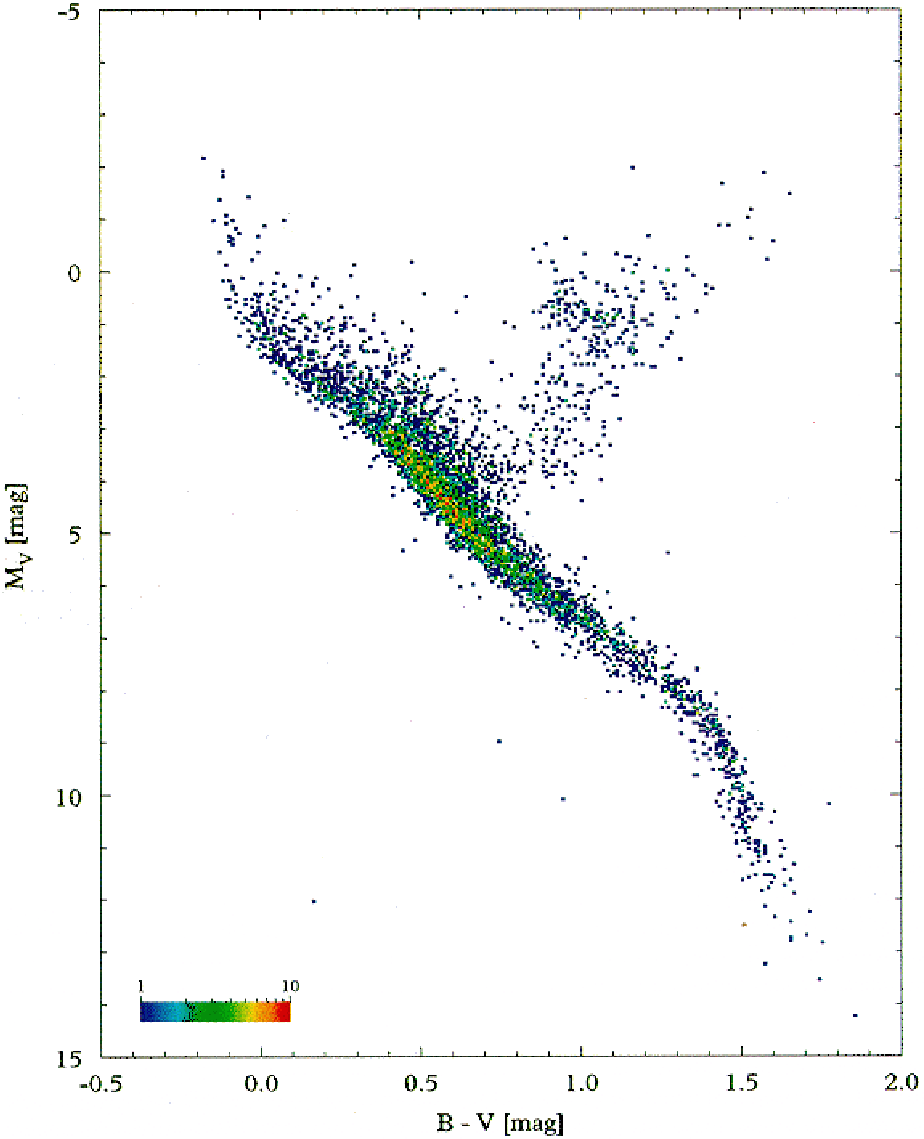
Structure and Evolution of Stars

Lecture 18: Post-Main Sequence Evolution of Low Mass and High Mass Stars

- Very low mass stars $M<0.7M_{sun}$ long lives and truncated evolution
- Evolution of $5M_{sun}$ star
  - Schonberg-Chandrasekhar Limit
  - Ascent of the red giant branch
  - Ignition of Helium burning in core
  - Horizontal Branch and ascent of the asymptotic giant branch
  - Mass Loss-planetary nebula -white dwarf
- View from the centre of the star
- Evolution of $>10M_{sun}$ stars
  - onion-skin model
  - burning timescales
  - Type II supernova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Post-Main Sequence Evolution: <0.7M_{sun} Stars

- Chose mass range 0.7-2M_{sun} in Lecture 17 as essentially everything happens – degenerate core; shell-burning; giant-branch; Helium-flash; horizontal-branch; double shell burning; asymptotic giant branch; extreme mass loss; planetary nebula; remnant white dwarf

- For very low mass stars, convective mixing occurs down to H-burning core, making more fuel available. Lifetime on main sequence even longer than time indicated by $\tau = M/L \sim M^{-2}$ scaling

- No star with M<0.7M_{sun} has actually evolved off the main sequence – $\tau_{ms} > \tau_{universe}$!

- Core will become exhausted eventually and shell H-burning phase surrounding degenerate core initiated and L increases

Post-Main Sequence Evolution: <0.7M_{sun} Stars

- As envelope is already convective and star already close to Hayashi forbidden zone, star moves essentially vertically off the main sequence, ascending the giant branch $(L = 4\pi R\sigma T_{eff}^4)$

- Even at tip of the giant branch, mass of the degenerate He-core is not sufficient that $T_{\rm core}$ reaches 10^8K for initiation of triple-α reaction

- No horizontal branch phase, or any phase, in which He-burning occurs

- Believed that stars will undergo extensive mass loss and suffer similar instabilities (as H-burning in shell begins to give out) as more massive stars at the tip of the asymptotic giant branch – creation of low mass Helium white dwarf, revealed as stellar envelope ejected

Post-Main Sequence Evolution: $5M_{sun}$ Star

- On the ZAMS, energy generation rate is so large (high $T_{core} \approx 20$ million K), that core is convective (Point A on plot)

- End of main sequence life due to depletion of Hydrogen in core (Point B on plot)

- As with lower mass stars, lack of energy source in the Helium core results in contraction of core and T and ρ in surrounding shell rise to point where shell H-burning occurs

- Important difference for the core is that T_{core} in the isothermal core is high enough that degeneracy pressure does not dominate and $P=nkT$

- How massive can an isothermal Helium core be and remain stable?

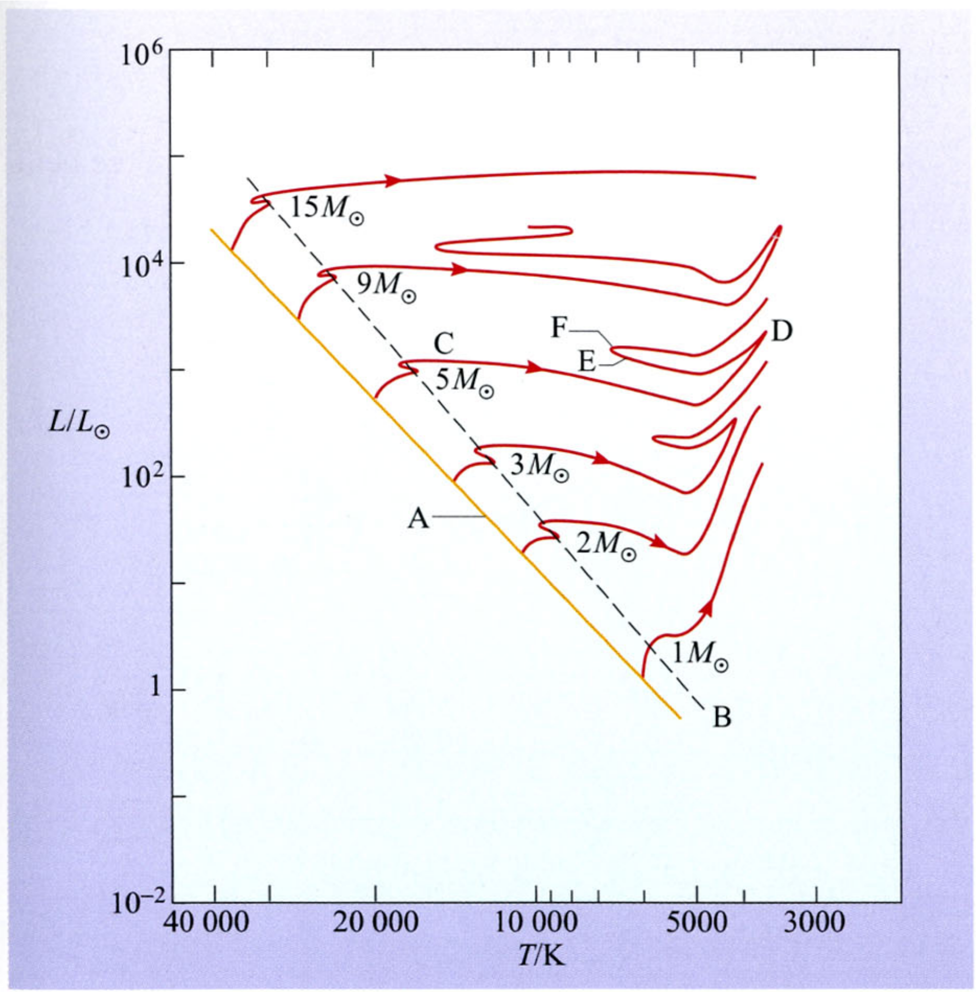


Figure 7.2 The predicted paths of stars on the H–R diagram as they evolve off the main sequence to the red giant (or supergiant) phase. The letters on the $5M_\odot$ track refer to different stages of nuclear reactions in the star. The line marked A denotes the onset of hydrogen core fusion – the start of main sequence life. The dashed line B denotes the cessation of hydrogen core fusion – the end of main sequence life, and the onset of hydrogen shell fusion. Subsequent stages are labelled on the $5M_\odot$ track only: (C) hydrogen shell fusion continues; (D) helium core fusion starts; (E) helium core fusion continues; (F) helium shell fusion starts. The small loops to the left for the $1M_\odot$ and $2M_\odot$ stars have been omitted for clarity.

Schonberg-Chandrasekhar Limit

- Used virial theorem in Lecture 6 to derive relation between KE & PE at radius *s* in a star

$$P_s V_s - \int_0^{M_s} \frac{P}{\rho} dm = \frac{1}{3} E_{grav}(r < s)$$

- Denoting core properties by *c* and *P* at the core boundary by *s*

$$\int_0^{V_c} P dV = P_s V_c + \frac{1}{3} \alpha \frac{G M_c^2}{R_c}$$

- For perfect gas, *P=nkT*:

$$\int_0^{V_c} P dV = \frac{k}{\mu_c m_H} T_c \int \rho dV = \frac{k}{\mu_c m_H} T_c M_c$$

- Also have relation between *V* and *R*:

$$V_c = 4\pi R_c^3 / 3$$

- Substituting last 2 equations in the second equation:

$$\Rightarrow P_s(R_c) = \frac{3}{4\pi} \frac{k T_c}{\mu_c m_H} \frac{M_c}{R_c^3} - \frac{\alpha G}{4\pi} \frac{M_c^2}{R_c^4}$$

Schonberg-Chandrasekhar Limit

- Find minimum and maximum pressure at fixed core mass by differentiating

$$dP_s / dR_c = 0$$

$$\Rightarrow P_s = 0 \text{ at } R_0 = \frac{\alpha \mu_c m_H G}{3k} \frac{M_c}{T_c}$$

core with radius $<R_0$ would collapse under own gravity

and

$$\Rightarrow P_{s,\max} \text{ at } R_1 = \frac{4\alpha \mu_c m_H G}{9k} \frac{M_c}{T_c}$$

- Substitute for R_1 in last equation on previous slide to find maximum P at boundary of core as a function of core mass

$$\Rightarrow P_{s,\max}(M_c) = const \frac{T_c^4}{M_c^2 \mu_c^4}$$

Schonberg-Chandrasekhar Limit

- Core is small, treat as a point mass, and use constraint on central pressure from hydrostatic equilibrium (Lecture 6)

$$R_c << R; \quad P_{env} > \frac{GM^2}{8\pi R^4}$$

- Obtain limit on maximum pressure for stability:

$$P_{s,\max}(M_c) = const\,\frac{T_c^4}{M_c^2 \mu_c^4} \geq \frac{GM^2}{8\pi R^4}$$

- Use homology relation derived in Lecture 10 to eliminate *T* and *R*

$$T_c \propto \frac{\mu_{env} m_H G}{k}\frac{M}{R}$$

- Gives limit to the fractional mass of the core in terms of the composition of the core and the envelope. Full derivation has *const*=0.37

$$\Rightarrow \frac{M_c}{M} \leq const\left(\frac{\mu_{env}}{\mu_c}\right)^2$$

- Once limit reached, core collapses

$$\mu_{env} \approx 0.6; \;\; \mu_c \approx 1 \Rightarrow \frac{M_c}{M} \leq 0.13$$

Post-Main Sequence Evolution: $5M_{sun}$ Star

- Main sequence stars with masses $>2M_{sun}$ possess Helium cores that exceed the Schonberg-Chandrasekar limit and it is the violation of the limit that produces rapid contraction (c.f. behaviour of degenerate core in low mass stars)

- Core shrinks on a dynamical timescale until temperature gradient capable of balancing gravity is achieved.

- Initially envelope expands, and star moves to the right in the HR-diagram, with radius increasing and T_{eff} dropping (from Point C)

- Shell H-burning starts to undergo large increase in L, even greater than for low-mass stars because CNO cycle dominates

- Hayashi forbidden zone limits rightward motion and ascent of giant branch begins (towards Point D)

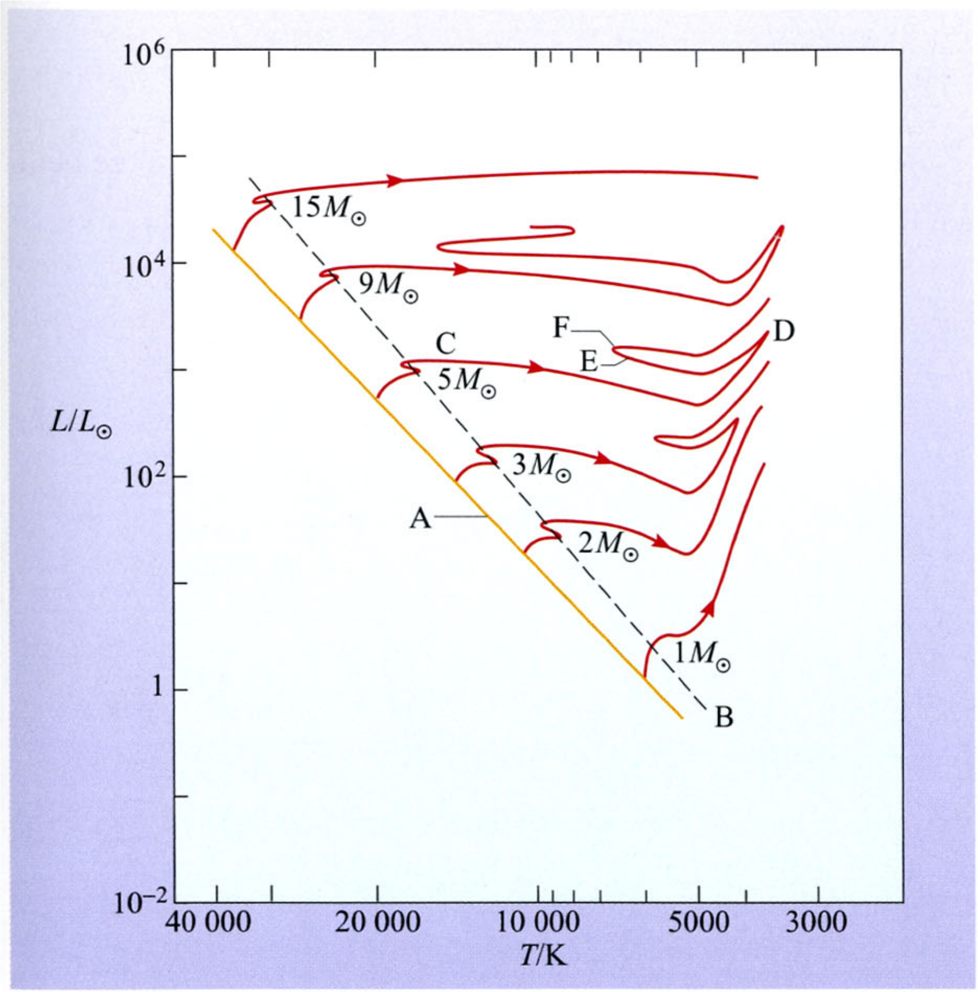


Figure 7.2 The predicted paths of stars on the H–R diagram as they evolve off the main sequence to the red giant (or supergiant) phase. The letters on the $5M_\odot$ track refer to different stages of nuclear reactions in the star. The line marked A denotes the onset of hydrogen core fusion – the start of main sequence life. The dashed line B denotes the cessation of hydrogen core fusion – the end of main sequence life, and the onset of hydrogen shell fusion. Subsequent stages are labelled on the $5M_\odot$ track only: (C) hydrogen shell fusion continues; (D) helium core fusion starts; (E) helium core fusion continues; (F) helium shell fusion starts. The small loops to the left for the $1M_\odot$ and $2M_\odot$ stars have been omitted for clarity.

Post-Main Sequence Evolution: $5M_{sun}$ Star

- Evolution from Points C to D is very rapid and HR-diagram is very sparsely populated – *Hertzsprung Gap*

- Ascent up the giant branch is ended by core temperature reaching 10^8K and core Helium burning commencing (Point D)

- Unlike $<2M_{sun}$ stars the ignition does not take place under degenerate conditions and is much more controlled! – $P=nkT$ thermostat

- Effect on star is much the same though, core expands, now providing a second energy source, but H-burning shell decreases output, envelope shrinks, and star descends onto the horizontal branch (Point E)

- Core becomes depleted of He with increasing C+O content

Post-Main Sequence Evolution: $5M_{sun}$ Star

- Increasing mass of C+O in core from He-burning

- Temperature cannot initiate C-burning, He-burning decreases but radius of C+O core decreases and *T* rises allowing He-burning rate to increase and H-burning shell output increases

- Envelope expands and star begins ascent of the asymptotic giant branch. H-burning shell weakens somewhat

- Core becomes degenerate and shrinks causing Temperature in surrounding shell to reach 10^8K and Helium-burning in shell commences

- Double shell burning phase with shrinking core, increase in *L* and expansion of envelope

Post-Main Sequence Evolution: $5M_{sun}$ Star

- Star with unstable double shell burning configuration, exceeding the Eddington Luminosity. Mass loss takes place and Helium-shell flashes occur, causing thermal pulses

- Planetary nebula phase ensues and bare C+O white dwarf revealed

- The lifecycle for a $5M_{sun}$ star is applicable over the main sequence mass range 2-$10M_{sun}$ with increasingly shorter lives

- Instructive to consider evolution not just from the surface – location on the HR-diagram – but from the centre, where the location of the core in the Log ρ vs Log T, determines the behaviour of the star

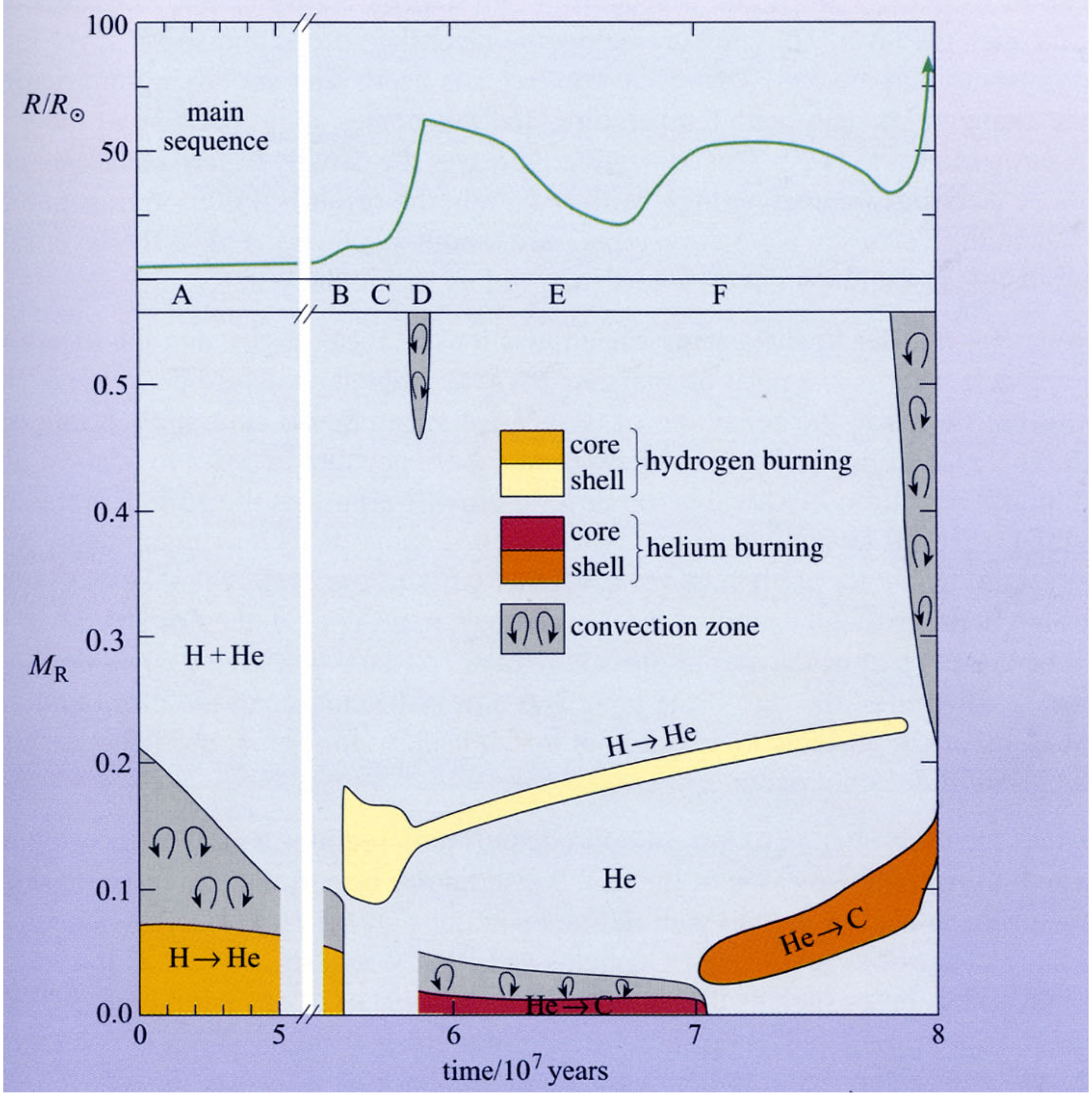


Figure 7.5 Schematic representation of the internal structure of a star of mass $5M_{\odot}$ during and after its main sequence lifetime. The upper panel shows the change in radius of the star with time. Note there is a change of scale in the time axis between 5 and 6×10^7 years to reflect the faster evolution of the star after it leaves the main sequence. The lower panel shows the change in composition and nuclear reactions in the star as it evolves. The vertical axis is the mass fraction M_R (the fraction of the total mass inside a given radius as we move outwards from the centre of the star), with the centre of the star at the bottom. The coloured regions indicate the locations of nucleosynthesis and the grey zones are convection zones. The labels A to F indicate the times of significant changes in the nuclear reactions as shown on the evolutionary track on the H–R diagram in Figure 7.2.

The log ρ – log T Plane

Where does He-core in 0.7-2M$_{sun}$ star lie just before Helium flash?

Helium core in 5M$_{sun}$ star?

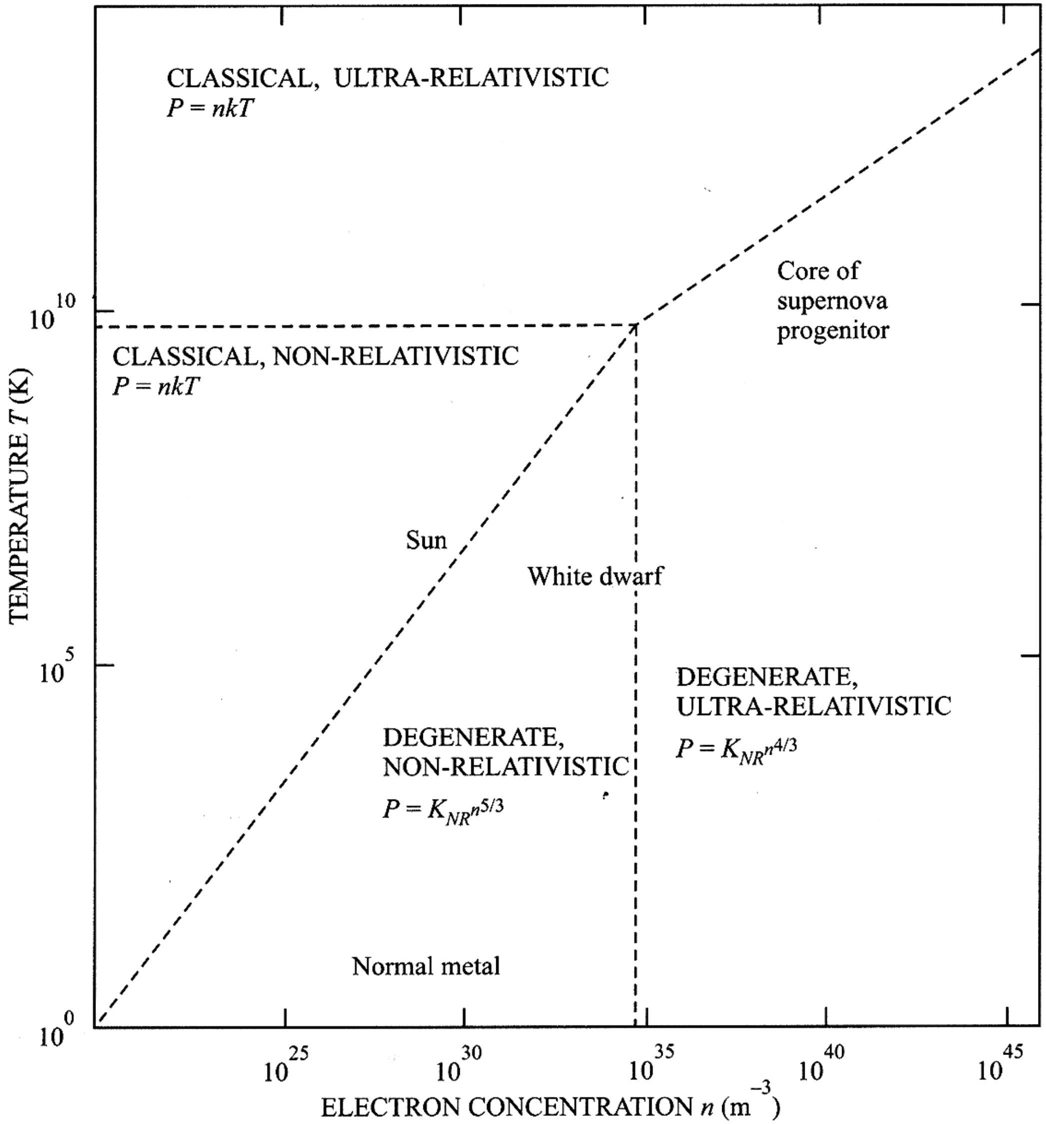
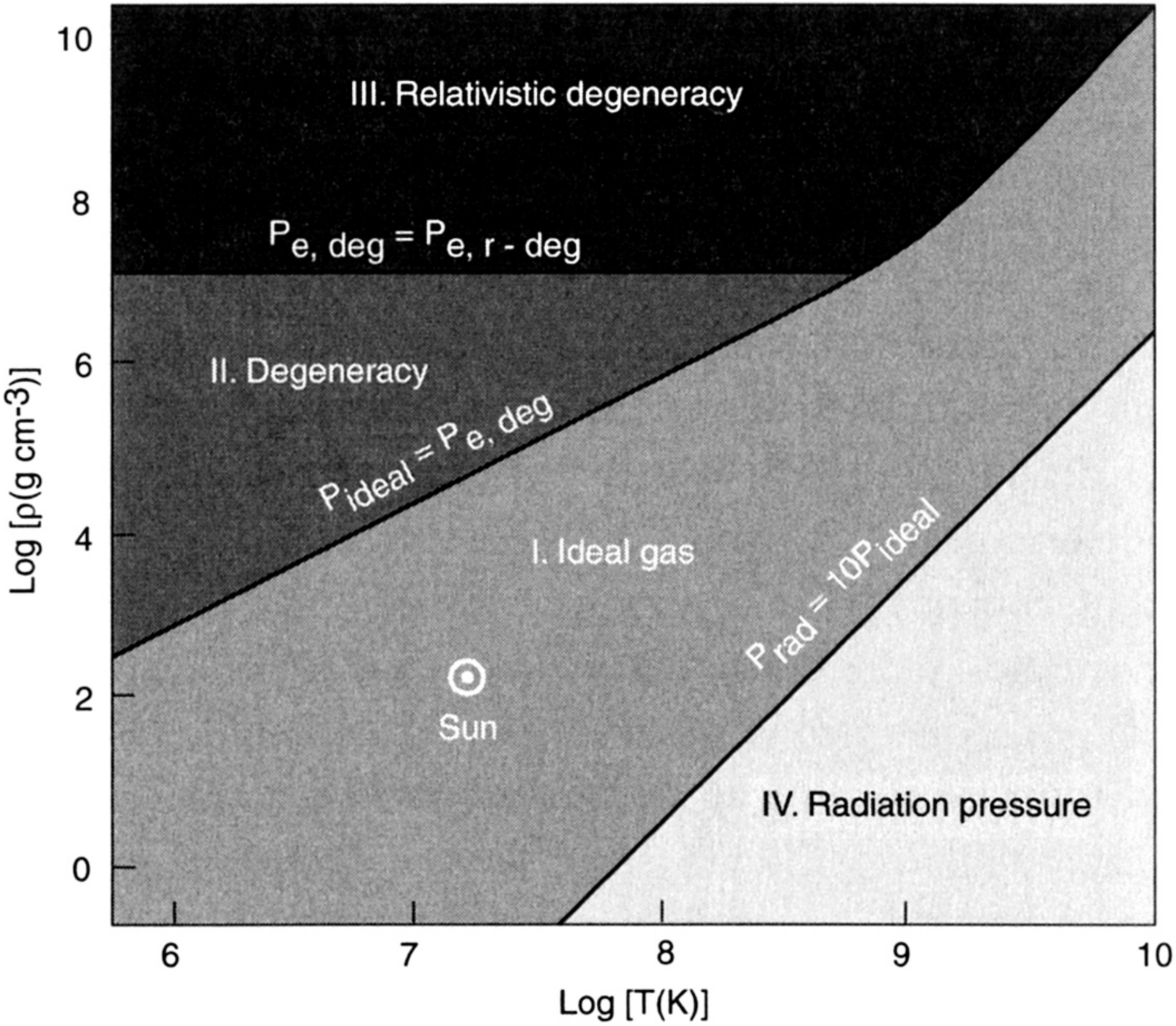


Fig. 2.2 Equation of state regimes for an ideal electron gas at a temperature T and at a density of n electrons per cubic metre. Typical values are shown for the temperature and density for electrons in a normal metal, in the sun, in a white dwarf and in the iron core of an evolved star just prior to a supernova


III. Relativistic degeneracy
$P_{e,\,deg} = P_{e,\,r\text{-}deg}$
II. Degeneracy
$P_{ideal} = P_{e,\,deg}$
I. Ideal gas
$P_{rad} = 10P_{ideal}$
Sun
IV. Radiation pressure
Log [ρ(g cm-3)]
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6
4
2
0
Log [T(K)]
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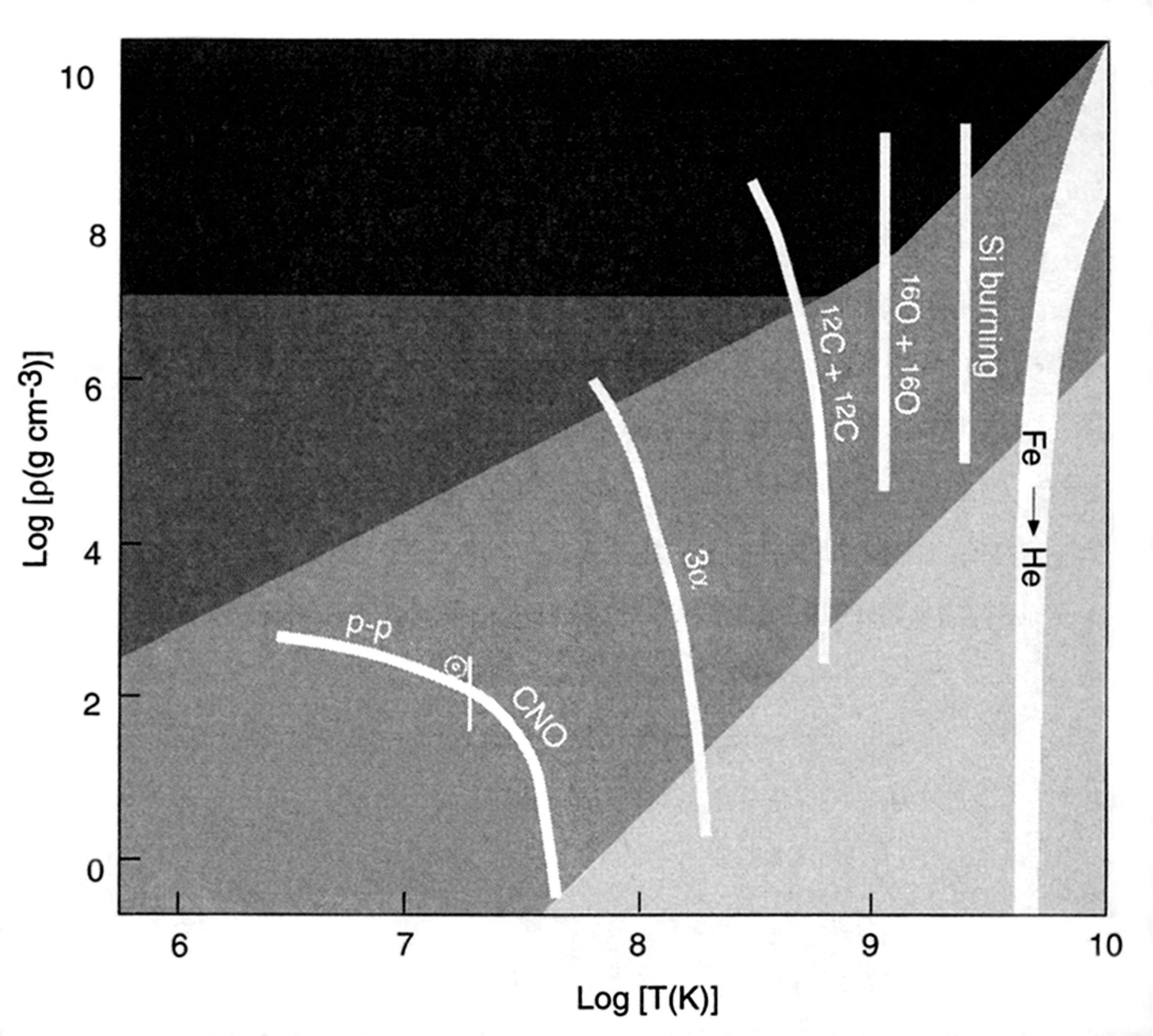
7
8
9
10


10
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Log [ρ(g cm-3)]
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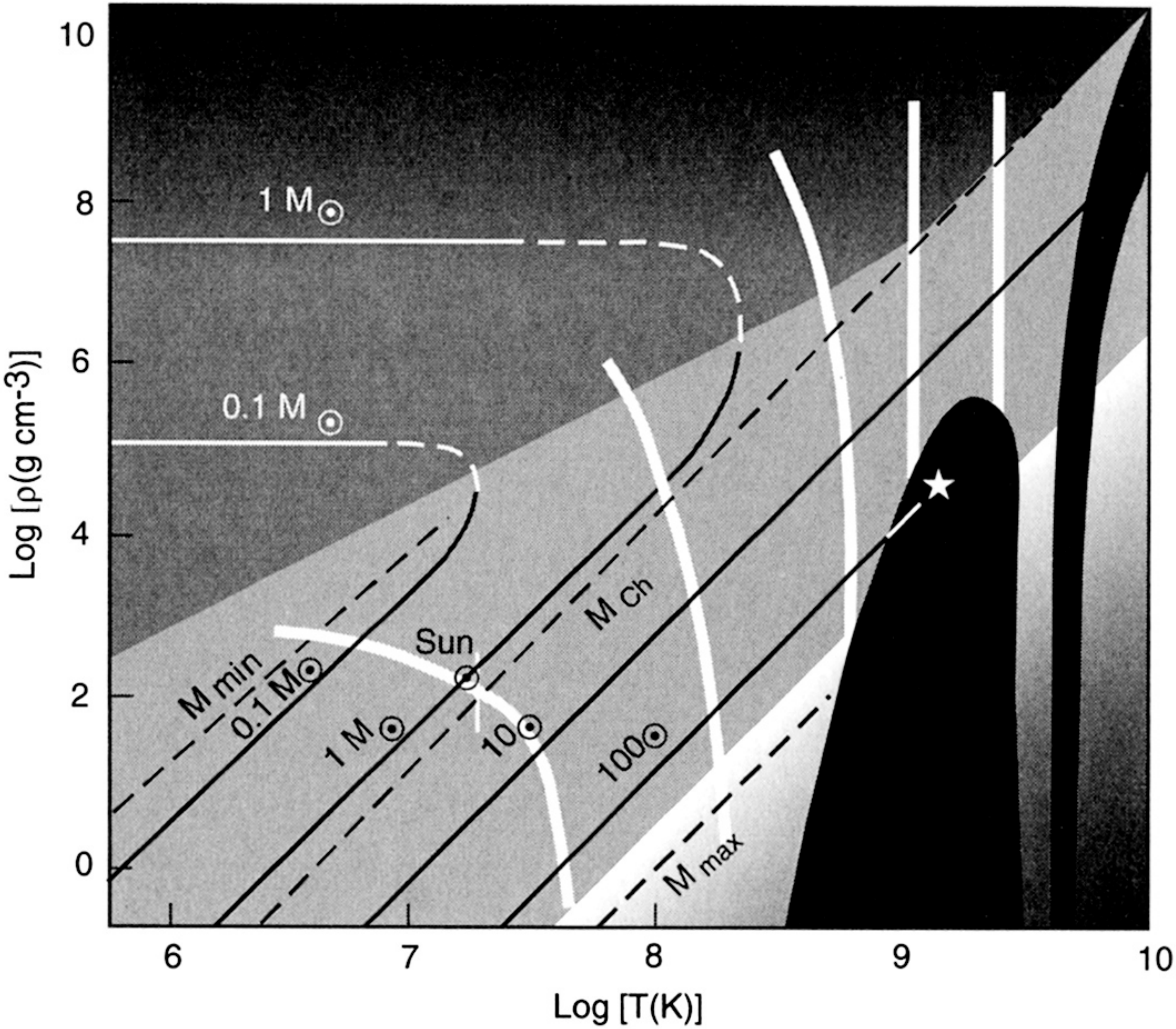
Log [T(K)]
p-p
⊙
CNO
3α
12C + 12C
16O + 16O
Si burning
Fe → He


1 M⊙
0.1 M⊙
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Sun
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100⊙
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Log [T(K)]
10
8
6
4
2
0
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7
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10

Post-Main Sequence Evolution: >10M_{sun} Stars

- Important differences in the evolution of the most massive stars are evident compared to their less massive counterparts

- On the main sequence, the stars are at, or very close to their Eddington Luminosities, so mass loss is important at a much earlier stage in their evolution

- The luminosity remains almost constant throughout lifecycle – increase over Eddington Luminosity would blow star apart. Motion in HR-diagram after leaving main sequence is essentially horizontal

- Motion across HR-diagram is slow when stable nuclear burning occurs and fast when core contracts and envelope expands

Post-Main Sequence Evolution: >10M_{sun} Stars

- Temperature in core of massive stars is so high that degeneracy pressure is not a factor until the very final stages of evolution

- Nuclear burning simply runs through the fusion of higher and higher mass elements as outlined in Lecture 11

- H-burning in core via CNO cycle
- He-burning in core with H-burning shell
- H & He-burning shell sources followed by start of C-burning core
- Ne-burning core plus shell sources…
- O-burning core plus shell sources…
- Si-burning core plus shell sources…

- Eventual buildup of inert Fe-core – maximum binding energy per nucleon and unable to extract energy via fusion

Onion-skin model for nuclear burning in massive star

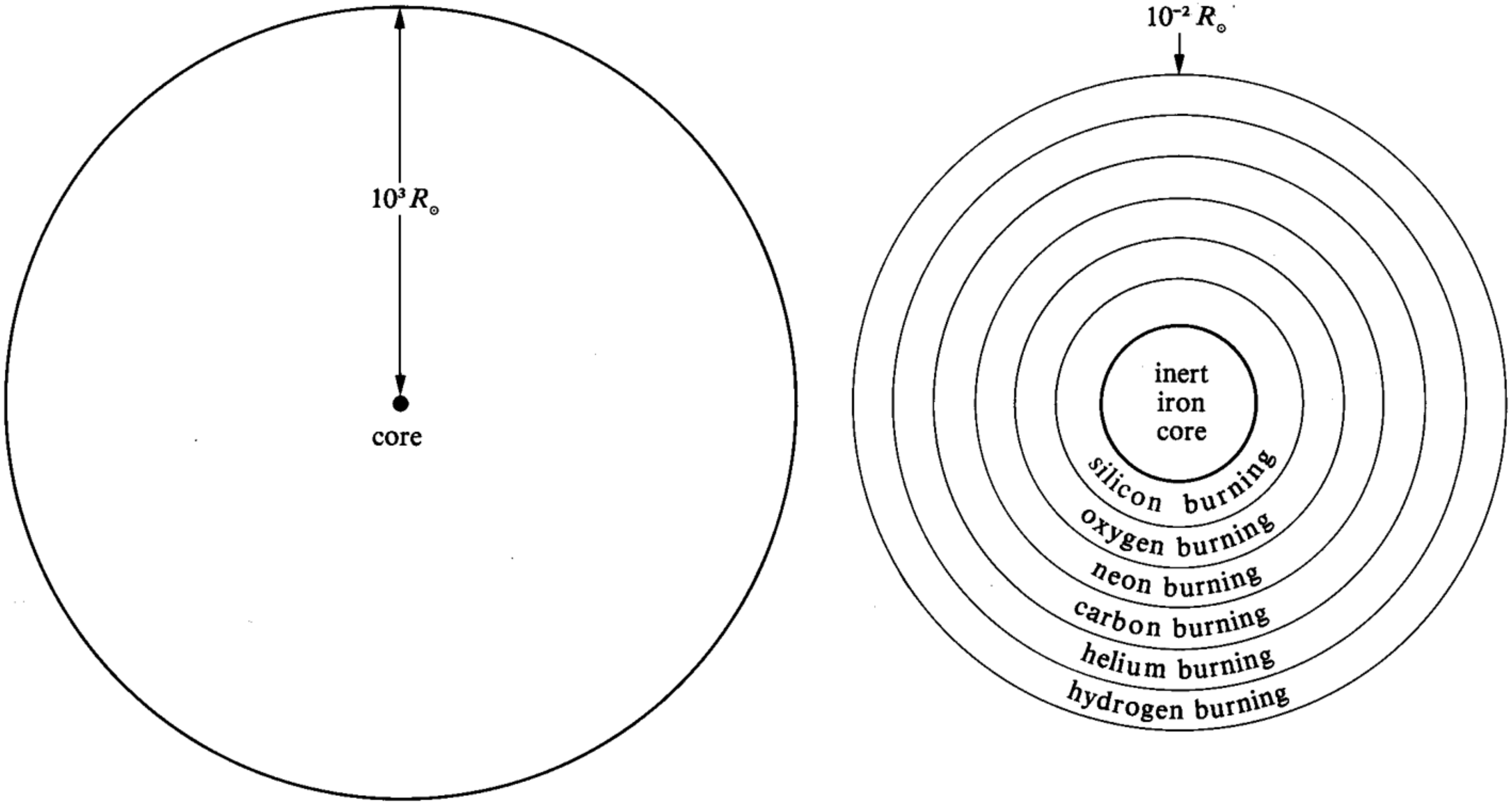
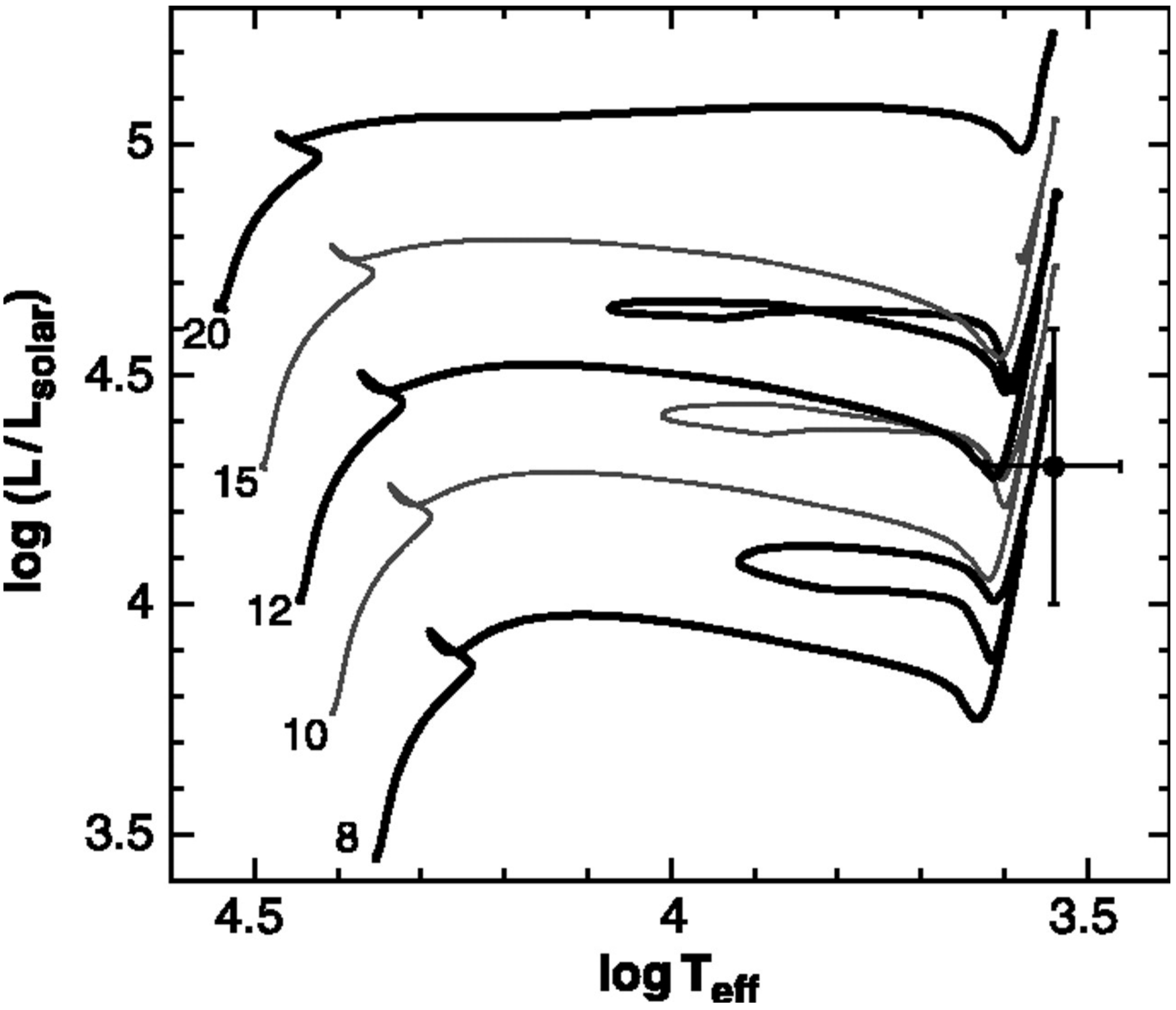


Figure 8.13. The onion-ring structure of a pre-supernova star (a very evolved star of high mass). The diagram on the left shows the dimensions of the entire star, a red supergiant, from core to photosphere. The diagram on the right shows the nuclear-burning regions near the inert iron core.



Modern evolutionary tracks (from *Star of the Week* #2)

Note left-right excursions corresponding to core contraction and onset of new burning phase

Nuclear Burning Timescales for a 25M_{sun} Star

Fuel Source	Timescale	$T(10^9 K)$	$\rho(\mathrm{kg\ m^{-3}})$
H	7×10^6 years	0.06	5×10^4
He	5×10^5 years	0.23	7×10^5
C	600 years	0.93	2×10^8
Ne	1 year	1.7	4×10^9
O	6 months	2.3	1×10^{10}
Si	1 day	4.1	3×10^{10}

Post-Main Sequence Evolution: >10M_{sun} Stars

- Life of high-mass star can end in a Type II Supernova explosion when degenerate Fe-core without an energy source collapses on a dynamical timescale – 10^{-3}s. *Star of the Week* #2 is example of status of pre-supernova star.

- Recent observations (including gravitational waves) suggest very massive stars collapse to black hole with no luminous supernova

- Onset of collapse aided by loss of energy due to neutrinos and photodisintegration

- Will look at conditions for stability of degenerate cores and the range of remnants left at the end of stellar evolution in later lecture

Lecture 18: Summary

- Low mass stars will ascend giant branch but no ignition of He-core

- For mass range 2-10M_{sun}, He-core is non-degenerate and He-core ignites gently but, overall, the evolution is very similar to the 0.7-2M_{sun} range, only much faster

- At high masses >10M_{sun}, luminosities are at or close to Eddington and mass loss is important on the main sequence. Core temperatures sufficient to initiate burning all the way up through Silicon without formation of a degenerate core. Burning takes the form of an onion-skin model with motions in the HR-diagram essentially horizontal. Creation of inert iron core leaves star without any energy source, leading to collapse on a dynamical timescale and a Type II supernova

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