# WEEK 4

**ANNUITIES: ORDINARY AND DUE**

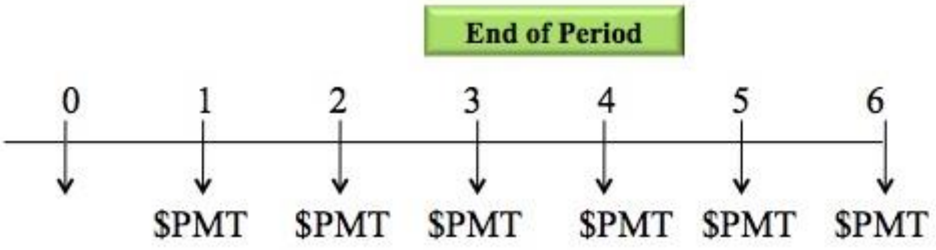
**Defining annuities**

In finance we commonly encounter situations which call for payments of **equal amounts** of cash at regular intervals of time over several time periods.

Any financial contract that calls for **equally spaced and level cash flows** (consecutive, regular, equal payments) over a finite number of periods is called an **annuity**.

**Ordinary annuities**

Most annuities are structured so that cash payments are made or received at the **end** of each period. Because, this is the most common structure these annuities are called **ordinary** annuities. Ordinary annuities are also referred to as **annuity in arrears**.

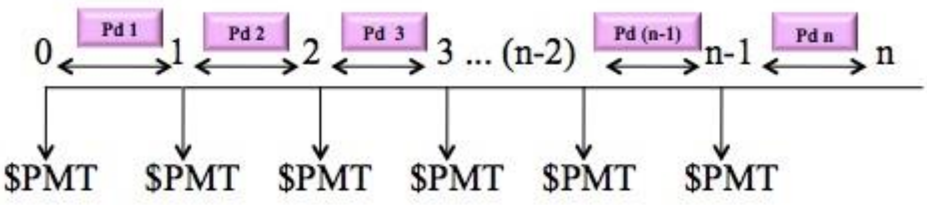



**MPORTANT!! Always assume the ordinary!**

If nothing is said about the timing of cash flows ALWAYS assume that  the cash flow occurs at the END of each period.

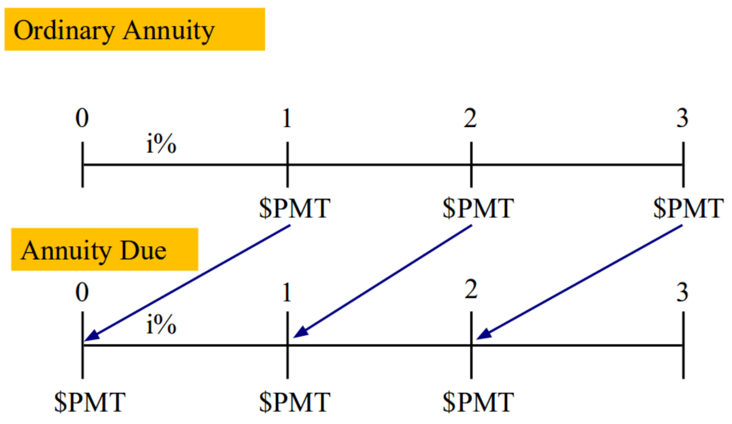
**Annuities due**:

If the cash payments are made or received at the **beginning (start)** of each period then the annuity is referred to as an **annuity due**.




Note: **Due annuities** are also referred to as **annuities in advance**.

**Ordinary annuities vs annuities due**




**Future and present values of an ordinary annuity**

Future value of an ordinary annuity

$$FV = \frac{PMT}{i}\left[(1+i)^n - 1\right]$$

Present value of an ordinary annuity

$$PV = \frac{PMT}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

Where:

FV = the accumulated or future value of the annuity

PMT = the cash flow received/paid under the annuity

n = the time period over which the annuity occurs

i = the per-period interest rate

**Present and future value of an annuity due**

**FV of Annuity Due = FV of Ordinary Annuity × (1+i)**

$$FV = \frac{PMT}{i}\left[(1+i)^n - 1\right] \times (1+i)$$

**PV of Annuity Due = PV of Ordinary Annuity × (1+i)**

$$PV = \frac{PMT}{i}\left[1 - \frac{1}{(1+i)^n}\right] \times (1+i)$$

## APPLICATIONS OF ANNUITIES

### Deferred and equivalent annuities

**Deferred annuities:** Annuities don't always start in the first period of a given timeline (time 0 for an annuity due, time 1 for ordinary annuity). For example, one could start repaying a loan in 5 years' time in the form of equal payments at fixed regular points in time. This, by definition, constitutes an annuity. However, as the payments don't start at the beginning of the timeline, this is called a **deferred** annuity. You could be asked to calculate the PV or FV of such an annuity given the interest rate and payment amount. Extra care needs to be taken here and using a timeline is strongly encouraged.

**Equivalent annuities:** The ordinary annuity formula is also used to evaluate projects with unequal lives. For example, two projects could be generating cash flows over two different time periods-one could have a 5 year timeline while the other could generate cash flows over a 7 year timeline. To compare these two projects the Equivalent Annual Annuity (EAA) method can be used-this is covered in week 5.

### Perpetuities

A perpetuity is similar to an ordinary annuity or annuity due as it involves equally spaced cash flows of same amounts. The major difference is that in the case of a perpetuity these cash flows last **forever (in perpetuity).** For this reason, the PV and FV calculations of a perpetuity will differ from that of an ordinary annuity or annuity due to reflect **indefinite** cash flows.

The PV of an Ordinary Perpetuity is given as follows.

$$PV = \frac{PMT}{i}$$

where:

PMT = the cash flow per period,

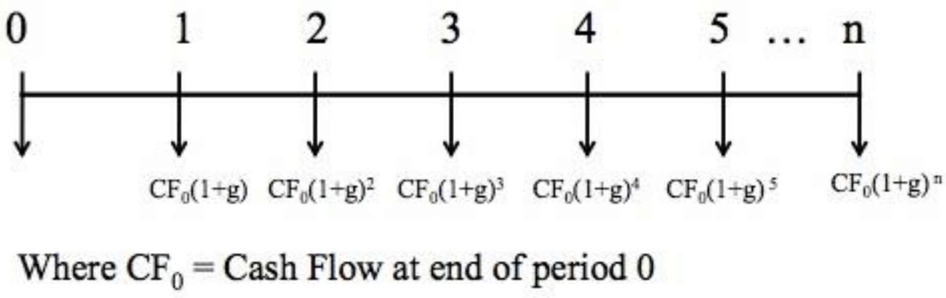
i = the interest rate per period.

**Note:** The formula values cash flows <u>ONE</u> period <u>BEFORE</u> the first cash flow.

## GROWING ANNUITIES AND PERPETUITIES

### Growing annuities

Financial managers may also need to evaluate the value of contracts with cash flows that increase each year at a constant rate. These are called **growing** annuities.




The **present value** of a growing annuity is calculated using this formula

$$PV = \frac{CF_0(1+g)}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]$$

Where

- PV = Present Value of a growing ordinary annuity with n-periods
- $CF_0$ = Cash flow at end of period -0
- i = Discount rate
- g = Constant growth rate per period

Note: The PV can be calculated so long as i > g

**Present value of a growing annuity**

The present value of a **growing annuity** is calculated using this formula:

$$PV = \frac{CF_1}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]$$

$$PV = \frac{CF_0(1+g)}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]$$

Where

PV = Present Value of a growing ordinary annuity with n-periods

$CF_0$ = Cash flow at end of period 0

i = Discount rate