A ball bouncing off a rigid floor is described by the potential energy function

$$V(x) = mgx \quad \text{for } x > 0$$
$$\quad\;\; = \infty \quad \text{for } x \leq 0$$

Which of the following schematic diagrams best represents the phase space plot of the ball?

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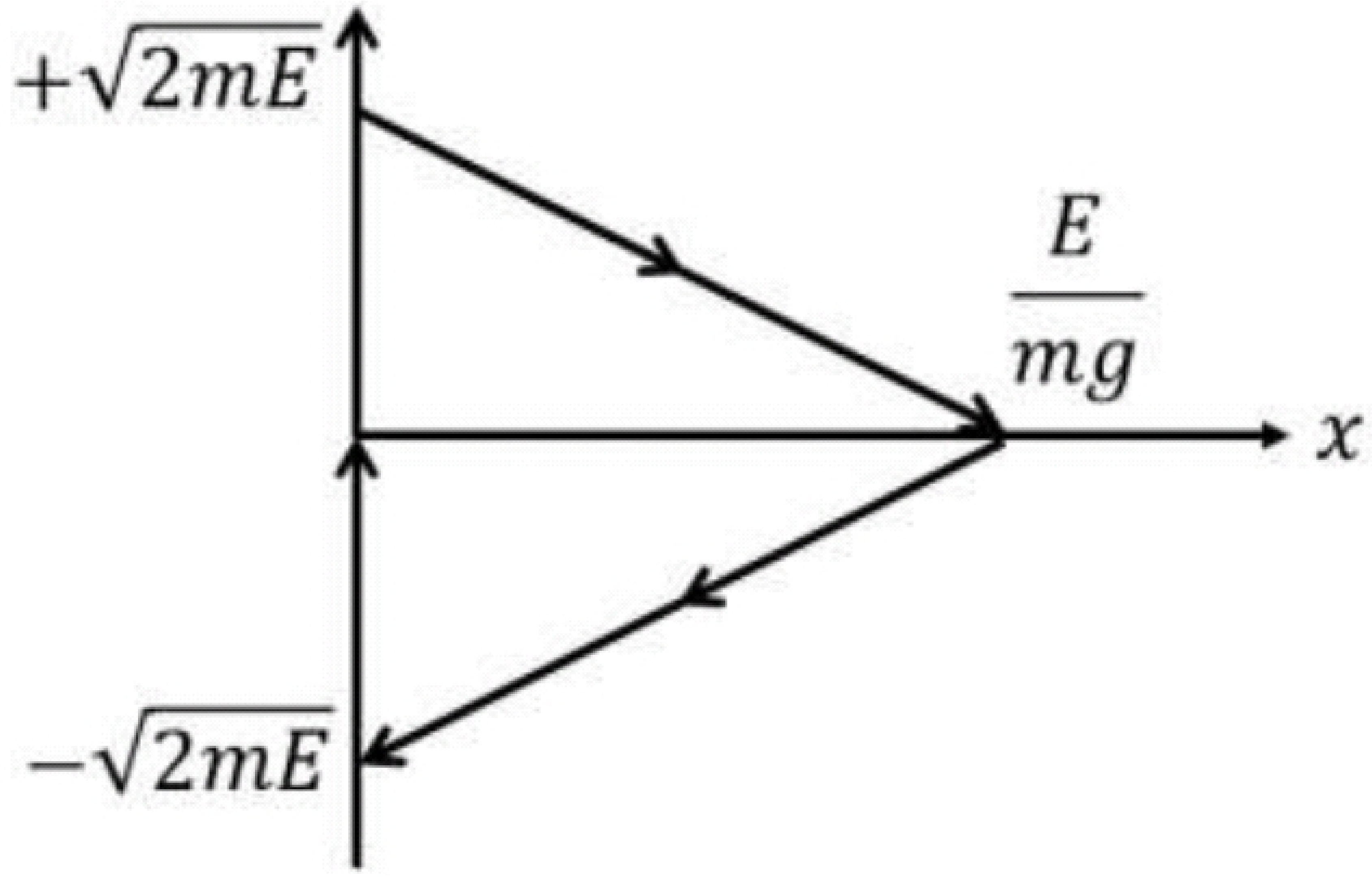
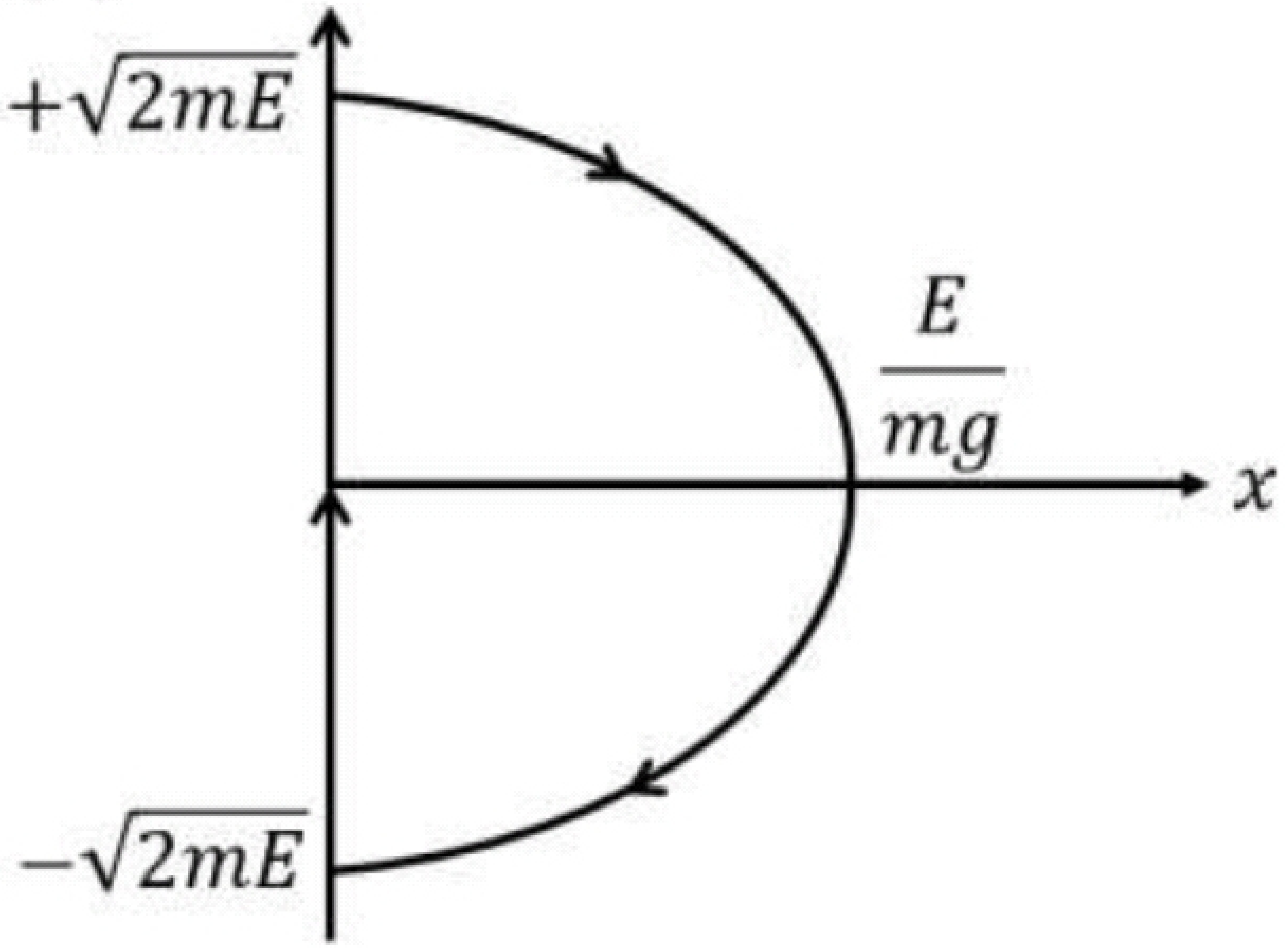
(A)



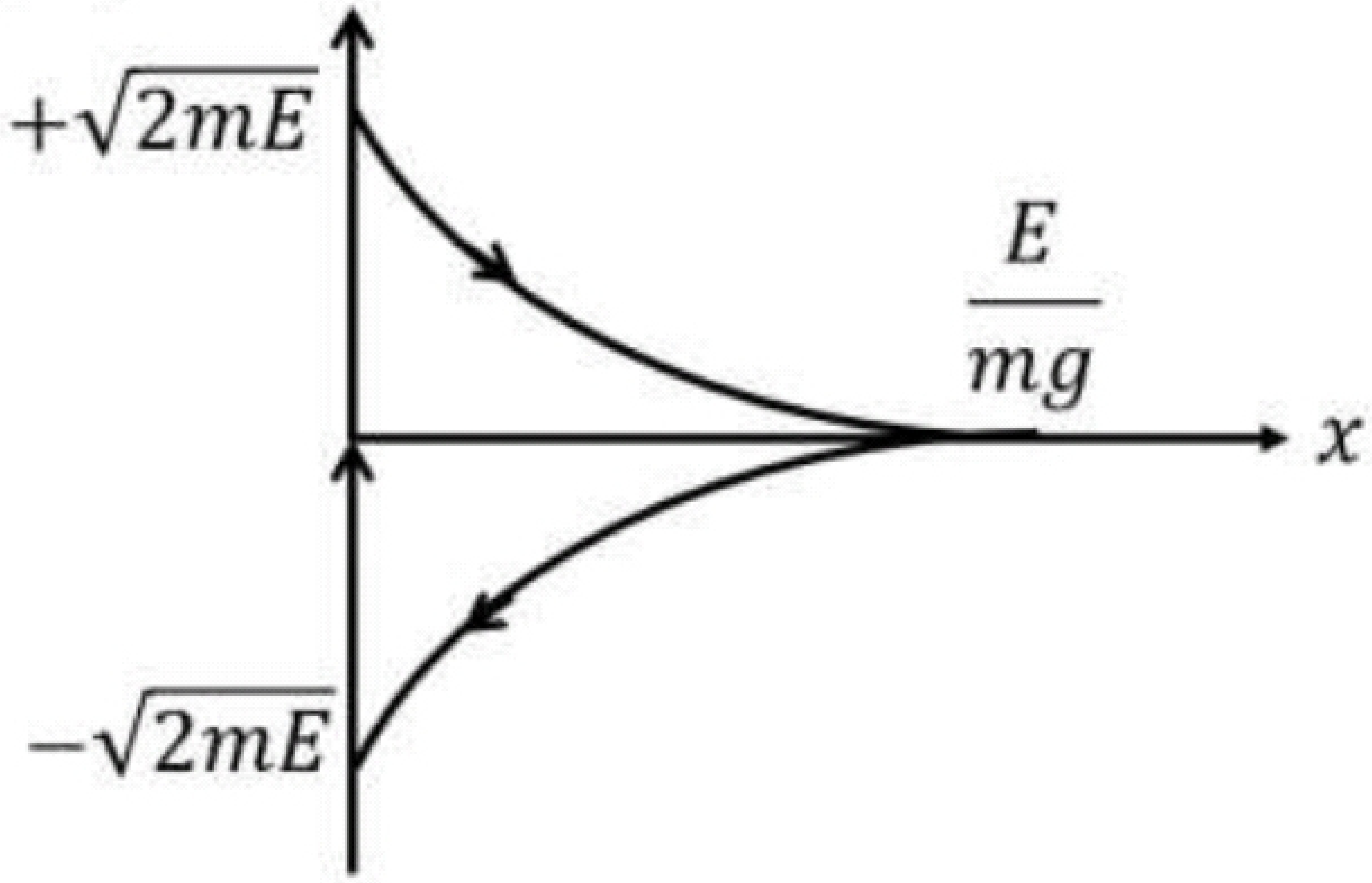
$+\sqrt{2mE}$
$\frac{E}{mg}$
x
$-\sqrt{2mE}$

(B)



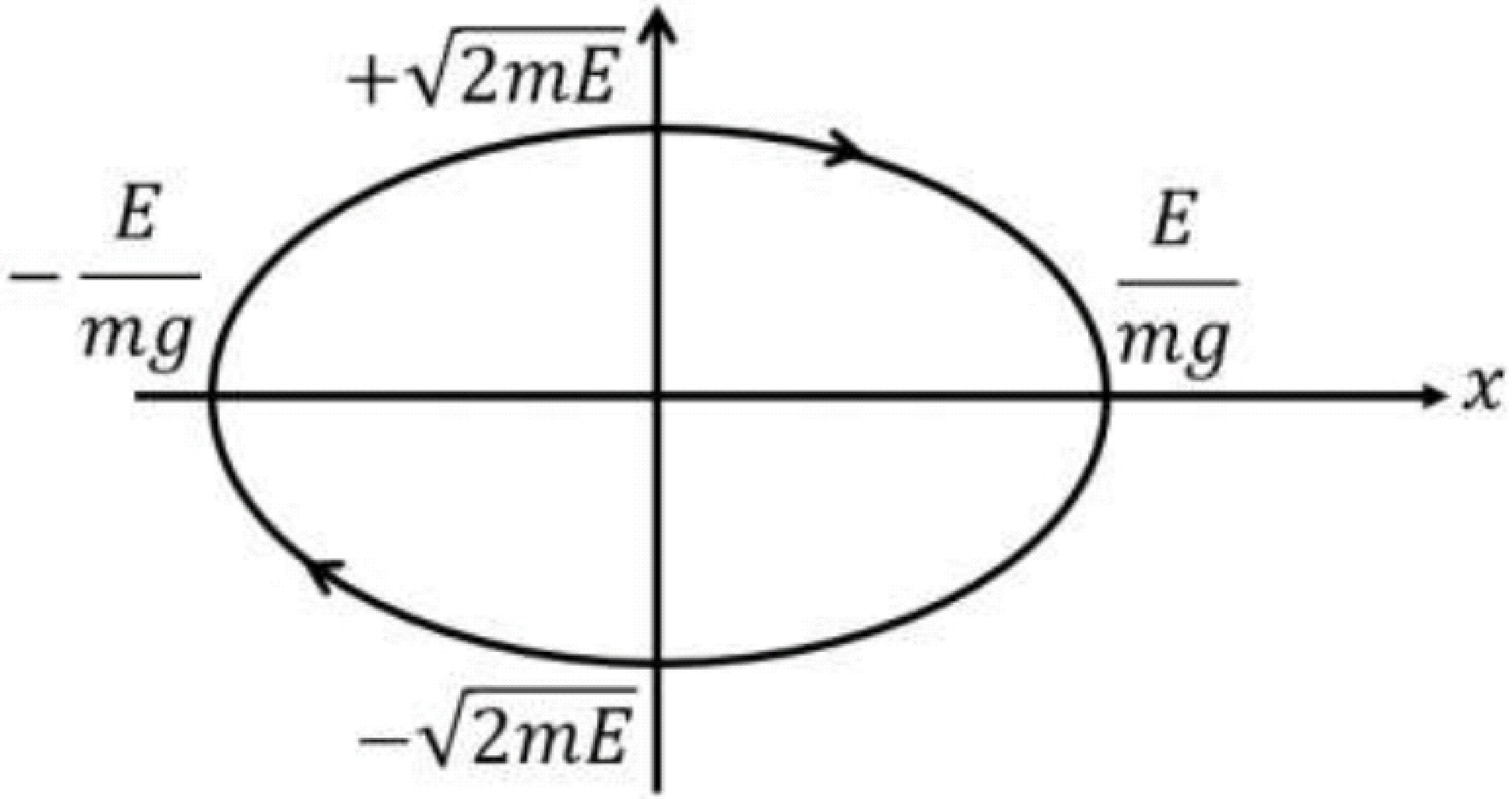
$+\sqrt{2mE}$
$\frac{E}{mg}$
x
$-\sqrt{2mE}$

(C)



$+\sqrt{2mE}$
$\frac{E}{mg}$
x
$-\sqrt{2mE}$

(D)



$+\sqrt{2mE}$
$-\frac{E}{mg}$
$\frac{E}{mg}$
x
$-\sqrt{2mE}$

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Phase-space diagram is a plot between p and x

It is between generalized coordinate and generalized momentum.

The lagrangian & the hamiltonian for the system is given as follows:

$$L = T - V$$

$$L = \frac{1}{2}m\dot{x}^2 - mgx$$

$$\boxed{H = \frac{p^2}{2m} + mgx}$$

$$E = \frac{p^2}{2m} + mgx = \text{constant} \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$\Rightarrow p = \pm\sqrt{2m(E - mgx)}$$

- At $x = 0$

$$p = \pm\sqrt{2mE}$$

- At $p = 0$

$$x = \frac{E}{mg}$$

- We can clearly see from the equation (1) that the potential energy and kinetic energy have inverse relation between them.

- For x<0 we have potential energy as infinite so, kinetic energy would be zero for the system. Thus momentum for this system is also zero.

- We have determine the nature of the curve in this graph.

- As we see the equation no. (1) we can see that the relation between p & x is in the form of parabola. Thus option (B) is the correct answer.

Consider the Hamiltonian $H(q,p) = \frac{\alpha p^2 q^4}{2} + \frac{\beta}{q^2}$, where α and β are parameters with appropriate dimensions, and q and p are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q,\dot{q})$ is

(A) $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} - \frac{\beta}{q^2}$ (B) $\frac{2}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ (C) $\frac{1}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ (D) $-\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$

Solution:

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$$L = \sum_i p\dot{q}_i - H$$

Using hamilton's equation we will find the generalized momentum for the given system.

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{q} = \frac{\partial}{\partial p}\left(\frac{\alpha p^2 q^4}{2} + \frac{\beta}{q^2}\right)$$

$$\dot{q} = \frac{2\alpha p q^4}{2}$$

$$\dot{q} = \alpha p q^4$$

$$\Rightarrow p = \frac{\dot{q}}{\alpha q^4}$$

$$L = \frac{\dot{q}^2}{\alpha q^4} - \left(\frac{\alpha p^2 q^4}{2} + \frac{\beta}{q^2}\right)$$

$$L = \frac{\dot{q}^2}{\alpha q^4} - \frac{\dot{q}^2}{2\alpha q^4} - \frac{\beta}{q^2}$$

$$L = \frac{\dot{q}^2}{2\alpha q^4} - \frac{\beta}{q^2}$$

Option (A) is the correct answer