Demographic Fluctuations, Generational Welfare and Intergenerational Transfers

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Abstract

This paper extends the Ramsey model's normative analysis to issues of generational welfare and intergenerational transfers. A planner, who maximizes the discounted welfare of an endless stream of generations, is intrinsically biased against larger cohorts, which are more costly to provide utility. Imperfect production substitutability produces a market bias against baby booms as well, lowering their lifetime income. The market bias, however, tends to be greater than that of the planner, who provides the baby boom cohort with more favourable lifetime transfers. Intuitively, the baby boom benefits from temporarily reduced elderly dependency, allowing greater lifetime consumption relative to lifetime income. Declining population growth leads to rising elderly dependency, which the planner supports with increasing intergenerational transfers. Secularly rising social security taxes, and declining lifetime returns, with a baby boom cohort receiving more favourable treatment than their heavily burdened successors, are consistent with the wishes of a social planner in an environment with declining population growth.

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I. Introduction

The baby boom generation has, since birth, elicited positivist inquiries into the impact of cohort size on general equilibrium and generational welfare. Early Malthusian discussions of resource depletion and food supplies (Davis 1953), were followed, as the baby boom reached maturity and entered the labour force, by more prosaic analyses of imperfect labour substitutability and the impact of cohort size on lifetime earnings (Freeman 1979, Welch 1979). The movement of the baby boom into middle age inspired studies of the demand for housing (Mankiw & Weil 1989, Poterba 1991), while, more recently, its approaching retirement has motivated discussions of rising asset prices and potential future meltdowns (Abel 2001, Brooks 2000, Poterba 2000). The impact of cohort size and congestion on educational attainment has also drawn the attention of economists, as long term postwar trends in lifetime educational attainment become apparent (Card & Lemieux 2000). While the magnitude and significance of these effects is a matter of some controversy, the overall direction is not:[1] as a consequence of belonging to a large cohort, the baby boom generation acquired less human capital, experienced lower earnings, and has and will enjoy less favourable asset prices and returns then would otherwise have been the case.[2]

On a seemingly orthogonal dimension, the postwar decades have seen a growing literature on the worsening generational return from social security (e.g. Myers and Schobel 1983, 1992; Caldwell et al 1998; Geanakoplos et al 1998). As birth rates have fallen and life spans increased, the tax rates necessary to support elderly benefits have risen dramatically, drawing attention to

[1] Even a skeptic such as Poterba (1991, 2000), who manages to reverse the predicted signs in non-structural multivariate correlations, repeatedly supports the direction of the effects implied by theory, interpreting his results as merely implying that demographic structure is, in the grand scheme of things, of little practical significance.

[2] Perhaps the strongest expression of the "demography as destiny" viewpoint was made by Easterlin (1980), who associated the size of the baby boom cohort with reduced lifetime incomes, higher unemployment, less fertility, higher female participation, increased divorce, crime and suicide rates and stagflation. Russell (1982) made a less ambitious review of traditional economic issues, reaching more muted conclusions.

the fact that an intergenerational transfer system in a dynamically efficient economy cannot, despite the largess heaped on prewar generations, assymptotically provide a market rate of return. Anticipated continued increases in elderly dependency have sparked repeated political debates on tax increases and benefit reductions. While all postwar generations will likely experience a substantial lifetime tax burden, the longer the tax increases or benefit reductions necessary to achieve indefinite fiscal soundness are postponed, the more that burden will be shifted away from the baby boom generation to later cohorts.

This paper looks to integrate these two literatures with a normative analysis of generational policy. Specifically, I ask how a social planner, or integrated household, would treat successive generations in an economy with a fluctuating birth rate. I find that the planner is intrinsically biased against large cohorts which, on a per capita basis, are more costly to give utility. Larger cohorts, however, benefit from a reduced burden of supporting their elderly parents. Consequently, while the large size of a baby boom diminishes its lifetime income, its consumption, aided by the income effect of reduced dependency, falls less than proportionately. Put differently, the bias of the market against large cohorts, described above, is greater than that of the planner. Secularly declining birth rates lead to rising elderly dependency, which the planner supports with increasing social security tax rates and lifetime tax burdens. Within this secular trend, however, a baby boom generation will be granted more favourable tax treatment than its successor cohorts. In sum, the model suggests that an economy with a declining birth rate should introduce a growing transfer system to the elderly, with positive net lifetime transfers to early recipients, but, in the face of a temporary baby boom, should postpone long term fiscal responsibility, granting that generation more favourable treatment than its heavily burdened successors.

The paper proceeds as follows: Section II motivates the formal analysis by reproducing, in the simplest general equilibrium framework, the market bias against large cohorts. Consideration of a social planner's response to market outcomes requires some measure of the planner's resources. To this end, Section III develops the concept of the "demographic gift", defined as the negative of the change in the capital stock necessary to keep the welfare of all current and future generations constant following a change in the birth rate. I show that the gift is related to both welfare outcomes, as it equals a marginal utility normalized sum of discounted future generational welfare changes, and intergenerational transfers, as any social optimum can be achieved by transferring the demographic gift of each generation to its parents. With this analytical tool in hand, Section IV reexamines the simple general equilibrium example from the perspective of a social planner that maximizes the discounted sum of generational utilities, deriving the results described above. Section V concludes.

II. Demographic Fluctuations and Generational Welfare: A Simple Example

Consider an economy populated with an infinite number of generations, indexed by their time of birth t, whose members live two periods and enjoy lifetime utility derived from consumption when young and when old:

$$(1) \quad U_t \;=\; \ln C_t^y + B \ln C_{t+1}^o \,.$$

Each individual in the economy inelastically supplies one unit of labour when young, and output is produced using that labour and capital, which depreciates completely in one period:

$$(2) \quad Q_t \;=\; K_t^{\alpha} L_t^{1-\alpha} \qquad K_{t+1} \;=\; s_t Q_t \,,$$

where L_t denotes the size of generation t and s_t the gross savings rate. The growth of cohort size from period to period, $n_t = L_t / L_{t-1}$, is an i.i.d. random variable.

The equilibrium of this overlapping generations economy, in the absence of government intervention, is well known. Each individual, seeking to maximize their expected lifetime utility, obeys the first order condition

$$(3) \quad \frac{1}{C_t^y} \;=\; E_t\left[\frac{BR_{t+1}}{C_{t+1}^o}\right],$$

where R_{t+1} is the gross return on capital. With the production structure laid out in (2) above, and the net present value of lifetime consumption limited by wage income when young (W_t), the solution to each individual's problem involves consuming and saving a fixed fraction of wage income:

$$(4) \quad C_t^y \;=\; \frac{W_t}{1+B} \qquad C_{t+1}^o \;=\frac{R_{t+1} W_t \mathrm{B}}{1+\mathrm{B}} \,.$$

The gross savings rate is a constant fraction of output, $s_t \;=\; (1-\alpha)B/(1+B)$, and capital per worker follows the stochastic difference equation

$$(5) \quad \frac{K_{t+1}}{L_{t+1}} \;=\; k_{t+1} \;=\; \frac{s k_t^{\alpha}}{n_{t+1}} \,.$$

With the consumption decisions given by (4), the lifetime utility enjoyed by each generation is a positive function of both the wage and the gross rental which, using (5), can be reduced to a function of the capital-labour ratio at birth and subsequent labour force growth:

$$
\begin{aligned}
(6) \quad U_t &= (1+B)\ln W_t + B\ln R_{t+1} + c \\
&= \alpha(1+B\alpha)\ln k_t + B(1-\alpha)\ln n_{t+1} + c',
\end{aligned}
$$

where c and c' are constants.

We now consider the distributional impact of a baby boom, i.e. an unusually high realization of n_t. A baby boom in period t lowers the value of capital per worker, k_t, raising the gross rental and lowering the wage rate. The unanticipated boom in capital income increases the welfare of the elderly ($U_{t-1}\uparrow$), but the reduction in wage income lowers the expected lifetime utility of the current young ($E_t[U_t]\downarrow$). In fact, given the constant savings rate, the expected value of the ln of all future capital-labour ratios is lowered, reducing the expected lifetime utility of all future generations ($E_t[U_{t+i}]\downarrow \forall\, i \geq 0$). Thus, the baby boom results in the most drastic of intergenerational redistributions. The welfare of the current elderly is increased, but at the cost of reducing the expected welfare of all their descendents. This example illustrates the market bias in favour of smaller cohorts.

One can hardly write down a problem such as that described above without, immediately, introspecting on how the redistribution implied by the OLG market equilibrium would be viewed by a social planner who cares about the utility of all generations. Can the enhancement of the utility of the elderly at the expense of all future generations possibly constitute a social optimum? If so, what factors in the problem encourage a planner to pursue such a "perverse" outcome? How might a planner want to modify the market outcome, and what form would the optimal intergenerational redistribution of resources take? These are the problems addressed in this paper.

III. The Demographic Gift

I begin the analysis by developing a measure of the incremental resources available to a planner following a change in cohort size. To this end, define the "demographic gift" as -1 times the adjustment to the capital stock or output necessary to hold the welfare of all current and future generations constant following a proportional change in the size of a given cohort. This "gift", which may be positive or negative, is akin to a compensating variation. As will be seen, it determines the welfare impact of demographic change and is intimately related to the intergenerational transfers imposed by any planner in pursuit of a social optimum.

To put some meat on the concept, consider a generalization of the economy of the previous section, in which each generation now has a maximum lifespan of T periods and derives utility from consumption and, perhaps, leisure. Output is a concave constant-returns-to-scale function of labour and capital inputs, with the dynamics of the capital stock given by

$$(7) \quad K_{t+1} = F(K_t, L_t^T, L_t^{T-1}, \ldots, L_t^0, t) + (1-\delta)K_t - \sum_{i=0}^{T} C_t^i L_t^i ,$$

where L_t^i denotes the number of individuals of age i alive at time t and C_t^i their associated consumption per capita. The listing of all generations, from age 0 to T, in the production function allows, implicitly, for varying labour supplies and lifecycle productivities. Expressed in intensive terms, the capital stock follows the equation

$$(8) \quad k_{t+1}n_{t+1} = F(k_t, l_t^T, l_t^{T-1}, \ldots, 1, t) + (1-\delta)k_t - \sum_{i=0}^{T} C_t^i l_t^i ,$$

where k_t is capital per newborn and l_t^i the relative size of surviving age cohorts

$$(9) \quad k_t = K_t/L_t^0 \qquad l_t^i = L_t^i/L_t^0 = p_{t-i}^i \prod_{m=0}^{i-1} n_{t-m}^{-1},$$

with p_{t-i}^i denoting the probability of a member of generation t-i surviving to age i and n_t the growth of cohort size between periods t-1 and t, i.e. the birth rate. I assume that the economy is Pareto efficient. My intent is to calculate the demographic gift associated with a proportional change in n_t, holding constant all subsequent birth and death rates.

If the economy is Pareto efficient, to hold the welfare of current and future generations constant one can do no better than keep all per capita consumption, and implicit labour supplies, unchanged. As such, for this exercise, the only variables in the economy are capital per newborn, k_t, and the relative size of surviving age cohorts, l_t^i. By time t+T, the influence of n_t on the relative size of age cohorts has disappeared (see (9) above), so that if the economy can reach period t+T with unchanged capital per newborn, k_{t+T}, it is guaranteed the ability to provide all future generations their original welfare. To this end, differentiate equation (8) to derive an equation of motion in the deviation of capital per newborn:

$$(10) \quad n_{t+i+1}\frac{\mathrm{d}k_{t+i+1}}{\mathrm{d}n_t} = R_{t+i}\frac{\mathrm{d}k_{t+i}}{\mathrm{d}n_t} + \sum_{j=i+1}^{T}\frac{(C_{t+i}^j - W_{t+i}^j)l_{t+i}^j}{n_t},$$

where W_{t+i}^j denotes the labour income of persons aged j at time t+i, $R_{t+i} = F_K^{t+i} + 1 - \delta$ the corresponding gross rental, and I have made use of the fact that

$$(11) \quad l_{t+i}^j = p_{t+i-j}^j \prod_{m=0}^{j-1} n_{t+i-m}^{-1}, \quad \text{so that} \quad \frac{\mathrm{d}l_{t+i}^j}{\mathrm{d}n_t} = \frac{-l_{t+i}^j}{n_t} \quad \forall\ j \geq i+1 .$$

Multiplying both sides of (10) by $n_t/n_{t+i+1}k_{t+i+1}$ allows the expression

$$(12) \quad \frac{\hat{k}_{t+i+1}}{\hat{n}_t} = R_{t+i}\left(\frac{K_{t+i}}{K_{t+i+1}}\right)\frac{\hat{k}_{t+i}}{\hat{n}_t} + \sum_{j=i+1}^{T}\frac{(C_{t+i}^j - W_{t+i}^j)L_{t+i}^j}{K_{t+i+1}},$$

where ^ denotes a proportional change. Leading this equation forward T periods, one finds that

$$(13) \quad \frac{\hat{k}_{t+T}}{\hat{n}_t} = \frac{\prod_{k=0}^{T-1} R_{t+k}}{K_{t+T}}\left[\sum_{i=0}^{T-1}\left(\frac{\sum_{j=i+1}^{T} C_{t+i}^j L_{t+i}^j - W_{t+i}^j L_{t+i}^j}{\prod_{k=0}^{i} R_{t+k}}\right) + \left(\frac{\hat{k}_t}{\hat{n}_t}\right)K_t\right].$$

Setting (13) equal to zero, and noting that $\hat{k}_t/\hat{n}_t = \hat{K}_t/\hat{n}_t - 1$, or $\mathrm{d}K_t/\hat{n}_t = K_t(\hat{k}_t/\hat{n}_t + 1)$, produces the increment to the capital stock necessary to keep welfare unchanged

$$(14) \quad \frac{\mathrm{d}K_t}{\hat{n}_t} = K_t + \sum_{i=0}^{T-1}\frac{\sum_{j=i+1}^{T} W_{t+i}^j L_{t+i}^j - C_{t+i}^j L_{t+i}^j}{\prod_{k=0}^{i} R_{t+k}}.$$

The demographic gift is the negative of this increment

$$(15)\quad \Delta K_t \;=\; \sum_{i=0}^{T-1} \frac{\sum_{j=i+1}^{T} C^j_{t+i} L^j_{t+i} - W^j_{t+i} L^j_{t+i}}{\prod_{k=0}^{i} R_{t+k}} \;-\; K_t \,,$$

or, when expressed in units of gross output

$$(16)\quad \Delta Q_t \;=\; R_t \Delta K_t \;=\; \sum_{i=0}^{T-1} \frac{\sum_{j=i+1}^{T} C^j_{t+i} L^j_{t+i} - W^j_{t+i} L^j_{t+i}}{\prod_{k=1}^{i} R_{t+k}} \;-\; R_t K_t \,.$$

As shown in the appendix, in a Pareto efficient economy the demographic gift equals the net present value of changes in real per capita expenditure or, equivalently, a marginal utility normalized sum of the present discounted value of changes in realized generational utilities

$$(17)\quad \Delta K_t \;=\; \sum_{i=0}^{\infty} \sum_{j=0}^{T} \frac{\frac{\mathrm{d}E^j_{t+i}}{\mathrm{d}\ln n_t} L^j_{t+i}}{\prod_{k=0}^{i} R_{t+k}} \;=\; \sum_{i=0}^{\infty} \frac{\frac{\mathrm{d}\overline{U}_{t+i}}{\mathrm{d}\ln n_t} L^0_{t+i} / \mathrm{MU}^0_{t+i}}{\prod_{k=0}^{i} R_{t+k}} \,,$$

where E^j_{t+i} denotes real expenditure, on consumption and leisure, by persons aged j at time t+i, $\overline{U}_{t+i}$ the average realized lifetime utility of cohort t+i and MU^0_{t+i} their marginal utility of real expenditure at birth. The demographic gift defines an implicit deviation of the economy’s capital endowment from its original value. When the gift is positive, the economy’s endowment has increased, and a Pareto improvement is possible. When the gift is negative, the welfare of at least one generation must be reduced.

As can be seen from (16) above, the gift associated with an increase in the size of the birth cohort at time t depends upon the difference between the net present value of the consumption of older cohorts alive at that time and the net present value of their corresponding labour and capital income. When this measure is positive, older cohorts are leaving a negative net bequest, i.e. they live at the expense of future generations, and a rise in the birth rate dilutes this tax across a broader base, raising welfare. Similarly, when pre-existing cohorts plan on leaving a positive net bequest, a rise in the birth rate dilutes this transfer across a broader base, lowering welfare. For an OLG economy such as that discussed in the previous section, the gift is identically zero. However, if one defines the demographic gift per new born as $\Delta k_t = \Delta K_t / L_t^0$, one can (albeit with some difficulty) show that the derivative of this gift with respect to $\ln n_t$ is given by

$$\text{(18)} \quad \frac{\mathrm{d}\Delta k_t}{\hat{n}_t} = \sum_{k=0}^{T-1} \left(\frac{\mathbf{x}_{t+k}{}' \mathbf{A}_{t+k} \mathbf{x}_{t+k}}{L_t^0 L_{t+k}^0 \prod_{m=0}^{k} R_{t+m}} \right),$$

where $\mathbf{A}_{t+k}$ is the matrix of second derivatives of F with respect to capital and the T labour inputs, evaluated at time t+k, and

$$\text{(19)} \quad \mathbf{x}'_{t+k} = (-K_{t+k} \hat{k}_{t+k} / \hat{n}_t, L_{t+k}^T, L_{t+k}^{T-1}, \ldots, L_{t+k}^{k+1}, 0, \ldots, 0)\,.$$

As the production function is assumed to be concave, the second term on the right-hand side is negative, a result akin to the diminishing marginal product of labour. It follows that for a Pareto efficient OLG economy the demographic gift associated with any non-infinitesimal increase *or* decrease in the size of a birth cohort is negative, i.e. must always result in a welfare loss for at least one generation.

The demographic gift can be related to the intergenerational transfers that might be imposed by a planner. With the demographic gift per newborn, in units of output, given by $\Delta q_t = \Delta Q_t / L_t^0$, one can see that

$$\text{(20)} \quad -\Delta q_t + \frac{n_{t+1}}{R_{t+1}} \Delta q_{t+1} = \sum_{i=0}^{T} \frac{C_{t+i}^i p_t^i}{\prod_{k=1}^{i} R_{t+k}} - \sum_{i=0}^{T} \frac{W_{t+i}^i p_t^i}{\prod_{k=1}^{i} R_{t+k}}\,.$$

The right-hand side of this equation is the difference between the net present value of the consumption of generation t and the net present value of its lifetime labour income. It follows that any social planner whose optimal plan respects individuals’ first order conditions can achieve his desired allocation in a decentralized OLG setting by imposing a lifetime tax on each generation equal to their demographic gift (calculated using the socially optimal path of consumption and factor returns), and paying it as benefits to their parents.

Before proceeding further, it might be advisable to repair some of the corners cut in the interest of brevity. The economy discussed in the previous section, as well as later in the paper, operates in an environment of aggregate uncertainty, whereas in the analysis of the demographic gift laid out above all future birth and death rates are deemed known. In an uncertainty framework, the expected welfare of all current and future generations can be held constant if, following a change in the size of cohort t, the planner can keep k_{t+T} unchanged in all realized states of the world in period t+T. When individuals live for only two periods, this collapses to the certainty case, as the planner only needs information on variables in period t.[3] As this will be the case in all of the examples pursued in this paper, I have skirted the issue of uncertainty in the presentation above. The appendix extends the results to a general uncertainty framework.

In keeping the future growth of cohort size ($n_{t+i} = L_{t+i}/L_{t+i-1}$) constant, the analysis of the demographic gift laid out above implicitly assumes that only the youngest cohort produces children, ruling out the possibilities of further fluctuations in relative cohort size induced by an initial shock (e.g. "baby boomlets"). A more realistic assumption would be to allow the general birth process

$$(21) \quad L_{t+1} = \sum_{i=0}^{T} b_t^i L_t^i \qquad \text{or} \qquad n_{t+1} = \sum_{i=0}^{T} b_t^i l_t^i \,,$$

where b_t^i is the fertility of persons of age i at time t. While this complicates the algebra immeasurably, it does not change the spirit of the results. For example, in these circumstances one can still show that the social planner can achieve a social optimum in a decentralized OLG setting by imposing a lifetime tax on each generation equal to their demographic gift per capita, in exchange for benefits equal to the demographic gift of their progeny:

[3] Consider (15) when T = 1.

$$(22) \quad -\Delta q_t + \sum_{i=0}^{T} \frac{b_{t+i}^i \Delta q_{t+i+1}}{\prod_{k=1}^{i+1} R_{t+k}} = \sum_{i=0}^{T} \frac{C_{t+i}^i p_t^i}{\prod_{k=1}^{i} R_{t+k}} - \sum_{i=0}^{T} \frac{W_{t+i}^i p_t^i}{\prod_{k=1}^{i} R_{t+k}}.$$

I leave such complications for later empirical applications, concentrating, in this paper, on the theoretical usefulness of the demographic gift as a means of comprehending a simple social planning problem.

IV. The Simple Example Reexamined

I now reconsider the simple economy of Section II from the perspective of a planner who maximizes the expected value of the discounted sum of generational utilities

$$(23) \quad E_t\left[\sum_{i=-1}^{\infty} U_{t+i}\gamma^i L_{t+i}^{\eta}\right], \quad \text{where } U_t = \ln C_t^y + B\ln C_{t+1}^o,$$

subject to the technological constraints and restrictions

$$(24) \quad Q_t = K_t^{\alpha}L_t^{1-\alpha}, \quad K_{t+1} = s_tQ_t = Q_t - C_t^yL_t - C_t^oL_{t-1}, \quad n_t = L_t/L_{t-1}$$

$$\text{and} \quad \gamma\in(0,1), \quad \eta\in[0,1], \quad \text{and} \quad E[n_t^{\eta}] < 1/\gamma.$$

The social planner discounts time at a rate γ, which may or may not equal the individual rate of discount B, and places a weight on generational size, η, which varies between 0 and 1. When $\eta = 0$, the planner, in the Samuelsonian [1958] tradition, cares about the average utility per generation, whereas when $\eta = 1$ he follows Lerner [1959] in seeking to maximize the total sum of generational utility. With a stochastic birth rate n_t, the effective discount rate, γn_t^{η}, is a random variable whose mean is constrained to be less than one. This planning problem integrates the generations of the economy in a manner similar to that achieved when individuals care about the welfare of their offspring, but goes further by allowing negative bequests, i.e. transfers from the young to the elderly.

The usual variational argument establishes the characteristics of the socially optimal consumption plan. At each date t, the planner can take an infinitesimal unit of consumption ΔC and reallocate it from the elderly to the youth, increasing the maximand by an amount

$$(25) \quad \left(\frac{\Delta C}{L_t}\right)\left(\frac{\gamma^tL_t^{\eta}}{C_t^y}\right) - \left(\frac{\Delta C}{L_{t-1}}\right)\left(\frac{B\gamma^{t-1}L_{t-1}^{\eta}}{C_t^o}\right).$$

Each member of the younger generation gains a consumption of $\Delta C/L_t$, which increases the maximand by the marginal utility of their consumption ($1/C_t^y$) times the weight the planner places on the per capita utility of that generation ($\gamma^tL_t^{\eta}$). The loss, to the planner, is the reduction in the per capita consumption of the elderly ($\Delta C/L_{t-1}$) multiplied by their marginal utility (B/C_t^o) and their weight in the planner's objective function ($\gamma^{t-1}L_{t-1}^{\eta}$). Along an optimal path, (25) must equal zero at all times, which allows one to derive the intratemporal first order condition

$$(26) \quad C_t^o = C_t^y \left(\frac{B}{\gamma}\right) n_t^{1-\eta} .$$

Along a socially optimal path the ratio of elderly to youth consumption depends upon the ratio of private to social discount rates and the rate of population growth. With regards to the latter, in particular, a rise in the rate of population growth will raise the relative consumption of the elderly if $\eta < 1$. The cost of providing utility to each member of a generation is increasing in the size of the cohort. This automatically imparts a bias in favour of providing utility to smaller cohorts, a bias which is offset to the degree that the planner places weight on the size of each generation. In the extreme, when $\eta = 1$, the two forces exactly offset each other and the relative consumption of the elderly and the young is constant. For the purposes of the succeeding analysis, it is useful to define a measure of dependency as the total consumption of the elderly relative to that of the young

$$(27) \quad \frac{C_t^o L_{t-1}}{C_t^y L_t} = \frac{B}{\gamma n_t^{\eta}} = X_t .$$

When $\eta = 0$ the planner responds to a baby boom and the increased cost of providing utility to the young by drastically shifting consumption in favour of the elderly, so that, despite the rise in the relative number of young, overall dependency does not fall. However, a positive weight on generational size ($\eta > 0$) reduces the substitution in favour of the elderly and ensures that dependency declines with the rate of population growth.

Turning to intertemporal considerations, at any point in time the planner can consider the possibility of taking an infinitesimal unit of consumption ΔC and reallocating it from the youth of the day to the youth of tomorrow, for a net gain of:

$$(28) \quad -\left(\frac{\Delta C}{L_t}\right)\left(\frac{\gamma^t L_t^{\eta}}{C_t^y}\right) + E_t\left[\left(\frac{\Delta C R_{t+1}}{L_{t+1}}\right)\left(\frac{\gamma^{t+1} L_{t+1}^{\eta}}{C_{t+1}^y}\right)\right] .$$

Requiring that (28) equal zero along an optimal path produces the intertemporal first order condition

$$(29) \quad \frac{1}{C_t^y} = E_t\left[\left(\frac{\gamma n_{t+1}^{\eta}}{C_{t+1}^y}\right)\left(\frac{R_{t+1}}{n_{t+1}}\right)\right].$$

The planner equates the expected marginal utility of the young to that of their successors, adjusted for the effective discount placed on the per capita utility of later cohorts (γn_{t+1}^{η}) and the rate of intertemporal transformation between generations (R_{t+1}/n_{t+1}), as capital accumulated from one period to another is diluted by labour force growth. An optimal path of consumption will satisfy (29), as well as the intertemporal budget constraint

$$\begin{aligned}(30) \quad n_{t+1}k_{t+1} \;=\; s_t k_t^{\alpha} \;&=\; k_t^{\alpha} - C_t^y - C_t^o/n_t \\ &=\; k_t^{\alpha} - C_t^y(1+X_t)\,.\end{aligned}$$

To complete the solution, I rely upon intuition derived from standard models. Define $\bar{n}$ as the value of n_t at which the random variable $n_t^{\eta} = E[n_t^{\eta}]$. When n_t equals $\bar{n}$, the planner's discount factor equals its expected value and his problem is very similar to that of an infinitely lived consumer with a constant discount factor. Accordingly, one can conjecture that the solution will follow the standard problem and, with the savings rate equal to the discount factor times the share of capital, define

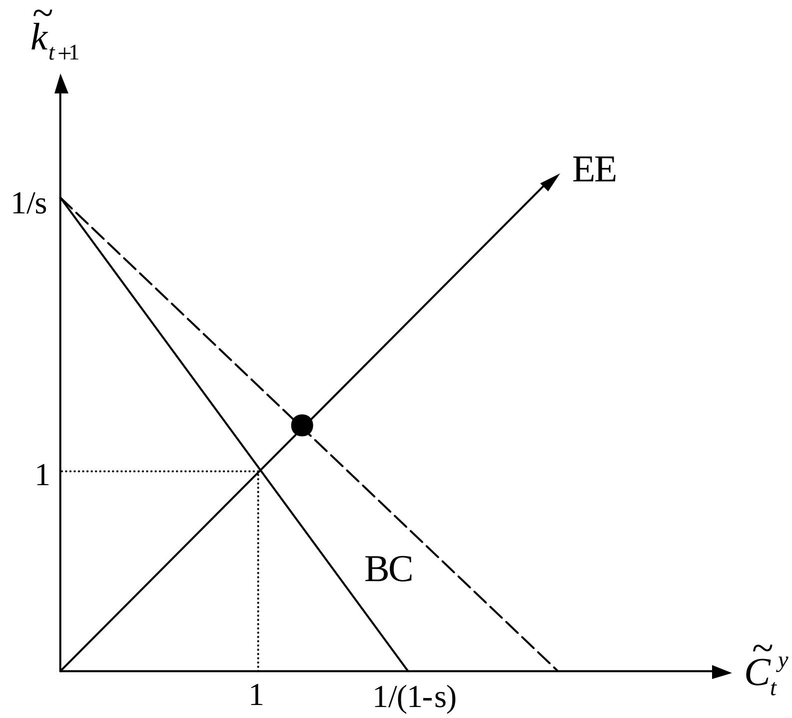
$$(31) \quad \bar{k}_{t+1} \;=\; \frac{s k_t^{\alpha}}{n_{t+1}} \quad \text{and} \quad \bar{C}_t^y \;=\; \left(\frac{1-s}{1+X}\right)k_t^{\alpha},$$

$$\text{where} \quad s \;=\; \alpha\gamma E[n_t^{\eta}] \quad \text{and} \quad X \;=\frac{B}{\gamma \bar{n}_t^{\eta}} \;=\; \frac{B\alpha}{s}.$$

For deviations of n_t away from $\bar{n}$, define corresponding deviations in the consumption of the young and the future capital stock

$$(32) \quad \tilde{k}_{t+1} \;=\; \frac{k_{t+1}}{\bar{k}_{t+1}} \qquad \text{and} \qquad \tilde{C}_t^y \;=\; \frac{C_t^y}{\bar{C}_t^y}\,.$$

Figure I: Distributing the Benefits of Reduced Dependency



Dividing both sides of the economy's intertemporal budget constraint (30) by $n_{t+1}\bar{k}_{t+1} = sk_t^{\alpha}$, one finds

$$(33) \quad \text{BC:} \quad \tilde{k}_{t+1} = \frac{1}{s}\left(1 - \frac{\tilde{C}_t^y(1+X_t)(1-s)}{1+X}\right).$$

Equation (33) appears as the downward sloping line BC in Figure I above. Provided $\eta > 0$, a rise in n_t lowers dependency (X_t) rotating the budget constraint outward. The planner's Engel curve is drawn as the line EE in Figure I. This line is derived by conjecturing that the ln-ln structure of the model delivers certainty equivalence, with future values of n_t^{η} treated as being known, and equal to their expected value, so that the intertemporal first order condition, (29), can be manipulated as follows

$$(34) \quad \frac{1}{C_t^y} = \left(\frac{\gamma n_{t+1}^{\eta}}{C_{t+1}^y}\right)\left(\frac{\alpha k_{t+1}^{\alpha-1}}{n_{t+1}}\right) = \frac{sk_{t+1}^{\alpha}}{\bar{C}_{t+1}^y n_{t+1}k_{t+1}} = \frac{s(1+X)}{(1-s)n_{t+1}k_{t+1}}.$$

Inverting both sides of the equation, and dividing by $\bar{C}_t^y = (1-s)k_t^{\alpha}/(1+X)$, one finds the Engel curve

$$(35) \quad \text{EE:} \quad \tilde{C}_t^y = \tilde{k}_{t+1}.$$

Combining BC and EE, one can solve for

$$(36) \quad \tilde{C}_t^y = \frac{1+X}{s(1+X)+(1+X_t)(1-s)}.$$

In sum, one can conjecture that the optimal path of youth consumption, and associated gross savings rate, is given by

$$(37) \quad C_t^y = \frac{(1-s)k_t^{\alpha}}{s(1+X)+(1-s)(1+X_t)} \qquad \text{and} \qquad s_t = \frac{s(1+X)}{s(1+X)+(1-s)(1+X_t)}.$$

It is easily confirmed that (37) satisfies the intertemporal first order condition (29) above, and completes the solution of the model.

A rise in n_t depletes capital per worker (k_t) and may lower the dependency rate (X_t). In the typical infinitely lived consumer model, when production and preferences are ln-linear and the depreciation rate is 100%, the income and substitution effects of any movement in capital per worker exactly cancel so that the savings rate is constant and consumption responds ln-linearly to changes in capital per worker brought about by fluctuations in the birth rate. The model described above shares a similar preference and technological structure and the income and substitution effects of a change in capital per worker similarly cancel, so that the savings rate is independent of capital per worker, while consumption responds ln-linearly to movements in that variable. Unlike the standard model, however, the framework delineated above explicitly considers changes in the relative consumption of different age groups. Provided $\eta > 0$, a rise in n_t lowers the dependency rate ($X_t \downarrow$), a pure income effect which allows for both greater consumption today ($C_t^y \uparrow$) and greater consumption tomorrow ($s_t \uparrow$). This effect plays an important role in the analysis of intergenerational transfers further below.

(a) Distributing the Demographic Gift

While the preceeding demonstrated how the stochastic difference equation implicitly defined by the intertemporal first order condition can be solved by isolating the effects of capital dilution and dependency, the concept of the demographic gift allows a simpler, more direct, solution method. To this end, consider the demographic gift derived from a proportional deviation of n_t from $\overline{n}$, expressed as a share of the current capital stock, i.e. (using (15) earlier above)

$$(38) \quad \frac{\Delta K_t}{K_t} = \frac{C_t^o L_{t-1} - R_t K_t}{R_t K_t} = \frac{\theta_{old}(1-s)}{\alpha} - 1 \, ,$$

where $\theta_{old} = X/(1+X)$ is the consumption share of the elderly. An increase in n_t, for a given level of youth consumption, raises the consumption of the elderly. This spends part of the capital of the demographic gift

$$(39) \quad \frac{\hat{C}_t^o}{\hat{n}_t}\left(\frac{C_t^o L_{t-1}}{R_t K_t}\right) = \frac{(1-\eta)\theta_{old}(1-s)}{\alpha} \, ,$$

leaving a net gift of

$$(40) \quad \frac{\Delta K_t(net)}{K_t} = \frac{\eta\theta_{old}(1-s)}{\alpha} - 1 .$$

In a model with ln-linear preferences and production, and without considerations of demographic structure, a planner typically responds to a change in capital per worker by changing the consumption of the young and capital next period by an equivalent, ln-linear, amount ($\hat{C}_t^y = \hat{k}_{t+1} = \alpha\hat{k}_t$). The net demographic gift is defined so as to keep k_{t+1} constant, after allowing for $\hat{n}_t$, and also adjusts for any increase in the consumption of the elderly. As such, it represents the change in resources available to the central planner, i.e. the "true" change in capital per worker. The planner responds accordingly, with:

$$(41) \quad \frac{\hat{C}_t^y}{\hat{n}_t} = \frac{\hat{k}_{t+1}}{\hat{n}_t} = \alpha * \frac{\Delta K_t(net)}{K_t} = \eta\theta_{old}(1-s) - \alpha .$$

Simple differentiation of (37) with respect to $\ln n_t$ confirms that this solution agrees the one derived in the previous section.

To extend the analysis to discrete changes in n_t away from $\bar{n}$, define K_t^* as the capital stock needed to keep k_{t+1} unchanged following a deviation in the birth rate, holding constant youth consumption but allowing substitution in favour of the consumption of the elderly, i.e.

$$(42) \quad n_{t+1}k_{t+1} = s\left(\frac{K_t}{\bar{n}L_{t-1}}\right)^{\alpha} = \left(\frac{K_t^*}{n_t L_{t-1}}\right)^{\alpha} - C_t^y(1+X_t)$$
$$= \left(\frac{K_t^*}{n_t L_{t-1}}\right)^{\alpha} - \frac{1-s}{1+X}\left(\frac{K_t}{\bar{n}L_{t-1}}\right)^{\alpha}(1+X_t) ,$$

where I have substituted for k_{t+1} and C_t^y using their equilibrium values for a realized birth rate of $\bar{n}$. The net demographic gift, expressed as a fraction of the current capital stock, is the inverse of K_t^*/K_t

$$(43) \quad NG = \frac{K_t}{K_t^*} = \left(\frac{\bar{n}}{n_t}\right)\left(\frac{1+X}{s(1+X)+(1-s)(1+X_t)}\right)^{1/\alpha} .$$

The planner leaves the "true" savings rate unchanged, and sets youth consumption ln-linearly proportional to the "true" capital stock:

$$(44) \quad C_t^y \;=\; \frac{1-s}{1+X}\left(\frac{NG^*K_t}{\bar{n}L_{t-1}}\right)^{\alpha} \;=\; \frac{(1-s)k_t^{\alpha}}{s(1+X)+(1-s)(1+X_t)},$$

which, of course, agrees with (37) earlier. In sum, while the impact of a change in the birth rate on the consumption of the young, savings and the future capital stock can be understood in terms of capital dilution and dependency effects, it can also be summarized as a problem involving the distribution of the demographic gift. After substituting in favour of the elderly, the planner simply follows his usual policy of using a constant savings rate to distribute real resources between current and future generations.

(b) Generational Welfare

Table I below summarizes the welfare implications of a proportional change in the birth rate along a socially optimal consumption path and in the OLG equilibrium. Focusing first on the upper left-hand quadrant, the impact of a baby boom on the socially optimal welfare of generation t depends upon the proportional change in their consumption when young and when old, which is a simple function of the net demographic gift

$$(45) \quad \frac{\mathrm{d}U_t}{\mathrm{d}\ln n_t} \;=\; \frac{\hat{C}_t^y}{\hat{n}_t} + B\,\frac{\hat{C}_{t+1}^o}{\hat{n}_t} \;=\; \frac{\hat{C}_t^y}{\hat{n}_t} + \alpha B\,\frac{\hat{k}_{t+1}}{\hat{n}_t} \;=\; (1+\alpha B)\,(\eta\theta_t^{old}(1-s_t)-\alpha)\,.$$

Comparing this result with the impact of a change in the birth rate on the utility of the young in the OLG equilibrium as presented in the Table (and calculated by straight-forward differentiation of equation (6) earlier), one sees that the two are the same when $\eta = 0$. When $\eta = 0$ the planner cares only about the per capita welfare of each generation and in response to a baby boom, which raises the cost of providing welfare to later generations, dramatically raises the consumption of the elderly at the expense of the young, leaving overall dependency unchanged. In such circumstances, the net demographic gift equals capital dilution ($-\alpha$) which, as in the OLG economy, lowers the welfare of the young. More generally, however, a positive weight on generational

Table I: Distributional Implications of a Baby Boom		
	Social Optimum	OLG
$\frac{\mathrm{d}U_t}{\mathrm{d}\ln n_t}$	$(1+\alpha B)(\eta\theta_t^{old}(1-s_t)-\alpha)$	$-(1+\alpha B)\alpha$
$\frac{\mathrm{d}U_{t+i}}{\mathrm{d}\ln n_t}$	$\alpha^i(1+\alpha B)(\eta\theta_t^{old}(1-s_t)-\alpha)$	$-\alpha^i(1+\alpha B)\alpha$
$\frac{\mathrm{d}U_{t-1}}{\mathrm{d}\ln n_t}$	$B(1-\alpha+\eta[\theta_t^{old}(1-s_t)-1])$	$B(1-\alpha)$

size ($\eta > 0$) reduces the planner's substitution in favour of the elderly, leading to a fall in dependency. This ameliorates the impact of capital dilution, raising the net gift. As shown in the second row of the Table, the impact of a baby boom on later generations is a proportional multiple of its impact on generation t, as the i.i.d. characteristic of the savings rate in the social optimum produces similar propagation mechanisms.[4] Finally, with regards to the utility of the generation t-1 elderly, calculated in the third row of the Table using (6) and (26) above, one sees that as η increases above zero, their gain under the social optimum falls short of that achieved in the OLG equilibrium, as the planner's substitution in their favour is diminished.

To summarize, the fact that it is more costly to provide utility to larger generations imparts an automatic bias against the baby boom and their (enlarged) successor cohorts. When the planner places no weight on generational size, the social optimum mimics the OLG equilibrium, with strong substitution in favour of the elderly at the expense of the current youth and all future generations. As the planner's weight on generational size is increased, resources are transferred

[4] In both models, the evolution of the capital stock is given by

$$\ln k_{t+i} = \alpha^i \ln k_t + \sum_{j=0}^{i-1} \alpha^j (\ln s_{t+i-1-j} - \ln n_{t+i-j}) .$$

to larger cohorts. In the extreme, when $\eta = 1$, the consumption of the young and the old is proportional and all generations are treated equally. In this case, the impact of a baby boom on the utility of all generations depends on the sign and magnitude of the demographic gift

$$(46) \quad \theta_t^{old}(1 - s_t) - \alpha = \frac{C_t^o L_{t-1} - R_t K_t}{Q_t}.$$

More generally, however, the elderly do better, while the young and future generations consume a residual, net, demographic gift.

(c) Social Security

I now consider how a pay-as-you-go social security system allows the planner to achieve his desired allocation of consumption within a decentralized OLG economy. Let each atomistic individual maximize their expected lifetime utility

$$(47) \quad \text{Max} \quad E_t[\ln C_t^y + B \ln C_{t+1}^o]$$

subject to the budget constraint

$$(48) \quad C_{t+1}^o = R_{t+1}[W_t(1 - \tau_t) - C_t^y] + \tau_{t+1} W_{t+1} L_{t+1}/L_t$$
$$= \frac{R_{t+1}K_{t+1} + \tau_{t+1} W_{t+1} L_{t+1}}{L_t},$$

where τ_t is the proportional tax on labour income, which is paid out in full as benefits to the elderly. Individuals earn labour income and pay taxes when young, consuming the value of their capital assets, plus social security benefits, when elderly. The reader can easily confirm that the planner's consumption plan satisfies the individual's intertemporal first order condition (compare (26) and (29) with (3)). Consequently, a sequence of tax rates consistent with the individual's budget constraint (48) and the planner's allocation of consumption will ensure that the decentralized economy delivers the social optimum.

The desired ratio of elderly to youth consumption is given by

$$(49) \quad \frac{C_t^o L_{t-1}}{C_t^y L_t} = X_t = \frac{\alpha Q_t + (1-\alpha)\tau_t Q_t}{(1-s_t)Q_t/1+X_t},$$

where I have incorporated the individual budget constraint (48) in the numerator and the optimal level of youth consumption in the denominator. Rearranging the equation, one arrives at an expression for the tax rate

$$(50) \quad \tau_t = \frac{\theta_t^{old}(1-s_t)-\alpha}{1-\alpha}.$$

This expression can also be derived, more directly, by making use of the concept of the demographic gift. As noted in Section III, any social optimum can be achieved by a transfer scheme in which each generation pays their demographic gift to their parents. The equilibrium tax rate, in (50), is nothing other than generation t's demographic gift divided by its lifetime labour income.

The comparative statics of the tax rate are eminently intuitive. A rise in the consumption share of the elderly raises the equilbrium tax on the young. As social security discourages saving, an increase in the desired savings rate is associated with a reduction in the tax rate. Finally, an increase in the share of capital (α), i.e. the income share of the elderly, lowers the equilibrium tax rate.

Provided $\eta > 0$, a baby boom lowers the consumption share of the elderly and raises the desired savings rate, allowing the conclusion

$$(51) \quad \frac{d\tau_t}{dn_t} \leq 0 \quad \text{as} \quad \eta \geq 0.$$

As proven in the appendix, this result can be extended to the relationship between the unconditional expectation of the tax rate, $\bar{\tau}$, and $\mu = E[n_t^{\eta}]$:

$$(52) \quad \frac{d\bar{\tau}}{d\mu} < 0.$$

If $\eta > 0$, a rise in the rate of population growth raises the expected discount factor, increasing the average savings rate, while lowering the relative consumption of the elderly. Not surprisingly, the average social security tax falls. In sum, the model is broadly consistent with the historical experience of the United States, as a declining birth rate has been associated with a steadily rising social security tax with, however, the baby boom generation enjoying more favourable tax rates than is forecast for their successors.[5]

The preceeding should not be taken too literally. Private intergenerational transfers occur on a regular basis throughout the world. Without taking a position on the motivation behind these transfers (e.g. bequest motives, mutual insurance, etc.), and modelling their evolution through time, one cannot precisely determine how a social planer would evaluate any given public scheme.[6] Nevertheless, if one accepts that although intergenerational linkages may exist, they are imperfect, then there is a role for state transfers, a view apparently taken by the American political system at large. In this regard, one may note that a declining population growth rate is reasonably, and fairly obviously, associated with a rising consumption share of the elderly and increasing transfers in their favour. Similarly, a baby boom generation, burdened with less dependants than their successors, *should* be engaged in transferring less to, or receiving more from, their parents.

(d) Lifetime Tax Burden

As noted in the introduction, the large lifetime tax burden, i.e. the net present value of taxes paid minus benefits received as a share of lifetime income, imposed by social security on

[5] Increasing life-expectancy, another element of the American experience, could be incorporated into the model by introducing a probability p of survival to old age. A secular rise in p would be associated with increasing transfers to the elderly.

[6] In this regard, I should note that social security would have to be evaluated net of private transfers, so that the optimal payroll tax on the young could be positive, even when overall net transfers to the elderly are still negative (i.e. there are net bequests to the young).

postwar generations has been the subject of a number of studies. In the model of this paper, the lifetime tax burden of generation t is given by

$$(53)\quad T_t = \frac{\tau_t W_t - \tau_{t+1} n_{t+1} W_{t+1}/R_{t+1}}{W_t} = \tau_t - \tau_{t+1}\frac{n_{t+1}k_{t+1}}{\alpha k_t^{\alpha}}$$

$$= \tau_t - \frac{\tau_{t+1} s_t}{\alpha},$$

while the unconditional expectation of the tax burden equals

$$(54)\quad \bar{T} = \bar{\tau}\left(1 - \frac{\bar{s}}{\alpha}\right), \quad \text{where } \bar{x} \text{ denotes } E[x_t].$$

As proven in the appendix, $\bar{s} < s < \alpha$. It follows that if the expected tax rate is positive the social security system cannot be actuarially fair, i.e. deliver an expected net present value of benefits equal to payments made. This is not surprising, as the economy is dynamically efficient. A reduction in $\mu = E[n_t^{\eta}]$, as noted earlier, lowers the expected savings rate and increases the expected tax rate. If the expected tax rate is already positive, this can only raise the expected lifetime tax burden. An actuarially unfair social security system, which worsens as the rate of population growth falls, is completely sensible and compatible with the wishes of a social planner.

Turning to transitional issues, a temporary increase in n_t raises s_t and lowers τ_t (provided $\eta > 0$). From (53), it follows that a baby boom generation enjoys a lower lifetime tax burden than its successors. The intuition behind this result is made clearer if one focuses not on taxes, but on consumption and income, using the relation

$$(55)\quad 1 - T_t = \frac{C_t^y + C_{t+1}^o/R_{t+1}}{W_t}.$$

A baby boom dilutes capital per worker and lowers the dependency ratio. The reduction in capital per worker, however, lowers both incomes and consumption. In the ln-ln model these effects cancel exactly, leaving only the influence of dependency

$$(56) \quad 1 - T_t = \frac{\tilde{C}_t^y \overline{C}_t^y + \frac{n_{t+1} X_{t+1} \tilde{C}_{t+1}^y \overline{C}_{t+1}^y}{R_{t+1}}}{W_t} = \frac{1 - s}{(1-\alpha)(1+X)} \left[\tilde{C}_t^y + \frac{s_t X_{t+1} \tilde{C}_{t+1}^y}{\alpha} \right].$$

With a lower dependency ratio, the baby boom generation is able to enjoy greater youth ($\tilde{C}_t^y \uparrow$) and elderly ($s_t \uparrow$) consumption relative to its lifetime income.

Finally, with regards to the impact of a baby boom on the lifetime tax burden of the elderly, one sees from (53) that a fall in τ_t raises T_{t-1}. In response to a baby boom the planner tips consumption in favour of the elderly, but typically, unless $\eta = 0$, gives them less than the increase in capital returns (R_t). This result, together with that of the previous paragraph, conveniently highlights the distinction between welfare and lifetime tax burdens, which measure the shift of resources *relative* to those imparted by market outcomes. The planner favours smaller generations, whose welfare is more easily increased, at the expense of larger cohorts. If $\eta > 0$, however, the planner's bias is less than that of the market. Consequently, while favouring smaller generations in terms of welfare, the planner favours larger generations in terms of transfers.

Once again, these results are broadly in agreement with the historical and forecasted American experience. In response to declining population growth, a planner would institute a social security system, with positive transfers to the elderly. Persons who are elderly or middle aged when the system is instituted earn a positive return, by default, but lifetime participants face a negative expected tax burden. As the expected population growth rate falls, the tax burden becomes greater. A baby boom generation, because of declining expected population growth, might pay a higher tax rate than its predecessors (perhaps granting even early lifetime participants a positive return), but, temporarily burdened with less than expected elderly dependents, it will definitely enjoy a lower lifetime tax burden than is expected by its successors. Future cohorts will shoulder an onerous tax burden, as, from the point of view of the planner, well they should.

(e) Summary and Discussion

The preceeding illustrates how a social planner allocates the demographic gift of a baby boom across current and future generations. In distributing the demographic gift, the planner is intinsically biased in favour of smaller generations, whose per capita welfare is more easily increased. This bias is somewhat offset to the degree that the planner places weight on generational size, but persists, as long as the weight is less than linear. Consequently, there is an inherent welfare bias against a baby boom and their progeny in favour of earlier, smaller, cohorts. Market outcomes are also biased in favour of smaller cohorts. As shown by the simple example considered in this paper, this bias may easily exceed that of the social planner. This allows for outcomes where, in response to a baby boom, a planner lowers the welfare of a baby boom generation and raises that of their predecessors while, in a seemingly paradoxical fashion, lowering the lifetime tax burden of the former and raising that of the latter. Intergenerational differences in transfer tax burdens reflect a bias relative to that of the market, and are not necessarily informative about differences in welfare outcomes.

In regards to specific results, the analysis suggests that baby boom generations should pay a lower payroll tax rate and enjoy a lower lifetime tax burden than is expected by their successors. A secular decline in birth rates will give rise to a rising tax rate and (once transfers are positive) a worsening lifetime tax burden. These results follow from the negative relationship between the elderly share of consumption and the birth rate. As the birth rate rises, the elderly's share of total consumption falls. Holding constant factors such as the savings rate and the capital share of output (broadly construed as the elderly's claim on output), this automatically implies a decrease in transfers to, or an increase in transfers from, that group. More generally, a reduction in elderly "dependency" provides an income effect, allowing the current youth to enjoy higher consumption relative to their lifetime income. A falling birth rate leads to rising elderly dependency, which imposes an increasing burden on the youth. This mechanism is embarassingly simple.

While the simplicity of the example eases exposition, it is important to consider the robustness of the results to an allowance for more general functional forms and lifetime income structure. The predicted unambiguously negative relationship between the birth rate and the elderly share of total consumption is not, for instance, independent of the choice of functional forms. With general CES utility the optimal plan involves an allocation such that

$$(57) \quad C_t^o = C_t^y \left(\frac{n_t^{1-\eta} B}{\gamma} \right)^{\sigma},$$

where σ is the elasticity of intertemporal substitution. The greater the elasticity of substitution, the greater the desired shift in favour of the elderly so that if $\sigma(1-\eta) > 1$ the consumption share of the elderly will, paradoxically, rise with an increase in the birth rate. Empirical estimates of the elasticity of substitution [e.g. Hall 1988] are substantially less than one, suggesting that one can reasonably argue that a rise in the number of youth should, from a planner's perspective, lower the ratio of total elderly to youth consumption. Nevertheless, it is important to acknowledge that a large elasticity of substitution would reverse the relationship between dependency and population structure, changing the sign of the model's predictions, but leaving the analytical intuition unchanged.

As one moves away from the specialized assumptions of ln-linear consumption and production and 100% depreciation used in the analysis above, variations in capital per worker will affect the dynamics and steady state values of both the savings rate and the share of capital, influencing the optimal choice of tax rate and lifetime tax burden. The timing, sign and magnitude of these effects, however, depends upon a number of parameters, making their exploration less an exercise in theory and more a problem in the practical, empirical, implementation of the household model.[7] The dependency effects highlighted above are present, regardless of functional forms. Whether or not they are overturned by other issues, such as capital share and savings dynamics, is a matter of empirical investigation.

Finally, the reader might wonder whether, with all the emphasis on elderly dependency, the author has forgotten that newborns are also dependents. Although a baby boom might ultimately give rise to a fall in dependency, when a large working cohort supports the elderly, it would seem that this must be preceeded by a rise in dependency, when the large birth cohort is supported by their working parents. This would suggest that a more complicated lifetime income structure might reverse, or at least hopelessly muddle, the results. This intuition is actually incorrect. The childhood consumption needs of a large cohort are, indeed, a negative income shock to society. Unlike changes in elderly dependency, however, these consumption needs deplete the capital stock *before* the baby boom cohort begins work. One can easily show that, for the benchmark ln-linear case, childhood dependency needs lower lifetime consumption and

[7] Thus, in the analysis of the ln-linearized certainty-equivalent model, with general functional forms, let λ denote the slope of the consumption-capital saddlepath (with $n_t = \overline{n}$) and b the speed of convergence, i.e. $\hat{C}_t^y = \lambda\hat{k}_t$ and $\hat{k}_{t+1} = b\hat{k}_t$, where a ^ denotes a proportional deviation from steady state values. The response of the lifetime tax burden along the saddlepath to a proportional deviation of the capital stock equals

$$[\lambda NPV_y^C + b(\lambda + \theta_L/\varepsilon)NPV_o^C - \theta_K/\varepsilon]\hat{k}_t\,,$$

where NPV_i^C denotes the share of stage i in the net present value of lifetime consumption, while θ_i is the share of factor i and ε the elasticity of substitution in production, both calculated using the gross rental and gross output function. It is impossible to sign the term in brackets. In fact, even its derivative with respect to the elasticity of substitution, σ, locally, around a point where $\varepsilon = \sigma = 1$, may be positive or negative. To complicate matters further, if the net demographic gift is positive, while a baby boom leads to a negative $\hat{k}_t$, the increased resources available lead to a positive $\hat{k}_{t+1}$ (which then converges back to the steady state value), confounding, yet further, the analysis of the dynamics of the lifetime tax burden. Amidst all these complicated transition path dynamics, however, the model retains the temporary dependency effects (following each movement in the birth rate) described above.

lifetime income equiproportionally,[8] leaving only the positive benefits of reduced elderly dependency (later in the baby boomer's lifecycle), which raises the large cohorts' lifetime consumption relative to its income. In sum, the preceeding analysis can easily be extended to a T period framework, with any arbitrary lifetime income profile, without prejudice to its predictions.[9]

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[8] To see this most quickly, the reader can rework the ln-linear two period model discussed above under the assumption that the elderly work and the youth do not, so that a baby boom generates an increase in youth dependency, with no subsequent reduction in elderly dependency. In this case, the analysis will show that, regardless of η, a generation's lifetime tax burden is independent of its size (the lifetime tax burden is still increasing in the size of the subsequent generation). This demonstrates the offsetting effects alluded to above.

[9] In other words, allowing persons to live from age 0 to T, specify the general Cobb-Douglas production function:

$$Q_t = K_t^{\alpha}\left(\prod_{i=0}^{T} L_{t-i}^{\theta_i}\right)^{1-\alpha}, \quad \text{where} \quad \sum_{i=0}^{T}\theta_i = 1$$

and where some of the θ_i may be zero. With 100% depreciation and ln-linear utility, the predictions of this model are exactly the same as that of the two period example, as any early consumption needs in excess of income reduce both lifecycle income and consumption equiproportionately.

V. Conclusion

First year graduate students, stupefied by the infinite lifetime assumed in the typical analysis of the Ramsey model, are quickly assured that the utility function represents the welfare of an integrated household composed of an endless stream of mortal generations. In this paper I take the comments of our early instructors and textbooks seriously, exploring the household's behavior in an environment with a fluctuating birth rate and explicit life cycle labour supply. In so doing, I extend the standard model's normative analysis to the impact of demographic change on generational welfare and intergenerational transfers. The model's predictions are hearteningly intuitive and commonsensical. A dilution of bequested assets across more children reduces the offspring's welfare, while the dilution of bequested debts does the opposite. Societies with falling birth rates and rising elderly dependency should, indeed, transfer more resources to that group, at the cost of a higher lifetime tax burden for the youth. Baby boom cohorts may be economically disadvantaged, but enjoy less than forecasted elderly dependency during their lifetime, allowing for higher consumption relative to income, i.e. a lower lifetime transfer tax burden. Just as the standard Ramsey model has provided a normative perspective to positivist analyses of savings, so an extension of that model, with a fuller consideration of demographic structure, might reexamine the extensive literature on the intergenerational effects of demographic change.

VI. Bibliography

Abel, Andrew B., "Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?", NBER Working Paper #8131, February 2001.

Brooks, Robin, "Life Cycle Portfolio Choice and Asset Market Effects of the Baby Boom," Manuscript, International Monetary Fund, 2000.

Caldwell, Steven, Melissa Favreault, Alla Gantman, Jagadeesh Gokhale, Thomas Johnson and Laurence J. Kotlikoff, "Social Security's Treatment of Postwar Americans," NBER Working Paper #6603, June 1998.

Card, David and Thomas Lemieux, "Dropout and Enrollment Trends in the Post-War Period: What Went Wrong in the 1970s?," NBER Working Paper #7658, April 2000.

Davis, Joseph S., "The Population Upsurge and the American Economy, 1945-80," Journal of Political Economy 61 (1953), 369-388.

Easterlin, Richard A., Birth and Fortune: The Impact of Numbers on Personal Welfare (New York: Basic Books, 1980).

Freeman, Richard B., "The Effect of Demographic Factors on Age-Earnings Profiles," Journal of Human Resources 14 (1979), 289-318.

Geanakoplos, John, Olivia S. Mitchell and Stephen P. Zeldes, "Social Security Money's Worth," NBER Working Paper #6722, September 1998.

Hall, Robert E., "Intertemporal Substitution in Consumption," Journal of Political Economy 96 (1988), 339-357.

Lerner, Abba P., "Consumption-Loan Interest and Money," Journal of Political Economy 67 (1959), 512-518.

Mankiw, Gregory N. and David N. Weil, "The Baby Boom, the Baby Bust and the Housing Market," Regional Science and Urban Economics 19 (1989), 235-258.

Myers, Robert J. and Bruce D. Schobel, "A Money's-Worth Analysis of Social Security Retirement Benefits," Transactions of Society of Actuaries 35 (1983), 533-545.

________, "An Updated Money's-Worth Analysis of Social Security Retirement Benefits," Transactions of Society of Actuaries 44 (1992), 247-270.

Poterba, James M., "Demographic Structure and Asset Returns," Manuscript, MIT, December 2000.

________, "House Price Dynamics: The Role of Tax Policy and Demography," Brookings Papers on Economic Activity 1991, Issue 2, 143-183.

Russell, Louise B., The Baby Boom Generation and the Economy (Washington, D.C.: The Brookings Institution, 1982).

Samuelson, Paul A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy 66 (1958), 467-482.

Welch, Finis, "Effects of Cohort Size on Earnings: The Baby Boom Babies' Financial Bust," Journal of Political Economy 87 (1979), S65-S97.

VII. Appendix A: Aspects of the Demographic Gift

This appendix supplements the analysis of Section III. In regards to the derivation of equation (17), I begin by exploiting the constant returns to scale characteristic of the output function to rewrite it as:

$$\text{(A.1)}\quad F(K_t, S_t^T L_t^T, S_t^{T-1} L_t^{T-1}, \ldots, S_t^0 L_t^0, t) + (1-\delta)K_t \;=\; R_t K_t + \sum_{j=0}^{T} W_t^j S_t^j L_t^j\,,$$

where I have departed from the simplified exposition in the text by explicitly identifying labour supply in S_t^j, the hours of work supplied by workers of age j, and reinterpreting W_t^j as the corresponding hourly wage. Differentiating and dividing by L_t^0 yields the convenient result

$$\text{(A.2)}\quad 0 \;=\; (\mathrm{d}R_t)k_t + \sum_{j=0}^{T} (\mathrm{d}W_t^j) S_t^j l_t^j\,.$$

Substituting (A.1) into equation (7) in the text, leading the equation infinitely forward, and dividing by L_t^0, one derives the social intertemporal budget constraint

$$\text{(A.3)}\quad \sum_{i=0}^{\infty}\sum_{j=0}^{T} \frac{C_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} \;=\; \sum_{i=0}^{\infty}\sum_{j=0}^{T} \frac{W_{t+i}^j S_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} \;+\; k_t \qquad \left[\text{provided}\;\; \lim_{i\to\infty} \frac{K_{t+i}}{\prod_{k=0}^{i} R_{t+k}} = 0\right],$$

which, when differentiated with respect to $\ln n_t$, produces

$$\text{(A.4)}\quad \sum_{i=0}^{\infty}\sum_{j=0}^{T} \frac{\frac{\mathrm{d}C_{t+i}^j}{\mathrm{d}\ln n_t} l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} - \sum_{i=0}^{\infty}\sum_{j=0}^{T} \frac{\frac{\mathrm{d}S_{t+i}^j}{\mathrm{d}\ln n_t} W_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} = \left[\sum_{i=0}^{\infty}\sum_{j=0}^{T} \frac{\frac{\mathrm{d}W_{t+i}^j}{\mathrm{d}\ln n_t} S_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} + \sum_{i=0}^{\infty} \frac{\frac{\mathrm{d}R_{t+i}}{\mathrm{d}\ln n_t} k_{t+i} \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}}\right]$$

$$+ \left[\sum_{i=0}^{T-1}\sum_{j=i+1}^{T} \frac{C_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} - \sum_{i=0}^{T-1}\sum_{j=i+1}^{T} \frac{W_{t+i}^j S_{t+i}^j l_{t+i}^j \prod_{k=1}^{i} n_{t+k}}{\prod_{k=0}^{i} R_{t+k}} - k_t\right].$$

The first term in brackets on the right hand side is, by (A.2), equal to zero, while the second term, when multiplied by L_t^0, is none other than the demographic gift. As $\mathrm{d}C_{t+i}^j - W_{t+i}^j \mathrm{d}S_{t+i}^j$ equals the change in real expenditure, in units of consumption, on persons of age j at time t+i, it follows that (A.4) produces the first part of equation (17) in the text. As for the second part, in a

Pareto efficient economy the ratio of the marginal utility of real expenditure across periods for each individual equals the cumulative interest factor, so that the utility impact of a stream of real expenditure changes on the average utility of a member of cohort t can be reduced to

$$\text{(A.5)} \quad \frac{\mathrm{d}\overline{U}_t}{\mathrm{d}\ln n_t} = \sum_{i=0}^{T} \frac{\mathrm{d}E_{t+i}^i}{\mathrm{d}\ln n_t} \mathrm{MU}_t^i p_t^i = \mathrm{MU}_t^0 \sum_{i=0}^{T} \frac{\frac{\mathrm{d}E_{t+i}^i}{\mathrm{d}\ln n_t} p_t^i}{\prod_{k=1}^{i} R_{t+k}},$$

where MU_t^i is the marginal utility of real expenditure of the surviving members of cohort t at age i and p_t^i denotes the corresponding probability of survival to that age. Substitution into (A.4) produces the second part of equation (17).

To extend the concept of the demographic gift to an environment with aggregate uncertainty, I consider the increment to the capital stock necessary to keep the expected utility of all current and future generations constant following a proportional change in the size of generation t. As in the certainty environment, by period t+T the influence of n_t on the dynamics of the economy disappears. Let i index the universe of possible states of nature at that time and π_i their associated probabilities. Further, let $\mathrm{d}K_i$ denote the increment to the capital stock at time t necessary to keep k_{t+T} unchanged in realized state i. This can be calculated in the manner laid out in the text, as the concept of a realized state eliminates all aggregate uncertainty between periods t and t+T. Finally, let $P_{K_t}^i$ denote the equilibrium price in units of period t capital of an asset which delivers one unit of capital in state i in period t+T. The equilibrium price of this state contingent claim is given by the first order condition for members of generation t

$$\text{(A.6)} \quad P_{K_t}^i R_t \mathrm{MU}_t^0 = \pi_i R_{t+T}^i \mathrm{MU}_t^T(i),$$

where $\mathrm{MU}_t^T(i)$ is the marginal utility of consumption expenditure of generation t in state i.

To maintain the expected welfare of all current and future generations, the planner must buy a sufficient volume of capital claims to ensure his ability to replenish the capital stock in each realized state of nature i. This requires an expenditure, in units of current capital, of

$$\text{(A.7)} \quad \mathrm{d}K^* = \sum_i P^i_{K_t} \mathrm{d}K^i \prod_{k=0}^{T-1} R^i_{t+k}\,,$$

where R^i_{t+k} equals the interest rate realized in period t+k along the path to state i. Since the planner need only replenish k_{t+T} in a present value sense, not in an immediate physical sense, the purchase of such assets does not perturb the expected utility of anyone in the economy, as they can be bought in infinitesimal amounts from all current and future generations. The demographic gift equals $-\mathrm{d}K^*$ or, applying (A.6),

$$\text{(A.8)} \quad \Delta K_t = \frac{-\sum_i \left(\mathrm{d}K_i \prod_{k=0}^{T} R^i_{t+k} \right) \mathrm{MU}_t^T(i)\pi_i}{R_t \mathrm{MU}_t^0}\,.$$

VIII. Appendix B: Expected Savings and Tax Rates

This appendix establishes some results concerning the unconditional expectation of the savings and payroll tax rates. Let $z_t = n_t^\eta - \mu$, where $\mu = E[n_t^\eta]$, and, recalling that $X = B\alpha/s$ and $s = \gamma\alpha\mu$, manipulate (37) to produce

$$\text{(B.1)} \quad s_t = \frac{(s + B\alpha)(z_t + \mu)}{(1 + B\alpha)z_t + \mu + B/\gamma}.$$

Taking a first order expansion around $E[z_t] = 0$, with remainder evaluated at some z^*

$$\text{(B.2)} \quad s_t = s + \left[\frac{B\alpha(1-s)}{\mu + B/\gamma}\right] z_t - \left[\frac{(s + B\alpha)(B/\gamma)(1-s)(1+B\alpha)}{((1+B\alpha)z^* + \mu + B/\gamma)^3}\right] z_t^2$$

$$< s + \left[\frac{B\alpha(1-s)}{\mu + B/\gamma}\right] z_t,$$

one sees that $\bar{s} = E[s_t] < s$, as claimed in the text.

The unconditional expectation of s_t is given by

$$\text{(B.3)} \quad \bar{s} = \int \frac{(s + B\alpha)(z_t + \mu)}{(1 + B\alpha)z_t + \mu + B/\gamma} p(z_t)\, \mathrm{d}z_t.$$

where $p(z_t)$ is the probability density function of z_t. Taking the derivative with respect to μ, holding the distribution of z_t (i.e. the central moments of n_t^η) constant, produces the result $\mathrm{d}\bar{s}/\mathrm{d}\mu > 0$. Finally, with regards to the expected tax rate, straightforward manipulation of equation (50), substituting using (37), allows the expression

$$\text{(B.4)} \quad \tau_t = 1 - \frac{(1 + B\alpha)s_t}{(s + B\alpha)(1-\alpha)}, \quad \text{so that} \quad \bar{\tau} = 1 - \int \frac{(1 + B\alpha)(z_t + \mu)}{(1-\alpha)[(1 + B\alpha)z_t + \mu + B/\gamma]} p(z_t)\, \mathrm{d}z_t.$$

Differentiating, holding the distribution of z_t constant, confirms that $\mathrm{d}\bar{\tau}/\mathrm{d}\mu < 0$, as claimed in the text. I should note that the last two results can also be derived holding constant the distribution of the normalized random variable $z_t = n_t^\eta/\mu - 1$.