THE RECOGNITION PROBLEM FOR TOPOLOGICAL MANIFOLDS: A SURVEY

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Abstract

This is a survey of recent work on the problem of recognizing topological manifolds among topological spaces, including the results of J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger in higher dimensions, and M.V. Brahm, R.J. Daverman and D. Repovš in dimension three.

1. Introduction

The definition of a *topological n-manifold* M ($n \in \mathbf{N}$) is quite simple to state – besides the separability and metrizability we require (and this is the key *geometric* property of manifolds) that every point $x \in M$ should possess a neighbourhood $U \subset M$ which is *homeomorphic* to $\mathbf{R}^n$. (We shall only consider *closed* topological manifolds M, i.e. M is connected, compact and $\partial M = \emptyset$.) However, in practice, it is precisely the *verification* of the existence (or nonexistence) of such homeomorphism $h : U \to \mathbf{R}^n$, which is the biggest problem. So a natural question arises: *Is it possible to find a characterization of topological manifolds which does not mention homeomorphisms, which is reasonably simple to state but which is also not too difficult to verify?*

This is the so-called *Recognition problem for topological manifolds*, one of the most important problems of *geometric topology*, i.e. of its branch called *Bing* (or *Texas*) *topology*. In the present paper we plan to survey the most recent work on this problem, including the results due to J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger [10] in higher dimensions and M.V. Brahm [7], R.J. Daverman and D. Repovš [18] [19] in dimension three. For an account of the work done earlier see the surveys of J.W. Cannon [13] and D. Repovš [34] [35] (the survey [35] also contains an extensive bibliography on this subject). For the closely related topics — the topology of cell-like maps — the reader may wish to consult the latest survey by W.J.R. Mitchell and D. Repovš [26].

2. Generalized manifolds

Topological manifolds of dimensions 1 and 2 have very simple characterizations. For example, S^1 is the only compact, connected metric space containing at least 2 points, which is separated by every pair of its points [28], and S^2 is the only nondegenerate locally connected, connected, compact metric space which is separated by no pair of its points but is separated by each of its simple closed curves [3]. Nothing so simple is known to characterize higher dimensional manifolds.

The first question that arises in dealing with higher dimensions (≥ 3) is the following one: In which *class of topological spaces* do we want to detect topological manifolds? The most appropriate seems to be the class of so-called *generalized manifolds*. For these are the spaces which possess all the basic *algebraic* topology properties of manifolds, i.e. from the point of view of homology and homotopy theory they behave much like genuine manifolds (e.g. they satisfy the Poincaré duality and they are locally contractible). Also, they have all the required general topology properties – separability and metrizability. The main *difference* between topological n-manifolds and generalized n-manifolds is that the latter may fail to possesses sufficient *general position properties* (although they do possess some – cf. the work of J.J. Walsh [39] and W.J.R. Mitchell, D. Repovš and E.V. Ščepin [27]). In fact, there are examples due to R.J. Daverman and J.J. Walsh [20] of *ghastly* generalized manifolds, for all dimension ≥ 3, illustrating just how *wild* this class of spaces can be.

Generalized manifolds were first introduced into topology in the 1930's – one of the major motivations was the discovery by R.L. Wilder [40] that they were the proper framework to generalize classical theorems of the *Jordan-Schoenflies* type from dimension 2 to higher dimensions (since the examples like the *Alexander horned sphere* [1] makes a direct generalization impossible). Since then they have played an important role in various parts of topology, e.g. theory of transformation groups [4], theory of cell-like decompositions of manifolds [17], taming theory [12], suspensions of homology spheres [14], compactifications of open topological manifolds [8], manifold factors [15], etc.

Through the decades the definitions of a generalized manifold were changing, depending upon the particular interests of the people involved. The following is the geometric topologist's version:

DEFINITION 2.1.. A locally compact Hausdorff space X is said to be a generalized n-manifold ($n \in \mathbf{N}$) if X satisfies the following properties:

  (i) X is an Euclidean neighbourhood retract (ENR), i.e. for some integer m, X embeds in $\mathbf{R}^m$ as a retract of an open subset of $\mathbf{R}^m$ (equivalently, X is a locally compact, finite-dimensional separable, metrizable ANR); and

  (ii) X is a $\mathbf{Z}$-homology n-manifold, i.e. for every point $x \in X$,

$$H_*(X, X \setminus \{x\}; \mathbf{Z}) \cong H_*(\mathbf{R}^n, \mathbf{R}^n \setminus \{0\}; \mathbf{Z}).$$

Let X be a generalized n-manifold. If $n \leq 2$ then it follows by classical results that X is a topological n-manifold. On the other hand, if $n \geq 3$ then X need not be a genuine n-manifold anymore, in fact, it may fail to possess Euclidean n-dimensional neighbourhoods at all points $x \in X$. Such points are called *singularities* of X and they form the *singular set* $S(X)$ of X, i.e. $S(X) = \{x \in X \mid x$ does not have any neighbourhood in

X homeomorphic to $\mathbf{R}^n$}. Its complement, $M(X) = X \setminus S(X)$, is called the *manifold set* of X and, if $S(X) \neq X$, it is clearly an open n-manifold. (It is interesting to observe that for many *totally singular* generalized manifolds X, i.e. $S(X) = X$, the singularities completely vanish upon multiplication of X by the real line, i.e. $X \times \mathbf{R}$ is a genuine manifold – see [15].)

3. Resolutions of generalized manifolds

A *resolution* of an n-dimensional ANR X is a proper, cell-like map $f : M \to X$ from a topological n-manifold M onto X. It follows by classical results that if X admits a resolution, X *must* be a generalized n-manifold [24]. A resolution $f : M \to X$ is said to be *conservative* if it is *one-to-one* over the manifold set $M(X)$ of X, i.e. for every point $x \in M(X)$, $f^{-1}(x) = \text{point}$. It is not too difficult to show that if a generalized manifold has a resolution then it also has a conservative one.

Cell-like maps $f : M \to X$ are those whose point-preimages $f^{-1}(x)$ are *cell-like sets*, i.e. conti-nua with the (Borsuk) shape of a point [5]: $\text{Sh}(f^{-1}(x)) = \text{Sh}(\text{pt})$. Equivalently (for finite-dimensional sets), $f^{-1}(x)$ is cell-like if it embeds into some Euclidean space $\mathbf{R}^m$ as a *cellular* set in $\mathbf{R}^m$, i.e. the embedding $f^{-1}(x) \hookrightarrow \mathbf{R}^m$ can be represented as the intersection of a properly nested decreasing sequence of closed m-cells [9]

$$\mathbf{R}^m \supset B_1^m \supset \text{int } B_1^m \supset B_2^m \supset \text{int } B_2^m \supset \cdots \supset f^{-1}(x) = \bigcap_{i=1}^{\infty} B_i^m .$$

Roughly speaking, the concept of a cell-like map is a generalization of the idea of a homeomorphism since point-preimages of homeomorphism are *points* whereas point-preimages of a cell-like map only have *homotopical* and *geometrical* properties very much like a point.

Cell-like maps play an important role in topology and they have been significantly applied in solutions of several very difficult problems, e.g. the 4-dimensional Poincaré Conjecture [22]. It thus comes as no surprise that they also play a key role in the Recognition problem. (For more on topology of cell-like maps see the survey [26].)

The strategy for attacking the Recognition problem which J.W. Cannon [13] had in mind when he proposed the problem at the 1977 AMS Summer Meeting in Seattle, consisted of two key steps: *resolution* and *shrinking*. More precisely, given a generalized n-manifold X, one first seeks to build a resolution of X, $f : M \to X$, by blowing up the singularities of X into cell-like sets. We thus at once have the following problem:

RESOLUTION PROBLEM.   *Does every generalized manifold have a resolution?*

Second, given the resolution $f : M \to X$, one considers the associated cell-like, upper semicontinuous decomposition $G_f = \{f^{-1}(x) | x \in X\}$ of M, consisting of the preimages of the map f, and tries to establish some *general position properties* of X which would allow the controlled, simultaneous shrinking of the elements of the decomposition G_f to arbitrary small sizes. If such a manipulation can be carried out then the classical Shrinking theorem of R.H. Bing [25] tells us that f is a *near-homeomorphism*, i.e. f can be approximated arbitrarily closely by homeomorphisms $h : M \to X$. In particular, X must be homeomorphic to M, hence itself a topological n-manifold. So the other key

question we must address is the following one:

GENERAL POSITION PROBLEM. *Which general position property for a finite-dimensional ANR X, where X is the image of a cell-like map $f : M \to X$ on an n-manifold M, implies that f is a near-homeomorphism?*

In this paragraph we address the first problem. The best result so far in dimensions ≥ 5 is the following Resolution theorem (for a complete history of previous the work on this topics see the survey [35]):

THEOREM 3.1 (F.S. Quinn [33]–[31]). *Let X be a connected generalized n-manifold, $n \geq 5$. Then there is an integral invariant $I(X) \in H_0(X; \mathbf{Z})$ of X such that:*
  (i) $I(X) \equiv 1 \pmod 8$;
  (ii) *For every open subset $U \subset X$, $I(X) = I(U)$;*
  (iii) *For every generalized m-manifold Y, $m \geq 5$, $I(X \times Y) = I(X) \times I(Y)$; and*
  (iv) $I(X) = 1$ *if and only if X admits a resolution.*
  Moreover, if $f_i : M_i \to X$, $i \in \{1,2\}$, are any two conservative resolutions of X, $n \geq 4$, and $U \subset X$ is a neighbourhood of $S(X)$, then there is a homeomorphism $h : M_1 \to M_2$ such that $f_1(x) = f_2 h(x)$, for every point $x \in X \setminus U$.

For several years it was unknown whether Quinn's local surgery obstruction $I(X)$ can ever be *nontrivial*, i.e. whether there perhaps exist *nonresolvable* generalized manifolds. Finally, last year the following surprising results were announced:

THEOREM 3.2 (J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger [10]). *For every integers $n \geq 6$ and $m \geq 1$, and for every simply connected, closed n-manifold M, there exists a generalized n-manifold X such that:*
  (i) $I(X) = m$ *(hence X does not admit a resolution and is totally singular); and*
  (ii) X *is homotopy equivalent to M.*

THEOREM 3.3 (J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger [10]). *For every integer $n \geq 6$, there exists a generalized n-manifold X such that*
  (i) X *does not admit a resolution; and*
  (ii) X *is not homotopy equivalent to any topological manifold.*

Essentially nothing is known in dimension 4, except for the fact that a generalized 4-manifold X has a resolution if and only if $X \times \mathbf{R}$ has one. This follows from the following result:

THEOREM 3.4 (F.S. Quinn [30]). *Let X be a generalized n-manifold ($n \geq 4$). Then the following statements are equivalent:*
  (i) X *has a resolution;*
  (ii) *For some $k \in \mathbf{N}$, $X \times \mathbf{R}^k$ has a resolution; and*
  (iii) $X \times \mathbf{R}^2$ *is a topological $(n+2)$-manifold.*

In dimension 3, the Resolution problem is entangled with the Poincaré conjecture,

e.g. if there exist *fake* 3-cells it's easy to construct a *nonresolvable* generalized 3-manifold X, homotopy equivalent to S^3, with just one singularity [34]. The following is the best result so far (for earlier work see the survey [34]):

THEOREM 3.5 (T.L. Thickstun [38]). *If the Poincaré conjecture is true then every generalized 3-manifold X with $\dim S(X) = 0$ admits a (conservative) resolution.*

Finally, in the lowest dimension ≤ 2, there is no problem since we've already observed that the algebraic properties of generalized (≤ 2)- manifolds are strong enough to imply also the geometric properties and hence there can be no singularities at all.

4. General Position Properties

Higher dimensional (≥ 5) topological manifolds possess the following simple general position property:

DEFINITION 4.1. A metric space X is said to have the disjoint disks property (DDP) if for every pair of maps $f, g : B^2 \to X$ of the closed 2-cell B^2 into X and every $\varepsilon > 0$ there exist maps $f', g' : B^2 \to X$ such that $d(f, f') < \varepsilon$, $d(g, g') < \varepsilon$ and $f'(B^2) \cap g'(B^2) = \emptyset$.

It turns out that this property is also, to a large extent, *characteristic* for manifolds in this dimension range. This follows from the beautiful R.D. Edwards Cell-like approximation theorem:

THEOREM 4.2 (R.D. Edwards [21]). *Let M be a topological n-manifold, $n \geq 5$, and let $f : M \to X$ be a surjective cell-like map of M onto an ANR X. Then X is a topological n-manifold if and only if X has the DDP.*

As a corollary we immediately get the solution of the Recognition problem for dimensions ≥ 5:

COROLLARY 4.3. *For every $n \geq 5$, the class of topological n-manifolds $\mathcal{M}_n$ is equal to the class of generalized n-manifolds $\mathcal{G}_n$ with the DDP and vanishing Quinn's local surgery obstruction $I(X)$.*

Let X be any generalized n-manifold, $n \geq 6$, which doesn't admit a resolution. Then by Theorem (3.1), the product $X \times T^2$ is also a generalized $(n + 2)$-manifold without a resolution. However, by a theorem of R.J. Daverman [16], $X \times T^2$ has the DDP, so Theorem (3.2) implies that there exist generalized m-manifolds, $m \geq 8$, which are *not* topological m-manifolds *although* they do possess the DDP.

In dimension 3 the solution is unfortunately not so elegant. This is hardly surprising since there is less room for moving objects apart in this dimension. Here are the appropriate versions of DDP for 3-manifolds introduced by R.J. Daverman and D. Repovš in [18] [19] (for other properties which were used earlier to solve special cases of the General position problem see the survey [34]):

DEFINITION 4.4. A metric space X is said to have the Weak simplicial approximation property (WSAP) if for every map $f : B^2 \to X$ of the closed 2-disk B^2 into X, and for every $\varepsilon > 0$, there exists a map $f' : B^2 \to X$ such that for every $t \in B^2$, $d(f(t), f'(t)) < \varepsilon$ and $f'(B^2) \subset \bigcup_{i=1}^{n} D_i$, for some family $\{D_i\}_{i=1}^{i=n}$ of 1-LCC embedded 2-cells $D_i \subset X$.

Recall that a subset $Z \subset X$ of space X is *locally simply coconected* (1- LCC) if every $x \in X$ and every neighbourhood $U \subset X$ of x, there is a neighbourhood $V \subset U$ of x such that the inclusion-induced homomorphism $\Pi_1(V \setminus Z) \to \Pi_1(U \setminus Z)$ is trivial.

DEFINITION 4.5. A metric space X is said to have the Simplicial approximation property (SAP) if for every map $f : B^2 \to X$ and every $\varepsilon > 0$, there exist a map $f' : B^2 \to X$ and a finite topological 2-complex $K_{f'} \subset X$ such that
  (i) For every $t \in B^2$, $d(f(t), f'(t)) < \varepsilon$;
  (ii) $f'(B^2) \subset K_{f'}$; and
  (iii) $X \setminus K_{f'}$ is 1-FLG (free local fundamental group) in X.
  X is said to have the Spherical simplicial approximation property (SSAP) if the above holds when B^2 is replaced by S^2 throughout.

We define that $X \setminus K_{f'}$ is 1-FLG in X if for every $y \in K_{f'}$ and for every sufficiently small neighbourhood $U \subset X$ of y, there exists another neighbourhood $V \subset U$ of y, such that for every connected open neighbourhood $W \subset V$ of y, for each nonempty component $W' \subset W$ of $W \setminus K_{f'}$, the inclusion-induced image of $\Pi_1(W') \to \Pi_1(U')$ is a free group on $m - 1$ generators, where $U' \subset U$ is the component of $U \setminus K_{f'}$ containing W' and m is the number of components of $\mathrm{st}(y) \setminus y$ that meet $\mathrm{Cl}(W')$ [29].

The definition of the 1-FLG property is admittedly a bit technical. However, note that for any finite, connected 2-complex K, having no local separating points and lying in a generalized 3-manifold X, the following are equivalent:
  (i) $X \setminus K$ is 1-FLG in X;
  (ii) K is 1-LCC in X; and
  (iii) Each 2-simplex of K is 1-LCC in X (cf. [19]).
  It is easy to see that every topological (≥ 3)-manifold has WSAP, SAP and SSAP. We shall need another one (its verification for 3-manifolds is less trivial [18]):

DEFINITION 4.6. A metric space X is said to have the Light map separation property (LMSP) if for every $\varepsilon > 0$, every $k \in \mathbf{N}$, and every map $f : B \to X$ of a collection of k standard 2-cells $B = \coprod_{i=1}^{k} B_i^2$ into X such that:
  (i) $N_f \subset \mathrm{int}\, B$, where $N_f = \{y \in B | f^{-1}(f(y)) \neq y\}$;
  (ii) $\dim N_f \leq 0$; and
  (iii) $\dim Z_f \leq 0$, where $Z_f = \{x \in X | x \in f(B_i^2) \cap f(B_j^2) \text{ for some } i \neq j\}$;
there exists a map $F : B \to X$ such that
  (1) $d(F, f) < \varepsilon$;
  (2) $F|\partial B = f|\partial B$; and
  (3) For every $i \neq j$, $F(B_i^2) \cap F(B_j^2) = \emptyset$.

We can now state the two main results from [19] which solve the General position problem for 3-manifods:

THEOREM 4.7 (R.J. Daverman and D. Repovš [19]). *A resolvable generalized 3-manifold is a topological 3-manifold if and only if it possesses the SSAP.*

THEOREM 4.8 (R.J. Daverman and D. Repovš [19]). *A resolvable generalized 3-manifold X is a topological 3-manifold if and only if X possesses the WSAP and LMSP.*

One defines the property LMSP* to be LMSP without the condition (iii). A conjecture from [18] that all 3-manifolds possess this (stronger) property was recently verified (using very technical argument) by M.V. Brahm:

THEOREM 4.9 (M.V. Brahm [7]). *Every topological 3-manifold has the LMSP*.*

5. Epilogue

Let's make a review of the major questions concerning the Recognition problem. In dimension three the Resolution problem remains open (modulo the Poincaré conjecture) for generalized 3-manifolds X with $\dim S(X) \geq 1$:

QUESTION 5.1. *Suppose that there exist no fake cubes. Does there exist a nonresolvable generalized 3-manifold X? (Note that for such an example X, $\dim S(X) \geq 1$ is necessary.)*

In attemping to find a resolution, one may well wish to try the simplest unknown case (note that if $S(X)$ lies in a set $Z \subset X$ which is 1-LCC embedded in X and $\dim Z = 0$ then $S(X) = \emptyset$ [11]):

QUESTION 5.2. *Suppose that there exist no fake cubes. Let X be a generalized 3-manifold and suppose that $S(X) \subset Z$, where Z is an arc which is 1-LCC embedded in X. Does X admit a resolution?*

Note that in order to resolve a generalized 3-manifold X it suffices to find an almost $\mathbf{Z}_2$-acyclic resolution of X:

THEOREM 5.3 (D.Repovš and R.C. Lacher [37]). *Let $f : M \to X$ be a closed, monotone map from a 3-manifold M onto a locally simply connected $\mathbf{Z}_2$-homology 3-manifold X. Suppose that there is a 0-dimensional (possibly dense) set $Z \subset X$ such that for every point $x \in X \setminus Z$, $\check{H}^1(f^{-1}(x); \mathbf{Z}_2) = 0$. Then the set $C = \{x \in X | f^{-1}(x)$ is not cell-like $\}$ is locally finite in X. Moreover, X is a resolvable generalized 3-manifold.*

Before we proceed to dimensions ≥ 4, we wish to ask few more questions about generalized 3-manifolds. First, consider the following result (*the Dehn's lemma property* and the *Map separation property* are another kind of general position properties of 3-manifolds which were used earlier to shrink certain cell-like decompositions of 3-manifolds

— see [36]):

THEOREM 5.4 (W. Jakobsche and D. Repovš [23]). *Suppose that there exist fake cubes. Then there exists a compact homogeneous ANR X with the following properties:*
(1) X *is a generalized 3-manifold and* $S(X) = X$;
(2) X *does not admit a resolution;*
(3) X *has the Dehn's lemma property;*
(4) X *has the Map separation property;*
(5) $X \times S^1$ *is homeomorphic to* $S^3 \times S^1$.

QUESTION 5.5. *Does the example of W. Jakobsche and D. Repovš [23] also possess any of the following position properties:*
(1) *LMSP(∗)* ; *or*
(2) *(W)SAP* ; *or*
(3) *SSAP?*

The *natural* converse of Theorem (4.9) is still a conjecture:

CONJECTURE 5.6 (R.J. Daverman and D. Repovš [18]). *A resolvable generalized 3-manifold X is a topological 3-manifold if X possesses the LMSP*.*

Related to this is the following conjecture which would significantly strengthen Theorem (4.9) (for more on this conjecture and its equivalents see [6] and [7]):

CONJECTURE 5.7. *Let* $f : N \to M$ *be a continuous map of a 2-manifold N into a 3-manifold M such that* $\dim N_f = 0$ *and* $N_f \subset \operatorname{int} N$. *Then for every continuous function* $\varepsilon : \operatorname{int} N \to (0,1]$ *going to zero near* ∂N *there exists an embedding* $g : N \to M$ *such that for every* $x \in \operatorname{int} N$, $d(f(x), g(x)) < \varepsilon(x)$.

In dimension 4 very little is known (see [2] and [19] for partial results) both Resolution problem as well as General position problem are still open, while in dimensions ≥ 5 there also remain some questions, e.g.

QUESTION 5.8. *Does there exist a nonresolvable generalized 5-manifold?*

Also, there is still no written details available of the R.D. Edwards proof of the 5-dimensional case of Theorem (4.2) – the manuscript [21] deals only with the higher dimensions (≥ 6). It would be a worthwhile project to carefully rethink Edwards' argument for dimension 5 again and write it down, since there are some nontrivial engulfing problems one must know how to get around in this case.

Finally, the following is a related, very difficult problem from *cohomological dimension theory*, equivalent to the celebrated Cell-like mapping problem in dimension 4 (for more see the survey [26]):

QUESTION 5.9. *Suppose that* $f : M \to X$ *is a cell-like map of a topological 4-manifold M onto a space X. Is* $\dim X < \infty$ *(equivalently,* $\dim X = 4$*)?*

Note that by theorem of W.J.R. Mitchell, D. Repovš and E.V. Ščepin [27], $\dim X <$ ∞ if and only if X has a certain kind of general position property, called the *disjoint Pontryagin triples property*.

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