Energy-based shape regulation of soft robots with unactuated dynamics dominated by elasticity

Borja Rosales, L.P.; Dabiri, A.; Della Santina, C.

DOI
10.1109/RoboSoft54090.2022.9762071

Publication date
2022

Document Version
Final published version

Published in
Proceedings of the 5th International Conference on Soft Robotics (RoboSoft 2022)

2022 IEEE 5th International Conference on Soft Robotics (RoboSoft)
April 4-8, 2022, Edinburgh, UK

Energy-based shape regulation of soft robots with unactuated dynamics dominated by elasticity

Pablo Borja[1], Azita Dabiri[2], Cosimo Della Santina[1,3]

2022 IEEE 5th International Conference on Soft Robotics (RoboSoft) | 978-1-6654-0828-8/22/$31.00 ©2022 IEEE | DOI: 10.1109/ROBOSOFT54090.2022.9762071

Abstract— **This paper proposes a model-based control design approach for a broad class of soft robots, having their elastic field dominating gravity in the unactuated coordinates. To this end, we consider finite-dimensional dynamic models obtained from approximations of the system's energy. Then, we propose a general control architecture that can stabilize soft robots based on potential energy shaping. We discuss three specializations of this general architecture: a PD with mixed feedback-feedforward gravity compensation, a PD with feedforward compensation, and a saturated version of the latter. We provide a physical interpretation of the controllers, and we illustrate their applicability through simulations.**

I. INTRODUCTION

Continuum soft robots can be understood as nonlinear infinite-dimensional mechanical systems. Indeed, contrary to standard rigid robots, these systems are built with continuously deformable materials [1]. This makes even basic control problems extremely hard to solve [2]. Still, despite their complexity, their behavior is determined by their energy and dissipation, as for any other mechanical system. In this regard, the port-Hamiltonian (pH) framework has proven suitable to capture such physical quantities in infinite-dimensional mathematical models. See, for instance, [3], [4]. However, for control implementation purposes, the infinite-dimension nature of the resulting models can be a challenge. Thus, researchers have proposed approximation modeling methods to describe soft robots as finite-dimensional mechanical systems to overcome this problem. See, for example, piecewise constant strain models and functional parameterizations [5], [6]. In particular, these approaches are suitable for finding approximations of the system's damping and kinetic and potential energy. Hence, they are ideal to obtain energy-based dynamic models of soft robots, i.e., Euler-Lagrange (EL) or pH representations.

Due to their close relation to concepts like energy and dissipation, passivity theory and passivity-based control (PBC) are powerful tools for the analysis and control of complex nonlinear systems. See, for instance, [7]–[9]. In particular, PBC techniques have proven suitable to deal with underactuated mechanical systems [8], [10], [11]. Hence, energy-based design approaches and PBC for model-based control design have gained interest in the soft robotics community in recent years. For instance, in [12] and [13], the authors propose energy-based modeling approaches for soft robots; in [14], the authors provide an energy-based design approach that eases the selection of adequate actuators for applications involving soft robots; in [15] and [16], the authors validate, through simulations, the effectiveness of energy-shaping controllers for a cyberoctopus soft arm represented by the *Cosserat rod model*; in [17], a PBC approach is used to control a soft robot, which is approximated as a rigid link system; in [18] and [19], the authors provide experimental results that corroborate the effectiveness of PBC techniques for stabilizing soft robot manipulators approximated as rigid link systems. Additionally, [20], [21] are other examples of applications with soft continuum systems, where approximations, energy-based models, and PBC techniques are used to solve the stabilization problem.

However, despite these substantial advancements, general results that apply to all soft robots—no matter the modeling technique—are still lacking [2]. The goal of this paper is to contribute to solving this challenge. To this end, we focus on a generic class of soft robots, where elasticity dominates the undesired gravity effects in the unactuated coordinates. Then, we propose a general control design approach for these systems using PBC. The main contributions of this work are:

- **(C1)** A vast family of regulators suitable to stabilize underactuated soft robots with dominant elasticity in the unactuated coordinates. Notably, the proposed controllers encompass and generalize recently introduced control approaches such as the PD+feedforward [2].
- **(C2)** Straightforwardly verifiable conditions for the stabilization of underactuated soft robots via collocated feedback. Remarkably, such conditions do not hinge on a particular approximation method.

The remainder of this paper is organized as follows. Section II revisits energy-based modeling, and provides a class of interest of these systems. Then, Section III is devoted to the control design, where we establish the main results of this work. We provide examples of instances of the proposed strategy in Section IV. In Section V, we report simulation results. We present the concluding remarks in Section VI.

Notation: we denote the $n \times n$ identity matrix as I_n. The symbol $\mathbf{0}$ denotes a vector or matrix of appropriate dimensions whose entries are zeros. Given $x \in \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}$, we consider that $\frac{\partial f(x)}{\partial x} = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} & \dots & \frac{\partial f(x)}{\partial x_n} \end{bmatrix}^\top$. Moreover, given the constant vector $x_\star \in \mathbb{R}^n$, we define $\left(\frac{\partial f}{\partial x}\right)_\star := \left(\frac{\partial f(x)}{\partial x}\right)\Big|_{x=x_\star}$. When they are clear from the context, we omit the arguments of functions to simplify the notation.

II. ENERGY-BASED MODELING

Without loss of generality, the approximated[1] kinetic energy of a soft robot is given by $\mathcal{T}(q,p) = \frac{1}{2}p^\top \mathcal{M}^{-1}(q)p$, where $q \in \mathbb{R}^n$ is the configuration variables vector, $\mathcal{M} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the *positive definite* inertia matrix, and

[1]Department of Cognitive Robotics, Delft University of Technology, Delft, The Netherlands {l.p.borjarosales,c.dellasantina}@tudelft.nl. [2] Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands {a.dabiri}@tudelft.nl. [3] Institute of Robotics and Mechatronics, German Aerospace Center (DLR), Oberpfaffenhofen, Germany.

[1]Henceforth, we omit the adjective *approximated* while referring to the finite-dimensional model.

$p \in \mathbb{R}^n$ is the momenta vector, which satisfies $p = \mathcal{M}(q)\dot{q}$. Moreover, the potential energy is given by $\mathcal{V}(q) = \mathcal{V}_{\mathsf{e}}(q) + \mathcal{V}_{\mathsf{g}}(q)$, where $\mathcal{V}_{\mathsf{e}} : \mathbb{R}^n \to \mathbb{R}^+$ is the elastic potential energy and $\mathcal{V}_{\mathsf{g}} : \mathbb{R}^n \to \mathbb{R}^+$ is the potential energy due to gravity. Hence, the total energy of the system, referred to as the system's Hamiltonian $\mathcal{H} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^+$, is given by

$$\mathcal{H}(q,p) = \mathcal{T}(q,p) + \mathcal{V}(q).$$

Moreover, the behavior of the system can be modeled as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & -\mathcal{D}(q,p) \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathcal{H}(q,p)}{\partial q} \\ \dfrac{\partial \mathcal{H}(q,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathcal{A}(q) \end{bmatrix} \tau; \tag{1}$$

where $\tau \in \mathbb{R}^m$, with $m \leq n$, denotes the input vector; $\mathcal{D} : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ is the damping matrix, which is *positive semi-definite*; and $\mathcal{A} : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is the input matrix, which has *full (column) rank.*

A. Assignable equilibria

While dealing with the set-point regulation (stabilization) problem, the desired equilibrium is often not an open-loop equilibrium point. Thus, it is essential to identify which equilibrium points can be assigned by the control input. Such points are characterized by

$$\mathcal{E} := \left\{ q \in \mathbb{R}^n \mid \mathcal{A}^\perp(q) \frac{\partial \mathcal{V}(q)}{\partial q} = \mathbf{0} \right\}, \tag{2}$$

where $\mathcal{A}^\perp(q)$ denotes the *full-rank left annihilator* of $\mathcal{A}(q)$, i.e., $\mathcal{A}^\perp(q)\mathcal{A}(q) = \mathbf{0}$. Equivalently, if $q_\star \in \mathcal{E}$, then there exists $\tau_\star \in \mathbb{R}^m$ such that $\mathcal{A}_\star \tau_\star = \left(\frac{\partial \mathcal{V}}{\partial q}\right)_\star$, implying that $(q_\star, \mathbf{0})$ is an equilibrium for (1).

B. Class of systems of interest

We restrict our attention to a class of systems that encompasses the approximation of a wide variety of soft robot dynamics. To characterize such systems, below we introduce three assumptions.

Assumption 1. *The input matrix is constant, i.e., $\mathcal{A}(q) = \mathcal{A}$, with $\mathcal{A} \in \mathbb{R}^{n \times m}$.*

Assumption 1 ensures that there exists a linear transformation such that the new coordinates can be split into actuated and unactuated ones, which is essential for presenting the results of Section III. This is formalized in the following proposition.

Proposition 1. *Consider the system* (1) *with $\mathcal{A}$ constant. There exists a linear invertible transformation $T \in \mathbb{R}^{n \times n}$ such that the dynamics of the coordinates*[2]

$$q := T^{-\top} q, \quad p := Tp. \tag{3}$$

are

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I_n \\ -I_n & -D(q,p) \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(q,p)}{\partial q} \\ \dfrac{\partial H(q,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ A \end{bmatrix} \tau \tag{4}$$

$$H(q,p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q),$$

where

$$V(q) := \mathcal{V}(q)\big|_{q=T^\top q}; \quad M(q) := T\mathcal{M}(q)T^\top\big|_{q=T^\top q};$$

$$D(q,p) := T\mathcal{D}(q,p)T^\top\big|_{\substack{q=T^\top q \\ p=T^{-1}p}}; \quad A := T\mathcal{A} = \begin{bmatrix} I_m & \mathbf{0} \end{bmatrix}^\top.$$

Proof: Consider

$$T = \begin{bmatrix} \left(\mathcal{A}^\top \mathcal{A}\right)^{-1} \mathcal{A}^\top \\ \mathcal{A}^\perp \end{bmatrix}.$$

Since $\mathcal{A}^\perp$ is the full-rank left annihilator of $\mathcal{A}$, T has full rank. Therefore, T is invertible. Moreover, note that

$$H(q,p) := \mathcal{H}(q,p)\big|_{\substack{q=T^\top q \\ p=T^{-1}p}}.$$

Accordingly, from (3) and the chain rule, we have that

$$\frac{\partial H(q,p)}{\partial q} = T \left. \frac{\partial \mathcal{H}(q,p)}{\partial q} \right|_{\substack{q=T^\top q \\ p=T^{-1}p}},$$

$$\frac{\partial H(q,p)}{\partial p} = T^{-\top} \left. \frac{\partial \mathcal{H}(q,p)}{\partial p} \right|_{\substack{q=T^\top q \\ p=T^{-1}p}}.$$

Hence,

$$\dot{q} = T^{-\top}\dot{q} = T^{-\top}\frac{\partial \mathcal{H}}{\partial p} = \frac{\partial H}{\partial p}$$

$$\dot{p} = T\dot{p} = T\left(-\frac{\partial \mathcal{H}}{\partial q} - \mathcal{D}\frac{\partial \mathcal{H}}{\partial p} + \mathcal{A}\tau\right)$$

$$= -\frac{\partial H}{\partial q} - D\frac{\partial H}{\partial p} + A\tau$$

Furthermore, the inertia and damping matrices are modified by congruence transformations. Accordingly, they remain positive definite and positive semi-definite, respectively. ■

Remark 1. *The transformation T such that* (1) *takes the form* (4) *is not unique.*

Consider the system (4) and define the degree of under actuation as $s := n - m$. Then, we can split the coordinates into actuated and unactuated, namely,

$$q_{\mathsf{u}} := A^\perp q; \quad q_{\mathsf{a}} := A^\top q; \quad p_{\mathsf{u}} := A^\perp p; \quad p_{\mathsf{a}} := A^\top p;$$

where $A^\perp = [\mathbf{0} \quad I_s]$, $q_{\mathsf{u}}, p_{\mathsf{u}} \in \mathbb{R}^s$, and $q_{\mathsf{a}}, p_{\mathsf{a}} \in \mathbb{R}^m$. Note that, given the desired configuration $q_\star \in \mathcal{E}$ for the system (1), we have the corresponding desired configuration $q_\star := T^{-\top} q_\star$ for the system (4).

The second assumption is related to the elasticity of the system, which plays a crucial role in the control design process.

Assumption 2. *Given the desired configuration $q_\star \in \mathcal{E}$, the elastic potential energy satisties*

$$K \preceq \left(\frac{\partial^2 V_{\mathsf{e}}}{\partial q^2}\right)_\star$$

where $V_{\mathsf{e}}(q) := \mathcal{V}_{\mathsf{e}}(T^\top q)$, and $K \in \mathbb{R}^{n \times n}$ is a positive semi-definite matrix.

We stress that Assumption 2 is satisfied by a large class of soft robots. In particular, it holds if the elastic potential energy is convex, for instance, soft robots with linear elasticity, i.e., the elastic potential energy has the form $V_{\mathsf{e}}(q) = \frac{1}{2} q^\top \bar{K} q$, where $\bar{K} \in \mathbb{R}^{n \times n}$ is positive semi-definite.

[2] Where $T^{-\top} = (T^\top)^{-1} = (T^{-1})^\top$.

Note that K can be expressed as

$$K := \begin{bmatrix} k_{\text{aa}} & k_{\text{au}} \\ k_{\text{au}}^\top & k_{\text{uu}} \end{bmatrix},$$

with $k_{\text{uu}} \in \mathbb{R}^{s\times s}$, $k_{\text{au}} \in \mathbb{R}^{m\times s}$, and $k_{\text{aa}} \in \mathbb{R}^{m\times m}$. Furthermore, K is positive semi-definite *only if* k_{uu} and k_{aa} are *positive semi-definite*.

The third and final assumption is that the elasticity dominates the forces resulting from the gravity in the unactuated coordinates.

Assumption 3. *Given the desired configuration $q_\star \in \mathcal{E}$, the potential energy related to the unactuated coordinates satisfies*

$$k_{\text{uu}} + \left(\frac{\partial^2 V_{\text{g}}}{\partial q_{\text{u}}^2}\right)_\star \succ 0; \quad V_{\text{g}}(q) := \mathcal{V}_{\text{g}}(q)\big|_{q=T^\top q}.$$

Assumption 3 is equivalent to assuming that the zero dynamics of (4) is *asymptotically stable*, considering as the output $y = q_{\text{a}}$ [22]. In other words, the system is *minimum phase* for the *collocated case*. For further details on minimum phase systems and zero dynamics, we refer the reader to [9]. Section III provides a general control design strategy for soft robots characterized by the three assumptions mentioned above.

III. CONTROL DESIGN

In this section, we suppose (1) satisfies Assumption 1. Then, we propose a control design approach to stabilize (4)—equivalently, (1).

The following proposition provides a general control law that stabilizes soft robots satisfying Assumptions 1–3.

Lemma 1. *Consider the system* (4) *and the desired configuration $q_\star \in \mathcal{E}$ satisfying Assumptions 2 and 3. Let $\Phi : \mathbb{R}^m \to \mathbb{R}$ be a $\mathcal{C}^2$ function such that*

$$\left(\frac{\partial V}{\partial q_{\text{a}}}\right)_\star + \left(\frac{\partial \Phi}{\partial q_{\text{a}}}\right)_\star = \mathbf{0} \tag{5}$$

$$\left(H_{\text{ua}}^{''}\right)^\top \left(H_{\text{uu}}^{''}\right)^{-1} H_{\text{ua}}^{''} - H_{\text{aa}}^{''} \prec \left(\frac{\partial^2 \Phi}{\partial q_{\text{a}}^2}\right)_\star, \tag{6}$$

where

$$H_{\text{aa}}^{''} := \left(\frac{\partial^2 V_{\text{g}}}{\partial q_{\text{a}}^2}\right)_\star + k_{\text{aa}},\ H_{\text{ua}}^{''} := \left(\frac{\partial^2 V_{\text{g}}}{\partial q_{\text{u}}\partial q_{\text{a}}}\right)_\star + k_{\text{au}}^\top,$$

$$H_{\text{uu}}^{''} := \left(\frac{\partial^2 V_{\text{g}}}{\partial q_{\text{u}}^2}\right)_\star + k_{\text{uu}}.$$

Hence, the controller

$$\tau = -\frac{\partial \Phi(q_{\text{a}})}{\partial q_{\text{a}}}, \tag{7}$$

(locally) stabilizes the system at $(q_\star, \mathbf{0})$.

Proof: Consider the Lyapunov candidate function

$$H_{\text{d}}(q,p) = H(q,p) + \Phi(q_{\text{a}}).$$

Since at the equilibrium $p = \mathbf{0}$, some simple computations show that

$$\left(\frac{\partial H_{\text{d}}}{\partial q}\right)_\star = \left(\frac{\partial V}{\partial q}\right)_\star + A\left(\frac{\partial \Phi}{\partial q_{\text{a}}}\right)_\star$$

$$\left(\frac{\partial H_{\text{d}}}{\partial p}\right)_\star = \left(M^{-1}(q)p\right)_\star = \mathbf{0}. \tag{8}$$

Furthermore, by applying the transformation T to (2), we have that $q_\star \in \mathcal{E}$ implies $\left(\frac{\partial V}{\partial q_{\text{u}}}\right)_\star = \mathbf{0}$. Therefore, (5) ensures

$$\left(\frac{\partial H_{\text{d}}}{\partial q}\right)_\star = \mathbf{0}. \tag{9}$$

On the other hand, from Assumption 2, we get that

$$\left(\frac{\partial^2 H_{\text{d}}}{\partial q^2}\right)_\star \succeq \begin{bmatrix} H_{\text{aa}}^{''} & \left(H_{\text{ua}}^{''}\right)^\top \\ H_{\text{ua}}^{''} & H_{\text{uu}}^{''} \end{bmatrix} \tag{10}$$

$$\left(\frac{\partial^2 H_{\text{d}}}{\partial q \partial p}\right)_\star = \left(\frac{\partial^2 H_{\text{d}}}{\partial p \partial q}\right)_\star^\top = \mathbf{0} \tag{11}$$

$$\left(\frac{\partial^2 H_{\text{d}}}{\partial p^2}\right)_\star = M^{-1}(q_\star) \succ 0. \tag{12}$$

Furthermore, a Schur complement analysis show that Assumption 3, together with (6), guarantees that

$$\begin{bmatrix} H_{\text{aa}}^{''} & \left(H_{\text{ua}}^{''}\right)^\top \\ H_{\text{ua}}^{''} & H_{\text{uu}}^{''} \end{bmatrix} \succ 0,$$

which implies

$$\left(\frac{\partial^2 H_{\text{d}}}{\partial q^2}\right)_\star \succ 0. \tag{13}$$

Accordingly, from (8)–(13), we conclude that $(q_\star, \mathbf{0})$ is an *strict minimum* of $H_{\text{d}}(q,p)$. Moreover,

$$\begin{aligned} \dot{H} &= \left(\frac{\partial H}{\partial q}\right)^\top \dot{q} + \left(\frac{\partial H}{\partial p}\right)^\top \dot{p} \\ &= \left(\frac{\partial H}{\partial q}\right)^\top \dot{q} + \dot{q}^\top \left(-\frac{\partial H}{\partial q} - D\dot{q} + A\tau\right) \\ &= -\dot{q}^\top D\dot{q} + \dot{q}_{\text{a}}^\top \tau. \end{aligned}$$

Hence,

$$\dot{H}_{\text{d}} = \dot{H} + \dot{\Phi} = -\dot{q}^\top D\dot{q} + \dot{q}_{\text{a}}^\top \tau + \dot{q}_{\text{a}}^\top \frac{\partial \Phi}{\partial q_{\text{a}}} = -\dot{q}^\top D\dot{q}. \tag{14}$$

Accordingly, $H_{\text{d}}(q,p)$ is *non-increasing*. Thus, $(q_\star, \mathbf{0})$ is a stable equilibrium for the closed-loop system.

To prove asymptotic stability, i.e., that the trajectories converge to the desired equilibrium, we invoke LaSalle's invariance principle. Hence, we identify the set of points such that $\dot{H}_{\text{d}} = 0$. To this end, note that

$$\begin{aligned} \dot{H}_{\text{d}} = 0 \iff \dot{q} = 0 \iff p = 0 \implies \dot{p} = 0 \\ \implies \frac{\partial \Phi}{\partial q} + \frac{\partial V}{\partial q} = \mathbf{0}. \end{aligned}$$

Furthermore, since $(q_\star, \mathbf{0})$ is an strict minimum, we have that (at least) in a neighboorhood of the equilibrium

$$\frac{\partial \Phi}{\partial q} + \frac{\partial V}{\partial q} = \mathbf{0} \iff q = q_\star. \tag{15}$$

Accordingly, the equilibrium is asymptotically stable. ■

Remark 2. *The controller* (7) *shapes the potential energy of the system. Indeed, the closed-loop system can be interpreted as another mechanical system with potential energy $V_{\text{d}}(q) = V(q) + \Phi(q_{\text{a}})$. Note that this new potential energy has a minimum at the desired equilibrium. Thus, the dissipation $-\dot{q}^\top D\dot{q}$ ensures that the system converges to this point.*

Nonlinear systems may have several open-loop equilibrium points. Furthermore, it is not possible to remove the non-desired equilibria for the closed-loop system in some cases. For this reason, the results of Lemma 1 are only *local*. However, the following corollary provides sufficient conditions to claim global convergence.

Corollary 1. *The controller* (7) *stabilizes the system* (4) *at* $(q_\star, \mathbf{0})$ *for any initial condition if* (15) *is true for all* $q \in \mathbb{R}^n$.

Proof: If (15) is true for all $q \in \mathbb{R}^n$, then the only solution to $\dot{H}_{\mathtt{d}} = 0$ is $(q, p) = (q_\star, \mathbf{0})$. Hence, LaSalle's invariance principle establishes that all the trajectories converge to this point. ∎

The result of Lemma 1 relies on (5) and (6). Nevertheless, the following proposition establishes that Assumptions 2 and 3 guarantee the existence of $\Phi(q_{\mathtt{a}})$ satisfying these conditions.

Proposition 2. *Consider the system* (4) *and the desired configuration* $q_\star \in \mathcal{E}$ *satisfying Assumptions 2 and 3. There exists* $\Phi(q_{\mathtt{a}})$ *such that* (5) *and* (6) *hold.*

Proof: Consider the function

$$\Phi(q_{\mathtt{a}}) = -q_{\mathtt{a}}^\top \left(\frac{\partial V}{\partial q_{\mathtt{a}}}\right)_\star + \frac{1}{2}(q_{\mathtt{a}} - q_{\mathtt{a}_\star})^\top K_{\mathtt{P}}(q_{\mathtt{a}} - q_{\mathtt{a}_\star}), \quad (16)$$

where $K_{\mathtt{P}} \in \mathbb{R}^{m \times m}$ is a *positive definite* matrix. Hence,

$$\left(\frac{\partial \Phi}{\partial q_{\mathtt{a}}}\right)_\star = -\left(\frac{\partial V}{\partial q_{\mathtt{a}}}\right)_\star .$$

Thus, (5) holds. Moreover, (6) takes the form

$$\left(H''_{\mathtt{ua}}\right)^\top \left(H''_{\mathtt{uu}}\right)^{-1} H''_{\mathtt{ua}} - H''_{\mathtt{aa}} \prec K_{\mathtt{P}}, \quad (17)$$

which holds for $K_{\mathtt{P}}$ large enough. ∎

While we propose a particular $\Phi(q_{\mathtt{a}})$ to prove the result of Proposition 2, other structures can be exploited for specific purposes. We report some examples in Sec. IV.

Remark 3. *Any fully actuated system can be stabilized by adopting the proposed approach. Note that* $q_{\mathtt{a}} = q$ *and the input matrix is square and invertible. Therefore, even if* $A(q)$ *is nonconstant, we can propose*

$$\tau = -A^{-1}(q)\frac{\partial \Phi(q)}{\partial q},$$

with $\Phi(q)$ *such that*

$$\left(\frac{\partial V}{\partial q}\right)_\star + \left(\frac{\partial \Phi}{\partial q}\right)_\star = \mathbf{0}, \quad \left(\frac{\partial^2 V}{\partial q^2}\right)_\star + \left(\frac{\partial^2 \Phi}{\partial q^2}\right)_\star \succ 0.$$

Remark 4. *Damping injection can improve the closed-loop system's transitory behavior. To do this, we add an extra term to* (7)*, i.e.,*

$$\tau = -\frac{\partial \Phi(q_{\mathtt{a}})}{\partial q_{\mathtt{a}}} - \psi(\dot{q}_{\mathtt{a}}), \quad (18)$$

where $\psi : \mathbb{R}^m \to \mathbb{R}^m$ *satisfies*

$$\psi^\top(\dot{q}_{\mathtt{a}})\dot{q}_{\mathtt{a}} \geq 0, \quad \forall\, \dot{q}_{\mathtt{a}} \in \mathbb{R}^m.$$

Therefore, (14) *takes the form*

$$\dot{H}_{\mathtt{d}} = -\dot{q}^\top D\dot{q} - \psi^\top(\dot{q}_{\mathtt{a}})\dot{q}_{\mathtt{a}} \leq 0.$$

Assumption 3 is the result of the system's under-actuation. If this assumption is not satisfied by the system to be controlled, then the controller (7) can assign the desired equilibrium but fails to render it stable. This problem can be overcome by also shaping the kinetic energy of the system. This process is often referred to as *total energy-shaping*. Some approaches that perform this process are the so-called interconnection and damping assignment (IDA) PBC [10] and the PID-PBC [11] method. Unfortunately, the former leads to partial differential equations that seem unfeasible to solve for soft robots, while the latter imposes conditions that are not satisfied by these kinds of systems.

IV. Examples of Regulators

We introduce four choices of functions $\Phi(q_{\mathtt{a}})$ and $\psi(\dot{q}_{\mathtt{a}})$ resulting in as many regulators. This should provide an idea of the generality of the control strategy (18) and how this can be specialized to implement essentially different controllers with similar stabilization properties. We use the symbol u instead of τ to present the different control architectures.

Define the error $\tilde{q}_{\mathtt{a}} := q_{\mathtt{a}} - q_{\mathtt{a}_\star}$, let $\varphi : \mathbb{R}^m \to \mathbb{R}$ be a function such that

$$\frac{\partial \varphi(q_{\mathtt{a}})}{\partial q_{\mathtt{a}}} = \left.\left(\frac{\partial V}{\partial q_{\mathtt{a}}}\right)\right|_{q_{\mathtt{u}} = q_{\mathtt{u}_\star}},$$

and define

$$\kappa := \left(\frac{\partial V}{\partial q_{\mathtt{a}}}\right)_\star .$$

Consider the positive gains $K_{\mathtt{P}}, K_{\mathtt{D}}, \alpha, \rho, \alpha_{\mathtt{d}}$, and $\rho_{\mathtt{d}}$. Hence, we can introduce the following controllers:

(i) by selecting $\Phi(q_{\mathtt{a}})$ as in (16) and

$$\psi_{\mathtt{PD}}(\dot{q}_{\mathtt{a}}) = K_{\mathtt{D}}\dot{q}_{\mathtt{a}},$$

we get

$$u_{\mathtt{PD}} = \kappa - K_{\mathtt{P}}\tilde{q}_{\mathtt{a}} - K_{\mathtt{D}}\dot{q}_{\mathtt{a}}. \quad (19)$$

Moreover, (6) reduces to (17).

(ii) by considering

$$\Phi_{\mathtt{PD}_+}(q_{\mathtt{a}}) = -\varphi(q_{\mathtt{a}}) + \frac{1}{2}\tilde{q}_{\mathtt{a}}^\top K_{\mathtt{P}}\tilde{q}_{\mathtt{a}}$$

and $\psi = \psi_{\mathtt{PD}}$, we obtain

$$u_{\mathtt{PD}_+} = \frac{\partial \varphi(q_{\mathtt{a}})}{\partial q_{\mathtt{a}}} - K_{\mathtt{P}}\tilde{q}_{\mathtt{a}} - K_{\mathtt{D}}\dot{q}_{\mathtt{a}}. \quad (20)$$

Moreover, (6) takes the form

$$\left(H''_{\mathtt{ua}}\right)^\top \left(H''_{\mathtt{uu}}\right)^{-1} H''_{\mathtt{ua}} - H''_{\mathtt{aa}} \prec K_{\mathtt{P}} - \left(\frac{\partial^2 \varphi}{\partial q_{\mathtt{a}}^2}\right)_\star .$$

(iii) the selection[3]

$$\Phi_{\mathtt{sat}}(q_{\mathtt{a}}) = -q_{\mathtt{a}}^\top \kappa + \frac{\alpha}{\rho}\ln\left(\cosh(\rho\tilde{q}_{\mathtt{a}})\right)$$

$$\psi_{\mathtt{sat}}(\dot{q}_{\mathtt{a}}) = \alpha_{\mathtt{d}}\tanh(\rho_{\mathtt{d}}\dot{q}_{\mathtt{a}})$$

yields

$$u_{\mathtt{sat}} = \kappa - \alpha\tanh(\rho\tilde{q}_{\mathtt{a}}) - \alpha_{\mathtt{d}}\tanh(\rho_{\mathtt{d}}\dot{q}_{\mathtt{a}}). \quad (21)$$

Furthermore, (6) takes the form

$$\frac{\left(H''_{\mathtt{ua}}\right)^2}{H''_{\mathtt{uu}}} - H''_{\mathtt{aa}} < \alpha\rho. \quad (22)$$

(iv) the choice

$$\Phi_{\mathtt{exp}}(q_{\mathtt{a}}) = -\varphi(q_{\mathtt{a}}) + K_{\mathtt{P}}\left(\mathrm{e}^{\tilde{q}_{\mathtt{a}}} + \mathrm{e}^{-\tilde{q}_{\mathtt{a}}}\right)$$

and $\psi = \psi_{\mathtt{PD}}$ results in

$$u_{\mathtt{exp}} = \frac{\partial \varphi(q_{\mathtt{a}})}{\partial q_{\mathtt{a}}} - K_{\mathtt{P}}\left(\mathrm{e}^{\tilde{q}_{\mathtt{a}}} - \mathrm{e}^{-\tilde{q}_{\mathtt{a}}}\right) - K_{\mathtt{D}}\dot{q}_{\mathtt{a}}. \quad (23)$$

Moreover, (6) takes the form

$$\left(H''_{\mathtt{ua}}\right)^\top \left(H''_{\mathtt{uu}}\right)^{-1} H''_{\mathtt{ua}} - H''_{\mathtt{aa}} \prec 2K_{\mathtt{P}} - \left(\frac{\partial^2 \varphi}{\partial q_{\mathtt{a}}^2}\right)_\star . \quad (24)$$

Note that the four choices of $\Phi(q_{\mathtt{a}})$ above presented satisfy (5). Furthermore, $u_{\mathtt{PD}}$ is a classical PD controller, $u_{\mathtt{PD}_+}$ is a PD controller with a compensation term, $u_{\mathtt{exp}}$ is a completely

[3]We consider the case $m = 1$ for simplicity. We refer the reader to [23] for more details about the case $m > 1$.

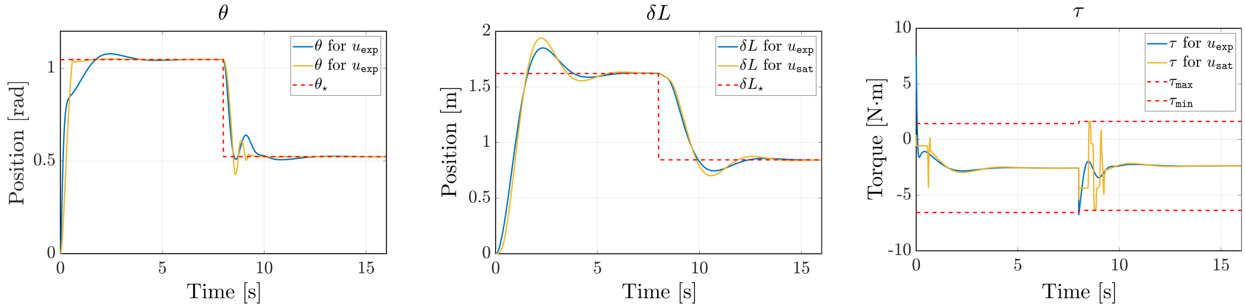

Fig. 1. Simulation results obtained for $u_{\mathtt{exp}}$ and $u_{\mathtt{sat}}$ depicted in blue and yellow, respectively. The plots show the evolution of the configuration variables and the control law, where the desired values and saturation limits are represented by dashed red lines.

nonlinear controller, and $u_{\mathtt{sat}}$ is a saturated controller. In particular, $u_{\mathtt{sat}} \in [\kappa - (\alpha + \alpha_{\mathrm{d}}), \kappa + \alpha + \alpha_{\mathrm{d}}]$.

V. Simulations

In this section, we apply the results of Lemma 1 to stabilize three soft robots. Note that the proposed strategies are applicable to robots with any number of degrees of freedom. However, we focus on simple low-dimensional examples to investigate the evolution of the relevant variables.

A. Constant curvature segment with variable length

Consider a robot consisting of a constant curvature segment with variable length. This system can be represented by (4), with[4]

$$q = \begin{bmatrix} \theta \\ \delta L \end{bmatrix} = \begin{bmatrix} q_{\mathrm{a}} \\ q_{\mathrm{u}} \end{bmatrix}, \; D = \beta \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix},$$

$V_{\mathrm{g}}(q) = -mg\,(q_{\mathrm{u}} + L_0)\,(q_{\mathrm{a}} - \sin q_{\mathrm{a}})\,/q_{\mathrm{a}}^2$, $V_{\mathrm{e}}(q) = k_{\mathrm{aa}} q_{\mathrm{a}}^2/2 + k_{\mathrm{uu}} q_{\mathrm{u}}^2/2$, where θ represents the curvature; δL the length variation; L_0 the length at rest; m the mass of the robot; and g the gravitational acceleration. Moreover, β k_{uu}, and k_{aa} are positive parameters. Note that Assumptions 1 and 2 are satisfied. Moreover, the assignable equilibria for this system can be parameterized in terms of the desired q_{a}, i.e., given $q_{\mathrm{a}_\star}$, we have $q_{\mathrm{u}_\star} = mg\,(q_{\mathrm{a}_\star} - \sin q_{\mathrm{a}_\star})\,/\,\left(k_{\mathrm{uu}} q_{\mathrm{a}_\star}^2\right)$. For this system, we have $\left(\frac{\partial^2 V}{\partial q_{\mathrm{u}}^2}\right)_\star = k_{\mathrm{uu}}$. Accordingly, Assumption 3 holds. Moreover,

$$\varphi(q_{\mathrm{a}}) = -\frac{mg}{q_{\mathrm{a}}^2}\,(q_{\mathrm{u}_\star} + L_0)\,(q_{\mathrm{a}} - \sin q_{\mathrm{a}}) + \frac{1}{2} k_{\mathrm{aa}} q_{\mathrm{a}}^2.$$

To corroborate the effectiveness of the control approach, we consider that the system starts at rest, i.e., $q_0 = \mathbf{0}$. Then, the control objective is to stabilize the system at $q_{\mathrm{a},1_\star} = \frac{\pi}{3}$. Once the system reaches the desired configuration, it is steered to the new desired configuration $q_{\mathrm{a},2_\star} = \frac{\pi}{6}$.

We consider $m = 1$[kg]; $L_0 = 1$[m]; $g = 9.81[\frac{\mathrm{m}}{\mathrm{s}^2}]$; $k_{\mathrm{uu}} = k_{\mathrm{aa}} = 1[\frac{\mathrm{N \cdot m}}{\mathrm{rad}}]$; and $\beta = 0.5[\frac{\mathrm{N \cdot m \cdot s}}{\mathrm{rad}}]$ for simulation purposes. Hence, for $q_{\mathrm{a},1_\star}$ we get

$$q_{\mathrm{u},1_\star} = 1.6207, \kappa = -2.5628, \left(\frac{\partial^2 \varphi}{\partial q_{\mathrm{a}}^2}\right)_\star = 2.2327,$$

while for $q_{\mathrm{a},2_\star}$ we have

$$q_{\mathrm{u},2_\star} = 0.8444, \kappa = -2.3694, \left(\frac{\partial^2 \varphi}{\partial q_{\mathrm{a}}^2}\right)_\star = 1.4635.$$

We test the performance of $u_{\mathtt{sat}}$, given in (21), and $u_{\mathtt{exp}}$, provided in (23). Therefore, (22) becomes $-0.3351 < \alpha\rho$ and $0.9957 < \alpha\rho$ for $q_{\mathrm{a},1_\star}$ and $q_{\mathrm{a},2_\star}$, respectively. Thus, $\alpha\rho$ must be greater than 0.9957. Furthermore, (24) reduces to

$$-0.3351 < 2K_{\mathrm{P}} - 2.2327, 0.9957 < 2K_{\mathrm{P}} - 1.4635$$

for $q_{\mathrm{a},1_\star}$ and $q_{\mathrm{a},2_\star}$, respectively. Accordingly, K_{P} must be greater than 1.2296. We select the gains $K_{\mathrm{P}} = 3$, $K_{\mathrm{D}} = 1$, $\alpha = 3$, $\rho = 50$, $\alpha_{\mathrm{d}} = 1$, and $\rho_{\mathrm{d}} = 20$. The results are shown in Fig. 1, where we observe that both controllers achieve the control task. However, for the selected gains, $u_{\mathtt{sat}}$ exhibits a better performance.

B. The soft inverted pendulum with affine curvature

In this section, we stabilize a soft inverted pendulum using controllers of the form (18). To this end, we consider the dynamic model provided in [24]. Therefore, we refer the reader to the mentioned reference for further details on the model. The configuration variables for this system are $q = [\theta_0 \; \theta_1]^\top$, which approximate the curvature at each point of the pendulum's main axis. This system can be represented as in (1) with[5]

$$\mathcal{V}_{\mathrm{e}} = \frac{1}{2} q^\top k \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} q, \quad \mathcal{D} = \beta \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix},$$

where k and β are positive constant parameters. The control objective is to stabilize the pendulum at its upward configuration, i.e., at $q_\star = (0, 0) \in \mathcal{E}$. Note that the desired configuration is an open-loop equilibrium for the system. Therefore, $\kappa = 0$ and $\tilde{q}_{\mathrm{a}} = q_{\mathrm{a}}$. Furthermore, Assumption 1 is satisfied and the transformation (3), with

$$T = \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix},$$

ensures that $A = [1 \; 0]^\top$. Moreover,

$$\left(\frac{\partial^2 V_{\mathrm{e}}}{\partial q^2}\right)_\star = k \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}, \left(\frac{\partial^2 V_{\mathrm{g}}}{\partial q^2}\right)_\star = \frac{mgL}{30} \begin{bmatrix} -34 & 33 \\ 33 & -36 \end{bmatrix}, \tag{25}$$

where m, g, L are the pendulum's mass, the gravity acceleration, and the pendulum's length, respectively. From (25), we conclude that Assumption 2 holds. We consider that the parameters m, g, L, k are such that $10k > mgL$ but $\left(\frac{\partial^2 V}{\partial q^2}\right)_\star$ is not positive definite. Thus, Assumption 3 is satisfied but $q_\star$ is not a stable equilibrium for the open-loop system.

We consider $m = 1$[kg], $L = 1$[m], $g = 9.81[\frac{\mathrm{m}}{\mathrm{s}^2}]$, $k = 1[\frac{\mathrm{N \cdot m}}{\mathrm{rad}}]$, and $\beta = 0.1[\frac{\mathrm{N \cdot m \cdot s}}{\mathrm{rad}}]$ for simulation purposes. Moreover, the controllers (19), (20), and (21) can be expressed in terms of the original coordinates through the

[4] We omit $M(q)$ due to space constraints.

[5] Due to space limitations, we omit the expressions of $\mathcal{M}(q)$ and $\mathcal{V}_{\mathrm{g}}(q)$.

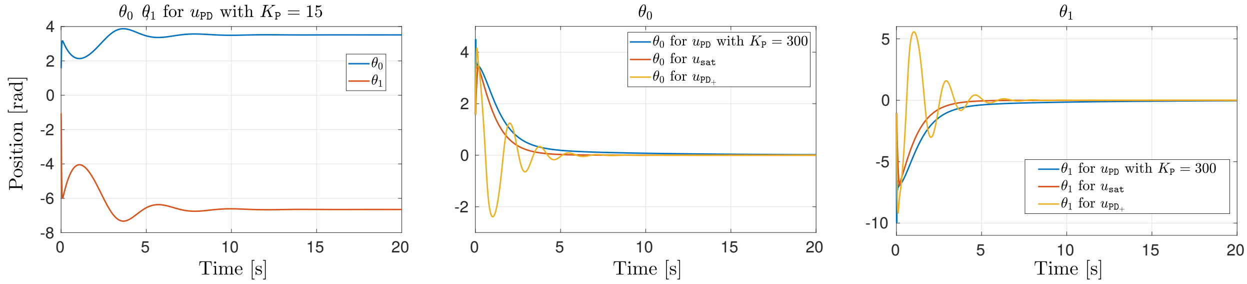

Fig. 2. Simulation results for the initial configuration $q = [\frac{\pi}{2} \;\; -\frac{\pi}{3}]^\top$. The plots show the evolution of the configuration variables for u_{PD} with $K_{\mathrm{P}} = 15$ and $K_{\mathrm{D}} = 0.005$; u_{PD} with $K_{\mathrm{P}} = 300$ and $K_{\mathrm{D}} = 0.005$; u_{PD_+} with $K_{\mathrm{P}} = 15$ and $K_{\mathrm{D}} = 0.005$; and u_{sat} with $\alpha = 4$, $\rho = 50$, $\alpha_{\mathrm{d}} = 0.5$, and $\rho_{\mathrm{d}} = 10$.

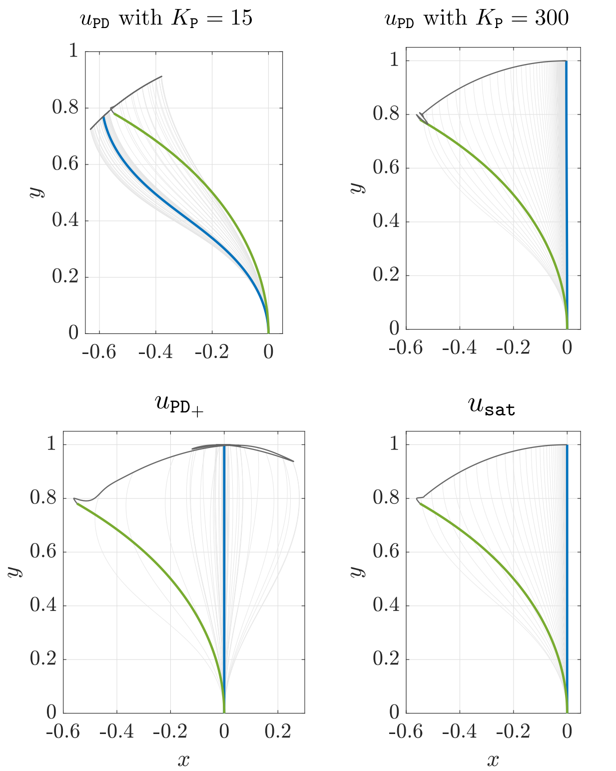

Fig. 3. Evolution of the soft inverted pendulum in the cartesian plane for different controllers.

expression $q_{\mathrm{a}} = \theta_0 + \frac{1}{2}\theta_1$. Moreover, (17) and (22) become $107.7920 < K_{\mathrm{P}}$ and $107.7920 < \alpha\rho$, respectively. Fig. 2 shows the simulation results considering the initial conditions $q_0 = [\frac{\pi}{2} \;\; -\frac{\pi}{3}]^\top$, $p_0 = \mathbf{0}$ and different controllers. Similarly, Fig. 3 shows the behavior of the robot in the cartesian plane, where the green line denotes the initial position and the blue line the final configuration. In particular, we observe in the leftt-hand plot of Fig. 2 that u_{PD} fails to stabilize the system for $K_{\mathrm{P}} < 107.7920$. To solve this problem, we drastically increase the proportional gain, as is illustrated in the middle and right-hand plots of Fig. 2. Nonetheless, this approach yields important peaks in the control signal, making it infeasible from a practical perspective. In order to avoid high gains, we use u_{PD_+} which includes a compensation term. Remarkably, this controller stabilizes the system at the desired equilibrium with the same gains as the PD used to obtain the left-hand plot of Fig. 2. Finally, we observe in Figs. 2 and 3 that u_{sat} has a similar performance than the high-gain PD controller. However, u_{sat} ensures that the control signal is constrained to $\pm 4.5[\mathrm{N}\cdot\mathrm{m}]$. Hence, the saturated controller has a better performance as it stabilizes the system without exhibiting an oscillatory behavior while keeping the control signals constrained. Fig. 4 shows the simulation results considering the initial conditions $q_0 = [\frac{\pi}{4} \; -\frac{\pi}{4}]^\top$ and $p_0 = \mathbf{0}$. In this case, we only consider u_{PD_+} and u_{sat}. We propose the same control gains as in the previous case. Notably, the system converges much slower with u_{sat}. Hence, comparing the performance of both controllers under both sets of initial conditions, we conclude that the saturated controller is more sensitive to the initial configuration.

C. Three-segments robot

We consider the 3D model of a soft robot consisting of three segments with variable lengths. We assume that the length variation is the unactuated coordinate for each segment. The configuration variables are given by the parameterization proposed in [25], i.e., $q_i = [\Delta_{\mathrm{x}_i}, \Delta_{\mathrm{y}_i}, \delta L_i]^\top$ with $i \in \{1,2,3\}$. Moreover, the robot is actuated through a pneumatic device, resulting in a nonconstant input matrix. In this case, Assumption 1 is satisfied by reordering the configuration variables and considering a modulation of the form $\tau = \mathcal{A}_{\mathrm{a}}^{-1}(q)u$, where $\mathcal{A}_{\mathrm{a}}$ is the part of the input matrix associated with the actuated variables, i.e., Δ_{x_i} and Δ_{y_i}. We consider linear elasticity with a diagonal stiffness matrix. Hence, Assumption 2 holds. In this case, $\left(\frac{\partial^2 V}{\partial q^2}\right)_\star$ is always positive definite, which guarantees that Assumption 3 is satisfied. We implement a PD controller and an exponential controller of the form $u_{\mathrm{exp}} = \kappa - K_{\mathrm{P}}\left(\mathrm{e}^{\tilde{q}_{\mathrm{a}}} - \mathrm{e}^{-\tilde{q}_{\mathrm{a}}}\right) - K_{\mathrm{D}}\dot{q}_{\mathrm{a}}$. Moreover, we consider that the length at rest is $0.2[\mathrm{m}]$ for each segment; the mass is $0.3[\mathrm{kg}]$; the stiffness coefficients are $1[\frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{rad}}]$ for the actuated variables and $10[\frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{rad}}]$ for the unactuated variables; and the damping matrix is diagonal with damping coefficients $0.1[\frac{\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}}{\mathrm{rad}}]$. Fig. 5 shows the simulation results considering the initial configuration $q = \mathbf{0}$ and the desired configuration $q_\star = (2, 2, 0.136, -1, -2-0.102, 1, 1, 0.066)^\top$. While both controllers stabilize the system, u_{PD} is physically infeasible because of the length variation. Moreover, u_{exp} results in faster convergence and fewer oscillations than u_{PD}, which is more notorius in the unactuated coordinates.

VI. CONCLUDING REMARKS

This paper introduced a general class of control strategies for soft robots having a constant input matrix, and elasticity dominating gravity in the unactuated coordinates. The resulting family of controllers encompasses PD regulators

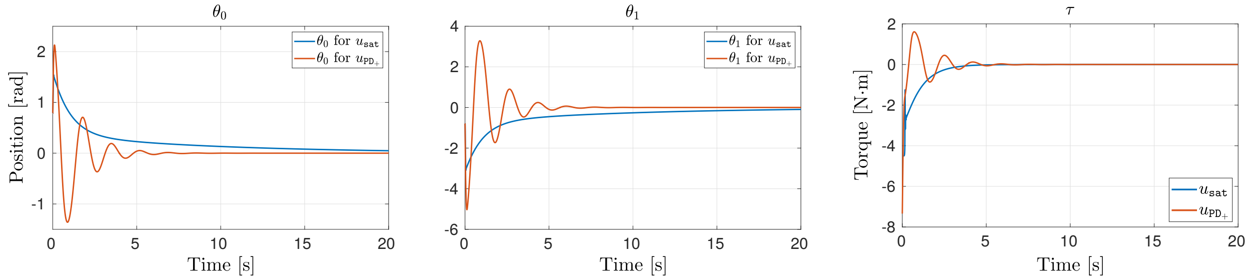

Fig. 4. Simulation results for the initial configuration $q = [\frac{\pi}{4} \; -\frac{\pi}{4}]^\top$. The plots depict the evolution of the configuration variables and the control signal for $u_{\mathtt{PD}_+}$ and $u_{\mathtt{sat}}$. The control gains are $K_{\mathtt{P}} = 15$, $K_{\mathtt{D}} = 0.005$, $\alpha = 4$, $\rho = 50$, $\alpha_{\mathtt{d}} = 0.5$, and $\rho_{\mathtt{d}} = 10$.

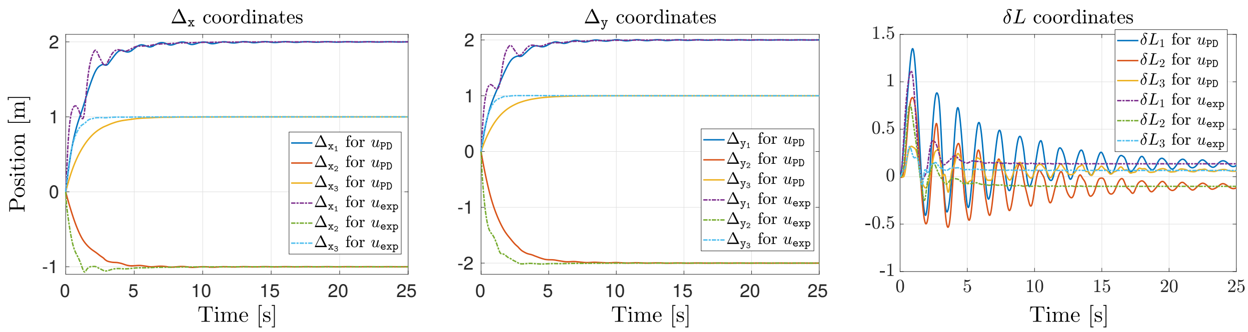

Fig. 5. Simulation results for the initial configuration $q = \mathbf{0}$. The plots depict the evolution of the configuration variables. The gains are $K_{\mathtt{P}_1} = \mathrm{diag}(15, 15)$, $K_{\mathtt{D}_1} = \mathrm{diag}(20, 20)$, $K_{\mathtt{P}_2} = K_{\mathtt{P}_3} = \mathrm{diag}(10, 10)$, and $K_{\mathtt{D}_2} = K_{\mathtt{D}_3} = \mathrm{diag}(15, 15)$ for $u_{\mathtt{PD}}$; and $K_{\mathtt{P}} = I_6$ and $K_{\mathtt{D}} = 1.5 I_6$ for $u_{\mathtt{exp}}$.

and more elaborated nonlinear feedback rules, e.g., saturated control laws and partially closed-loop gravity compensations. Future work will be devoted to exploring the space of possible choices of $\Phi(q_{\mathtt{a}})$ to shape the transient behavior and ensure the robustness of the closed-loop.

References

[1] C. Della Santina, M. G. Catalano, and A. Bicchi, "Soft robots," *Encyclopedia of Robotics*, 2020.

[2] C. Della Santina, C. Duriez, and D. Rus, "Model based control of soft robots: A survey of the state of the art and open challenges," *arXiv preprint arXiv:2110.01358*, 2021.

[3] A. Macchelli, Y. Le Gorrec, *et al.*, "Control design for linear port-Hamiltonian boundary control systems: An overview," in *Stabilization of Distributed Parameter Systems: Design Methods and Applications*, G. Sklyar and A. Zuyev, Eds. Springer International Publishing, 2021, pp. 57–72.

[4] R. Rashad, F. Califano, *et al.*, "Port-Hamiltonian modeling of ideal fluid flow: Part I. foundations and kinetic energy," *Journal of Geometry and Physics*, vol. 164, p. 104201, 2021.

[5] S. Grazioso, G. Di Gironimo, and B. Siciliano, "A geometrically exact model for soft continuum robots: The finite element deformation space formulation," *Soft robotics*, vol. 6, no. 6, pp. 790–811, 2019.

[6] F. Renda, C. Armanini, *et al.*, "A geometric variable-strain approach for static modeling of soft manipulators with tendon and fluidic actuation," *IEEE Robotics and Automation Letters*, vol. 5, no. 3, pp. 4006–4013, 2020.

[7] V. Duindam, A. Macchelli, *et al.*, *Modeling and control of complex physical systems: the port-Hamiltonian approach*. Springer Science & Business Media, 2009.

[8] R. Ortega, A. Loría, *et al.*, *Passivity-Based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications*. Communications and Control Engineering. Springer Verlag, London, 1998.

[9] A. J. van der Schaft, *L_2-Gain and Passivity techniques in nonlinear control*, 3rd ed. Berlin: Springer, 2016.

[10] G. Viola, R. Ortega, *et al.*, "Total energy shaping control of mechanical systems: simplifying the matching equations via coordinate changes," *IEEE Transactions on Automatic Control*, vol. 52, no. 6, pp. 1093–1099, 2007.

[11] R. Ortega, J. G. Romero, *et al.*, *PID Passivity-Based Control of Nonlinear Systems with Applications*. John Wiley & Sons, 2021.

[12] V. Falkenhahn, T. Mahl, *et al.*, "Dynamic modeling of bellows-actuated continuum robots using the Euler–Lagrange formalism," *IEEE Transactions on Robotics*, vol. 31, no. 6, pp. 1483–1496, 2015.

[13] I. S. Godage, R. J. Webster, and I. D. Walker, "Center-of-gravity-based approach for modeling dynamics of multisection continuum arms," *IEEE Transactions on Robotics*, vol. 35, no. 5, pp. 1097–1108, 2019.

[14] H.-T. D. Chun, N. K. Taylor, and A. A. Stokes, "Energy-based abstraction for soft robotic system development," *Advanced Intelligent Systems*, p. 2000264, 2021.

[15] H.-S. Chang, U. Halder, C.-H. Shih, *et al.*, "Energy shaping control of a cyberoctopus soft arm," in *2020 59th IEEE Conference on Decision and Control (CDC)*. IEEE, 2020, pp. 3913–3920.

[16] H.-S. Chang, U. Halder, E. Gribkova, *et al.*, "Controlling a cyberoctopus soft arm with muscle-like actuation," in *2021 60th IEEE Conference on Decision and Control (CDC)*, 2021, pp. 1383–1390.

[17] E. Franco, A. Garriga-Casanovas, *et al.*, "Model based adaptive control for a soft robotic manipulator," in *2019 IEEE 58th Conference on Decision and Control (CDC)*. IEEE, 2019, pp. 1019–1024.

[18] E. Franco, A. Garriga-Casanovas, and A. Donaire, "Energy shaping control with integral action for soft continuum manipulators," *Mechanism and Machine Theory*, vol. 158, p. 104250, 2021.

[19] E. Franco and A. Garriga-Casanovas, "Energy-shaping control of soft continuum manipulators with in-plane disturbances," *The International Journal of Robotics Research*, vol. 40, no. 1, pp. 236–255, 2021.

[20] P. S. Gandhi, P. Borja, and R. Ortega, "Energy shaping control of an inverted flexible pendulum fixed to a cart," *Control Engineering Practice*, vol. 56, pp. 27–36, 2016.

[21] A. Mattioni, Y. Wu, *et al.*, "Modelling and control of an IPMC actuated flexible structure: a lumped port Hamiltonian approach," *Control Engineering Practice*, vol. 101, p. 104498, 2020.

[22] P. Pustina, C. Della Santina, and A. De Luca, "Feedback regulation of elastically decoupled underactuated soft robots," *IEEE Robotics and Automation Letters*, vol. 7, no. 2, pp. 4512–4519, 2022.

[23] T. C. Wesselink, P. Borja, and J. M. A. Scherpen, "Saturated control without velocity measurements for planar robots with flexible joints," in *2019 IEEE 58th Conference on Decision and Control (CDC)*. IEEE, 2019, pp. 7093–7098.

[24] C. Della Santina, "The soft inverted pendulum with affine curvature," in *2020 59th IEEE Conference on Decision and Control (CDC)*, 2020, pp. 4135–4142.

[25] C. Della Santina, A. Bicchi, and D. Rus, "On an improved state parametrization for soft robots with piecewise constant curvature and its use in model based control," *IEEE Robotics and Automation Letters*, vol. 5, no. 2, pp. 1001–1008, 2020.