Introduction to Game Theory and Its Application in Electric Power Markets

HARRY SINGH

Game theory is a discipline that is used to analyze problems of conflict among interacting decision makers. It may be considered as a generalization of decision theory to include multiple players or decision makers.

The concepts used in game theory can be traced back to the work of Cournot, Bertrand, and von Stackelberg. However, it was not until 1928 that von Neumann published the general theory for solving zero-sum games. The general theory for solving zero sum games became more widely known in 1944 with the work of von Neumann and Morgenstern. Many of the important developments in the field took place during the period from 1950 to 1960. The best known among these is the concept of Nash equilibrium. Game theory gained additional prominence as a subject in 1994, when the Nobel prize for economics was awarded jointly to John Harsanyi, John Nash, and Rienhard Selten for their contributions to the analysis of equilibria in non-cooperative games.

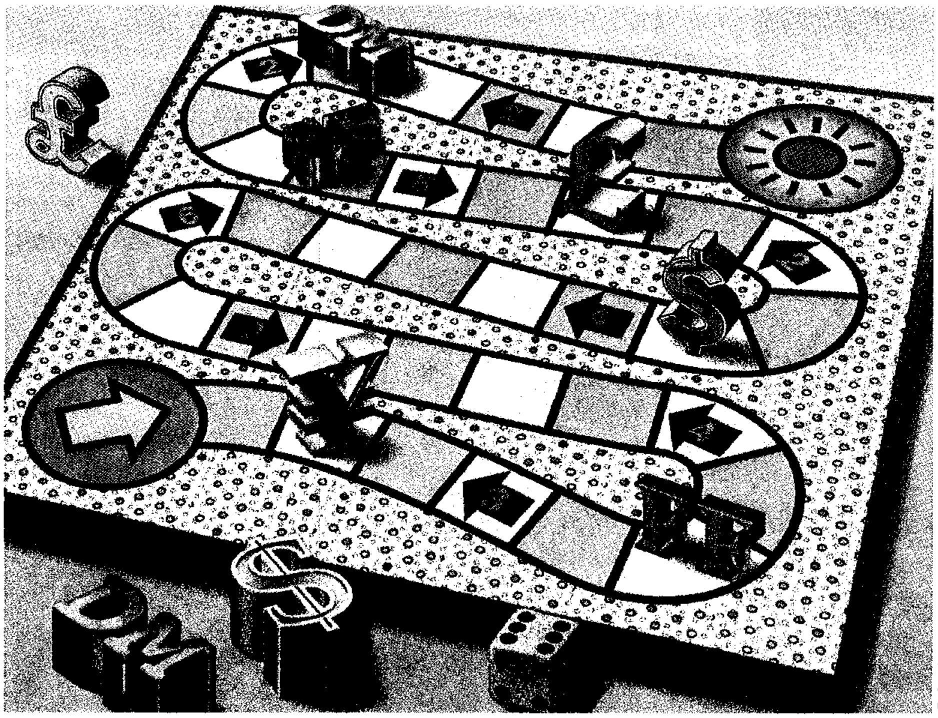
Game theory can be classified into two areas: *cooperative* and *noncooperative*. This tutorial provides a quick introduction to noncooperative game theory using applications in electric power markets.

Noncooperative Game Theory

Noncooperative games can be zero-sum games or nonzero-sum games. In *zero-sum games*, the gains of one player equal the losses of the other player. The solution of zero-sum games was first formulated by von Neumann and Morgenstern. In *nonzero-sum games*, the gains of one player do not equal the losses of the other player. The solution for nonzero-sum games was first formulated by John Nash, and the Nash equilibrium is now a universally used solution concept.

Noncooperative games can be described using two kinds for formats. The first format is the normal or strategic form, and the second is the extensive form. In the *strategic form*, one deals with a set of players, a set of choices or strategies available to the players, and a set of payoffs corresponding to these strategies. The payoff for a given player depends not only on the strategy chosen by that player but also on the strategies chosen by the other players. Additionally, it is assumed that the rules of the game, the strategies available to the players, and the payoffs are common knowledge. Each player is assumed to act rationally to maximize its profit.

Perhaps the best known problem in noncooperative game theory is the prisoner's dilemma. In the original version of the game, there are two players, A and B, who can either cooperate with each other and refuse to provide evidence or they can defect and implicate the other player. A concise representation of this game was provided by Aumann. Each player A and B must announce to a referee "Give me $1,000" or "Give the other player $3,000." The money under either strategy comes from a third party. The cooperate strategy for each player is to give the other player $3,000, while the defect strategy is to take $1,000. The payoffs for A and B can be represented as shown in Figure 1.



© Artville

The Nash equilibrium in this game involves each player choosing the defect strategy even though this is not the strategy that maximizes the payoff for a player. The payoffs for both players can be increased if they both choose the cooperate strategy. Howev-

Payoff ($)		Player B Cooperate	Player B Defect	
Player A	Cooperate	3000	0	A's payoff
		3000	4000	B's payoff
	Defect	4000	1000	A's payoff
		0	1000	B's payoff

Figure 1. Prisoner's dilemma

er, this is not a stable outcome as each player has an incentive to defect regardless of what the other player chooses. A Nash equilibrium exists if, for a given set of strategies chosen by other players, each player's strategy is an optimal response to those strategies. Thus, at a Nash equilibrium, a player's payoff decreases if it changes its strategy assuming all other players' strategies remain the same.

Finite nonzero-sum games are also called bimatrix games, given the notation used to represent the payoffs in the game. A bimatrix game Γ(A, B) consists of two players, each of whom has a finite number of actions called pure strategies. When player I chooses pure strategy i and player II chooses pure strategy j, their payoffs or gains are represented by a_{ij} and b_{ij}, respectively. A mixed strategy for player I is a vector x whose i-th component represents the probability of choosing pure strategy i. Thus $x_i \geq 0$ and $\Sigma\, x_i = 1$. A mixed strategy for player II is defined analogously. If x and y are a pair of mixed strategies for players I and II, their expected gains are $x'Ay$ and $x'By$, respectively. A pair of mixed strategies (x^*, y^*) is said to be a Nash equilibrium if

$$(x^*)'Ay^* \geq x'Ay^* \;\forall\; x \geq 0, \Sigma x_i = 1$$

and

$$(x^*)'By^* \geq (x^*)'By \;\forall\; y \geq 0, \Sigma y_i = 1.$$

In other words, (x^*, y^*) is a Nash equilibrium if neither player can gain by unilaterally changing its strategy.

A particularly interesting special case of a Nash equilibrium is a Nash equilibrium in pure strategies, i.e., one in which the probability of choosing a particular strategy is 1 for each player.

Noncooperative games are the foundation for some of the standard models in oligopoly. The study of oligopoly models is essential to study market power.

Cournot Duopoly

A *Cournot model* involves a duopoly game in which two firms produce an identical product and must decide how much to produce without knowing the output decision of the other. For convenience, assume that each firm's cost is 0. Assume that x_1 and x_2 represent the output decisions of each firm. The market price is represented by $p(x_1+x_2)$, where $p(x)$ is the inverse demand curve. The profits or payoffs for each firm are $\lambda_i = p(x_1 + x_2)x_i$. The strategy of each firm is to choose x_i in order to maximize its profit without knowing the decision of the other firm.

Bertrand Duopoly

Under a *Bertrand model*, each firm must choose the price at which it is willing to produce. Ignoring bounds on output, we can assume that the lower priced firm will capture market share and that both firms will have equal outputs at equal price. If $x(p)$ represents the market demand function, the payoff or profit of firm 1 can be represented as

$$\lambda_1(p_1,p_2)=\begin{cases} p_1x(p_1) & \text{if} \quad p_1<p_2 \\ p_1x(p_1)/2 & \text{if} \quad p_1=p_2 \\ 0 & \text{if} \quad p_1>p_2 \end{cases} \qquad (1)$$

A Bertrand game has a structure similar to that of the prisoner's dilemma. If both players cooperate, they can both charge the monopoly price. However, each player has an incentive to reduce its price slightly and capture market share, even though it knows that both players will be worse off if they both cut price.

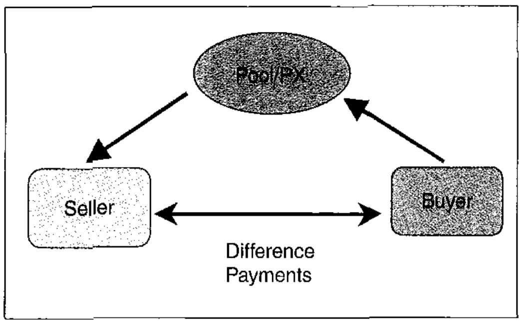


Figure 2. Payment streams in a CfD

Market Power Mitigation

Market power can be defined as the ability of a market participant to raise prices above the competitive level by restricting output or restricting new entrants. Horizontal market power is often associated with a single firm or a few firms controlling a large part of the supply.

Although generation divestiture has been used as a remedy for this problem in the electric power industry, it is not always a viable option. In such instances, financial contracts such as contracts for differences (CfD) can be used to accomplish what might be termed as virtual divestiture. Game theory can be used to study the effects of CfDs on bidding incentives. The purpose of a CfD is to insulate the supplier against the temporal price variations in the market. One counter-party in the CfD agrees to pay the other the difference between the contract price and the prevailing market or pool price, as depicted in Figure 2.

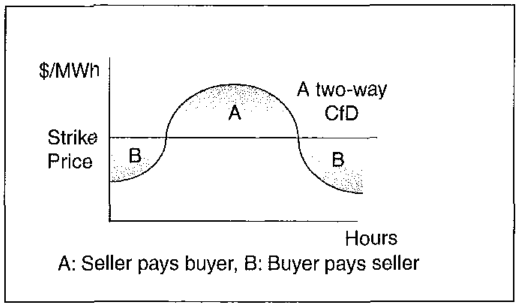


Figure 3. Two-way CfD

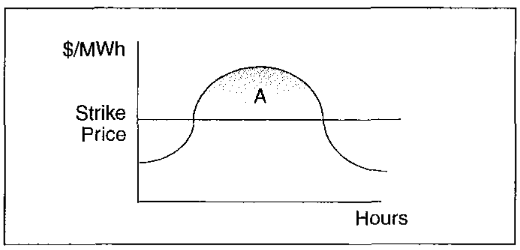


Figure 4. One-way CfD

A CfD can be either two-way or one-way. A *two-way CfD* is similar to a financial futures contract and is defined in terms of a strike price ($/MWh), and a quantity (MWh). As shown in Figure 2, for the defined quantity, the seller pays the buyer if the pool price rises above the strike price, and the buyer pays the seller if the pool price falls below the strike price. A *one-way CfD* is similar to a financial option contract and also includes an option fee in addition to the strike price and contract quantity. Under a one way contract, difference payments are made only if the pool price rises above the strike price, as shown in Figure 3.

The effect of a CfD is to fix or bound the revenue for a generator. In the extreme, where the entire output of a generator is contracted under a CfD, the generator's revenue will be completely insulated from market price variations, and, consequently, the generator should have no incentive to raise prices. Ideally, one would like to contract just the appropriate fraction of output required to mitigate market power.

To illustrate how CfDs can eliminate incentives to raise prices, we will set up a simple Cournot model with two generators (A and B) and one load, as shown in Figure 5. Each of the generators has an incremental cost of $10/MWh and a maximum output of 75 MW. The strategic decision for the generators is to choose a level of output that maximizes their profits. The price is set by the demand curve, which is also shown in Figure 5. We will assume that each generator chooses between two levels of output, a high output of 75 MW and a low output of 20 MW, as shown in Figure 6. The low output may be interpreted as

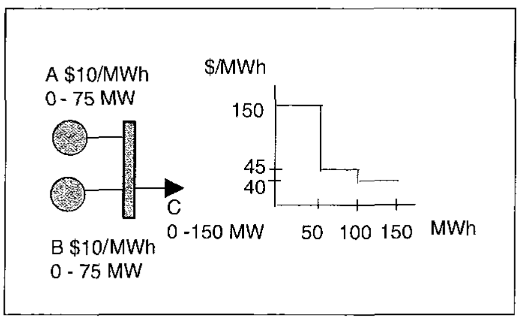


Figure 5. A two generation game

Output (MW)		Generator B High	Generator B Low	
Generator A	High	75	75	A's output
		75	20	B's output
	Low	20	20	A's output
		75	20	B's output

Figure 6. Output decisions of A and B

Price ($/MWh)		Generator B High	Generator B Low
Generator A	High	40	45
	Low	45	150

Figure 7. Prices corresponding to output decisions

Profit ($)		Generator B High	Generator B Low	
Generator A	High	2250	2625	A's profit
		2250	700	B's profit
	Low	700	2800	A's profit
		2625	2800	B's profit

Figure 8. Profits without CfD

withholding of capacity with a motivation to increase prices. If prices increase sufficiently, the generator can make a higher profit at the low output.

Profit ($)		Generator B High	Generator B Low	
Generator A	High	2250	2475	A's profit
		2250	550	B's profit
	Low	550	-500	A's profit
		2475	-500	B's profit

Figure 9. Profits with CfD for 30 MW

Profit ($)		Generator B High	Generator B Low	
Generator A	High	2250	2575	A's profit
		2250	650	B's profit
	Low	650	1700	A's profit
		2575	1700	B's profit

Figure 10. Profits with CfD for 10 MW

There are four possible cases to consider, depending on the decision of each generator. The prices corresponding to these cases are shown in Figure 7. Figure 8 shows a Nash equilibrium for the case when both generators choose low levels of output to maximize their profits. However, if a CfD is applied to 30 MW of the generators output, the Nash equilibrium changes, as shown in Figure 9. The strike price in the CfD is assumed to equal the competitive price of $40/MWh. In this case, profits are maximized at the competitive price corresponding to the high output by each generator. Similarly, Figure 10 shows the profits if a CfD is applied to 10 MW of the output.

Cooperative Game Theory

Cooperative game theory, which is quite different from noncooperative game theory, is generally applied to solve allocation problems. The various solutions proposed for cooperative games can be interpreted as alternative solutions to an allocation problem. The key ideas involve the concept of coalitions or groups that are formed to benefit from economies of scale.

Equity arguments call for solutions that allocate costs to coalitions in a manner that guarantees that all coalition members are at least as well off as they would be if they were not a part of the coalition. This is sometimes called the stand-alone test. Solutions that exactly allocate the total costs and satisfy the stand-alone test, are called *core solutions*. Alternative solution concepts such as the Shapley Value are also possible.

The emphasis in cooperative game theory is on solutions that are equitable. In contrast, noncooperative game theory helps us study efficient solutions under new market designs. Just as we study the stability of an engineering system, we can study how efficient a market design might be by using game theory.

The examples in this article were highly simplified. There are many other problems that deal with the behavior of market participants in transmission networks under congestion that the reader will find of interest.

Acknowledgment

This tutorial is based on excerpts from the *Game Theory Applications in Electric Power Markets* tutorial, which was prepared for presentation at the 1999 IEEE PES Winter Meeting. The 103-page multiauthor tutorial book (99TP-136) can be ordered through IEEE Customer Service. Chapters include: Introduction; Analyzing Strategic Bidding Behavior in Transmission Networks; Using Game Theory to Study Market Power in Simple Networks; Bidding Strategies for Lagrangian Relaxation Based Power Auctions; Risk Management Using Game Theory in Transmission Constrained Unit Commitment; Market Power Evaluation in Power Systems with Congestion; Market Power Mitigation; Some Things Experiments Reveal About Market Power Opportunities Offered by a Constrained Transmission System; The Best Game in Town: NERC's TLR Rules; and Hacking with Megawatts: Gaming via Governor Control in a Competitive Generation Environment.

For Further Reading

Game Theory Applications in Electric Power Markets, IEEE tutorial, 99TP-136, 1999.

R. Aumann, "Game Theory," in *The New Palgrave*, eds. J. Eatwell, M. Milgate, P. Newman, McMillan, London, 1987.

D. Fudenberg, J. Tirole, *Game Theory*, The MIT Press, Cambridge, 1991.

D. Fudenberg, D.K. Levine, "The Theory of Learning in Games," The MIT Press, Cambridge, 1998.

D.A. Kreps, Game Theory and Economic Modeling, Clarendon Press, Oxford, 1990.

H.R.Varian, Microeconomic Analysis, W.W. Norton and Co., New York, third edition, 1992.

J. von Neumann, O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton NJ, 1944.

H.P. Young, "Cost Allocation," in *Handbook of Game Theory with Economic Applications*, volume 2, eds. R. Aumann, S. Hart, North Holland, Elsevier, Amsterdam, 1994.

Biography

Harry Singh is manager of Electricity Economics at PG&E Energy Services (*PG&E Energy Services is not the same company as Pacific Gas and Electric Company, the utility. PG&E Energy Services is not regulated by the California Public Utilities Commission, and you do not have to buy PG&E Energy Services' products in order to continue to receive quality regulated services from Pacific Gas and Electric Company.*) Prior to joining PG&E Energy Services, he worked with the Pacific Gas and Electric Company in San Francisco, where he was a part of the team responsible for setting up the California ISO and PX. He received a PhD in electrical engineering from the University of Wisconsin at Madison.