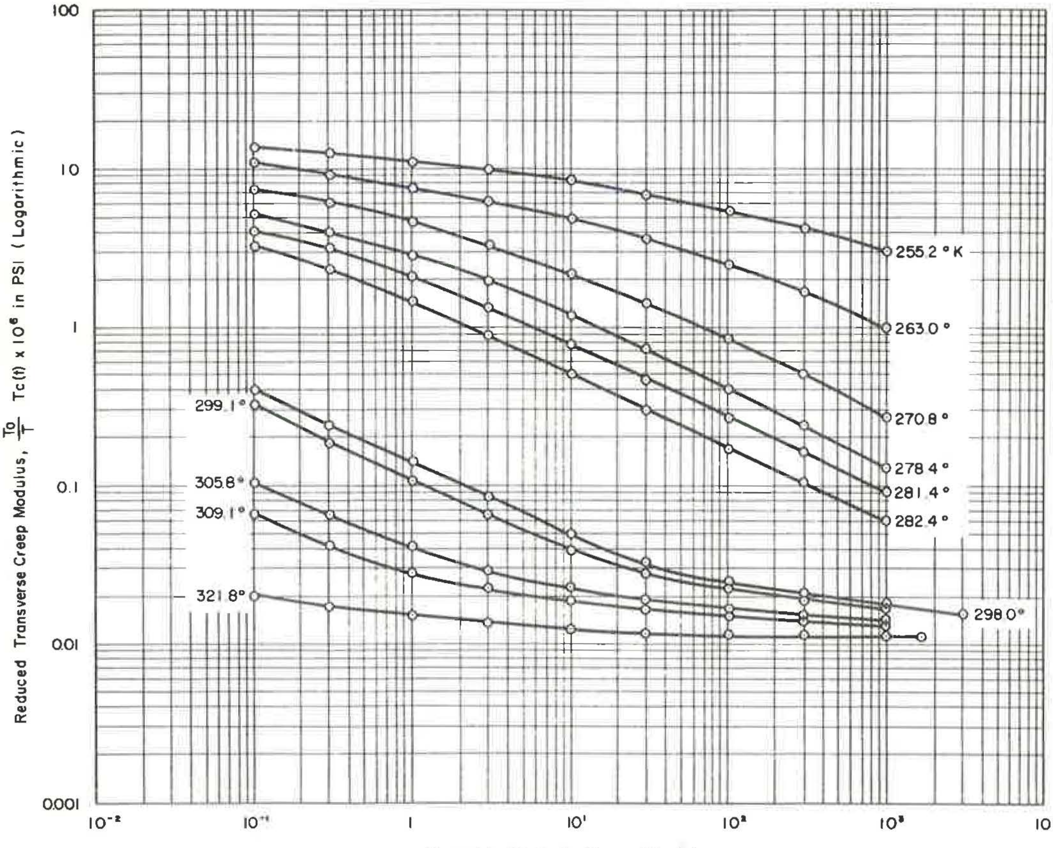

Figure 12. Transverse creep function reduced to 77 F vs time at eleven temperatures.

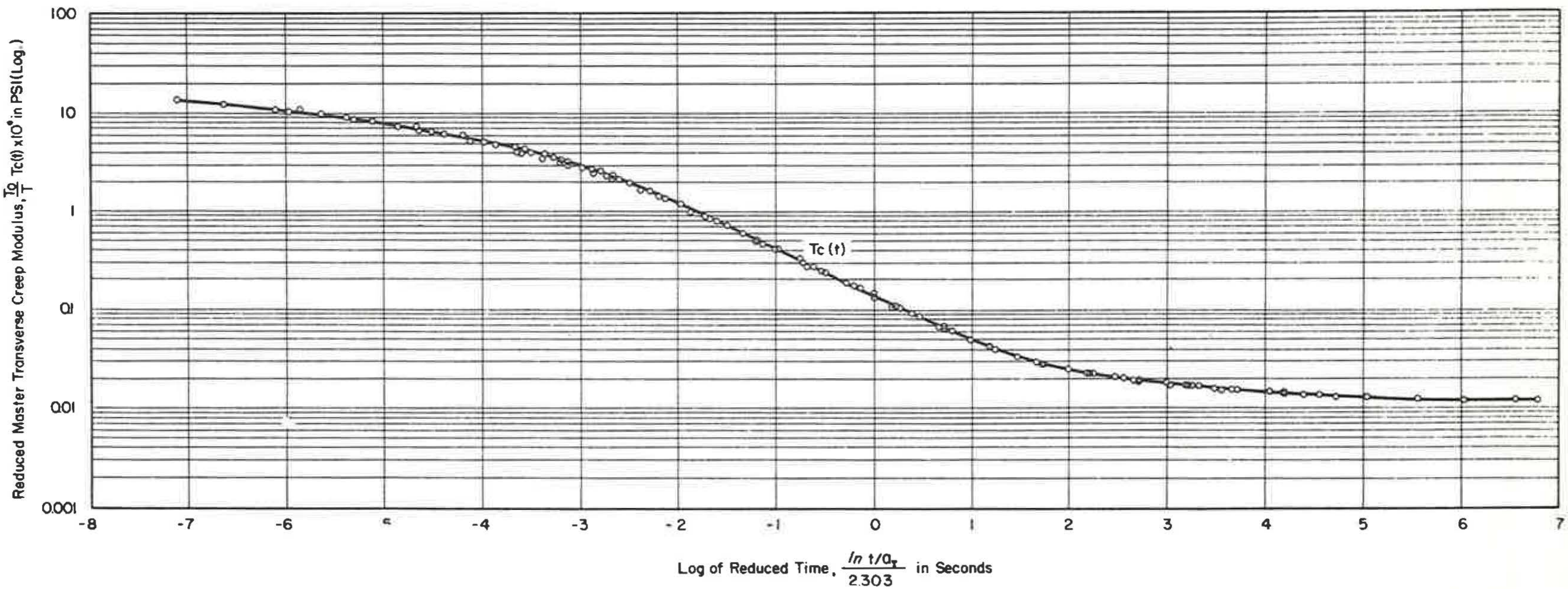

Figure 13. Composite master transverse creep function at 77 F.

obtained by neglecting the T_o/T and P_o/P factors, these curves were not as smooth or continuous. Perhaps this procedure of treating the data may be applied in many cases. However, due to the limitations of the experimental equipment and magnitude of the T_o/T shift, more refined experimentation is necessary at this time to evaluate this interesting concept.

Temperature Dependence of Shift Factor, a_T

The temperature dependence of the shift factor, a_T, was evaluated by plotting the relative horizontal shifts to obtain coincidence of adjacent creep modulus curves vs the experimental temperature differences of the viscoelastic curves. These values were obtained from Figures 10 and 12. The temperature reduction function, f (T), has been constructed in Figure 14 by plotting the time shifts between experimental curves vs temperature intervals of Figures 10 and 12 for $T_o/T\ E_c$ (t) and $T_o/T\ T_c$ (t), respectively.

The choice of 298 K as the standard reference temperature, T_o, is purely arbitrary and is based on convenience. It is interesting to note that by selecting the lowest temperature in the experimental range as T_o, the master curve is developed for exceedingly long loading times. However, if the short time response of the material is of interest, the master curve in this range can be developed by designating 120 F as T_o.

In Figure 14 the experimental results from E_c (t) and T_c (t) have been plotted and a straight line used to approximate f (T) for the 500 series bituminous concrete mixture. It well be shown later that the same f (T) will also be applicable to shift the magnitude and phase of E* and T* evaluated in this study. The good correlation of f (T) evaluated from both viscoelastic creep moduli and used to shift the absolute value and phase of the complex moduli in the frequency domain seems to indicate that if one characteristic viscoelastic function is controlled by the time-temperature relations, then all other viscoelastic functions of the same material are controlled by the same relations.

The temperature shift function has also been evaluated from the experimental data of E_c (t) and T_c (t) that were not first reduced vertically by the temperature factor, T_o/T. A straight line was again used to approximate f (T). An equation was developed for the shift factor, $a_T = 10^{40.6\ -\ 0.1360\ T}$, which is only slightly different from the corresponding equation in Figure 14.

The master creep functions, E_c (t) and T_c (t), were determined at 298 K by plotting $T_o/T\ E_c$ (t) vs ln t/a_T and $T_o/T\ T_c$ (t) vs ln t/a_T, respectively. Once the temperature shift function and master creep functions are evaluated for a reference temperature, the master curves can be readily shifted to any other temperature from 0 to 120 F by application of the appropriate values of ln a_T and vertical temperature factor.

Thus, starting from a complicated dependence on both temperature and time as illustrated in Figures 10 and 12, these two independent variables can be separated to yield a viscoelastic function of time alone at a standard reference temperature, and a temperature function. The temperature reduction function, f (T), and the viscoelastic functions, E_c (t) and T_c (t), can be used to completely define the material at any time and experimental temperature within the tested range. Figures 11 and 13 represent E_c (t) and T_c (t) as they would have been measured at temperature T_o over a large range of the time scale. In a rheologic study, the temperature and time dependent response of a material may also be dealt with by including the independent variable of temperature as well as time in the rheological equation of state used to depict the quantitative mechanical behavior of the material.

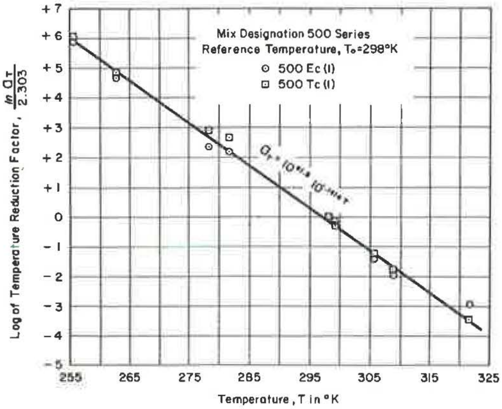

Figure 14. Temperature dependence of shift factor, a_T, evaluated from reduced axial and transverse creep moduli.

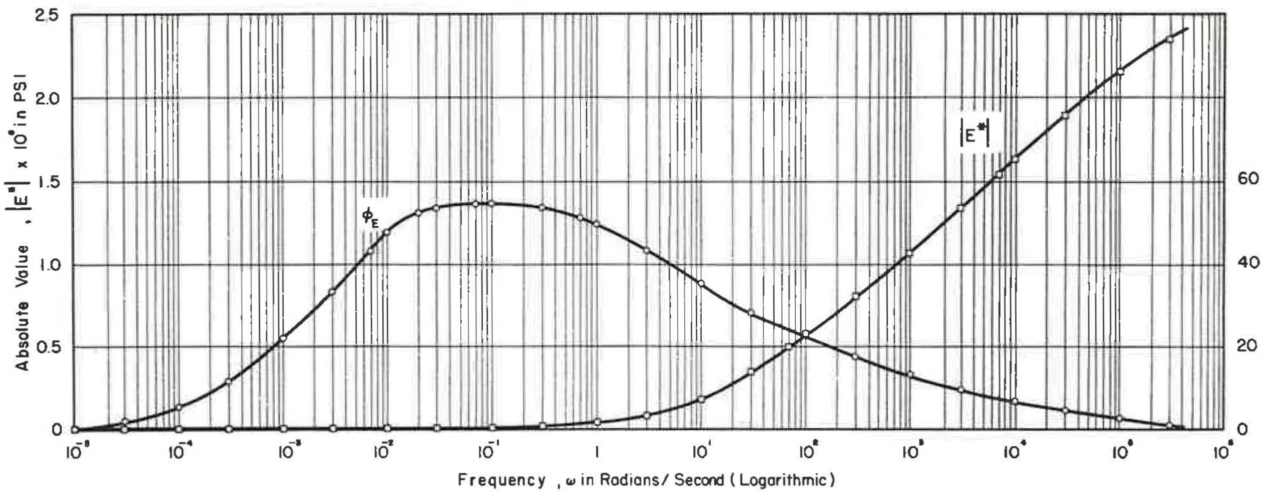

Figure 15. Magnitude and phase angle of complex elastic modulus, E*, as function of angular frequency at 77 F.

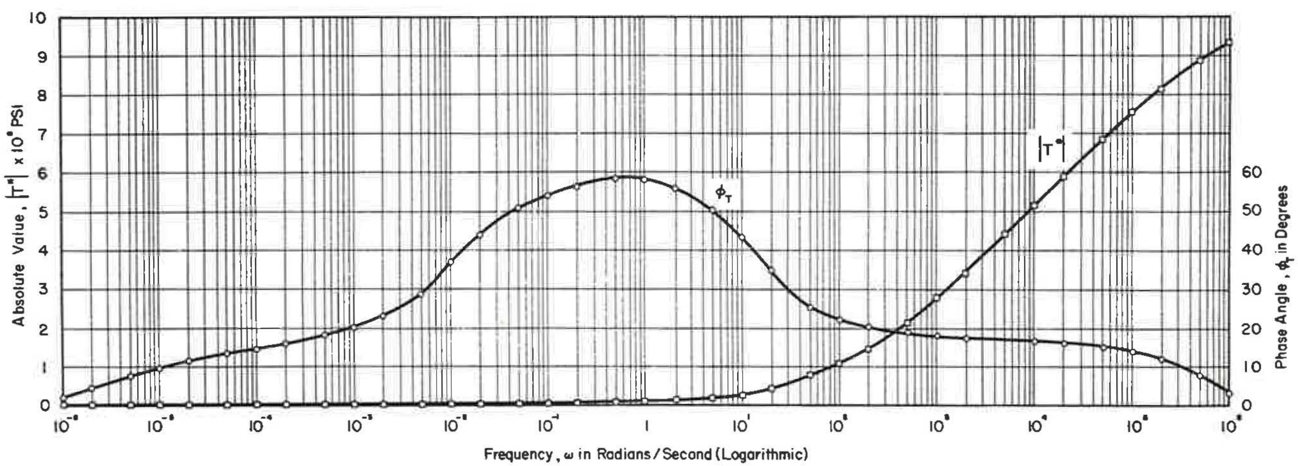

Figure 16. Absolute value and phase of complex transverse modulus, T*, at 77 F.

Prediction of Complex Moduli

Generalized mechanical models were used in this study to represent the response of real materials because this method allows the mechanical response of bituminous concrete to be adequately represented, and it is possible to deal directly with the viscoelastic functions rather than using a model consisting of a finite number of elements only to approximate the complicated response of the material. The highest degree of accuracy in representing the viscoelastic response was obtained in this case by use of the infinite Voigt model.

The distribution functions of retardation time were calculated from the creep experimental results, and by use of the interrelations among the viscoelastic functions, a method to evaluate the complex moduli of the material from the compression tests is available. The real and imaginary parts of E* and T* were calculated using the first- and second-order approximation methods suggested by Leaderman and Widder (1, 6). The master creep functions were first used to evaluate retardation spectra, $L(\overline{\ln \tau})$, which were then used to determine the storage and loss components of the complex moduli and compliances by similar approximate methods. At any specified temperature and frequency, the dynamic response of a material is defined by either $J_1(\omega)$ and $J_2(\omega)$ or alternately by $|E^*|$ and ϕ_E. The two methods of representing the complex moduli are equivalent and related to each other. Figure 15 is a plot of the absolute value of the complex elastic modulus, $|E^*|$, and its phase angle, ϕ_E, vs angular frequency at 298 K obtained from the master creep curve of Figure 11 using the previous relations. To develop similar functions at any temperature in the static experimental range from 0 to 120 F, the appropriate value of $\ln a_T$ from Figure 14 and temperature factor may be selected and used to transform the dynamic viscoelastic function to the temperature desired. An identical result could be obtained by first developing the master creep viscoelastic function in the time domain at the desired temperature T_i, and then transforming the creep function to frequency domain at this temperature. Thus, the same values of $|E^*|$ and ϕ_E can be obtained by two methods using the same value a_T. Figure 16 is a plot of $|T^*|$ and ϕ_T vs angular frequency at 298 K. These viscoelastic functions were obtained from the master transverse creep curve in Figure 13 by the approximate interrelations between viscoelastic functions.

Correlation of Creep and Dynamic Data

By comparing the direct dynamic and sonic test data presented in Tables 5 and 6 and the values of the magnitude and phase of the complex moduli predicted by creep tests, the correlation of results may be observed between both types of experimentation. Values of $|E^*|$, $|T^*|$, ϕ_E, and ϕ_T evaluated by creep and dynamic tests may be compared at the common experimental temperatures and frequencies. By applying the appropriate values of a_T and temperature factor to either the dynamic or creep data or simultaneously to both, the magnitude and phase or the real and imaginary components of E* and T* evaluated by dynamic tests and the corresponding viscoelastic functions calculated by creep tests were compared directly in the frequency domain over a range of temperatures for the 500 series mix.

The agreement between the dynamic viscoelastic functions predicted by creep tests, which are continuous functions of frequency, and the values of the magnitude and phase of E* and T* over the range of temperatures was considered to be good in 90 percent of the analyses of the data performed.

The good agreement in this type of experimentation between the experimental values of the magnitude and phase of the complex moduli evaluated by creep and dynamic tests provides another verification of the ability of the linear viscoelastic assumption to describe the response of bituminous concrete mixtures. It also provides verification of the application of the time-temperature superposition concept to this material, because the creep and dynamic test data were obtained in entirely different time intervals.

Investigation of f (T) evaluated from the creep experimental data and used for the dynamic test results seems to indicate that f (T) determined from one characteristic function may be used to obtain the temperature dependence of the other characteristic creep and dynamic viscoelastic functions for a given material. A satisfactory correlation of f (T) evaluated from E_c (t) and T_c (t) and checked by the directly evaluated values of the magnitude and phase of the complex moduli E* and T* can be noted from the experimental data.

The proposed methods may now be applied to evaluate the response of bituminous concrete at any time or frequency and temperature in the experimental range by interpolation, and to extend the data to cover by extrapolation ranges of time not possible by conventional laboratory methods.

Analogous creep and dynamic experiments, calculations, and graphical procedures were performed on the 300 series bituminous concrete mixture. These calculations are not presented here in detail due to space limitations and may be found elsewhere (18). An analysis of this data also indicated the validity of the time-temperature superposition and linear viscoelastic concepts to the creep and dynamic experimental results of the 300 series mixture.

DISCUSSION

Significance of Results

The experimental results suggest that at low stress levels and for the conditions tested, the assumption of linear viscoelastic behavior is applicable to bituminous concrete. The experimental phase of this study also indicated that the time-temperature superposition concept can be applied to the mechanical properties of bitumen-aggregate compositions to a useful degree of accuracy. For a design engineer who is interested in the mechanical properties of bituminous mixes over a large range of environmental and loading conditions, the equivalence of time and temperature on the viscoelastic response, even when approximate, means an important simplification.

These concepts when properly applied, however, do provide a higher level of approximation than the present elastic theory. Whereas the elastic theory provides valuable information regarding stresses and strains in bituminous pavements under fast-moving wheel loads at freezing temperatures, the effects of slowly applied or static loads at relatively high temperatures cannot be accurately considered by the elastic idealization. In addition, the progressive accumulation of small irrecoverable deformations under repeated wheel loads and subsequent rutting of the bituminous pavement in the vehicle wheelpath cannot be accounted for by the elastic theory. Fatigue effects and plastic flow of flexible pavement surface courses parallel to the direction of traffic are other examples of the inelastic behavior of this engineering material. Thus, a more rational method of flexible pavement design must incorporate the effects of loading time or frequency and temperature on the stress-strain properties of the materials. If the concepts advanced in this study do not provide a sufficiently high degree of accuracy, more sophisticated approaches can be developed. However, the complexities due to the linear viscoelastic and time-temperature superposition concepts are now troublesome when applied to practical problems.

The relationship of original viscosity to temperature for two typical bitumens is shown in Figure 17.

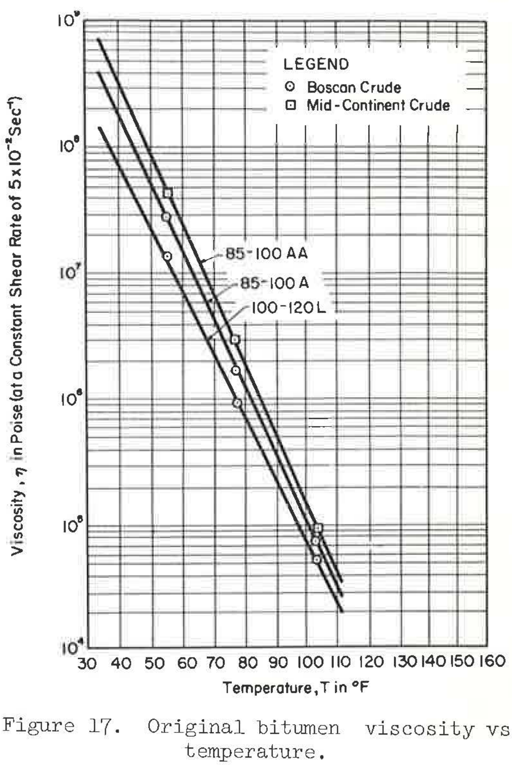

Figure 17. Original bitumen viscosity vs temperature.

Application of the previously discussed principles to the mechanical properties of pavement materials in thickness design procedures are more tentative at this time than their immediate application to bituminous concrete mixture strength evaluation and comparison. These concepts may, however, prove to be more significant in the future, and the elastic layered-system design theories which are available today may be supplemented by future viscoelastic theories which will also include the temperature-dependent properties of the material. Any analysis will be complicated by the many variables that enter into pavement design methods and evaluation.

When only one type of experimental method is available to evaluate mixture strength, covering only a limited portion of the time or frequency scale, the concepts investigated allow the viscoelastic functions to be traced out over a much larger effective range by varying the experimental temperature. For the bituminous concrete studied, an increase in temperature corresponds to an increase in the time or a decrease in the frequency scale in its effect on the viscoelastic functions of the material. This reduction scheme must be used judiciously with regard to the reservations of the theories which support it, and when properly applied, yields plots in terms of reduced variables which can be used with confidence to predict the viscoelastic functions.

Viscoelastic data obtained from one type of experimental measurement are, in principle, sufficient to define all time-dependent properties within the range of linear response, provided the experimental time scale is varied over an exceedingly wide range. In practice, however, it is usually not possible to cover at a single temperature the full time scale required to characterize the transitions of the material. By application of the time-temperature relations over a range of temperatures, the complete transitions of bituminous concrete have been evaluated.

Three-Dimensional Response

The concepts developed in previous sections defining the response of linear viscoelastic materials in one direction can also be extended to analyze their behavior in the three-dimensional case. The experimental data have indicated that if one fundamental property of a thermorheologically linear material follows the time-temperature relations, then all other fundamental properties obey the same relations.

The stresses or strains in two mutually perpendicular directions are related by a linear differential equation in the time domain and by algebraic relations in the frequency domain. Thus, having obtained E^* and T^* for a given material, the other complex moduli of the material ν^*, G^*, K^* can be easily obtained. In fact, all equations which are valid for the Hookean solid remain valid for the linear viscoelastic material when expressed in terms of complex moduli, and the three-dimensional response of the material in the frequency domain can be defined over a wide range of temperatures. To evaluate the viscoelastic behavior in the time domain, the inverse transform of the response in the frequency domain is taken (25, 26).

CONCLUSIONS

The major objective of this study was to obtain a better understanding of the mechical properties of bituminous concrete mixtures, which are used over an extensive range of environmental and loading conditions. The following are the major conclusions of the investigation:

1. The experimental data indicate the mechanical properties of the dense bituminous concrete mixtures investigated can be expressed by the linear viscoelastic theory to a useful degree of approximation under the conditions investigated.

2. Experimentation of the phenomenological level has demonstrated that the time-temperature superposition principle is valid for the bitumen-aggregate compositions tested to a satisfactory level of approximation.

3. Time and temperature were shown to have an equivalent effect on the viscoelastic properties of the materials studied, and the number of experiments needed to define the response of the material over a wide range of temperature and time can now be greatly reduced. By use of the time-temperature relations, it was possible to project the ex-

perimental viscoelastic functions to loading times both shorter and longer than can normally be obtained experimentally, and for any intermediate temperature within the tested temperature range.

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REFERENCES

1. Stuart, H. A., Ed., "Die Physik der Hochpolymeren." Springer-Verlag, Berlin (1956).
2. Payne, A. R., "Sinusoidal-Strain Dynamic Testing of Rubber Products." ASTM Mater. Res. Std. (Dec. 1961).
3. Leaderman, H., "Elastic and Creep Properties of Filamentous Materials and Other High Polymers." Textile Foundation, Washington, D. C. (1943).
4. Alfrey, T., "Mechanical Behavior of High Polymers." International Pub., New York (1948).
5. Baker, R. F., "Pavement Design Using Rheologic Concepts." Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, Ann Arbor (1962).
6. Eirich, F. R., Ed., "Rheology, Theory and Applications." Academic Press, New York (1961).
7. Papazian, H. S., "The Response of Linear Viscoelastic Materials in the Frequency Domain." Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, Ann Arbor (1962).
8. Pister, K., and Monismith, C. L., "Analysis of Viscoelastic Flexible Pavements." HRB Bull. 269, pp. 1-15 (1960).
9. Ferry, J. D., "Viscoelastic Properties of Polymers." John Wiley and Sons, New York (1961).
10. Krokosky, E. M., Andrews, R. D., and Tons, E., "Rheological Properties of Asphalt/Aggregate Compositions." Paper presented at Annual Meeting of ASTM (163).
11. Brodnyan, J. G., "Use of Rheological and Other Data in Asphalt Engineering Problems." HRB Bull. 192, pp. 1-19 (1958).
12. Bergen, J. I., Ed., "Viscoelasticity-Phenomenological Aspects." Academic Press, New York (1960).
13. Papazian, H. S., Pagen, C. A., and Baker, R. F., "The Rheological Properties of Bituminous Concrete." Report 172-1, Ohio State Univ. (1961).
14. Burgers, J., and Scott Blair, C., "Report on the Principles of Rheological Nomenclature." Proc. Internat. Rheological Cong., Holland (1948).
15. Pickett, G., "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders." Proc. ASTM (1945).
16. Spinner, S., and Tefft, W., "A Method for Determining Mechanical Resonance Frequencies and for Calculating Elastic Moduli from These Frequencies." Proc. ASTM (1961).
17. Brown, A. B., and Sparks, J. W., "Viscoelastic Properties of a Penetration Grade Paving Asphalt at Winter Temperatures." Proc. AAPT (1958).
18. Pagen, C. A., "An Analysis of The Thermorheological Response of Bituminous Concrete." Ph. D. Diss., Ohio State Univ. (1963).
19. Reiner, M., "Deformation and Flow." H. K. Lewis and Co., London (1949).

20. Freudenthal, A. M., "The Inelastic Behavior of Engineering Materials and Structures." John Wiley and Sons, New York (1950).
21. Bland, D. R., "The Theory of Linear Viscoelasticity." Pergamon Press, London (1960).
22. Gross, B., "Mathematical Structure of the Theories of Viscoelasticity." Herman and Co., Paris (1953).
23. Ley, B. J., Lutz, S. G., and Rehberg, C. F., "Linear Circuit Analysis." McGraw-Hill, New York (1959).
24. Rouse, P. E., "A Theory of the Linear Viscoelastic Properties of Dilute Solution of Coiling Polymers." Jour. Chem. Phys. (1953).
25. Churchill, R. V., "Operational Mathematics." 2nd Ed., McGraw-Hill, New York (1958).
26. Truxal, J. G., "Automatic Feedback Control System Synthesis." McGraw-Hill, New York (1955).