4.5 The Traditional Square of Opposition

Recall that earlier we sided with Boole and against Aristotle, and claimed that universal propositions do NOT have existential import. For instance, if I say, "All unicorns are mammals," this does not commit us to the conclusion that there ARE such things as unicorns. And if I say, "No lazy students in my class are ones who will pass" does not imply that there ARE any lazy students in my class.

…But what if universal propositions—ones beginning with "all" or "no"—DID have existential import? Well, in that case, we could make a lot more valid inferences than we've been able to make so far.

Previously, under the Boolean interpretation, we were only able to make a few inferences; namely, certain inferences using **contradiction**, **conversion**, **obversion**, and **contraposition**. If we accept Aristotle's view that universal propositions about really existing things DO have existential import, we can add three more things to that list:

(1) Contrary: A relation that holds only between the "A" (All S are P) and "E" (No S are P") propositions, which says that, **if one is true, the other must be false**. Or, in other words, **they cannot BOTH be true**.

For instance, if "All kittens are cute" is true, then "No kittens are cute" MUST be false. However, it is still possible for contraries to BOTH be false. For example, "All humans are female" AND "No humans are female" are contraries, but they are both false.

(2) Subcontrary: A relation that holds only between the "I" (Some S are P) and "O" (Some S are not P) propositions, which says that, **if one is false, the other must be true**. Or, in other words, **they cannot BOTH be false**.

For instance, if "Some humans are able to fly" is false, then "Some humans are NOT able to fly" MUST be true. However, it is still possible for subcontraries to BOTH be true. For example, "Some humans are female" AND "Some humans are not female" are both true.

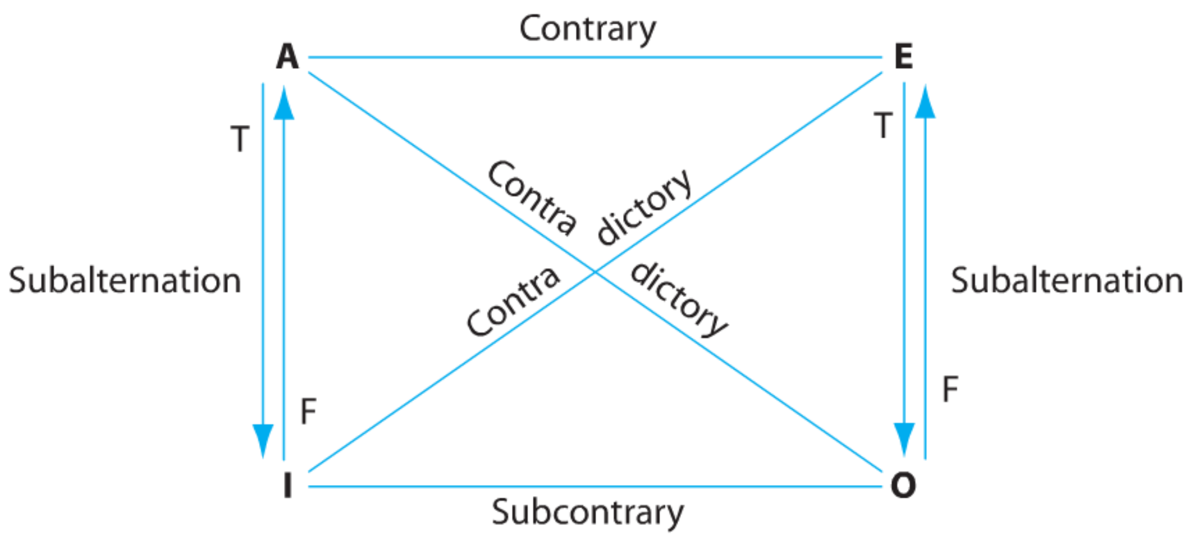
(3) Subalternation: A relation that holds only between "A" (All S are P) and "I" (Some S are P) propositions. It ALSO holds between "E" (No S are P) and "O" (Some S are not P) propositions. The relation has two components:

(a) If the UNIVERSAL proposition is TRUE, then the PARTICULAR proposition must ALSO be true.
(b) On the other hand, if the PARTICULAR proposition is FALSE, then the UNIVERSAL proposition must ALSO be false.

Put very simply, **truth flows from universal statements to particular ones**, while **falsity flows from particular statements to universal ones** (in the diagram of the square below, you can remember this as truth flowing "from above" and falsehood bubbling up "from below").

For instance, if "ALL pizza is delicious" is true, we know that "SOME pizza is delicious" is also true. Similarly, if "SOME people are able to fly" is false, then we know that "ALL people are able to fly" must also be false.

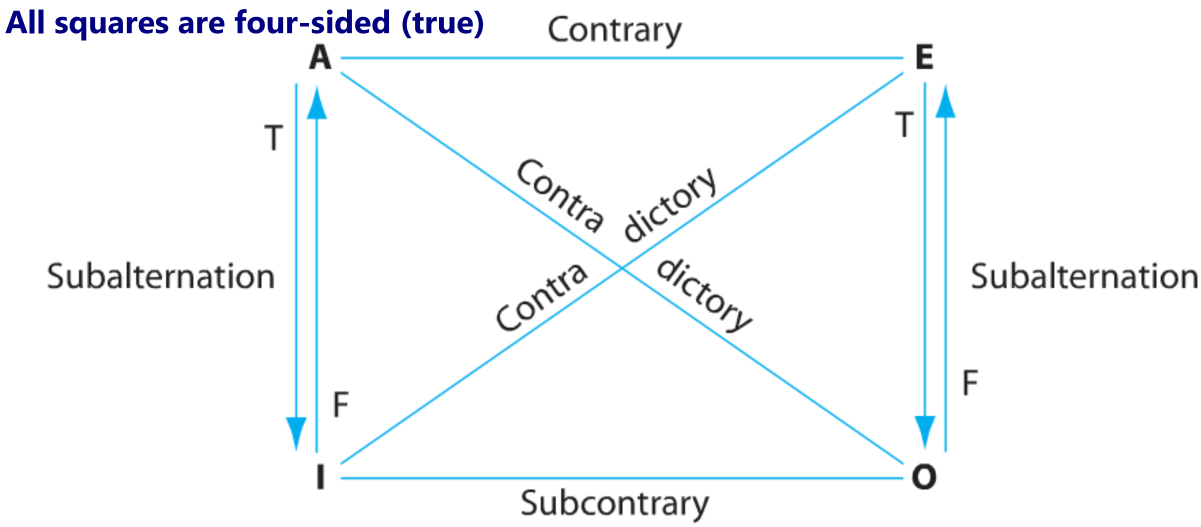
These three new inferences (along with contradiction) are represented here by what is called "**The Traditional Square of Opposition**". The blue lines represent relations between each of the four kinds of categorical proposition (A, E, I, and O).



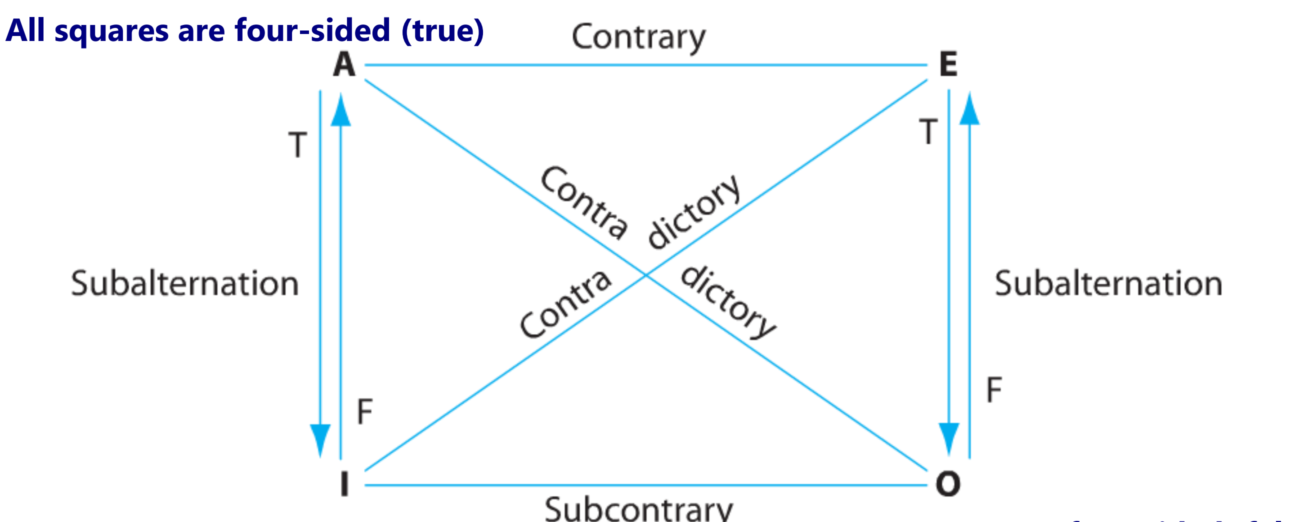
<u>Immediate Inferences:</u> The traditional square of opposition depicts the relation of contradiction, as well as the three new relations we just learned. The square is a useful tool: When we are given ONE single proposition and its truth value (i.e., whether it is true or false), we can then plug it into the square and figure out the truth values of a bunch of other propositions as well. For instance, consider:

- **<u>Example #1:</u>** "All squares are four-sided." This is clearly true.

<u>Step 1: What kind of proposition is it?</u> It is an "A" proposition (All S are P). So, we'll write it in the upper-left corner of the traditional square of opposition below (and since it is true, we can write "true" next to it).



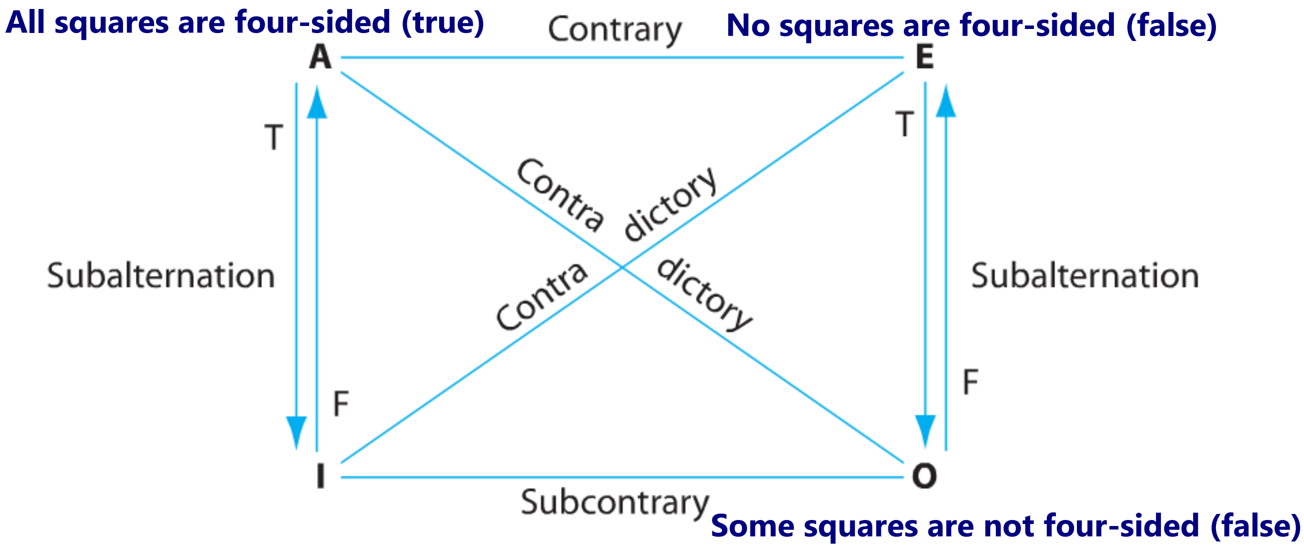
Step 2: Determining the contradictory: The next step should always be to determine what the contradictory is. Remember that contradictories always have OPPOSITE truth values. Now, since we're dealing with an "A" proposition, if we follow the diagonal line we see that its contradictory is an "O" proposition (Some S are not P). Since the "A" proposition was **true**, the "O" proposition must be **false**. Let's fill that in:



Step 3: Subalternation: Next, we can determine the truth value of the "E" proposition in one of two ways:

(a) *Method #1:* The **contrary** relation holds between "A" and "E" propositions. This relation states that, when one of them is true, the other MUST be false. So, since the "A" proposition is true, the "O" proposition must be false.
(b) *Method #2*: For the **subalternation** relation, falsity flows UPWARD (from below), from "O" to "E". So, since the "O" proposition is false, the "E" proposition (No squares are four-sided) must also be false.

Now we can fill this in on the traditional square:



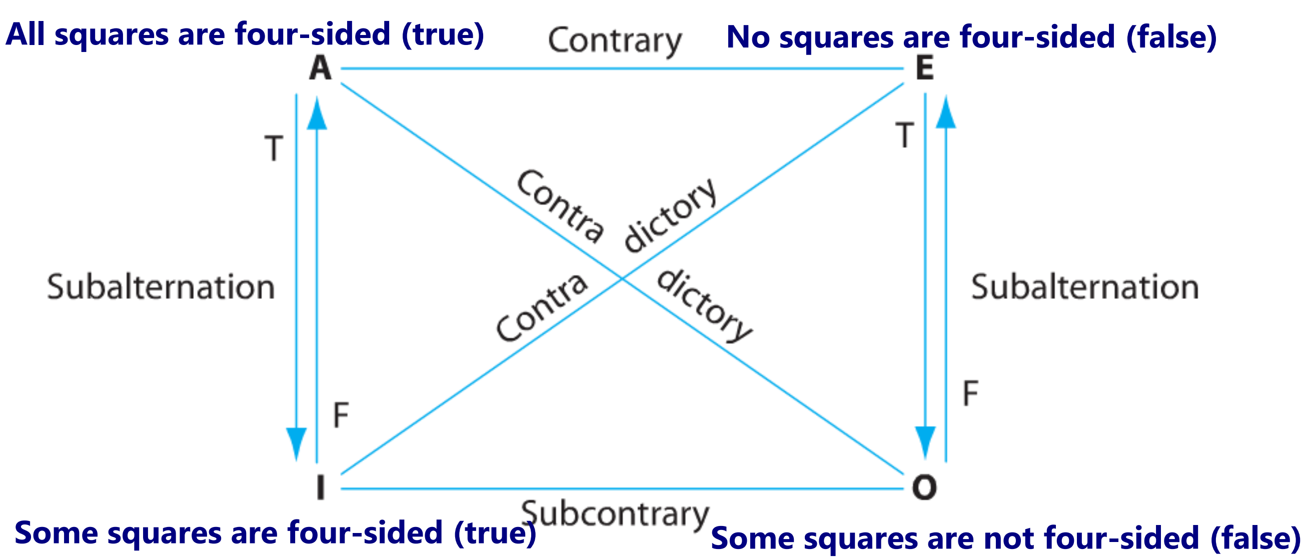
Step 4: Subcontrary and/or Contradictory: Finally, we can determine the truth value of the "I" proposition in any one of the following three ways:

(a) *Method #1:* Since "A" and "I" are subalternates, and the "A" proposition is true, truth flows DOWN (from above). So, the "I" proposition (Some squares are four-sided) must also be true.

(b) *Method #2:* Since "E" and "I" are contradictories, and the "E" proposition is false, it MUST be the case that the "I" proposition (Some squares are four-sided) is true.

(c) *Method #3:* Since "O" and "I" are subcontraries—such that ONLY ONE of them can be false—and we already know that the "O" proposition is false, it MUST be the case that the "I" proposition (Some squares are four-sided) is true.

In short, any one of these 3 methods for determining the truth value of the "I" proposition tells us that it is TRUE. We may complete the traditional square as follows:



Testing Inferences: We just used the traditional square in order to immediately draw several inferences from one proposition and its truth value. In the example above, given the fact that "All squares are four-sided" is true, we were able to immediately infer three other facts: (1) "No squares are four-sided" is false, (2) "Some squares are four-sided" is true, and (3) "Some squares are not four-sided" is false.

But, we can ALSO use the square in order to TEST inferences to see if they are valid or invalid (in much the same way as we did so in the unit on conversion, obversion, and contraposition).

Example #1: For instance, imagine that someone claims the following:

(1) It is not the case that some cockroaches are adorable.
(2) Therefore, it is not the case that all cockroaches are adorable.

Is this a valid inference? Does the conclusion follow from the premise? We start by writing down the FORM of this inference as follows:

(1) "Some S are P" is false.	("I" = false)
(2) Therefore, "All S are P" is false.	(Therefore, "A" = false)

So this inference claims that "I" and "A" propositions are connected in some way. Looking at our chart of the traditional square, we see that the relation which connects "I" propositions to "A" is **subalternation**. According to subalternation, if "I" is false, then "A" must also be false. But, this is exactly what the inference says. So, this is a **valid inference**.

Example #2: Let's try one more:

(1) No worms are delicious.
(2) Therefore, it is not the case that all worms are delicious.

Simplify this into its form as follows:

(1) "No S are P" is true.	"E" is true.
(2) Therefore, "All S are P" is false.	Therefore, "A" is false.

Looking at the square, we see that "E" and "A" are **contraries**. This means that, when one of them is true, then the other MUST be false. So, since the "E" proposition is true, the "A" proposition MUST be false. So, this is another **valid inference**.

Fallacies: Sometimes, the relations of **contrary**, **subcontrary**, and **subalternation** can be mis-applied in the wrong way in order to get conclusions that do NOT legitimately follow from the premises. In this case the argument is invalid because it commits the fallacy of **illicit contrary**, **illicit subcontrary**, or **illicit subalternation**.

Also: Even though Aristotle allows that universal propositions DO have "existential import," it IS still possible for the **existential fallacy** to be committed. This happens whenever the argument WOULD be valid if the universal premise had existential import—however, the premise is about something that DOES NOT EXIST. Here's an example of an inference that is invalid because it commits the existential fallacy:

(1) All unicorns are magical.
(2) Therefore, some unicorns are magical.

On Aristotle's interpretation, if "All S are P" is true, then "SOME S are P" must also be true—EXCEPT when the subject does not exist. In cases where the subject does not exist, we treat universal propositions just as Boole did; namely, as NOT having existential import. Here is one more example of an inference that commits the existential fallacy:

(1) No married bachelors are eligible men.
(2) Therefore, some married bachelors are not eligible men.

IF premise one had existential import, then this inference WOULD BE valid. And normally, under Aristotle's interpretation, "E" propositions in the form of "No S are P" DO have existential import—but NOT when the subject does not exist. Since there is no such thing as a married bachelor, they do not exist. So, the inference is invalid because it commits the existential fallacy.

Remember: The existential fallacy occurs under the Aristotelian interpretation ONLY WHEN the premise is universal ("A" or "E") and the premise is particular ("I" or "O"), AND the premise is about something that DOES NOT EXIST (e.g., unicorns).

Note: Do homework for section 4.5 at this time.