Dispersive Regime for Quantum Computation

Non-Resonant (Dispersive) Interaction

approximate diagonalization: $|\Delta| = |\omega_a - \omega_r| \gg g$:

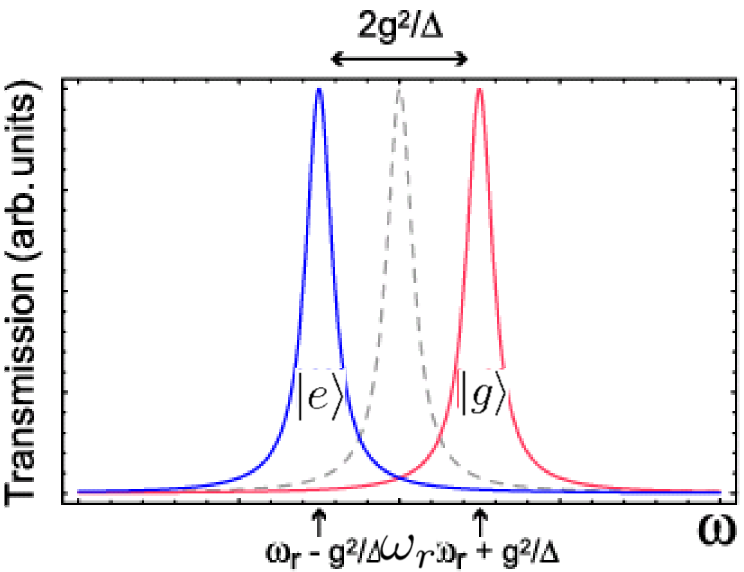
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{\hbar}{2}\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

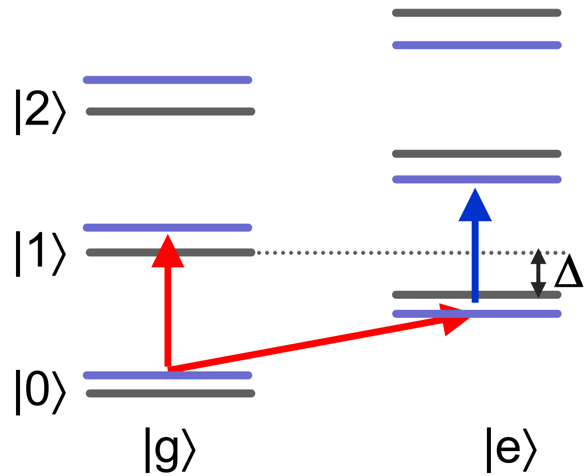
//

cavity frequency shift
and qubit ac-Stark shift

//

Lamb Shift





qubit detuned by Δ
from resonator

A. Blais *et al.*, *PRA* 69, 062320 (2004)

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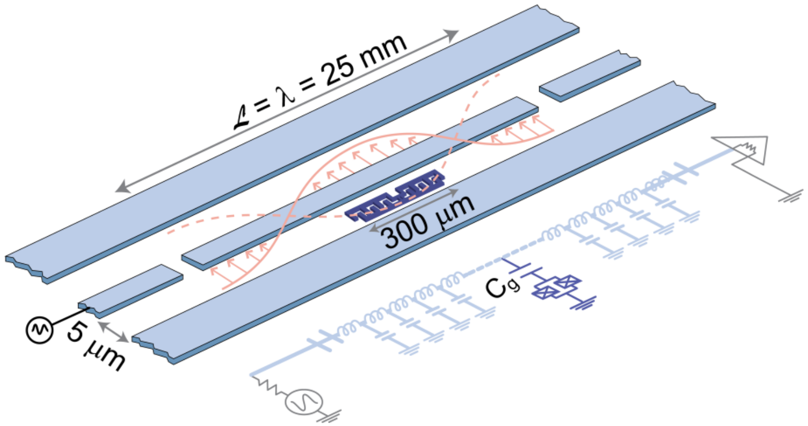
Swiss Federal Institute of Technology Zurich

Circuit QED – read out of qubit state

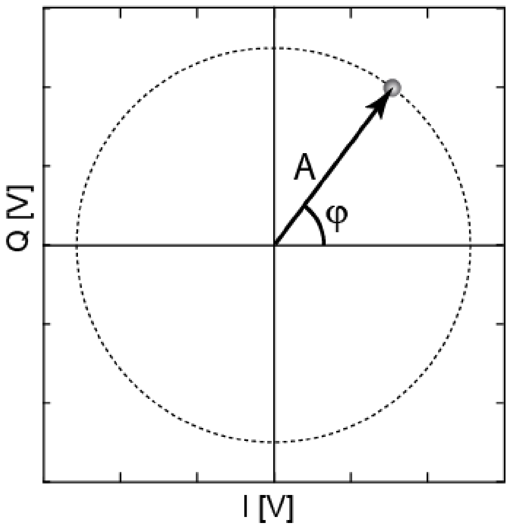
- transmission measurement to determine qubit state:



Phase sensitive measurement of transmitted microwave:

Voltage signal:

$$A(t)\sin(\omega_m t + \phi(t)) \equiv I(t)\sin\omega_m t + Q(t)\cos\omega_m t$$



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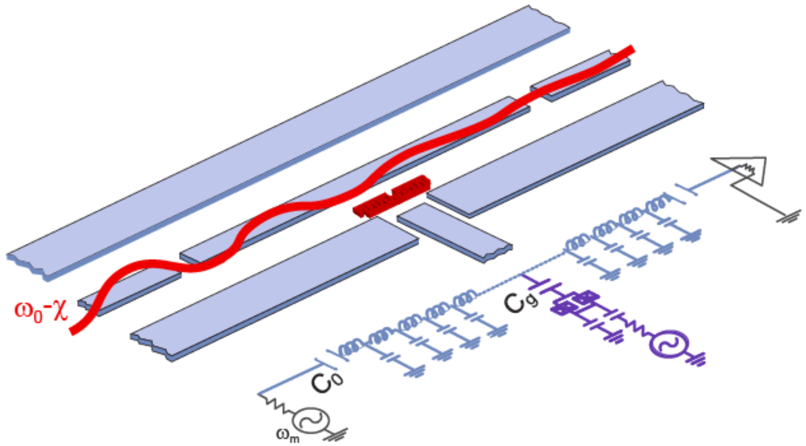
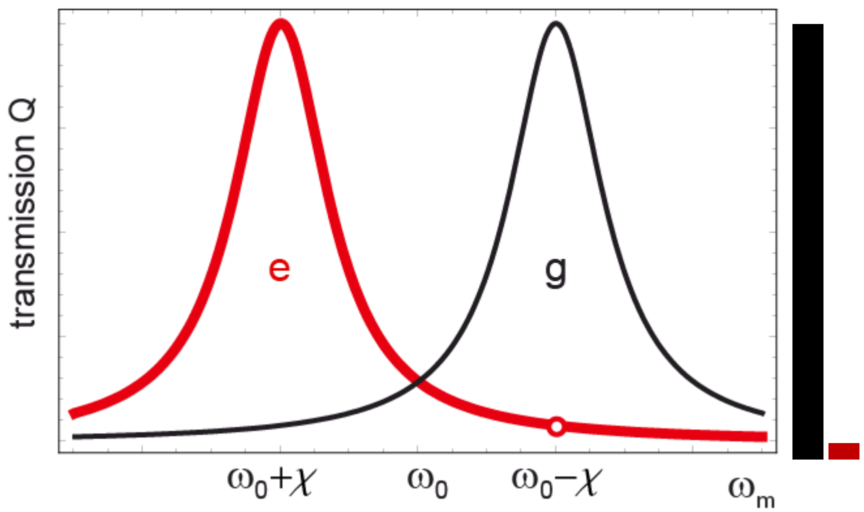




dispersive Hamiltonian:

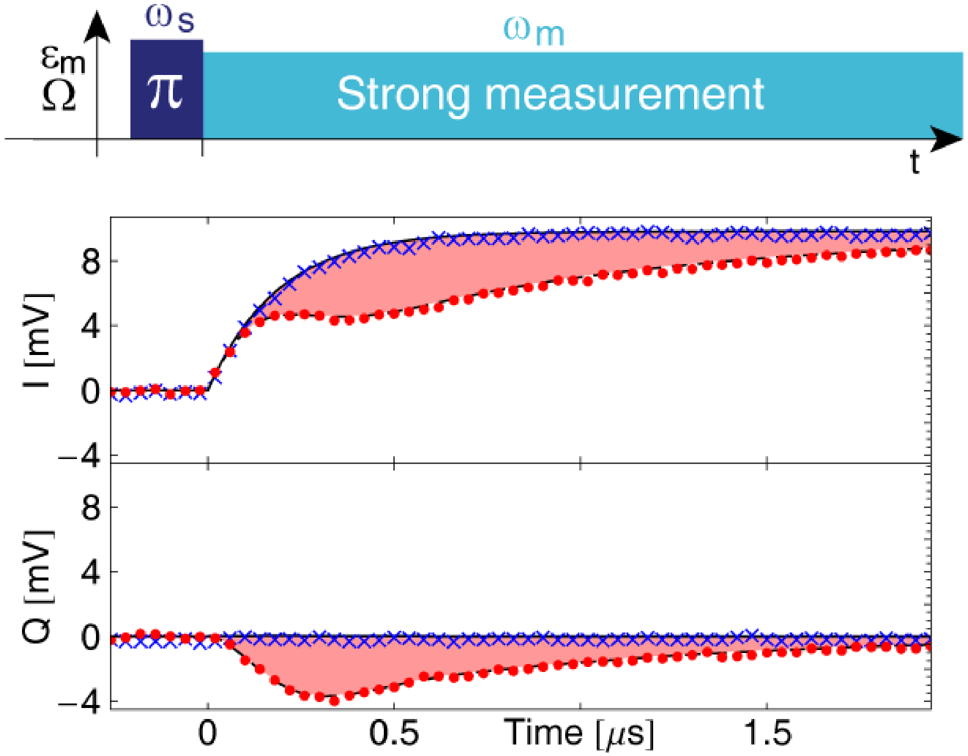
$$H = \hbar(\omega_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi)\sigma_z$$

state-dependent frequency shift -> σ_z determined

extendable to more qubits

Time dependent measurements

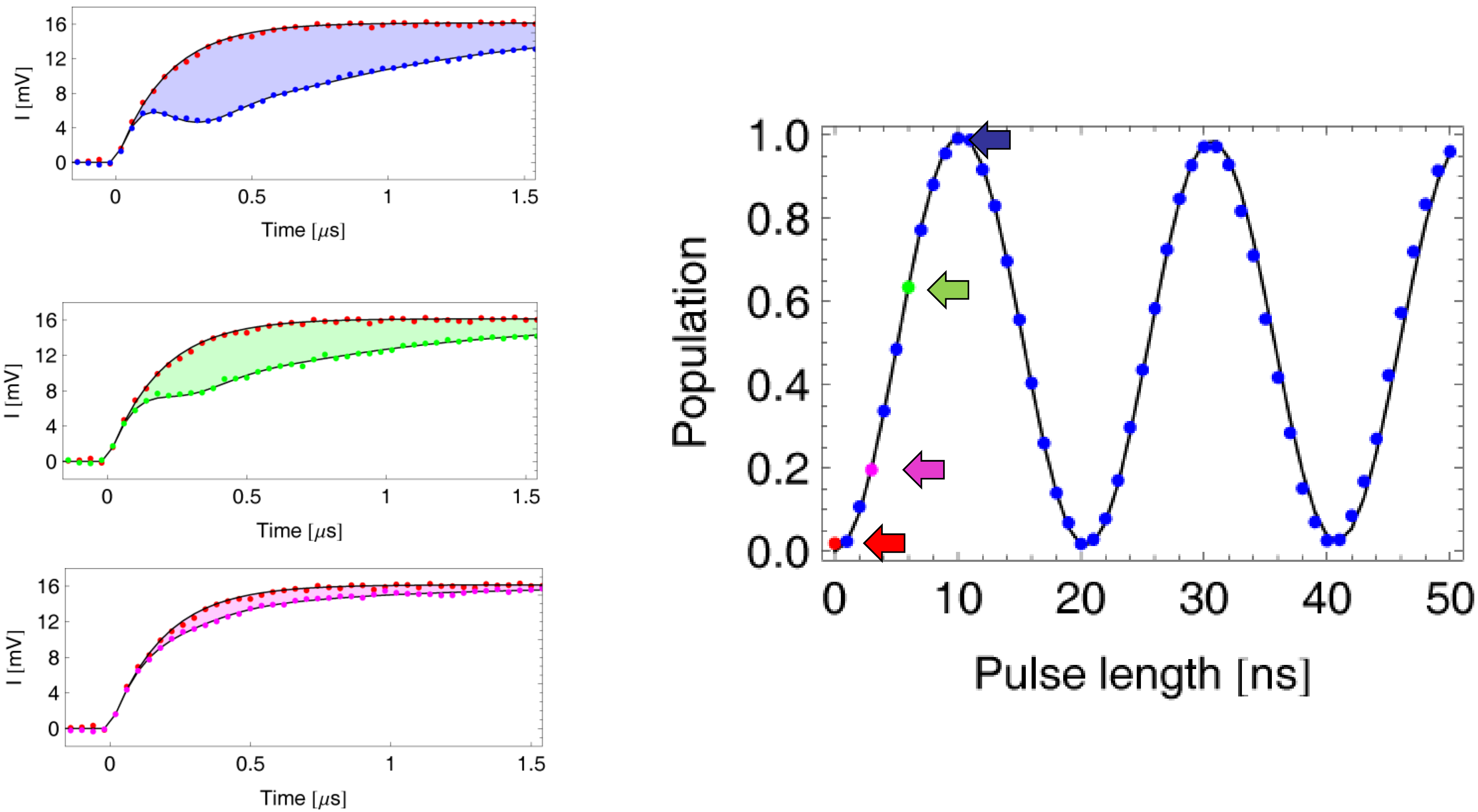
- excite qubit at t<o
- measure transmitted field quadratures (I, Q) with microwave drive at resonance ($\omega_m = \omega_r - \chi$)
- qubit in ground state: full resonator transmission (rise time given by κ)
- qubit in excited state: only partial transmission until qubit decays to ground state



[Bianchetti *et al*. PRA 80 (2009)]

Population reconstruction

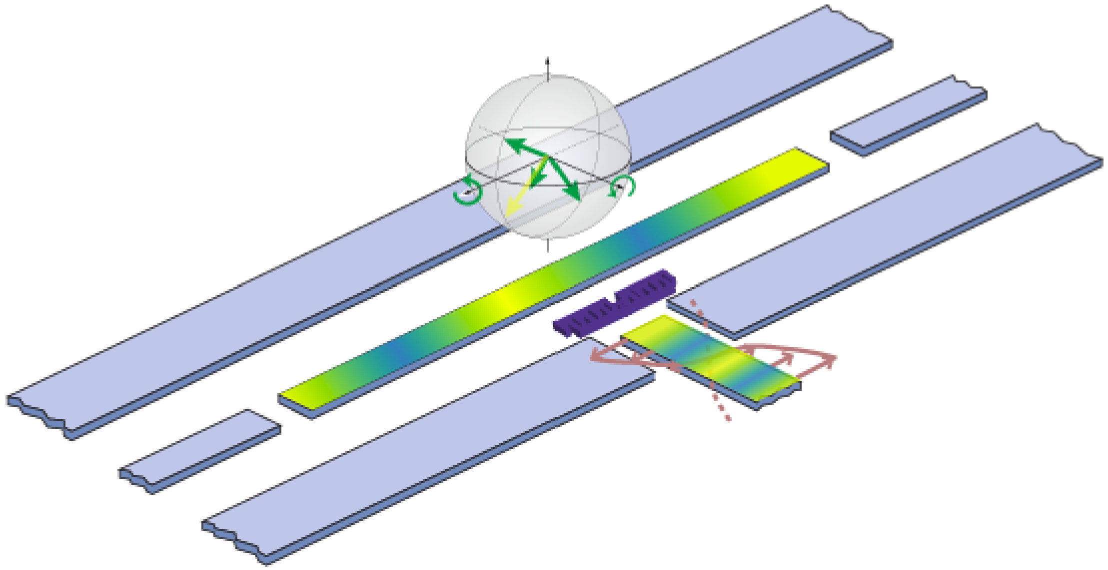
Area between curves is proportional to qubit state population:



Coherent Single Qubit Control

Qubit control

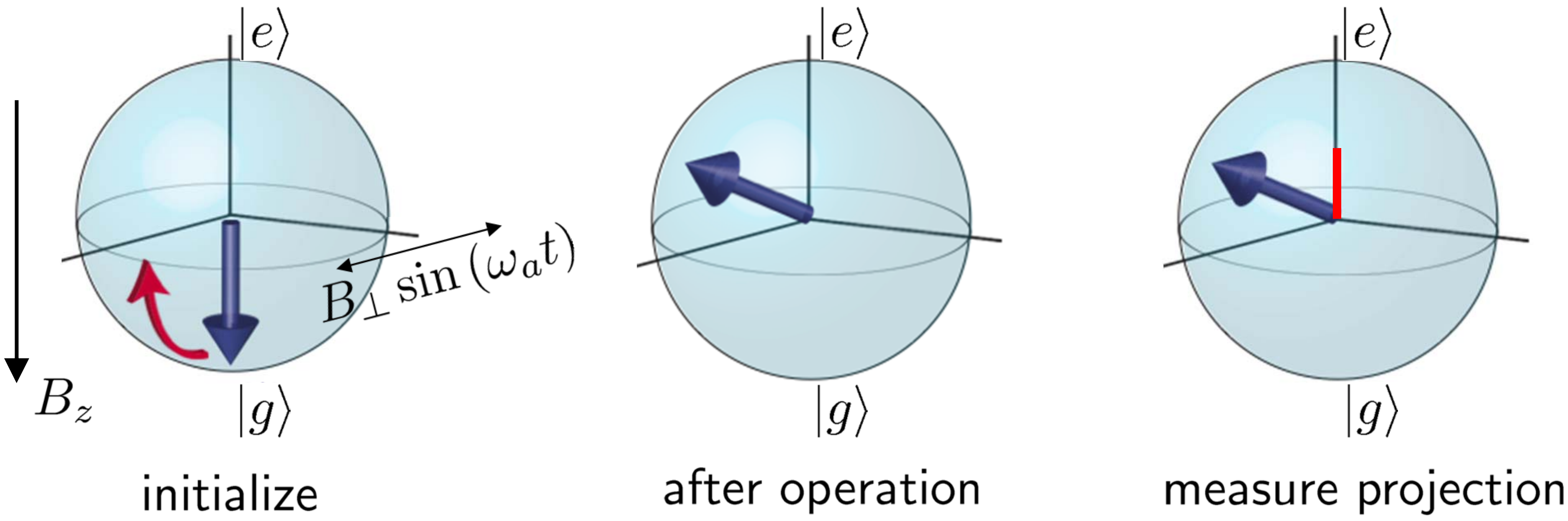
- apply microwave signal through resonator input
- or through side-gate



- time-dependent Hamiltonian for state manipulation

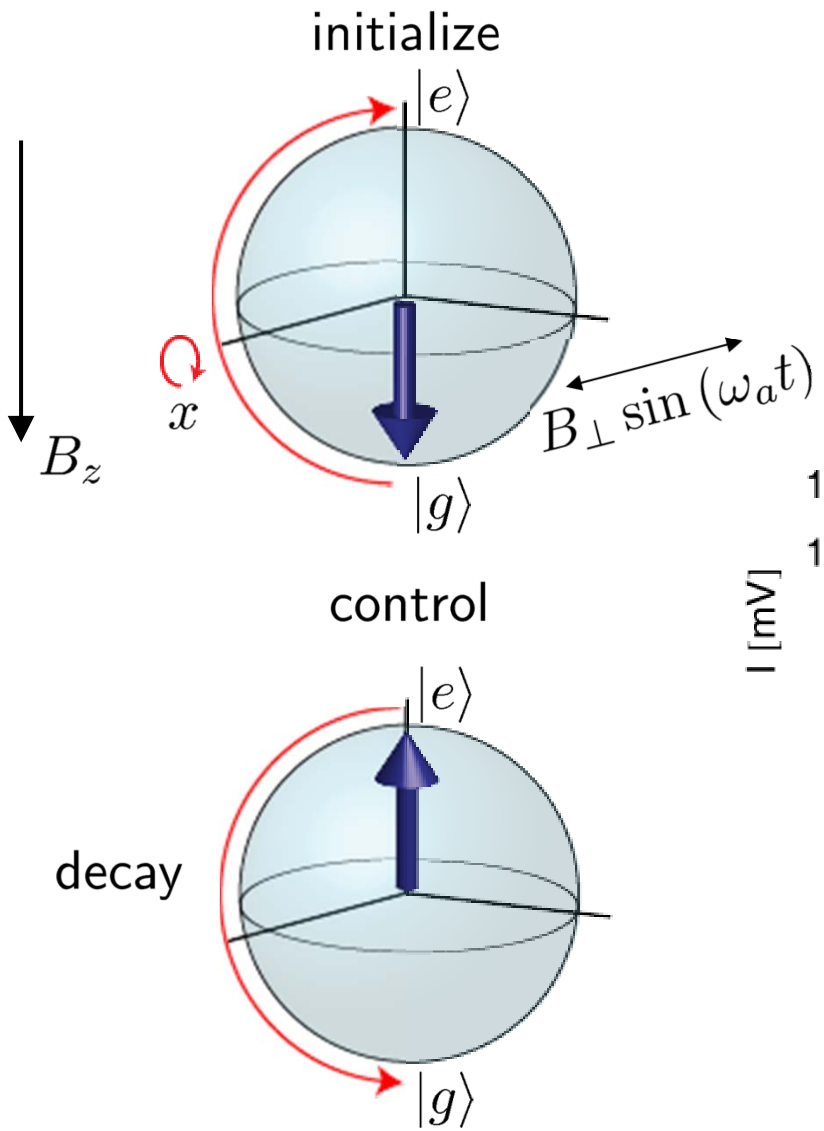
$$\hat{H} = \frac{1}{2}\hbar\omega_a\hat{\sigma}_z + \hbar\Omega_R\cos(\omega_b t + \phi_R)\hat{\sigma}_x$$

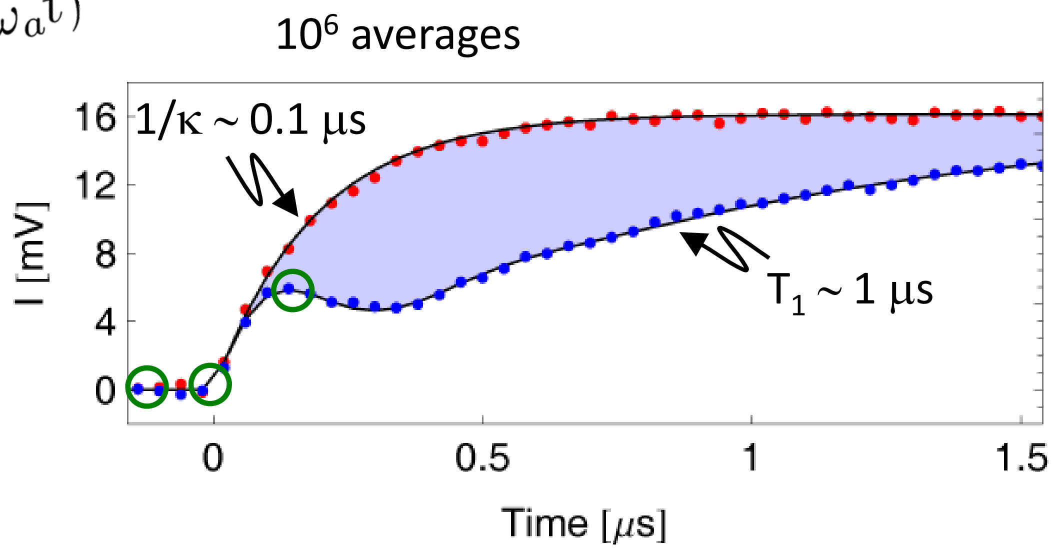
Coherent Control of a Qubit in a Cavity



- qubit state represented on a Bloch sphere
- vary length, amplitude and phase of microwave pulse to control qubit state

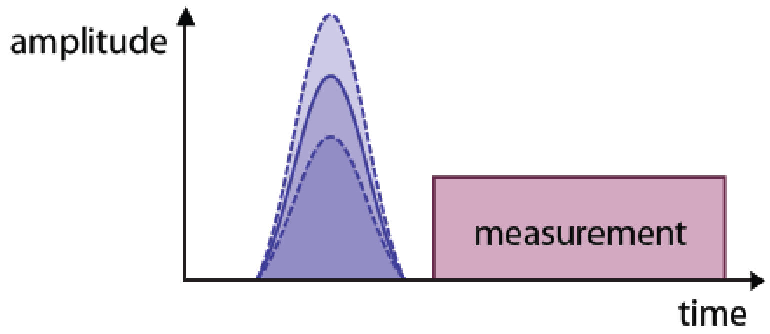
Qubit Control and Readout

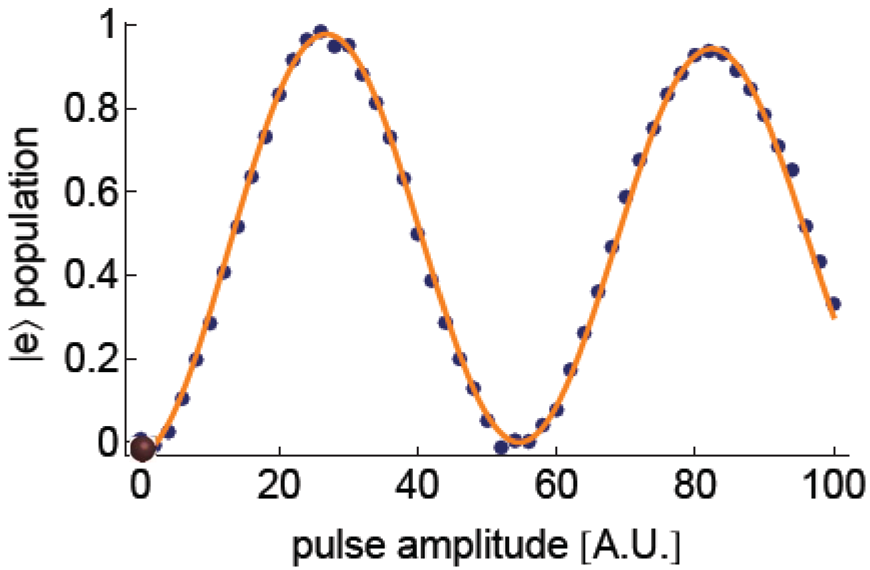


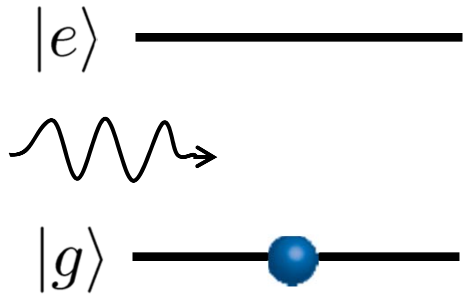


Coherent population transfer – Rabi Oscillations

drive system at its resonance frequency with varying drive strength:





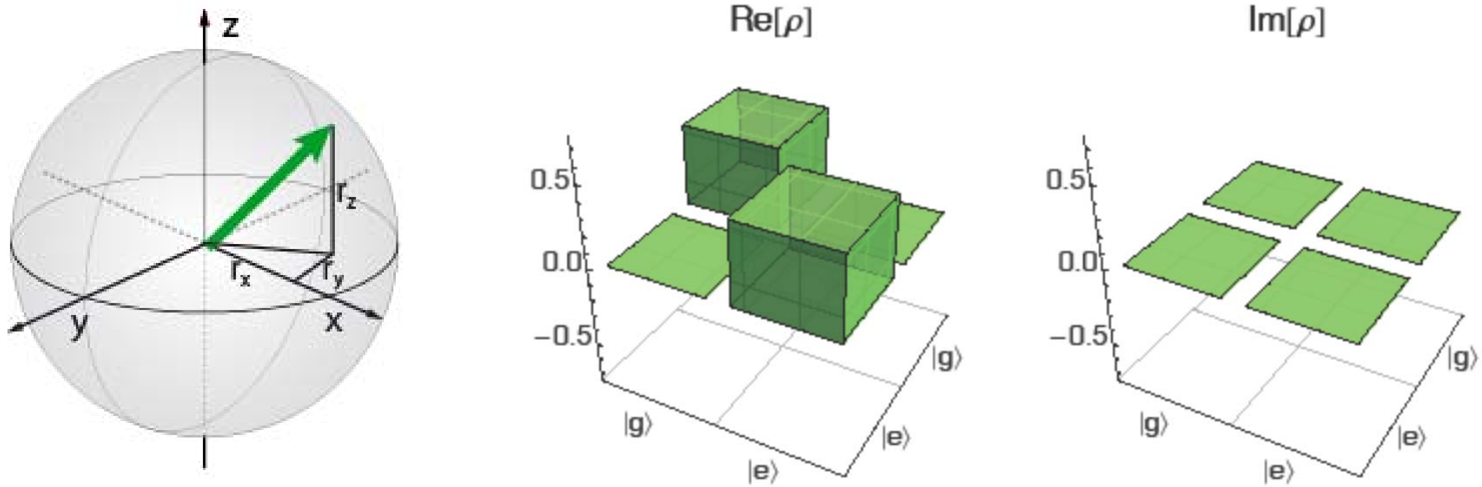


- high visibility (~ 99%)
- well characterized and understood measurement
- good control accuracy

State reconstruction single qubit

3 measurements for 3 coefficients r_x, r_y, r_z of

$$\rho = \frac{1}{2}(\mathrm{id} + r_x\sigma_x + r_y\sigma_z + r_z\sigma_z)$$



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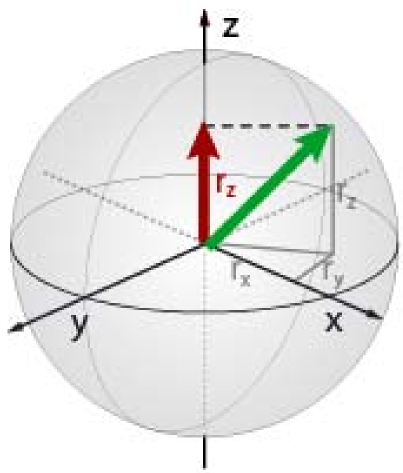
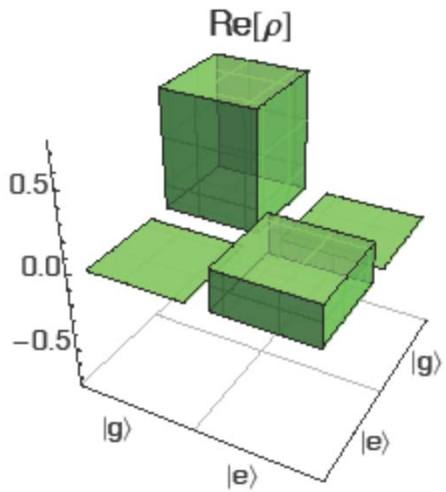
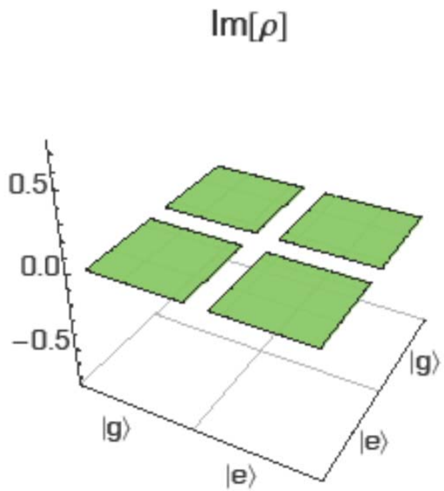
Measurement along z-axis: $r_z = \langle \sigma_z \rangle = \mathrm{Tr}[\rho\sigma_z]$







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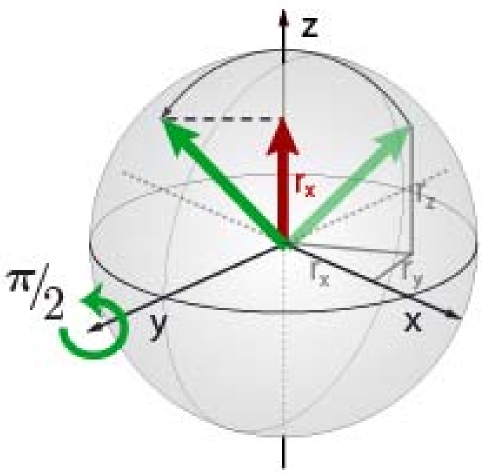

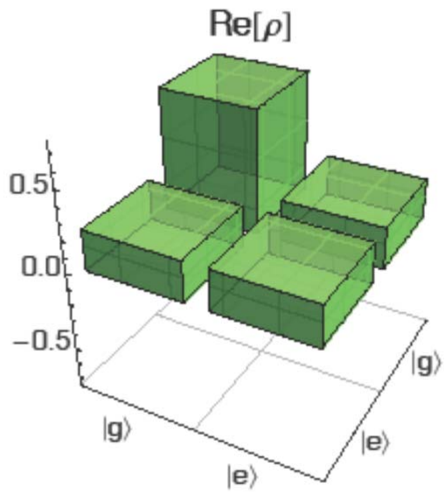

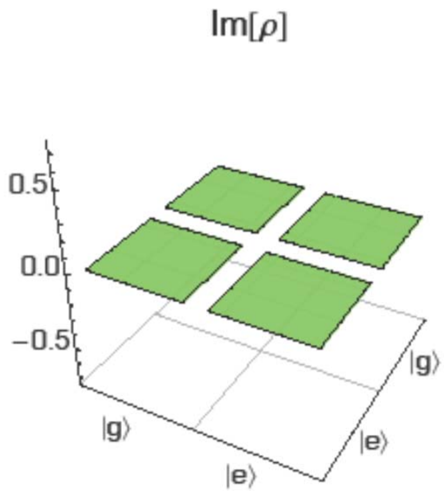

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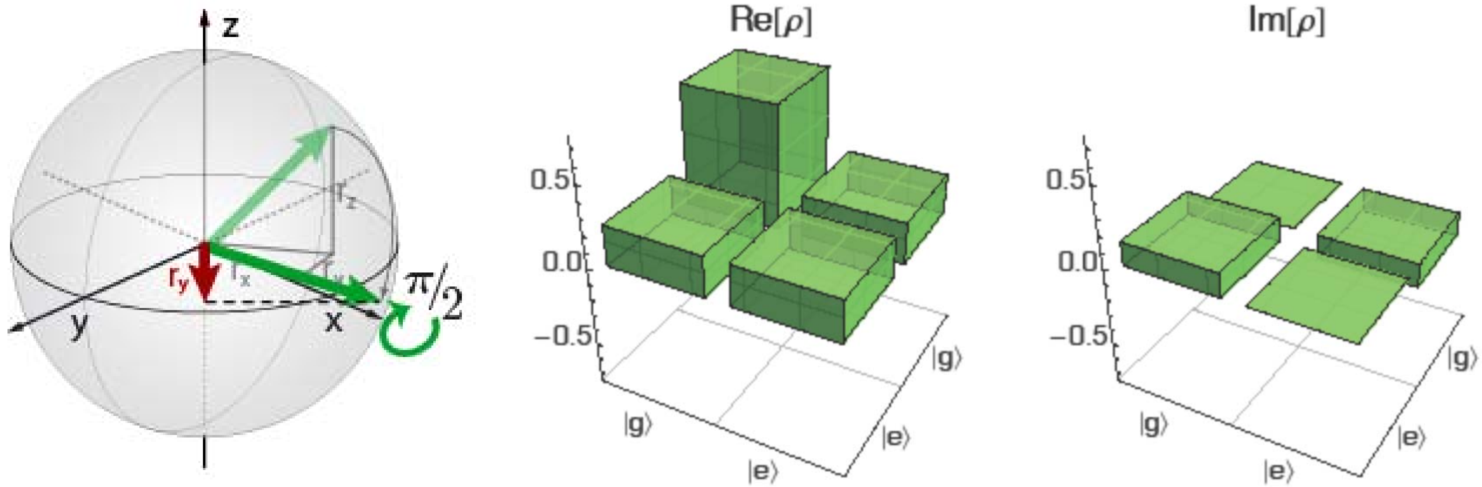
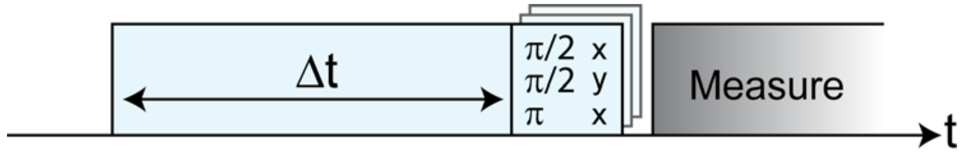
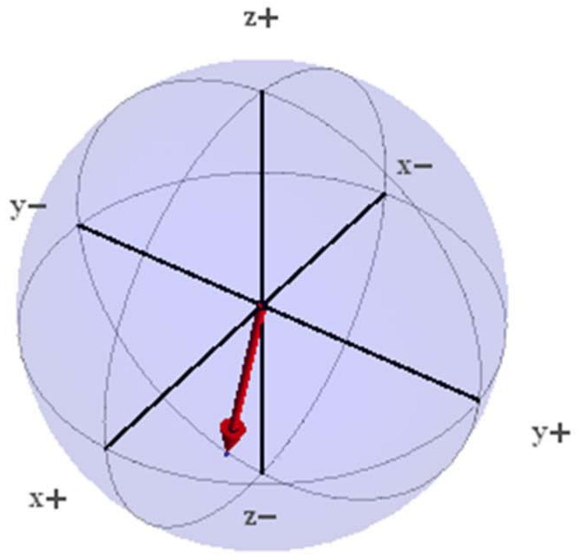


Control and Tomographic Read-Out of Single Qubit

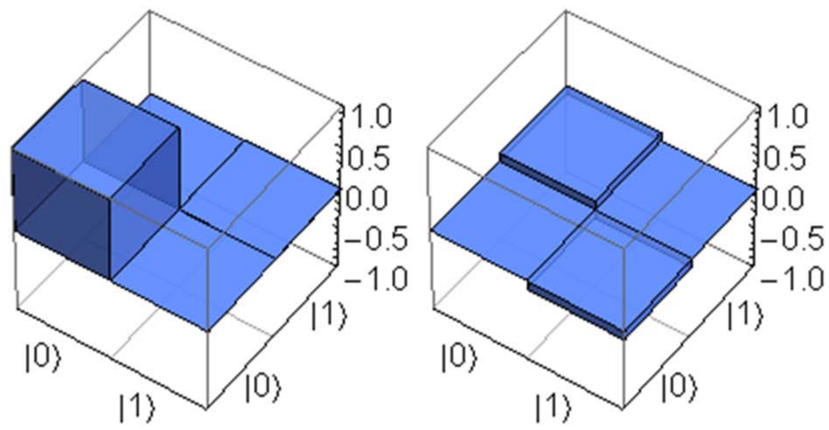
Rabi rotation pulse sequence:



experimental Bloch vector:



experimental density matrix:



$\Delta t = 0$ ns

L. Steffen *et al.*, Quantum Device Lab, ETH Zurich (2008)