to offset the cash flow gains by employers who submitted receipts early in the month.

CONCLUSIONS

The Jacksonville TFP demonstration has provided a very useful data set for quantifying the impacts that result from implementing an employer-based monthly transit pass program. The main conclusions of the demonstration are highlighted below.

Although the pure convenience aspect of being able to purchase a pass at one's place of employment resulted in few individuals switching from other modes to transit, convenience did play a major part in a transit user's decision to buy a pass at work. This was revealed when 58 percent of the pass buyers stated that they would discontinue buying a pass if it was sold only through JTA's regular pass outlets.

In contrast to passes available to the general public, and thus to the entire transit-dependent community, passes sold through employers were typically thought of and used as commuter passes. Few new transit trips were taken by pass purchasers during off-peak hours or on weekends. Consequently, pricing the pass to provide little or no discounts over cash fares, with employees bearing all the up-front risks (e.g., unexpected sick days), resulted in a low level of pass sales. However, providing modest pass discounts and encouraging employers to subsidize the pass as an employee fringe benefit, or in lieu of an employer-provided parking space, resulted in substantial increases in pass sales.

Soliciting employers to participate in the program was successfully accomplished by relying on a personal meeting with a high executive officer at each potential firm. Most of the employers recognized the benefits to their employees by participating in the JaxPASS program. In fact, by the eighteenth month, one-third of the employers (9 out of 28) were providing partial (usually $4.00) or full subsidies to their employees who bought a pass. Administrative costs borne by the employer were small, ranging from 0.5 to 4 h/month. No firms discontinued their involvement in the program because of the administrative requirements associated with selling and distributing passes to their employees.

Administrative resources expended by the transit operator consisted of 3 to 4 person-days/month. These activities were handled by existing staff members. Of course, much larger TFP programs would require full-time staff.

The JaxPASS program resulted in some new transit users and the new revenue from these individuals helped to offset the revenues lost to the more frequent transit users who also bought passes. On balance, the introduction of the program resulted in slight negative revenue loss (about 0.3 percent of total monthly revenues). However, as additional employers join in subsidizing the pass price, thereby encouraging more marginal transit users to buy a pass, revenue losses because of the pass should decrease further.

ACKNOWLEDGMENT

The work described in this paper was performed under contract to TSC as part of the UMTA SMD program. The support of those agencies is hereby acknowledged. Particular recognition goes to Larry Doxsey of TSC, who served as technical advisor and monitor for this evaluation, and Vince Milione, who was the UMTA project monitor. In addition, Don Pill and Ruth Sargent of JTA, Leo Hall of the Jacksonville Coach Company Lines, and John Mullis and Rose Ella Feagin of Paragon Productions were helpful in providing much of the data from the site. Daniel Brand of Charles River Associates provided important insight and comments throughout the course of the evaluation.

The opinions and conclusions expressed in the paper are mine and do not necessarily reflect the views or policy of TSC, UMTA, or JTA.

REFERENCES

1. Transit Fare Summary--Fare Structures in Effect on February 1, 1981. American Public Transit Association, Washington, DC, 1981.
2. Transit Fares Climb Again, 45¢ Average. Passenger Transport, Vol. 39, No. 17, April 24, 1981, p. 1.
3. Pre-Paid Transit Pass Program Study. Decision Research Corporation and Temple, Barker and Sloane, Boston, MA, March 1976.
4. Jacksonville Transit Fare Prepayment Demonstration--Final Report. Charles River Associates Inc., Boston, MA, 1982.
5. Sacramento Transit Fare Prepayment Demonstration--Final Report. SYSTAN, Inc., Los Altos, CA, 1980.
6. P. Bloomfield and J. Crain. Transit Fare Prepayment Demonstration in Austin, TX, and Phoenix, AZ. Crain and Associates, Menlo Park, CA, June 1979.

Publication of this paper sponsored by Committee on Transit Service Characteristics.

Abridgment

Graphical Person-Machine Interactive Approach for Bus Scheduling

AVISHAI CEDER AND HELMAN I. STERN

A highly informative graphical technique for the problem of finding the least number of buses required to service a given schedule of trips is described. The purpose is to develop the methodology for variable bus scheduling in which trip departure times can vary within acceptable tolerances. This is a continuation of a research project concerned with the problem of fixed scheduling where the timetable of trips and length of trip times are fixed. The motivation for this study comes from the Israel National Bus Carrier, Egged, which is responsible for scheduling an average of 54 400 daily trips by about 5200 buses. Consequently, the research takes on a practical nature. The approach used is based on the deficit-function theory where the deficit function at time t defines the total number of trips that have departed from a given terminal k less the number of trips that have arrived at k up to and including time t. The method developed is capable of aiding the scheduler to perform his or her tasks through a person-machine conversational mode. It allows

the scheduler to interject his or her own practical suggestions and to see immediately their effects on the final schedule. The method is based on the deficit-function graphical multiterminal display, which guides and advises the schedulers of possible directions in reducing fleet size. In addition, this methodology aids schedulers in evaluating the results of their own suggestions. The potential of this person-machine interactive bus-scheduling procedure is presented by using several practical examples.

The planner of bus work schedules is in charge of allotting vast resources, and naturally the aim is to allocate the buses in an optimal and feasible manner. A graphical approach to the problem of scheduling buses to trips is described. The approach is based on an algorithm capable of aiding the scheduler through a person-machine conversational mode. It allows the scheduler to select one of several computer-suggested directions, to interject his or her own suggestions, and to immediately see the effect on the final schedule through observation of a graphical representation on a cathode-ray tube (CRT) or computer-generated output.

This research is a continuation of an algorithm development described by Ceder and Stern (1,2). The motivation for the overall study comes from the Israel National Bus Carrier, Egged. Egged's operation (making up a fleet size of 5200 buses) is characterized by (a) a substantial number of deadheading (empty) trips in the schedule, (b) frequent changes in the schedule, and (c) a complex bus-route network with many different locations of trip departure and arrival points. Egged's buses perform an average of 54 400 daily trips, which are currently scheduled by a team of about 60 schedulers by using a trial-and-error Gantt-chart approach. The need to expedite many of the schedulers' tasks has led Egged's management to embark on investigating a more efficient procedure. The experience with Egged and the initial implementation of the approach proposed here are presented later.

The algorithms described previously (1,2) assume that the given timetable of trips is fixed. On the other hand, the experienced Egged schedulers consider variable trip departure times (within some acceptable tolerances).

The purpose of this paper is to develop the methodology for variable bus scheduling in order to achieve further reduction in the number of buses required. The main aim is to allow the schedulers to use a person-computer interactive procedure that will guide and advise them on evaluating the results of their own suggestions, which include practical considerations.

BACKGROUND AND DEFINITIONS

An early development of the approach used in this paper is based on the deficit-function theory. A deficit function is simply a step function that increases by 1 at the time of each trip departure and decreases by 1 at the time of each trip arrival. Such a function may be constructed for each terminal in a multiterminal bus system. To construct a set of deficit functions, the only information needed is a timetable of required trips. The main advantage of the deficit function is its visual nature (3,4). Let $d(k,t,S)$ denote the deficit function for terminal k at time t for the schedule S. The value of $d(k,t,S)$ represents the total number of departures less the total number of trip arrivals at terminal k up to and including time t. The maximal value of $d(k,t,S)$ over the schedule horizon $[T_1,T_2]$ is designated $D(k,S)$.

Let t_s^i and t_e^i denote the start and end times of trip i, $i \epsilon S$. It is possible to partition the schedule horizon of $d(k,t,S)$ into a sequence of alternating hollow and maximal intervals. The maximal intervals $M_r^k = [s_r^k, e_r^k]$, $r = 1,2,\ldots,n(k)$, define the interval of time over which $d(k,t)$ takes on its maximum value. Denote the length of M_r^k as $\bar{M}_r^k = e_r^k - s_r^k$. Note that the S will be deleted when it is clear which underlying schedule is being considered. The index r represents the rth maximal interval from the left and $n(k)$ the total number of maximal intervals in $d(k,t)$. A hollow interval H_ℓ^k, $\ell = 0,1,2,\ldots,n(k)$, is defined as the interval between two maximal intervals. Hollows may consist of only one point. In case a hollow consists of one point not on the schedule horizon boundaries (T_1 or T_2), the graphical representation of $d(k,t)$ is emphasized by a clear dot.

If we denote the set of all terminals as T, the sum of $D(k)$ $\forall$ $k\epsilon T$ is equal to the minimum number of buses required to service the set T. This is known as the fleet-size formula and was independently derived by Bartlett (5), Linus and Maksim (6), Gertsbach and Gurevich (3), and Salzborn (7,8). Mathematically, for a given fixed schedule S,

$$N = \sum_{k\epsilon T} D(k) = \sum_{k\epsilon T} \max_{t\epsilon[T_1,T_2]} d(k,t) \qquad (1)$$

where N is the minimum number of buses to service the set T.

When deadheading (DH) trips are allowed, the fleet size may be reduced below the level described in Equation 1. Ceder and Stern (1,2) describe a procedure based on the construction of a unit-reduction deadheading chain (URDHC), which when inserted into the schedule allows a unit reduction in the fleet size. The procedure continues to insert URDHCs until no more can be inserted or a lower bound on the minimum fleet is reached (1).

In order to understand the URDHC procedure, a three-terminal example is briefly explained. The example illustrated in Figure 1 is referred to a fixed schedule because at this point we do not allow trip and departure times to be varied. The schedule is made up of nine trips with a trip-time matrix for both potential service (in parentheses) and DH trips, as follows:

$$[t_{ij}(t_{ij}')] = \begin{array}{c|ccc} & & j & \\ i & k & m & u \\ \hline k & 0 & 2(6) & 2(3) \\ m & 2(7) & 0 & 1(4) \\ u & 2(3) & 1(5) & 0 \end{array}$$

where t_{ij} and t_{ij}' are the trip times of DH and service trips between terminals i and j, respectively. For the example in question, the minimum number of buses required (before insertion of DH trips) is $D(k) + D(m) + D(u) = 3 + 2 + 1 = 6$. The chain-construction method can be carried out by the first-in, first-out (FIFO) rule or by a chain-extraction procedure described by Gertsbach and Gurevich (3). The resultant six chains that use the FIFO rule are [1-8], [2], [3-9], [5-4], [6], and [7], according to the trip numbers indicated in the fixed-schedule part of Figure 1. These chains are assigned to individual buses.

By the insertion of DH trips, the scheduler is able to reduce the fleet size of the sample problem from six to five buses. Suppose that terminal k is selected as a candidate terminal for reducing its fleet requirement. A deadheading trip, DH_1, that departs from terminal m at $t = 5$ can arrive at k at $t = 7 = s_1^k$ based on the above trip-time matrix in

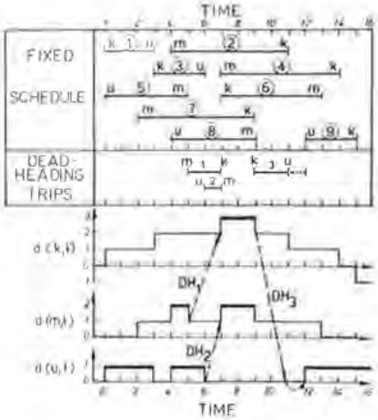


Figure 1. Three-terminal example of URDHC procedure using three DH trips to reduce fleet size by 1 at terminal k.

which t_{mk} = 2. This will have the effect of decreasing d(k,t) by one unit at and after t = 5. In order to eliminate the increase of D(m) at t = 7, another DH trip, DH_2, is inserted from terminal u to m at t = 6. Similarly, another DH trip is required, DH_3, to maintain the value of D(u) at and after t = 12. Note that a feasible insertion of a DH trip to link trip i at terminal u to trip j at terminal v (i,jϵS) requires the following:

$$t_e^i + t_{uv} \leq t_s^j \quad (2)$$

In the example shown in Figure 1, three DH trips are required to reduce D(k) from 3 to 2. An interesting observation is that instead of DH_3, another service trip can be inserted between terminals k and u from t = 9 to t = 12, since t_{ku}' = 3 (see the dotted line in Figure 1). In this way, the bus operator might increase the level of service for the passengers by using a technique to reduce its fleet size. After the DH trips have been inserted, the deficit functions are updated and the procedure is repeated until no further reductions of the fleet size are possible (1). The five chains can now be constructed through the FIFO rule: [1-8], [2], [3-DH_2-4], [5-DH_1-6], [7-DH_3-9].

DEFICIT-FUNCTION APPROACH WITH VARIABLE DEPARTURE TIMES

The following section is an analysis of bus scheduling through the deficit-function approach when variable departure times are allowed.

Variable Trip-Departure Times

A general description of a technique to reduce the fleet size for variable departure-time scheduling problems can be found in Gertsbach and Stern (4). This technique for job schedules uses the deficit-function representation as a guide for local minimization in maximal intervals, M_r^u $\forall$ uϵT. However, when variable departure times are considered along with a possible insertion of DH trips, the problem becomes more complex. The scheduler who performs shifting in trip departure times is not always aware of the consequences that could arise from these shifts. This section analyzes a method that will serve as a guide for the scheduler, particularly in a person-computer conversational mode.

Let us define $[t_s^i - \Delta_2^i, t_s^i + \Delta_1^i]$ as the tolerance time interval of the departure time of trip i, iϵS, where Δ_1^i is the maximum delay from the scheduled departure time (the case of a late departure) and Δ_2^i is the maximum advance of the trip scheduled departure time (the case of an early departure).

According to the definitions in the previous section, s_r^u and e_r^u, the start and end of the rth maximal interval M_r^u [r = 1,2,...,n(u)] at terminal u, uϵT, are associated with $t_s^{i_r}$ and $t_e^{j_r}$, respectively. That is, s_r^u refers to the departure time of a trip designated i_r and e_r^u to the arrival time of a trip designated j_r (where i_r, j_r can be selected from several trips that depart at time s_r^u and arrive at e_r^u, respectively). Now we can state a proposition that enables the scheduler to determine whether the fleet size can be reduced through shifts in trip departure times:

If $\bar{M}_r^u$ for all r [r = 1,2,...,n(u)] satisfies one or more of the conditions stated below, then by appropriate shifts (indicated in the conditions) of trip departure times, the fleet size at terminal u is decreased by 1 and remains unchanged for all other terminals. For the following four conditions, let $t_s^{i_r}$ be the departure time of trip i_r from terminal u to m and $t_e^{j_r}$ be the arrival time of trip j_r from terminal k to u.

Condition (a)

If $\bar{M}_r^u \leq \Delta_1^{i_r} + \Delta_2^{j_r}$, then $t_s^{i_r}$ and $t_e^{j_r}$ can be shifted in opposite directions so that the total shifting time is equal to $\bar{M}_r^u$ provided that neither D(m) nor D(k) increases as a result of this shift.

Condition (b)

If $\bar{M}_r^u < \Delta_1^{i_r}$, then $t_s^{i_r}$ can be shifted to the right by time $\bar{M}_r^u$, provided that D(m) does not increase because of this shift. The shifts in condition (a) can also be applied here.

Condition (c)

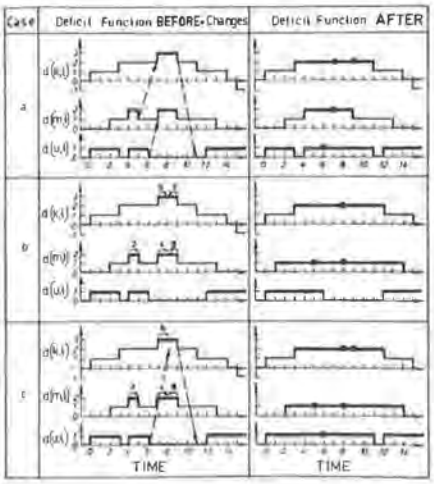
If $\bar{M}_r^u \leq \Delta_2^{j_r}$, then $t_e^{j_r}$ can be shifted to the left by time $\bar{M}_r^u$, provided that D(k) does not increase because of this shift. The shifts in condition (a) can also be applied here.

Condition (d)

If $\bar{M}_r^u \leq \Delta_i^{i_r}$ and $M_r^u \leq \Delta_2^{j_r}$, then condition (a), (b), or (c) could be considered.

Another possibility is to consider variable trip departure times along with the DH trip-insertion procedure. In that case, the feasibility

Figure 2. Before and after deficit-function representation for reducing fleet size through (a) URDHC procedure, (b) shifting trip departure times, and (c) mixed procedures.



requirement shown in Equation 2 is changed to the following:

$$t_e^i - \Delta_2^i + t_{uv} \leq t_s^j + \Delta_1^j \quad (3)$$

Multiobjective Criteria

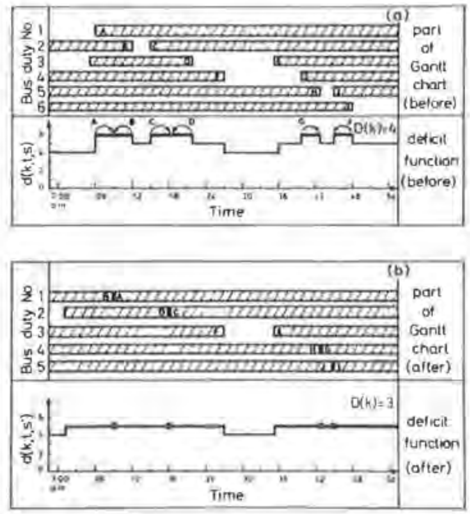
The main goal in the bus-scheduling problem is to reduce the fleet size, especially during major peak periods in a normal daily operation. This reflects the real possible saving in capital cost. However, when a person-machine conversational mode is incorporated into the scheduling process, secondary objectives can be adopted. In this way, the scheduler will be able to evaluate the results of his or her own suggestions in order either to reduce operating costs or to examine whether the resultant schedule follows a given policy.

In the proposition given above, four conditions are mentioned. If condition (b) or (c) or (d) is fulfilled, the scheduler might face an optional decision: to reduce the number of shifts or to keep the shift times as small as possible. For example, in the first option, a single trip departure time is shifted by time $\bar{M}_r^u$. In the second option, two trip departure times are shifted, each by time $\frac{1}{2}\bar{M}_r^u$. If the policy is to minimize changes in the timetables, the first option is preferable. If the policy is to adhere as closely as possible to a planned timetable, the second option is given priority.

A trade-off is also observed between insertion of DH trips and shifting trip departure time. For example, Figure 2 includes three scheduling cases for the fixed schedule presented in Figure 1. In case (a), as in Figure 1, the fleet size at terminal k is reduced by 1 through insertion of three DH trips. In case (b), the fleet size is reduced by 1 in both terminals k and m through shifts of trip departure times. In case (c), a mixed operation on the deficit function enables a reduction in the fleet size by 2 [the same result as that in case (b)] both through shifting trip departure times and inserting DH trips. Note that the indicated numbers of the shifted trips and the DH trip times are the same as the example in Figure 1. In addition, $\Delta_1^i = \Delta_2^i = 1$ time unit for all the nine trips in the schedule.

In this trade-off situation, there are two clear secondary objectives. The first is to minimize the changes in the planned timetables and to attempt the use of DH trips instead of shifting trip departure times. The second objective is to minimize the operational cost and to attempt shifting trip departure times rather than inserting DH trips. The first objective might be associated with the transport authorities' desire to maintain a highly reliable timetable for the passengers, whereas the second objective generally expresses the view of the bus operator.

Figure 3. Demonstration of superiority of deficit-function representation over Gantt-chart approach.



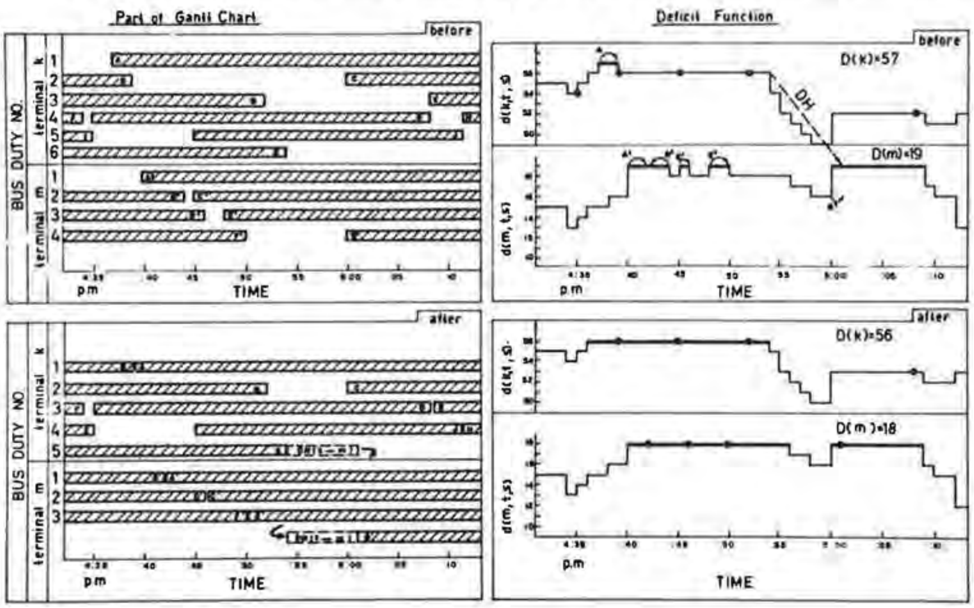
INITIAL EXPERIENCE WITH BUS COMPANY

As mentioned in the introduction, this research was motivated by Egged, the Israel National Bus Carrier. The need for a quicker response to timetable changes has led Egged management to investigate the use of a fully computerized scheduling system. This system is based on an optimization technique reported by Gavish, Schweitzer, and Shlifer (9) and discussed by Ceder and Gonen (10). Attempts to implement the computer-generated schedule have failed because of the inability to meet a number of necessary practical constraints. Such constraints include the need to plan for more than 2500 bus trips (the program maximum capability), consideration of drivers' meal breaks and relocation, constraints imposed by nonidentical bus types, etc. It was also felt that the optimal schedule provided had no advantage over the traditional methods. Furthermore, the schedulers were not confident in using the optimal technique because of their lack of knowledge of the operation of this method.

It was therefore decided to continue the search for an approach that would combine the advantages of modern electronic computers while at the same time allow the scheduler to make his or her own contribution to the scheduling task. Because of its visual nature, a deficit-function approach was selected to be used on a person-machine interactive system. The implementation of the deficit-function approach is now gradually being introduced so that the schedulers can gain confidence in this approach and reach the conclusion that this method is very useful in increasing the speed and accuracy of the scheduling tasks.

Two simple real-life examples are given here to demonstrate the implementation stage at Egged. In the first example, illustrated in Figure 3, the schedulers claimed at the beginning that it was

Figure 4. Two-terminal case in which two buses are saved through shifting departure times and modified URDHC procedures.



impossible to further reduce the fleet size from their Gantt-chart scheduling results. In Figure 3, only a small part of the Gantt chart and the corresponding deficit functions is shown, that part undergoing changes. The schedulers allow for an acceptable shift in trip departure time for all trips by $\Delta_1^i = \Delta_2^i = 3$ min. By illustrating $d(k,t,S)$, we saved a bus by shifting six trips, each by 3 min. In the Gantt chart, before changes (Figure 3a), trips are designated by letters for identification. This is for the sake of clarity in referring to shifts and reconstruction of the Gantt-chart chains in Figure 3b.

From Figure 3, the problem appears easy to handle. However, in Figure 3a only 6 out of 52 bus duties are shown, and those 6 rows in the Gantt chart were spread among the other 46 rows.

Following this demonstration, the schedulers were not wholly convinced. They argued that with a little more effort on their part, they too could have saved the bus as in Figure 3. Therefore, a more complex example was decided on, as shown in Figure 4. This second example refers to an afternoon schedule of two Egged branches, Ramle, terminal k, and Lod, terminal m. On the left-hand side of Figure 4, only those trips involved with changes are exhibited in the before and after Gantt-chart representation; trips are designated by letters. On the right-hand side, the deficit functions of the complete schedule are illustrated, which include trips not shown on the left-hand side. The schedulers again claimed that no further reductions could be achieved from the $D(k) + D(m) = 57 + 19 = 76$ fleet-size requirement. The information given was that $\Delta_1^i = \Delta_2^i = 2$ min and that the DH trip time between the terminals is $t_{km} = t_{mk} = 7$ min.

As seen in Figure 4, six shifts in trip departure times and a single DH trip are required in order to save two buses and to reduce the fleet requirement to 74 buses. It was only after this second demonstration that the schedulers began to take a serious interest in the deficit function. This was due particularly to its simplicity and visual nature. The schedulers expressed their positive feeling about the valuable aid of this gradual approach.

CONCLUDING REMARKS

This paper attempts to develop a methodology for variable bus scheduling in order to further reduce the fleet size in comparison with the developed algorithm (1,2) for a fixed bus-scheduling problem. The approach used here employs a highly informative graphical technique based on deficit-function theory and is designated primarily for operation in a conversational person-computer mode. For example, the fixed bus-scheduling algorithm (1,2) has been programmed for use with a PDP 11/40 video screen and light pen. This allows the scheduler to insert and delete trips quickly and to immediately see the effects on fleet size through the updated deficit-function display. In this way, the scheduler can use the light pen to shift a trip departure time and to see the effect on the number of buses required. The objective of the proposed approach is also to allow the scheduler to consider multiobjective criteria through evaluation of his or her own suggestions.

Work is continually in progress at Egged in three parallel directions: (a) providing the Gantt-chart schedulers with a computer-generated graphical representation of deficit functions, (b) preparing the ground for a person-machine interactive system, and (c) conducting further research to provide an algorithm with an enhanced flexibility to incorporate a large number of practical considerations.

REFERENCES

1. A. Ceder and H.I. Stern. A Deficit Function Procedure with Dead-Heading Trip Insertions for Fleet Size Reductions. Transportation Science, Vol. 15, No. 4, 1981, pp. 338-363.
2. H.I. Stern and A. Ceder. A Deficit Function Approach for Bus Scheduling. In Computer Scheduling of Public Transport (A. Wren, ed.), North-Holland, Amsterdam, Netherlands, 1981, pp. 85-96.
3. I. Gertsbach and Y. Gurevich. Constructing an Optimal Fleet for a Transportation Schedule. Transportation Science, Vol. 11, 1977, pp. 20-36.

4. I. Gertsbach and H.I. Stern. Minimal Resources for Fixed and Variable Job Schedules. Operations Research, Vol. 26, 1978, pp. 68-85.
5. T.E. Bartlett. An Algorithm for the Minimum Number of Transport Units to Maintain a Fixed Schedule. Naval Research Logistics Quarterly, Vol. 4, 1957, pp. 139-149.
6. V.K. Linus and M.S. Maksim. On the Problems of Constructing Routes. Proc., Institute of Civil Aviation Engineering, Vol. 102, 1967, pp. 36-45 (in Russian).
7. F.J.M. Salzborn. Optimum Bus Scheduling. Transportation Science, Vol. 6, 1972, pp. 137-148.
8. F.J.M. Salzborn. Minimum Fleet Size Models for Transportation Systems. In Transportation and Traffic Theory (D.J. Buckley, ed.), North-Holland/Elsevier, New York, 1974, pp. 607-624.
9. B. Gavish, P. Schweitzer, and E. Shlifer. Assigning Buses to Schedules in a Metropolitan Area. Computers and Operations Research, Vol. 5, 1978, pp. 129-138.
10. A. Ceder and D. Gonen. The Operational Planning Process of a Bus Company. UITP Review, Vol. 29(3), 1980, pp. 199-218.

Publication of this paper sponsored by Committee on Transit Service Characteristics.