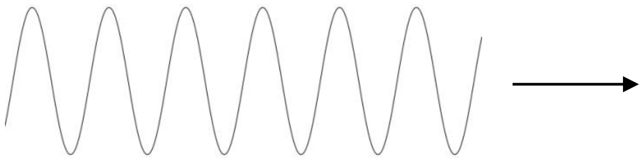
Pulsars and Supernovae II

3. DISPERSION AND SCINTILLATION

dispersion measure (DM)
structure of the interstellar medium
temporal and angular broadening from the ISM
the Fresnel scale and interstellar scintillation

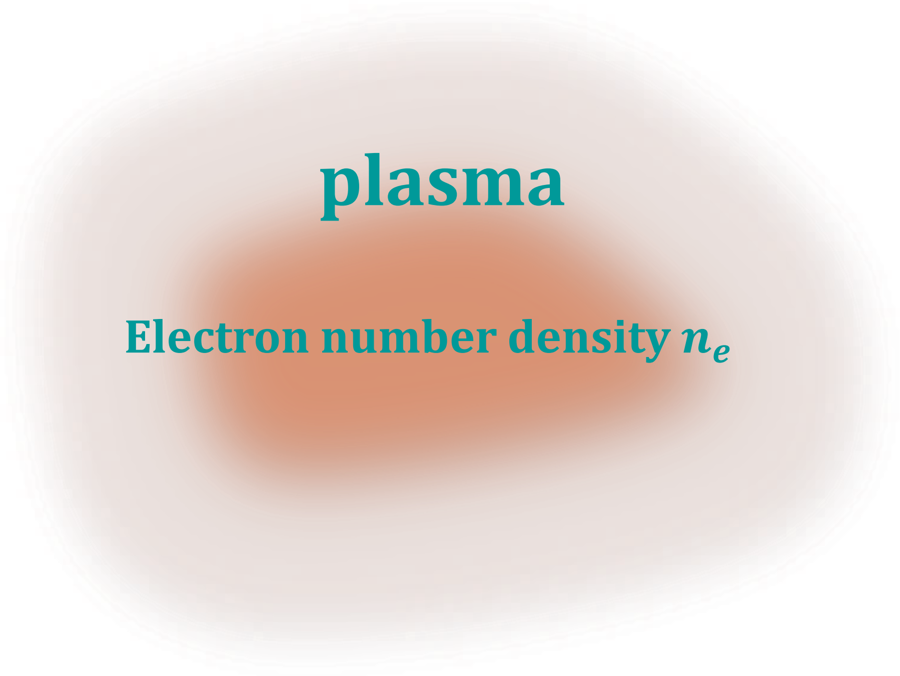
Propagation through plasmas – revision

- Propagation:



$$E(t) = E_0 \exp(i\omega t)$$

electromagnetic wave,
angular frequency ω



- Electrons oscillate around (quasi-stationary) protons as

$$x(t) = x_0 \exp(i\omega t).$$

- Equation of motion:

$$Ee = m_e\ddot{x} = -m_e\omega^2 x.$$

Propagation through plasmas – revision

- This charge separation appears as a bulk polarisation which defines the relative permittivity of the plasma, ϵ_{r} :

$$P = n_{\mathrm{e}}p = (\epsilon_{\mathrm{r}} - 1)\epsilon_0 E,$$

where $p = xe$ is the dipole moment for one electron/proton pair.

- Comparing with the equation of motion we see that

$$\epsilon_{\mathrm{r}} = 1 - \frac{n_{\mathrm{e}} e^2}{\epsilon_0 m_{\mathrm{e}} \omega^2},$$

so that the refractive index of the plasma, η, is

$$\eta = \epsilon_{\mathrm{r}}^{1/2} = \left(1 - \frac{f_{\mathrm{p}}^2}{f^2}\right)^{1/2}, \qquad \text{where} \quad f_{\mathrm{p}} = \frac{1}{2\pi}\left(\frac{n_{\mathrm{e}} e^2}{\epsilon_0 m_{\mathrm{e}}}\right)^{1/2}.$$

Propagation through plasmas – revision

$$f_{\mathrm{p}} = \frac{1}{2\pi}\left(\frac{n_{\mathrm{e}} e^2}{\epsilon_0 m_{\mathrm{e}}}\right)^{1/2} \simeq 9\left(\frac{n_{\mathrm{e}}}{\mathrm{cm}^{-3}}\right)^{1/2} \ \mathrm{kHz}$$

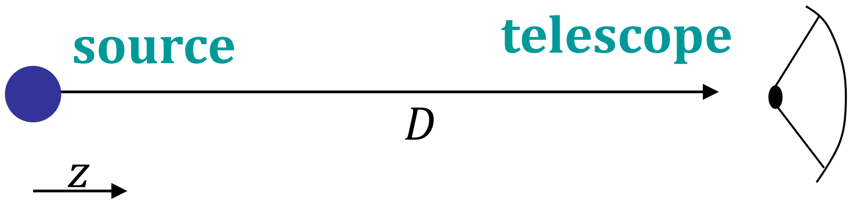
is the *plasma frequency* – the natural oscillatory frequency of the plasma.

- For $f < f_{\mathrm{p}}$ there are no TEM propagating modes.
- For $f > f_{\mathrm{p}}$, phase velocity $v_{\mathrm{p}} = c/\eta$ (which is $> c$)
  group velocity $v_{\mathrm{g}} = c\eta$ (which is $< c$)
- Spatial variations in electron density will give a non-uniform refractive index, leading to scattering of waves.

Pulsar dispersion

- The refractive index depends on frequency, so the travel-time of a signal is also frequency dependent:

$$\tau = \int_0^D \frac{\mathrm{d}z}{\eta(z)c}$$



- If $f_\mathrm{p} \ll f$ the extra delay relative to the travel time at c is

$$\tau_D = \tau - \frac{D}{c} = \frac{e^2}{8\pi^2\epsilon_0 m_\mathrm{e} c}\,\frac{1}{f^2}\int_0^D n_\mathrm{e}(z)\,\mathrm{d}z.$$

Pulsar dispersion

- This is usually written

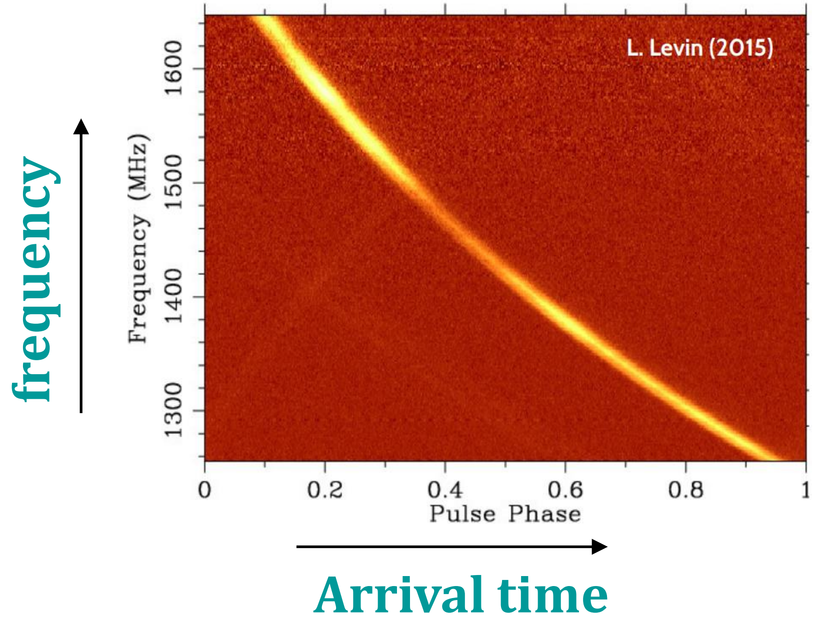
$$\tau_D = 4.15 \times 10^3 \frac{1}{f_{\mathrm{MHz}}^2} DM \text{ seconds}$$

where DM is the dispersion measure (note the conventional units for n and z):

$$DM = \int_0^D n_{\mathrm{e,cm^{-3}}}(z)\ \mathrm{d}z_{\mathrm{pc}}$$

- Radio dispersion transforms the pulses from pulsars into chirps:

  The highest frequencies arrive first.



Pulsar dispersion

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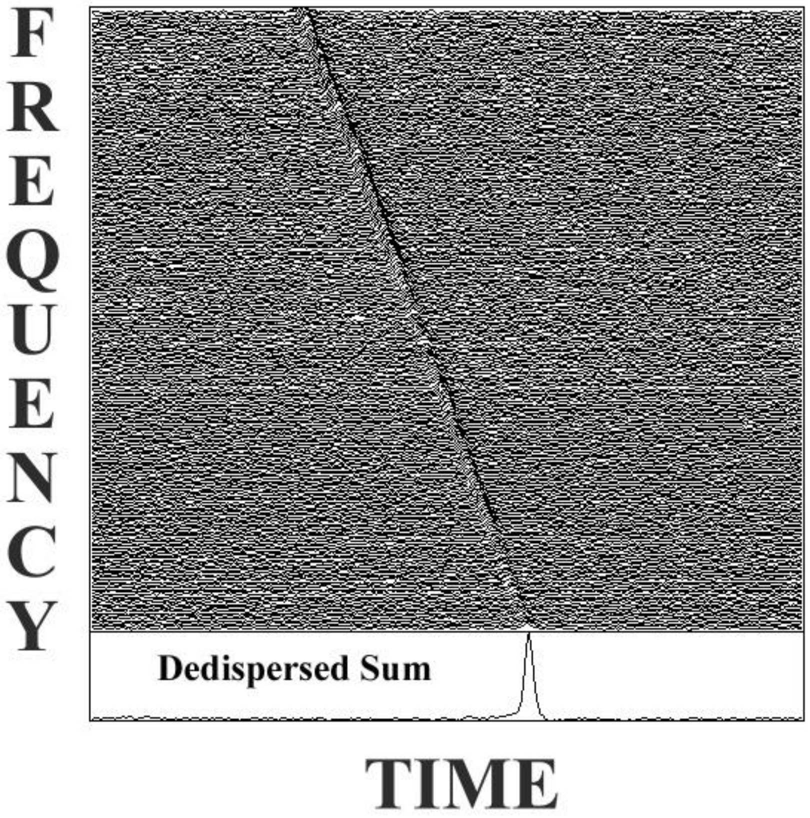
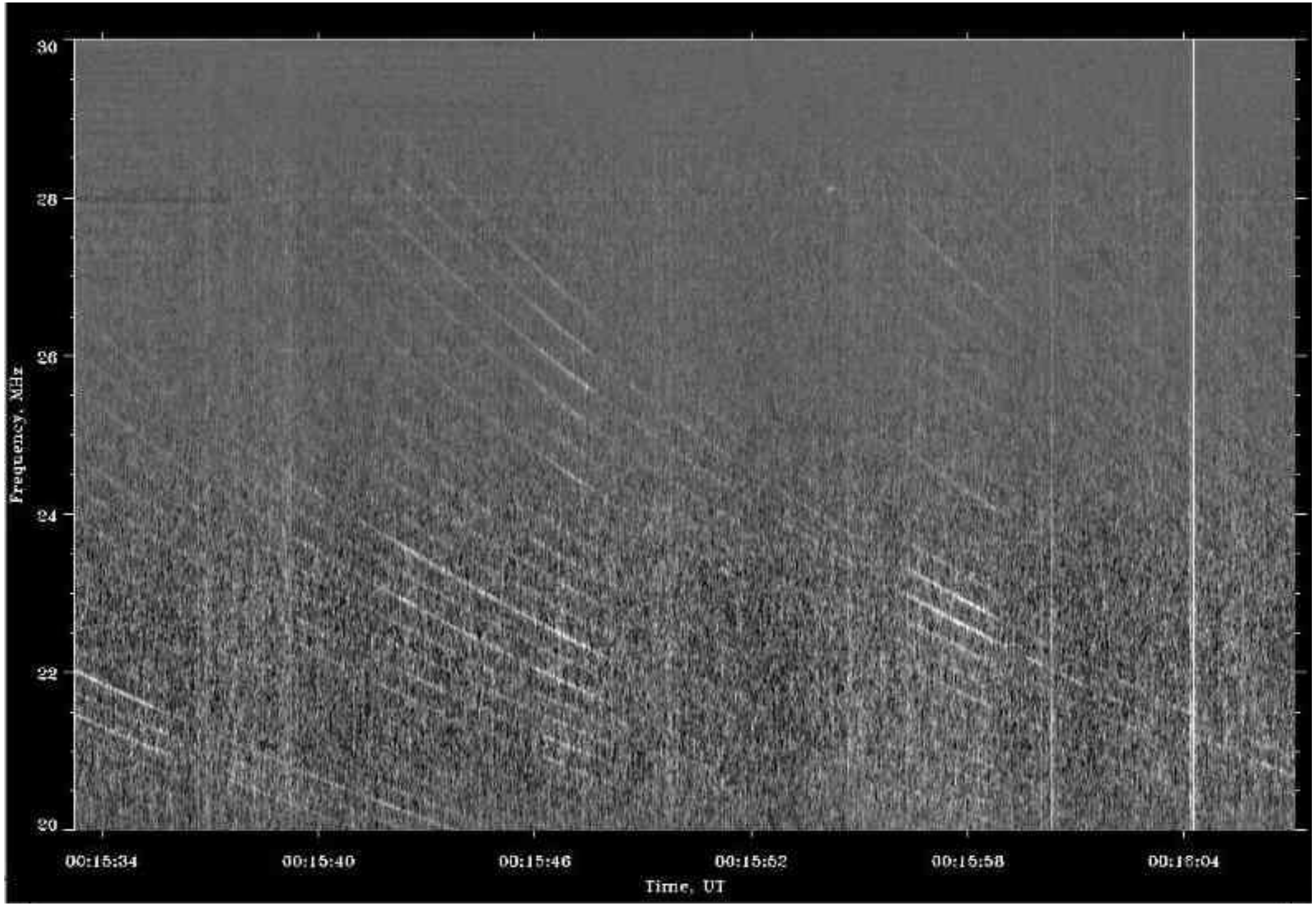


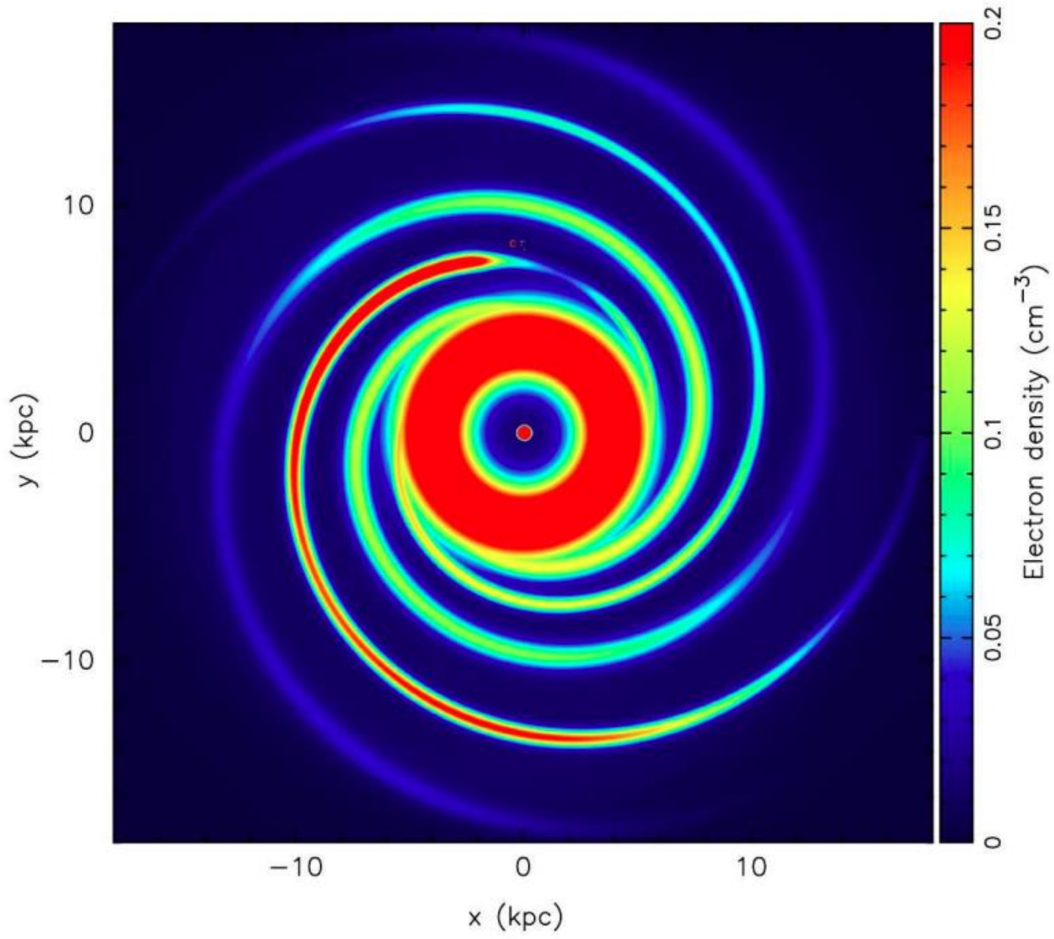
Low frequency dispersion

Over a wide bandwidth, the quadratic dependence on frequency becomes clear:



Pulsars as probes of the interstellar medium

- The dispersion measure to pulsars can be used to help map the galactic electron density:



Yao et al. YMW16: New Electron Density Model (2016)

Typically, $n_{\mathrm{e}} \simeq 0.03\ \mathrm{cm}^{-3}$, but there is much variation through the Galaxy.

The interstellar medium

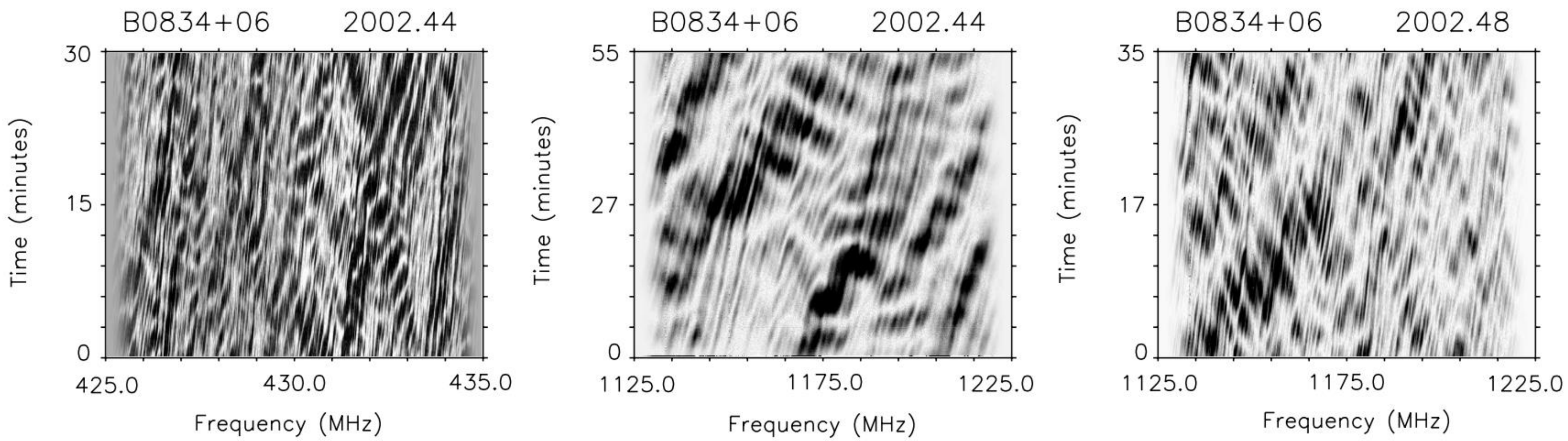
- The ISM is a complex mixture of neutral and ionised components (99% gas, 1% dust):

Component	Temp (K)	Volume fraction	Number density (cm^{-3})	Species
Molecular clouds	20-50	<1%	10^3-10^6	Molecular hydrogen
Cold neutral medium	50-100	1-5%	1-10^3	Atomic hydrogen
Warm neutral medium	10^3-10^4	10-20%	10^{-1}-10	Atomic hydrogen
Warm ionised medium	**10^3 –10^4**	**20–50%**	**10^{-2}**	**Electrons/protons**
HII regions	**10^4**	**10%**	**10^2–10^4**	**Electrons/protons**
Hot ionised medium	**10^6 –10^7**	**30–70%**	**10^{-4}–10^{-2}**	**Electrons/protons/ metallic ions**

Interstellar scintillation

- As well as dispersion, pulsars show slow (~mins) variations in flux density, called interstellar scintillation, caused by constructive and destructive interference between different propagation paths through the clumpy interstellar medium (time variations caused by the relative motion of source/observer/medium).

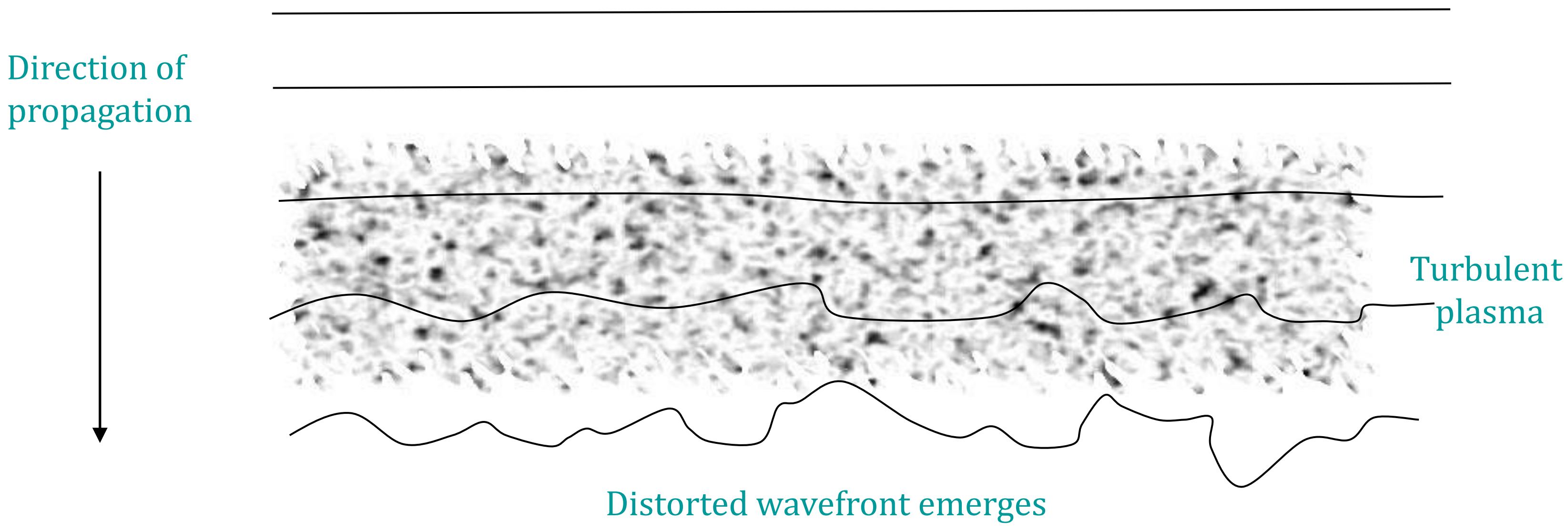
- A very informative probe of the ISM, but what does interstellar scintillation tell us about the ISM and pulsars?



Cordes et al. 2006

Non-uniform plasmas

- A plasma with refractive index variations (typically 0.1% in the ISM) will distort a plane wavefront propagating through it:



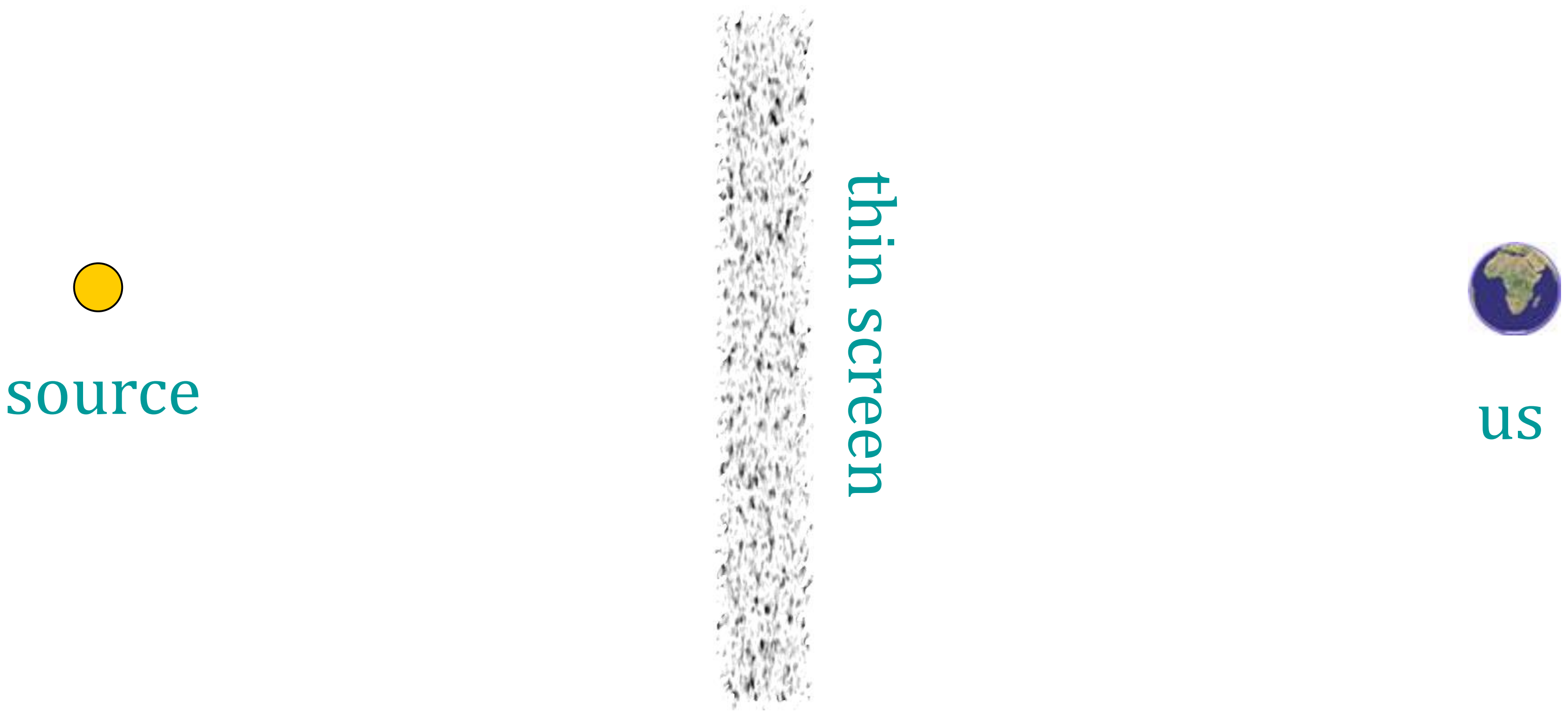
- Excess refractive index due to excess electron density at position $\boldsymbol{r}$ is

Classical radius of the electron

$$\Delta\eta(\boldsymbol{r}) = \frac{e^2}{8\pi^2\epsilon_0 m_\mathrm{e}} \frac{\Delta n_\mathrm{e}(\boldsymbol{r})}{f^2} = \frac{r_\mathrm{e}}{2\pi}\lambda^2 \Delta n_\mathrm{e}(\boldsymbol{r})$$

The thin screen approximation

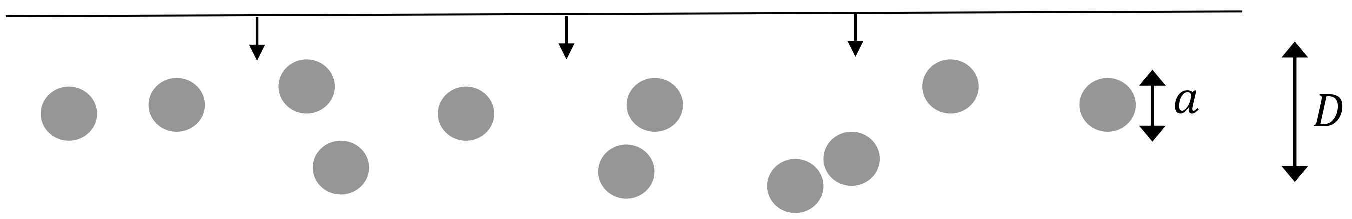
- It is usually a good approximation to imagine the plasma confined to a thin screen, about half way to the source:



Non-uniform plasmas – the blob approximation

- A simple and instructive way to model propagation through a random medium is to think of randomly placed, identical blobs of excess plasma density in the thin screen:

Randomly placed blobs of plasma with excess refractive index $\Delta\eta$



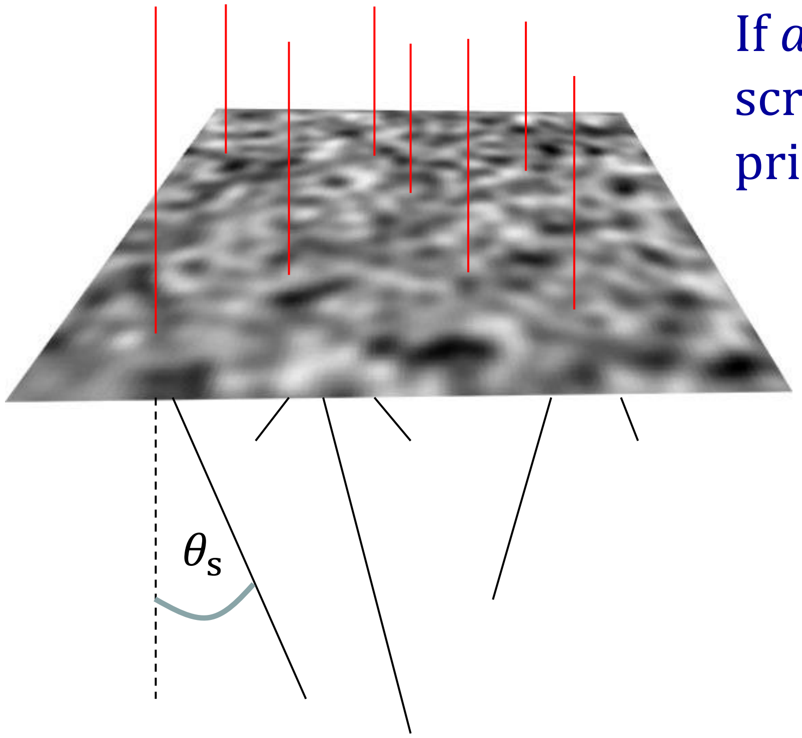
mean number of blobs encountered $= D/a$

rms variation in number encountered $= (D/a)^{1/2}$

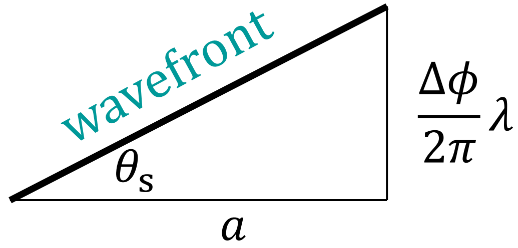
- Each introduces $2\pi\Delta\eta a/\lambda$ of phase, so phase perturbations across the wavefront are

$$\Delta\phi = r_e\lambda(Da)^{1/2}\Delta n_e$$

Simple phase screen – refractive scattering

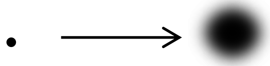


If $a \gg \lambda$, rays passing through the screen will be deflected (as if by little prisms)



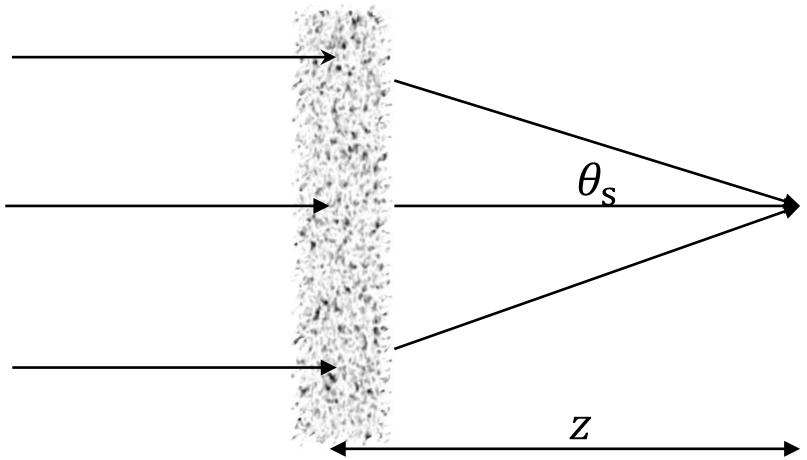
Scattering angle is $\theta_s \simeq \frac{\Delta\phi}{2\pi}\frac{\lambda}{a} = \frac{1}{2\pi} r_e \lambda^2 \left(\frac{D}{a}\right)^{\frac{1}{2}} \Delta n_e$.

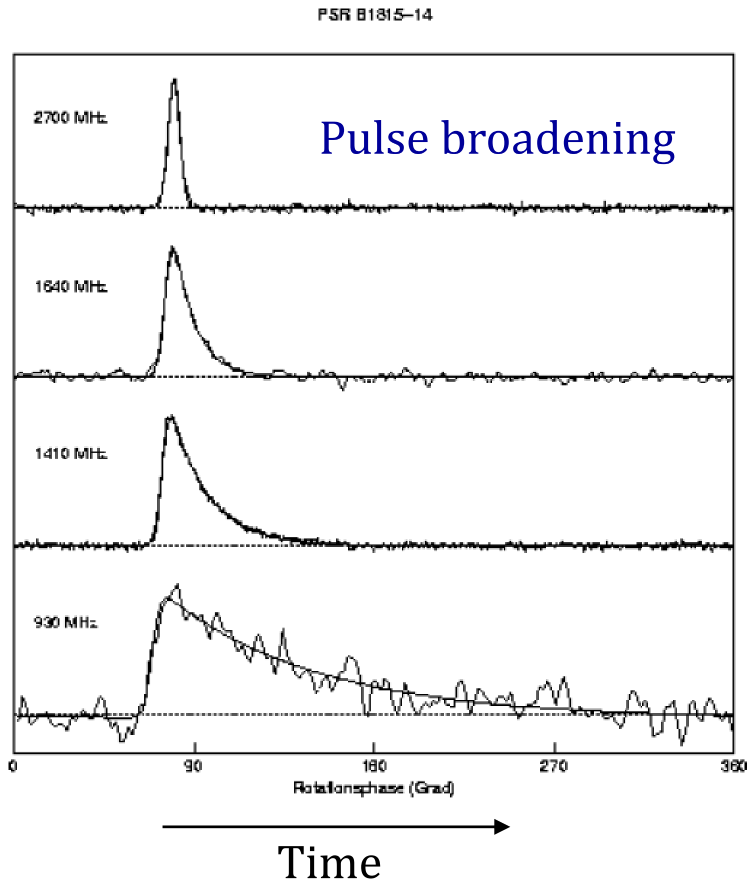
A point source therefore appears broadened.



Temporal broadening

- Different rays from the blurred source take different times to reach the observer:





Delay range $\tau_s = \frac{z}{c}(1 - \cos\theta_s) \simeq \frac{z\theta_s^2}{2c}$

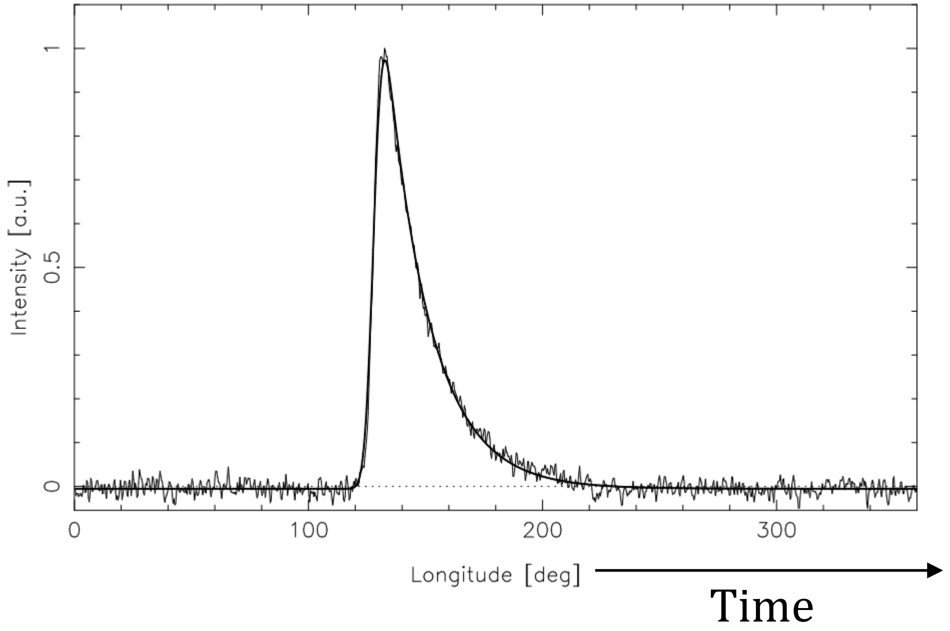
If $\theta_s \propto \lambda^2$ then $\tau_s \propto \lambda^4$.

Temporal broadening

- For a thin screen, and a gaussian shape to the scattered image, a short pulse is broadened to an approximately exponential decay

$$I(t) \propto \exp(-t/\tau_s).$$

Multiple scattering smooths this to something like this:



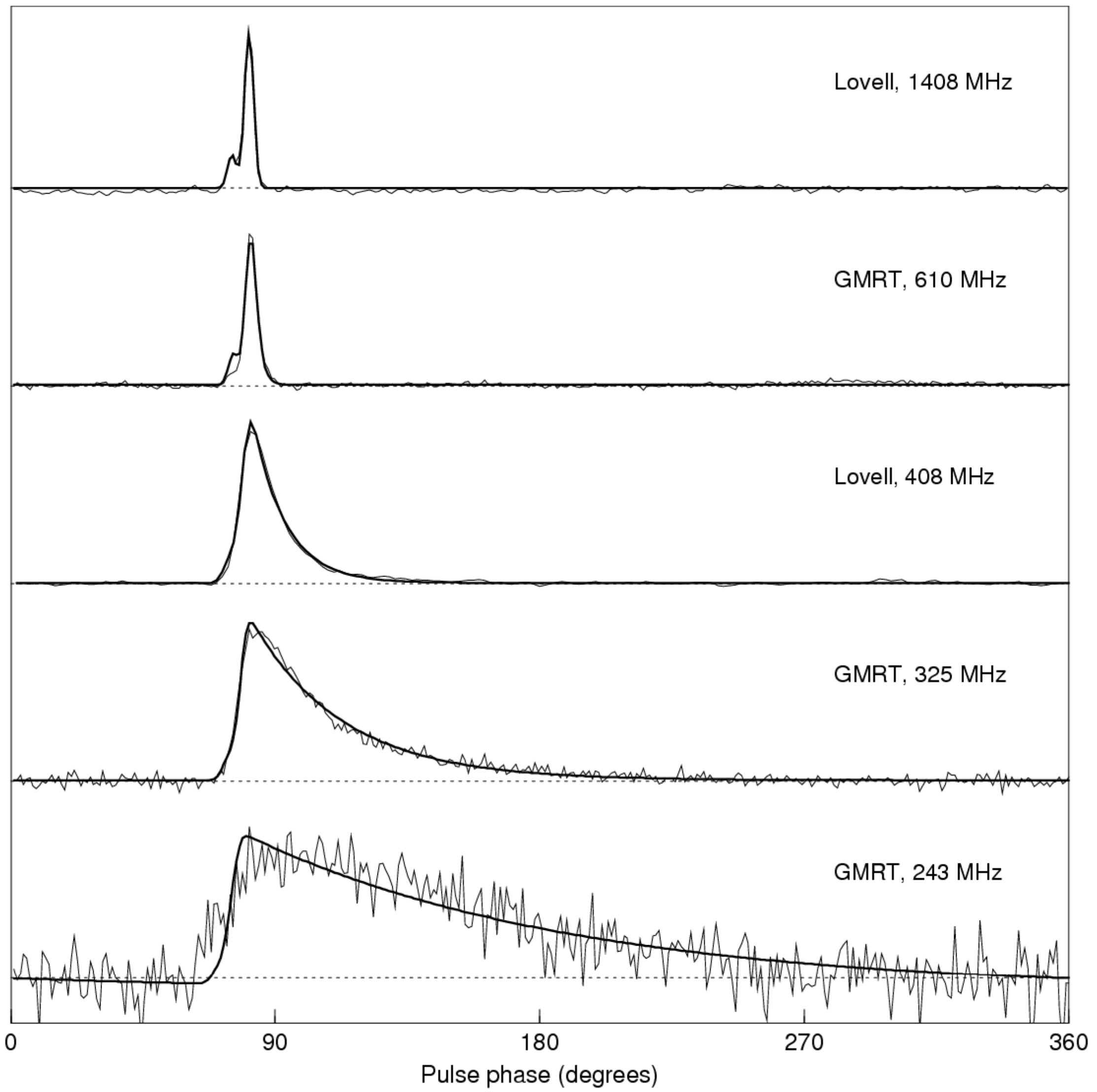
B1815-14 (Loehmer et al. 2001)

Note: $\tau_s \propto \lambda^4 z^2$, so scattering is severe at low frequencies and for distant pulsars.

[A z^2 rather than z dependence due to extended screen: $D \rightarrow z$]

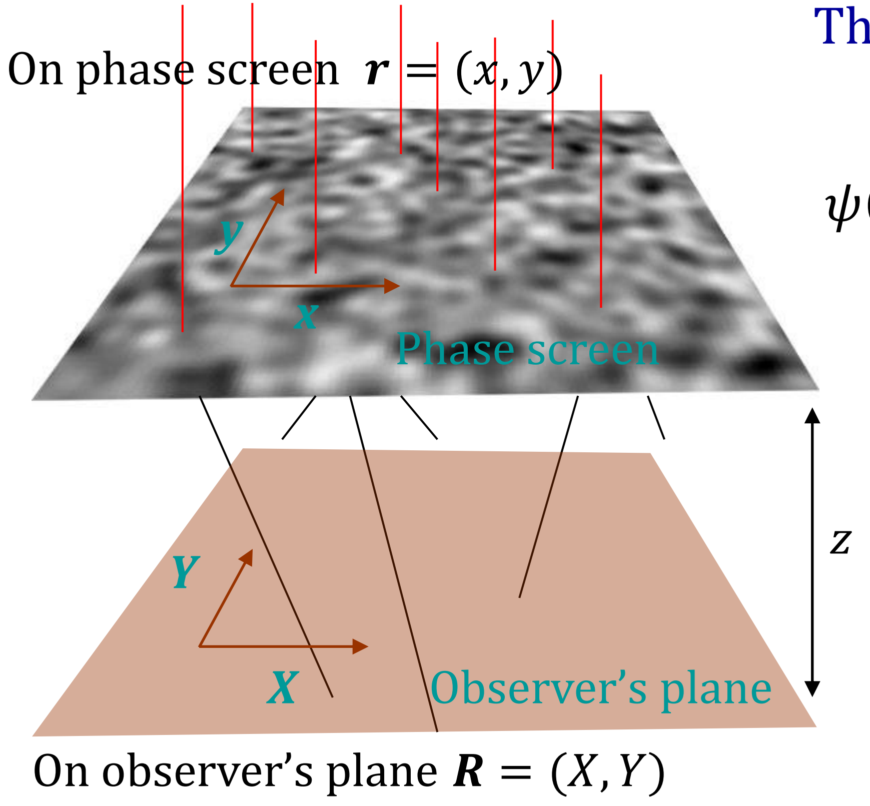
Temporal broadening

PSR B1831-03
(Lohmer et al. 2004)



Simple phase screen – scattering

- Point sources therefore appear broadened in angle and time
- The full evolution of the wave can be computed using the Fresnel diffraction formula:



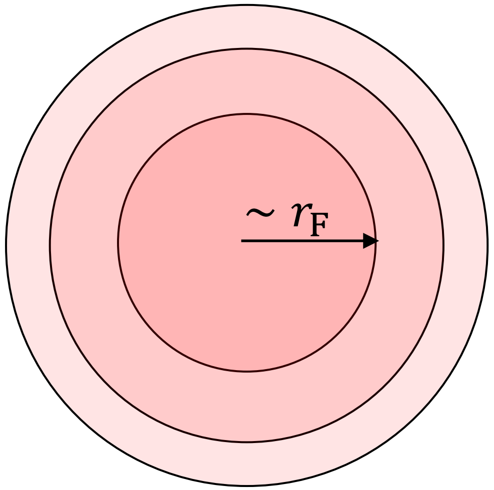
The wave amplitude and phase at $\boldsymbol{R}$ is

$$\psi(\boldsymbol{R}) = \frac{e^{-\frac{i\pi}{2}}}{r_F^2} \iint \exp\left[i\phi(\boldsymbol{r}) + i\pi \frac{|\boldsymbol{r}-\boldsymbol{R}|^2}{r_F^2}\right] d^2\boldsymbol{r}\,.$$

- $r_F = (\lambda z)^{1/2}$ is the Fresnel scale – the size of the centre patch of the screen within which all points are nearly equidistant from the observer.

Simple phase screen – scattering

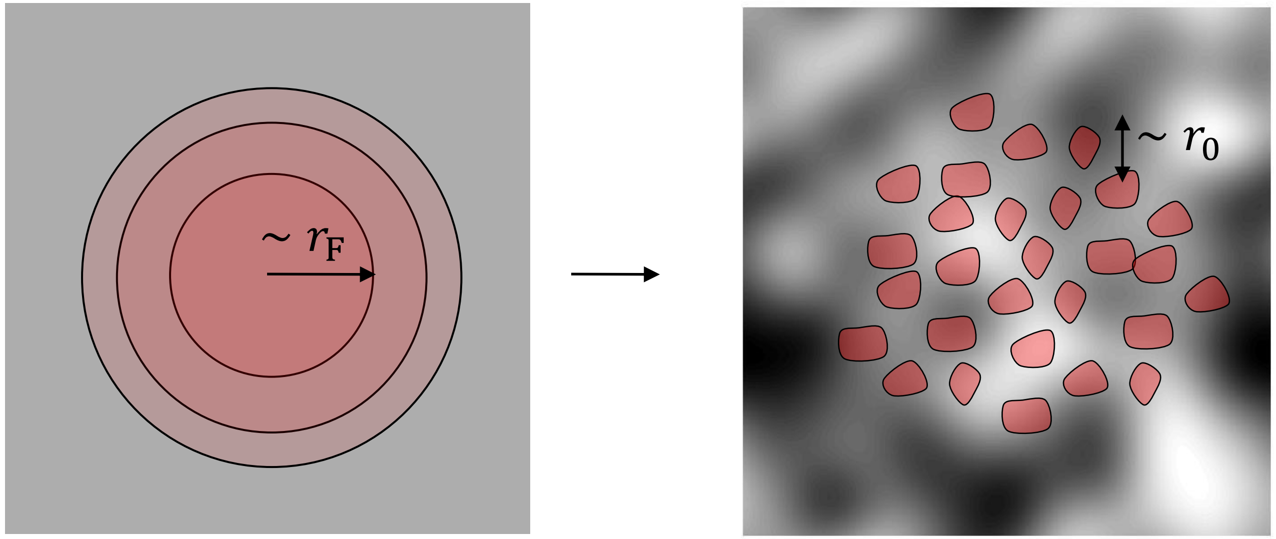
- In principle, the integral is over the whole screen, but in practice the contribution from the first Fresnel zone ($\sim r_{\mathrm{F}}$) dominates.



- If the phase disturbance changes only a little ($\ll \pi$) over the Fresnel zone, we have weak scattering.
- If there are large changes over the zone, we have strong scattering.

Strong scattering

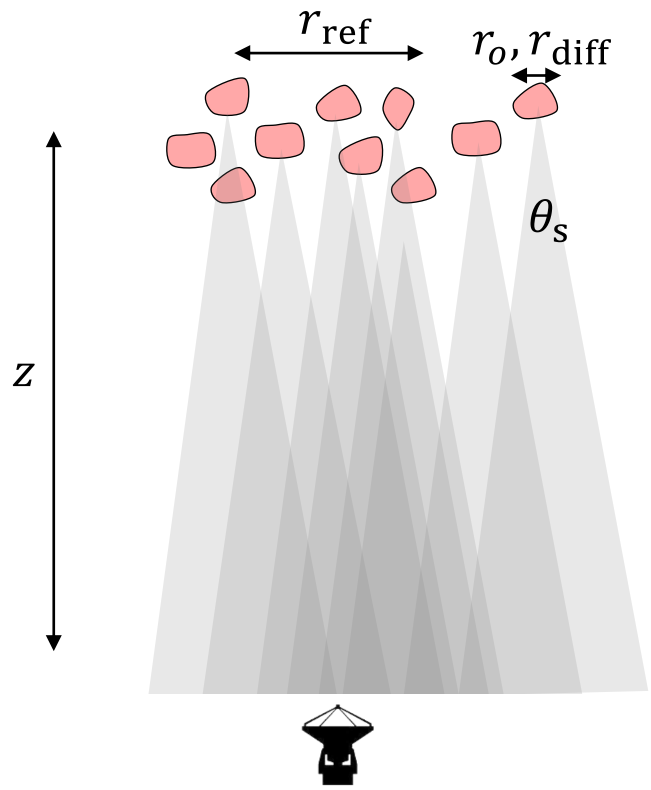
- Strong scattering corresponds to the situation where the screen generates a large variation in phase over the Fresnel scale (so destroying its importance)
- The new phase-stationary scale is called r_0.



Strong scattering

- Each patch diffracts radiation over a scattering angle $\theta_s \simeq \lambda / r_0$ and r_0 is sometimes called the diffractive scale, $r_{\rm diff}$.

- An observer sees radiation from patches over a scale $r_{\rm ref} = z\theta_s$, called the refractive scale.



- Note that $r_{\rm diff} r_{\rm ref} = r_{\rm F}^2$. In weak scattering we are restricted to one scintillation mode ('twinkling'), but in strong scattering we get diffractive scintillation and refractive scintillation.

Strong scattering– diffractive scintillation

- If the radio source is sufficiently small (and band-limited), the phase screen is illuminated with spatially coherent radiation and the overlapping scattered waves from each phase stationary patch create a strong, random, interference pattern on the ground with a scale size of r_{diff} (smeared out if $\theta_{\mathrm{source}} > r_{\mathrm{diff}}/z$).

- The radiation takes a range of paths to reach us. To maintain the interference pattern we must restrict the bandwidth to approximately the inverse of the temporal broadening time

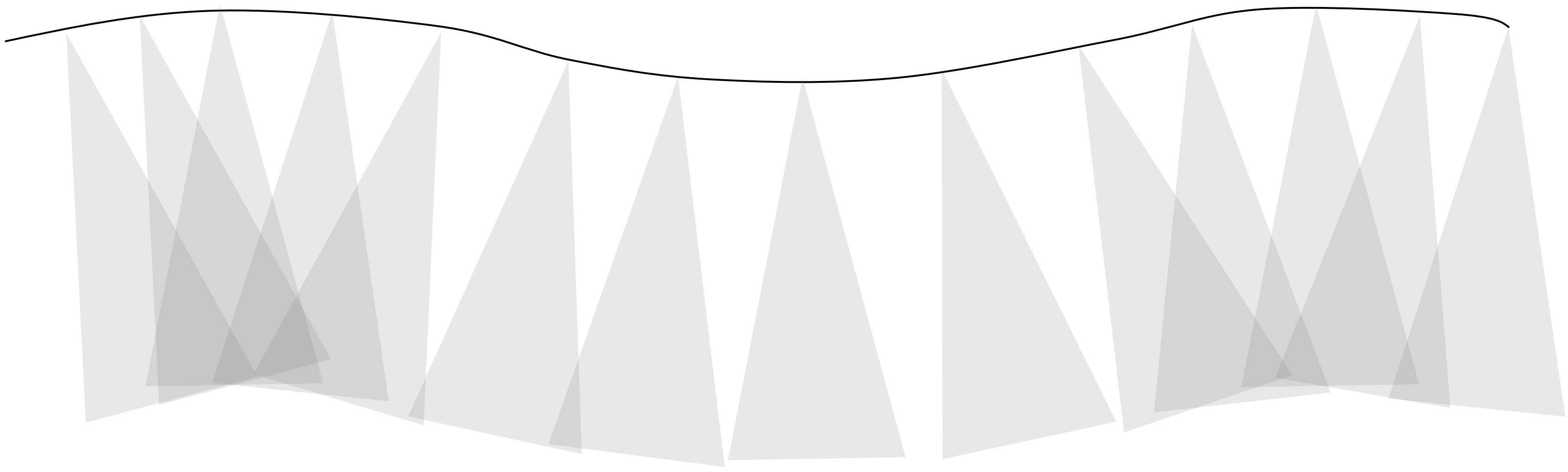
$$\Delta f \simeq \frac{1}{2\pi\tau_{\mathrm{s}}}.$$

This is the decorrelation bandwidth of the scintillations.

Strong scattering – refractive scintillation

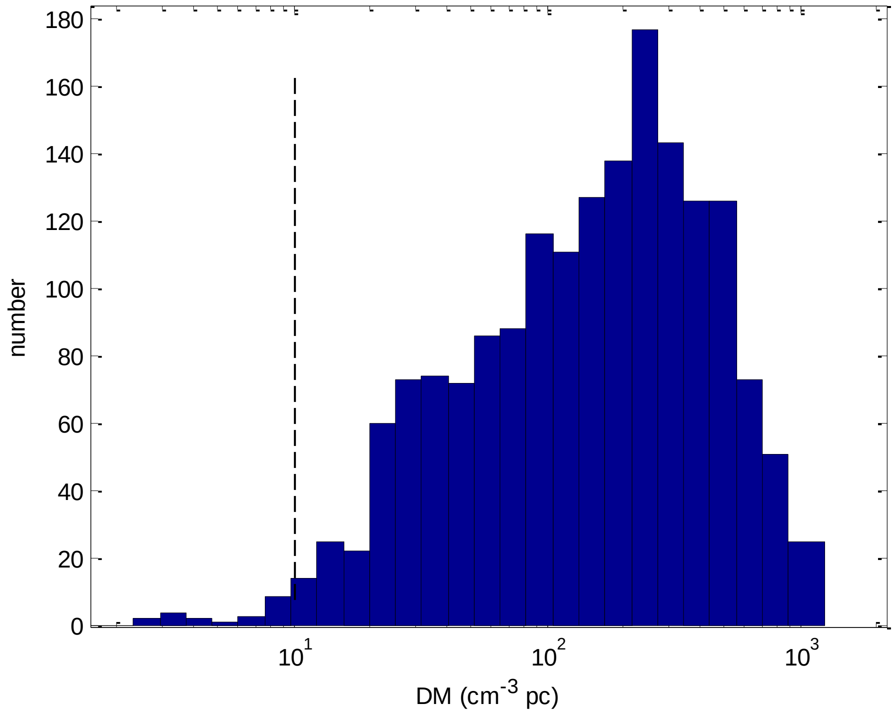
- The refractive scale defines the region of the phase screen that contributes to the intensity on the ground.

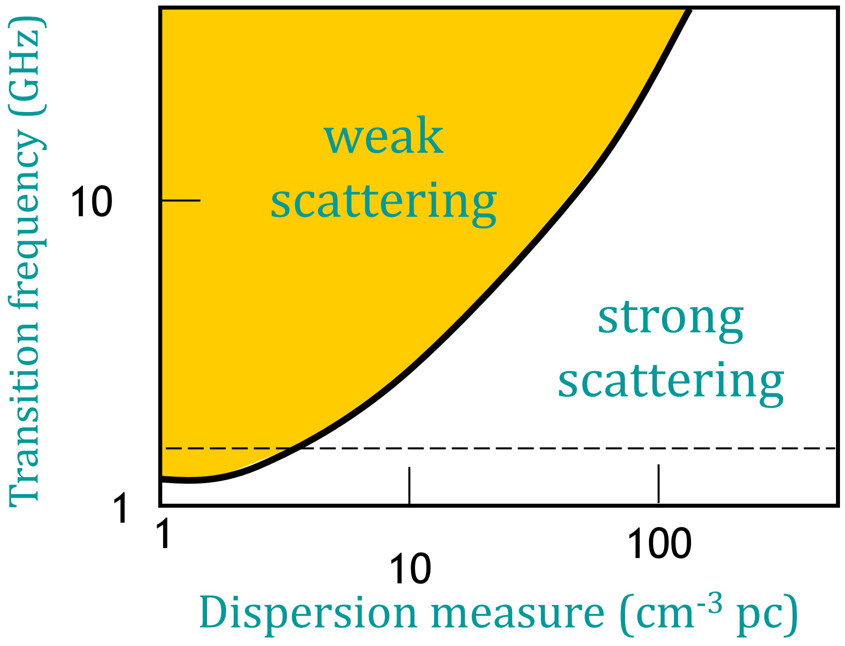
- Variations in the refractive index of the screen on > this scale will refract the scattering cones in/out of view, modulating the intensity. This is a broadband effect.



Pulsar scintillation

- Most pulsar observations fall into the strong scattering regime (dashed lines corresponding to 1.4 GHz)





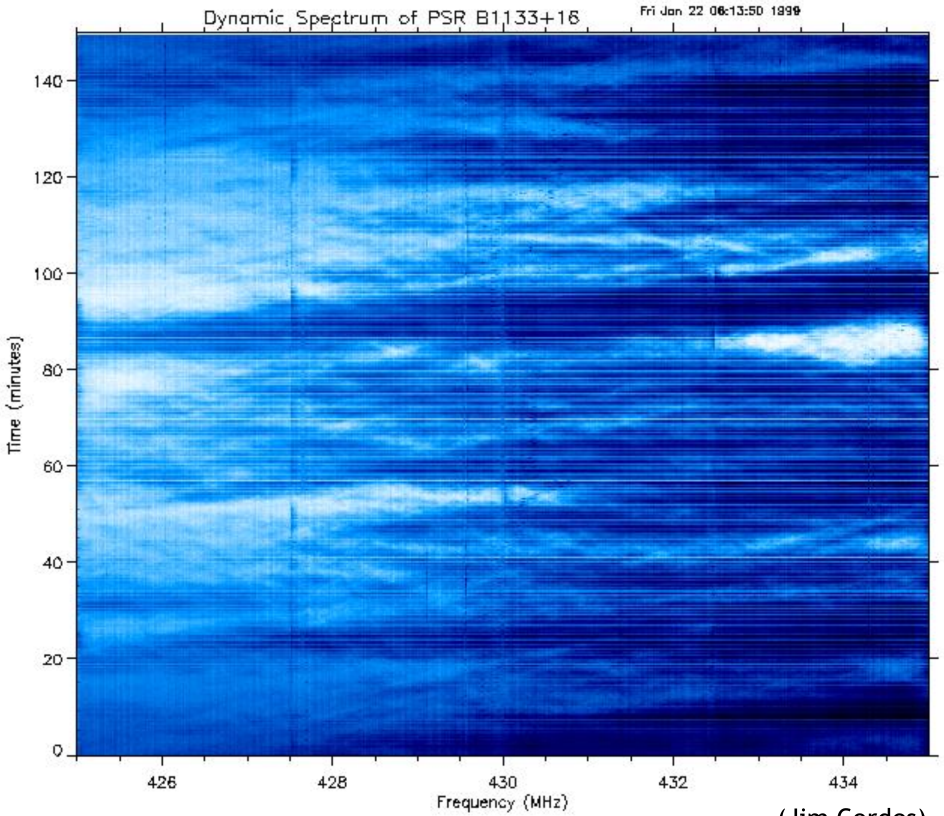
- For scintillation we need

$$\theta_{\text{source}} < \frac{r_{\text{diff}}}{z},$$

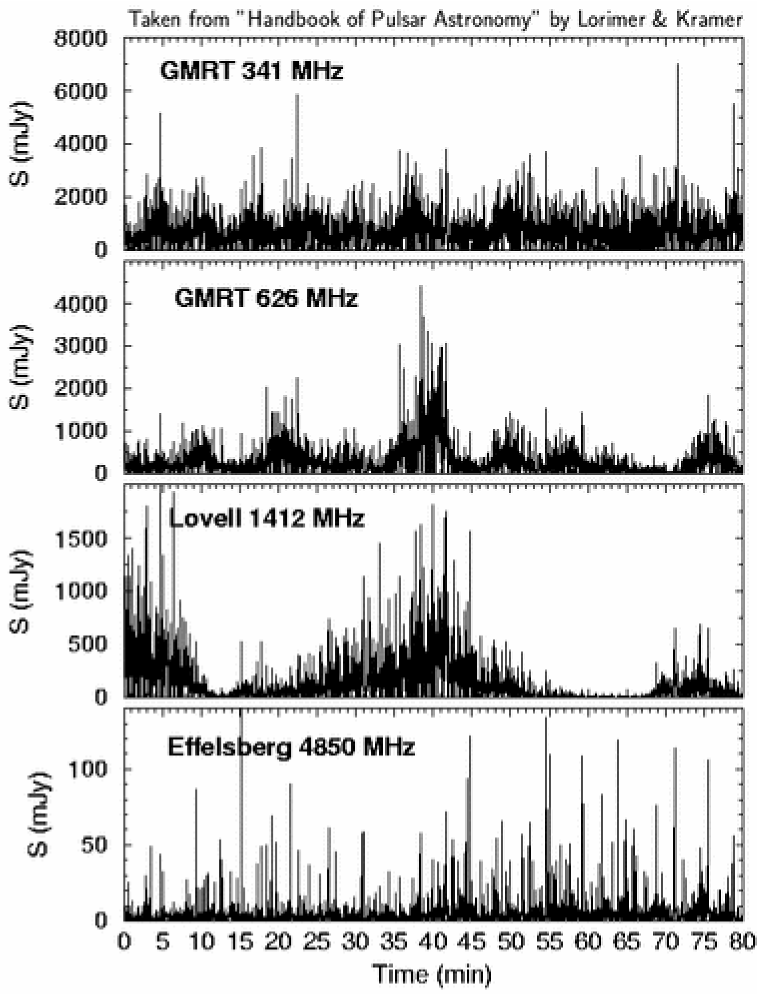
i.e., the source must be smaller than the diffractive scale ($\sim 10^4$ km at $\sim$0.5 GHz) – pulsars!

Pulsar diffractive scintillation

- Diffractive scintillation is clearly seen, probing the ISM on scales of $\sim 10^6$ to 10^8 m

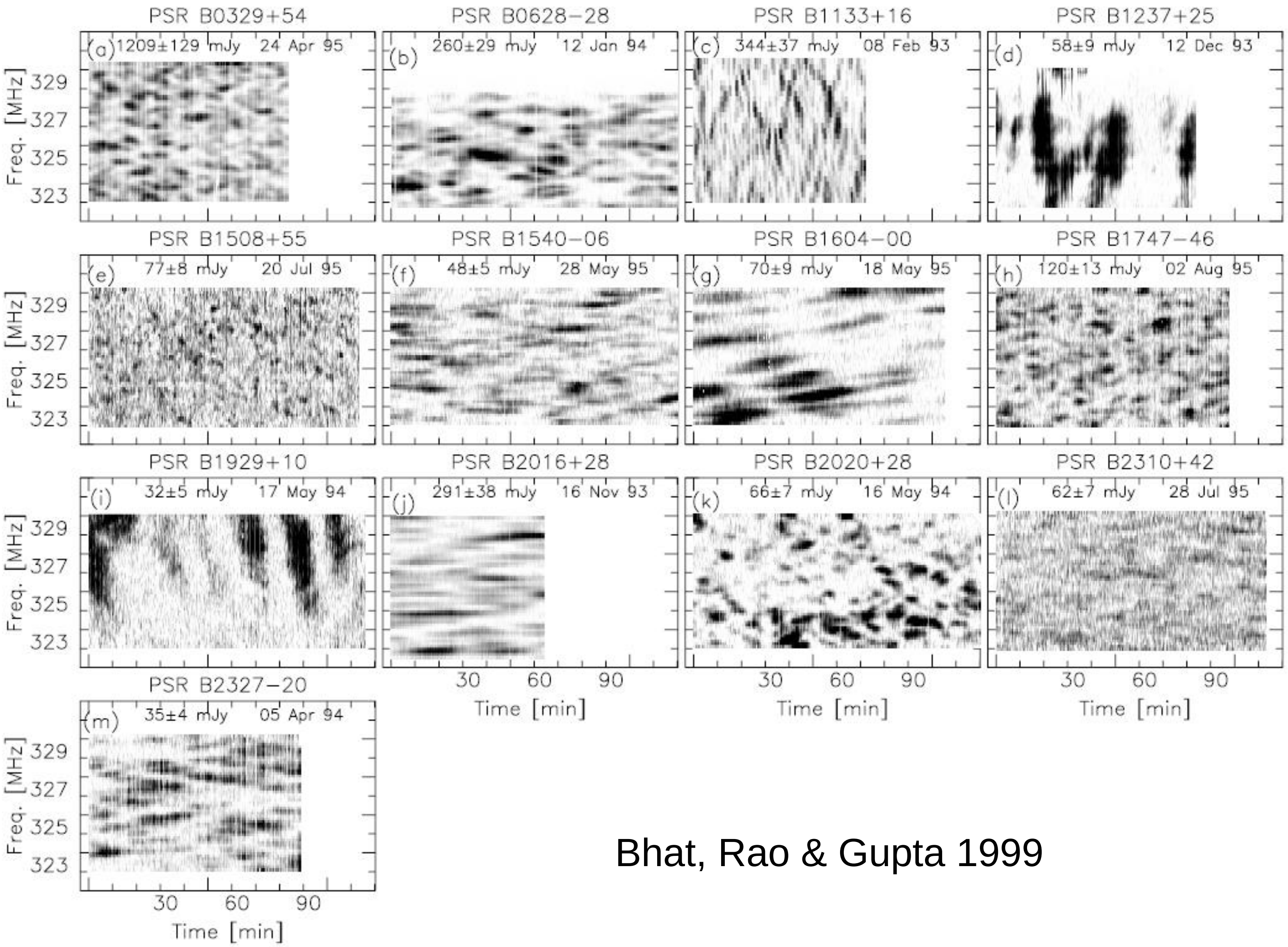


(Jim Cordes)



More diffractive scintillation…

- The ISM is a complex thing!



Bhat, Rao & Gupta 1999

Pulsar refractive scintillation

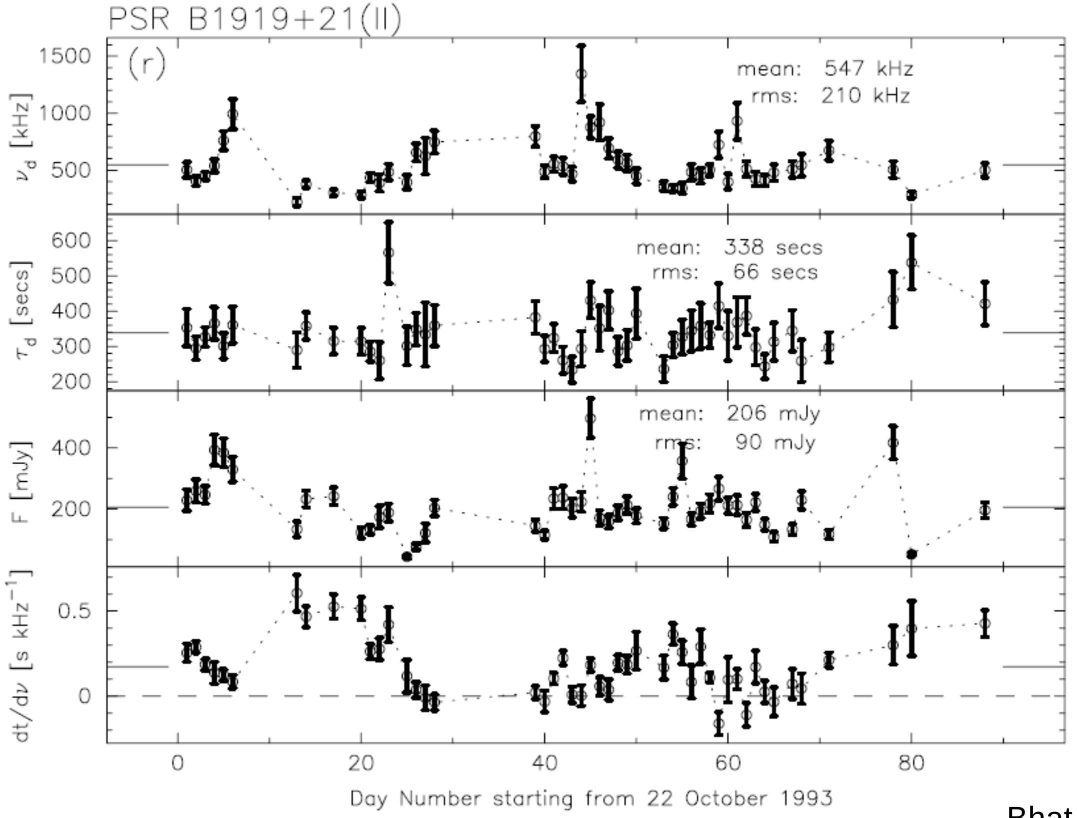
- Over timescales of days to months we see refractive scintillation, probing the ISM on scales of 10^{10} to 10^{12} m, though this can be hard to distinguish from intrinsic pulsar variability:

Decorrelation bandwidth

Diffractive scintillation timescale

Flux density

Pattern drift rate



Bhat, Rao & Gupta 1999