A New Course: Modeling Inhomogeneous Turbulence with a Historical Perspective

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APS Sorting Category: 41. Fluid Dynamics - Education, Outreach, and Diversity

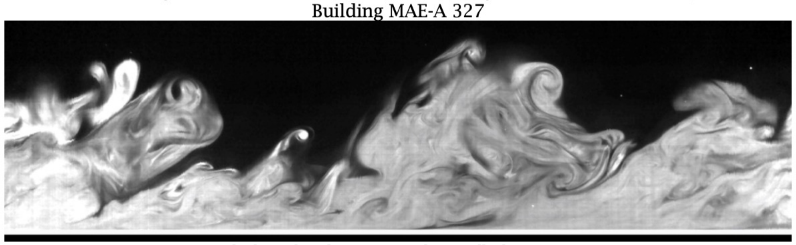
Course Overview

Theoretical Fluid Dynamics and Turbulence Group • Prediction via Theory •

Inhomogeneous Turbulence

A Spring 2022 Special Topic Class for Students who want to Predict Chaotic Flows

EGM6934, Class 19096, 3 Credits, Spring 2022,
Tuesday 5-6 (11:45 AM - 1:40 PM) & Thursday 6 (12:50 PM - 1:40 PM),
Building MAE-A 327



The boundary layer or a Jackson Pollock painting.

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Course Description

- A class that covers in depth concepts of the science and mathematics of turbulence modeling with a historical perspective. Examples are given as much as possible involving contemporary approaches.
- Statistical quantities, averages, correlations, coherence, the Russian school, law of the wall, chaos, compressible NSE, averaging relations, mean kinetic energy, Re stress transport eqn., boundary layer equations, two-dimensional in laminar and turbulent flows, mixing length concepts, Baldwin-Lomax, Cebeci-Smith, ½-equations, one-equation models, Prandtl's model, Spalart-Allmarus, k-ω and k-ε, Boussinesq, nonlinear relations, stress transport models, closure, applications and examples, physical considerations, Morkovin hypothesis, studies in particular flows.
- These topics will be related to turbulent flows that are observed in our daily lives and within various fields of engineering.

Course Material and Notes

- Each class students will receive a set of notes via PDF. These are used in class to discuss the modeling methods.
- There are no required or recommended textbooks for the class.

Grading Policy

- A four-part computer programming project will result in a stand-alone marching boundary layer solver. Students are expected to attend class and actively participate. A very short-term paper on a subject of the students choosing will be written.

Prerequisites

- Graduate class in fluid dynamics and/or turbulence, or permission of the instructor. Some programming knowledge.

- Turbulence modeling classes are now rare
- Science of turbulence modeling is being overcome with empirical nonsense (AI and big data)
- Design a new course to prepare researchers of the future to create physical models for prediction
- Purpose of theory is prediction
- Our graduates enter the research world with a unique perspective of problems with understanding of closure techniques
- Skepticism is central to the class

Course Objectives

A class that covers in depth concepts of the science and mathematics of turbulence modeling with a historical perspective. Examples are given as much as possible involving contemporary approaches. Statistical quantities, averages, correlations, coherence, the Russian school, law of the wall, chaos, compressible NSE, averaging relations, mean kinetic energy, Re stress transport eqn., boundary layer equations, two-dimensional in laminar and turbulent flows, mixing length concepts, Baldwin-Lomax, Cebeci-Smith, 1/2-equations, one-equation models, Prandtl's model, Spalart-Allmaras, k-ω and k-ε, Boussinesq, nonlinear relations, stress transport models, closure, applications and examples, physical considerations, Morkovin hypothesis, studies in particular flows. These topics will be related to turbulent flows that are observed in our daily lives and within various fields of engineering.

Course Delivery and Notes

- Delivered in person and recorded – *lets an international audience study the subject for free*
- Complete set of notes presented to students – *reduces cost to students and allows them to create their own set of notes*
- Approximately 20 journal articles reviewed – *integrates original contributions, increases critical review and comprehension*
- Integrated computer project – *let students develop their own solver and implement a simple model*
- Semester term paper with overview presentation at end of semester – *increases student writing and presentation abilities*

Course Material

- Handout of 351 pages, partially filled in notes
- Students complete handout during class

Cebeci-Smith model notes

4.5.1 Coefficients of Momentum Transport

Reynolds shear-stress term $-\varrho\overline{u'v'}$ can be expressed either by Prandtl's mixing-length theory in the form

$$-\varrho\overline{u'v'} = \qquad (4.22)$$

or by Boussinesq's eddy-viscosity concept in the form

$$-\varrho\overline{u'v'} = \qquad (4.23)$$

- Equations 4.22 and 4.23 are the definitions of mixing length and eddy viscosity, respectively.
  -
  -
  -
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Student handout

4.5.1 Coefficients of Momentum Transport

Reynolds shear-stress term $-\varrho\overline{u'v'}$ can be expressed either by Prandtl's mixing-length theory in the form

$$-\varrho\overline{u'v'} = \rho l^2 \left|\frac{du}{dy}\right| \frac{du}{dy} \qquad (4.22)$$

or by Boussinesq's eddy-viscosity concept in the form

$$-\varrho\overline{u'v'} = \rho \varepsilon_m (du/dy). \quad \text{closure eqn.} \qquad (4.23)$$

- Equations 4.22 and 4.23 are the definitions of mixing length and eddy viscosity, respectively.
  - l & ε_m form from experiments.
  - Inner region & outer region of the BL!
  -
  -

Notes – students fill in and make their own

Features my own terrible hand-writing!

Articles and Models

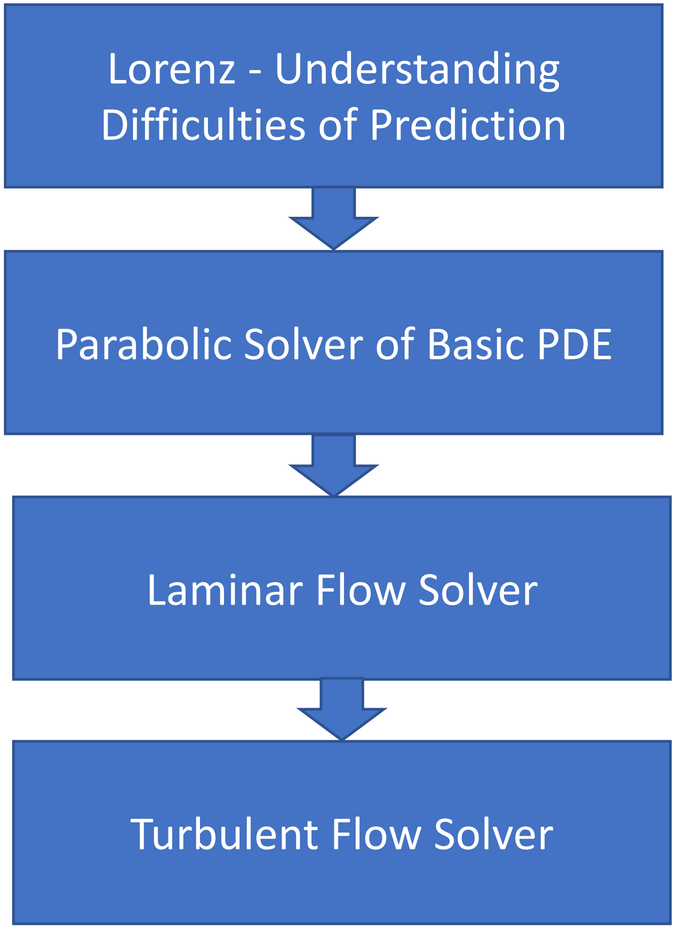
- As we increase modeling fidelity, we present key contributions
- Contributions are presented from accompany journal articles, which we review in class
- We review key equations in the articles and how they are implemented in CFD today

<u>Key Articles Reviewed in Class</u>

- Baldwin Barth, 1990
- Baldwin Lomax, 1978
- Cebeci, 1971
- Cebeci Smith, 1970
- Favre, 1965
- Johnson King, 1984
- Launder Sharma, 1974
- Lorenz, 1963
- Menter, 1992
- Prandtl, 1945
- Spalart Allmaras, 1994
- Wilcox, 1998

Computer Project Overview

- A four-part computer programming project results in a stand-alone marching
- Boundary layer solver. Submissions consist of a short description, results, and source code for each part
- Four-part project using student's favorite language
- Each part builds upon previous submission
- Assignments require short technical reports of results and source code



Computer Project: Part 1

- Reproduce time-domain predictions of Lorenz
- Become familiar with languages, compilers, and plotting

11.1 Chaos – The Butterfly Effect

The Lorenz system is governed by

$$X' = \quad -\sigma X + \sigma Y, \tag{11.1}$$

Assignment

$$Y' = -XZ + rX - Y, \tag{11.2}$$

and

$$Z' = XY - bZ, \tag{11.3}$$

where the prime indicates the time derivative with respect to τ. X, Y, and Z are the dependent variables. We will set the coefficients $\sigma = 10$, $b = 8/3$, and $r = 24.74$. March in time 3000 iterations using a non-dimensional time step of $\Delta\tau = 0.01$. The initial condition is $X_0 = 0$, $Y_0 = 1$, and $Z_0 = 0$. Use the 4th order Runge-Kutta integration method to solve the system of equations numerically. Do not call a library that contains the Runge-Kutta routine (it should be hand-coded).

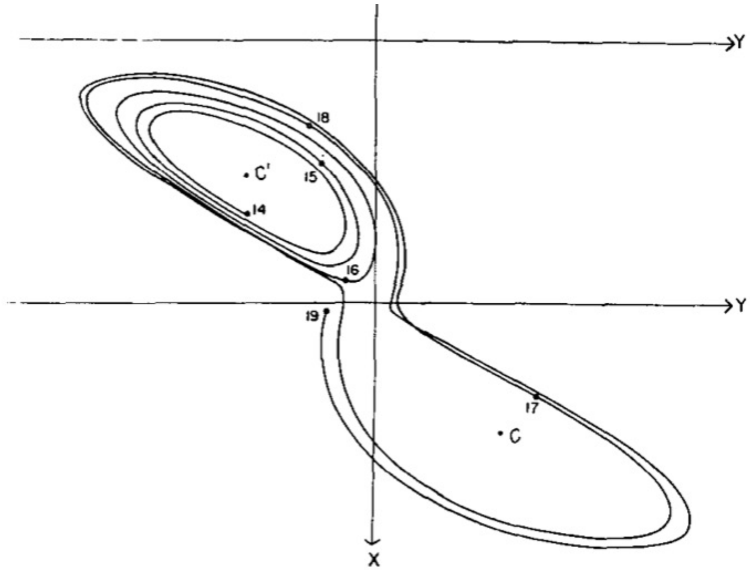


FIG. 2. Numerical solution of the convection equations. Projections on the X-Y-plane and the Y-Z-plane in phase space of the segment of the trajectory extending from iteration 1400 to iteration 1900. Numerals "14," "15," etc., denote positions at iterations 1400, 1500, etc. States of steady convection are denoted by C and C'.

From Lorenz, E. N, "Deterministic Nonperiodic Flow," J. Atm. Sci., 1963.

Computer Project: Part 2

- Create basic parabolic marching solver with representative equation
- Compare with analytical solution
- Prepare for laminar solver

$$\frac{du}{dx} - \frac{d^2u}{dy^2} = 1$$

Assignment

11.2 Development of a Basic Solver

Numerically solve the partial differential equation using a marching technique

$$\frac{du}{dx} - \frac{d^2u}{dy^2} = 1, \tag{11.7}$$

subject to boundary conditions

$$u(x,0) = u(x,1) = 0, \tag{11.8}$$

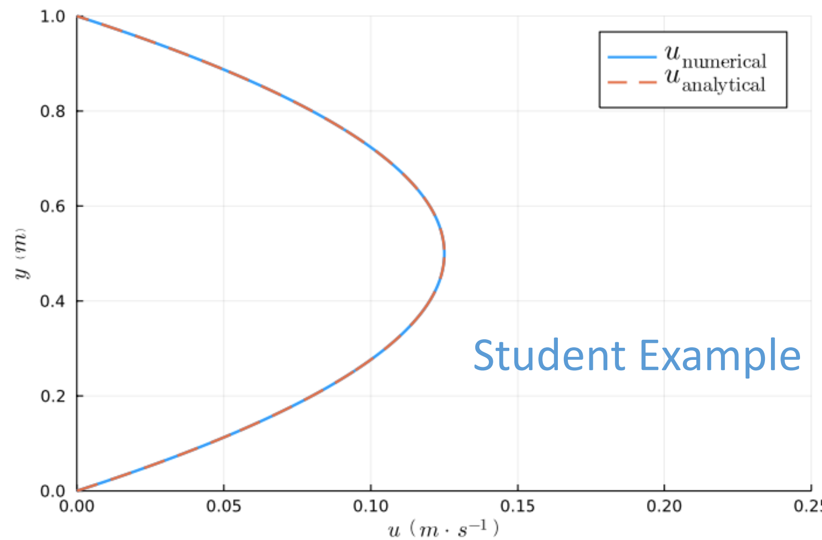
and initial condition

$$u(0,y) = 0. \tag{11.9}$$

The analytical solution is elegant and is

$$u(x,y) = \sum_{k=1}^{\infty} \frac{-2(\cos(k\pi)-1)\left(1-\exp\left[-k^2\pi^2 x\right]\right)}{k^3\pi^3} \sin(k\pi y). \tag{11.10}$$

The domain is restricted to $0 \leq y \leq 1$ and $x \geq 0$. Use the Crank-Nicolson marching scheme to advance in x. Construct a diagonal scheme using second order central differencing in the y direction.



Computer Project: Part 3

11.3 Laminar Flow

Predict the flow from a laminar boundary layer with no pressure gradient by numerical evaluating the equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{11.20}$$

and

Assignment

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{11.21}$$

With boundary conditions $u(x,0) = 0$, $v(x,0) = 0$, $\partial v(x, y = y_{\max})/\partial y = 0$. The initial condition is $u(0,y) = u_\infty$, where $u_\infty = 10$ m/s. Advance the momentum equation via spatial marching in the x direction using the Crank-Nicolson algorithm and a Thomas linear algebra solver. After finding the new solution for u, advance the cross-stream velocity component, v, via the conservation of mass equation using the Crank-Nicolson algorithm.

The discretization of the momentum equation is

$$\begin{aligned} u_{i-1,j}\frac{u_i - u_{i-1}}{\Delta x} &+ v_{i-1,j}\frac{1}{2}\left(\frac{\partial\eta}{\partial y}\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta\eta} + \frac{\partial\eta}{\partial y}\frac{u_{i-1,j+1} - u_{i-1,j-1}}{2\Delta\eta}\right) \\ &= \nu\frac{1}{2}\left(\left(\frac{\partial\eta}{\partial y}\right)^2\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta\eta^2} + \frac{\partial^2\eta}{\partial y^2}\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta\eta}\right) \\ &+ \nu\frac{1}{2}\left(\left(\frac{\partial\eta}{\partial y}\right)^2\frac{u_{i-1,j+1} - 2u_{i-1,j} + u_{i-1,j-1}}{\Delta\eta^2} + \frac{\partial^2\eta}{\partial y^2}\frac{u_{i-1,j+1} - u_{i-1,j-1}}{2\Delta\eta}\right) \end{aligned} \tag{11.22}$$

and the conservation of mass equation is

$$\frac{\partial v}{\partial\eta} = -\frac{\partial u}{\partial x}\frac{1}{\partial\eta/\partial y} \tag{11.23}$$

Here, the grid is transformed to the computational plane ξ, η from the physical plane x, y. For example, for a non-uniform grid spacing in the y direction we can find the relation

$$\frac{\partial}{\partial y} = \frac{\partial\eta}{\partial y}\frac{}{\partial\eta} \tag{11.24}$$

$\frac{\partial\eta}{\partial y}$, and similar terms can easily be found through finite differences of the grid point positions of η and y. Distribute the physical domain grid points uniformly $e.g.\Delta\eta = 1$ and choose $\xi = x$ and $\Delta\xi = \Delta x$ (e.g. no transform in the x-direction). Distribute the grid points in the physical domain to resolve the boundary layer as

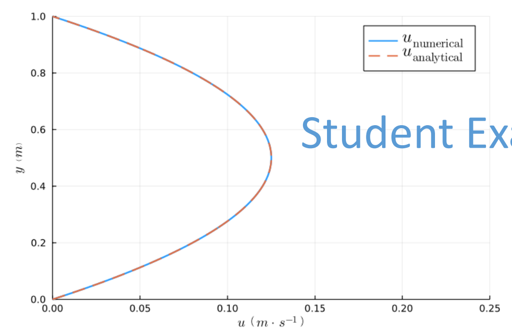
$$y_j = h_1\left(K^j - 1\right)/(K-1), \quad j = 1, 2, 3, \ldots, J, \quad 0 < K < 1$$

which is the distance to the jth y line. There are two parameters: h_1, the length of the first Δy step, and K, the ratio of two successive steps. The net in the y direction is a geometric progression having the property that the ratio of lengths of any two adjacent intervals is a constant that is, $h_j = Kh_{j-1}$. Typical values of h_1 and K might be ($K = 1.040$) with $h_1 = 0.0001$, $y_\infty = 301$, but please check for grid independence and meaningful solutions.

Convert general parabolic solver into laminar boundary layer solver.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$



Student Example

Figure 5: Numerical solution compared to the analytical solution at $x = 1.00$ m.

Computer Project: Part 4

10.4 Turbulent Flow

By modifying the laminar flow solver developed in the previous assignment, predict the flow of a turbulent boundary layer with no pressure gradient by numerical evaluating the equations

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \qquad (10.41)$$

and

Assignment

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right] \qquad (10.42)$$

with boundary conditions $\bar{u}(x,0) = 0$, $\bar{v}(x,0) = 0$, and $\partial \bar{v}(x, y = y_{\max})/\partial y = 0$. The initial condition is $\bar{u}(0,y) = \bar{u}_\infty$, where $\bar{u}_\infty = 34.0$ m/s.

Use the zero equation turbulence model closure of Baldwin and Lomax (1978), which is based on the Cebeci and Smith model. Eddy viscosity, ν_t, is formed from the calculation of the inner and outer eddy viscosity $\nu_{t,i}$ and $\nu_{t,o}$, respectively. The switch between the inner turbulent viscosity and the outer turbulent viscosity is at the y position where both values are equal, i.e. $\nu_{t,i} = \nu_{t,o}$. That is specifically, form ν_t from the calculation of $\nu_{t,i}$ and $\nu_{t,o}$ at each profile position x. The inner eddy viscosity is

$$\nu_{t,i} = l^2 |\omega|,$$

where l is modeled using Prandt and van Driest's model

$$l = \kappa y \left[1 - \exp\left(-y^+/A^+\right)\right]. \qquad (10.43)$$

Here, $\kappa = 0.41$ and $A^+ = 26$ for zero pressure gradient flows. The magnitude of vorticity is

$$|\omega| = \sqrt{\left(\frac{\partial \bar{u}}{\partial y} - \frac{\partial \bar{v}}{\partial x}\right)^2}. \qquad (10.44)$$

The outer eddy viscosity is

$$\nu_{t,o} = \alpha C_1 F_{\text{wake}} \gamma,$$

where $\alpha = 0.0168$ and $C_1 = 1.6$. F_{wake} is defined as

$$F_{\text{wake}} = \min\left[y_{\max} F_{\max}, C_2 y_{\max} U_{\text{diff}}^2 / F_{\max}\right]$$

with $C_2 = 0.25$ and

$$F(y) = y|\omega| \left[1 - \exp\left(-y^+/A^+\right)\right].$$

The quantity $F_{\max}$ denotes the maximum value of F. $y_{\max}$ is the value of y at $F_{\max}$. U_{diff} is the difference between the maximum and minimum total velocity $\left(\sqrt{\bar{u}^2 + \bar{v}^2}\right)$ at each profile location x. The intermittency is modeled using Klebonov's model as

$$\gamma = \left[1 + 5.5\left(\frac{C_3 y}{y_{\max}}\right)^6\right]^{-1},$$

where $C_3 = 0.3$.

Recall the basic properties of the law of the wall. The wall shear stress is $\tau_w = \nu \frac{d\bar{u}}{dy}$. The friction velocity is $u_\tau = \sqrt{\tau_w/\rho_\infty}$. The inner coordinate is $y^+ = y u_\tau/\nu$. The inner velocity is $u^+ = \bar{u}/u_\tau$.

Convert laminar flow solver into Baldwin-Lomax solver (turbulent). Compare with composite profiles.

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right]$$

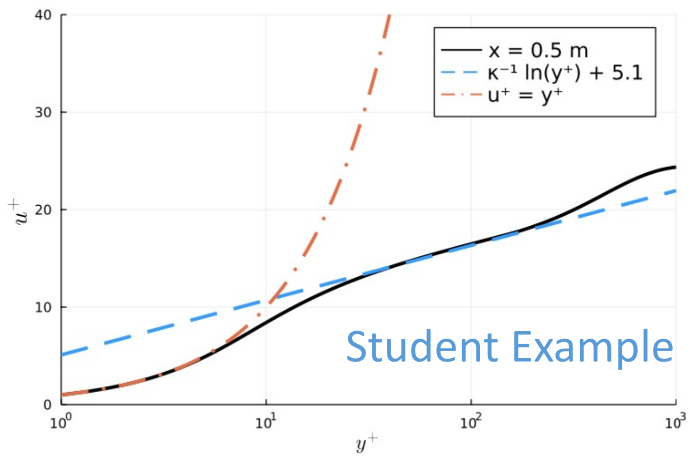


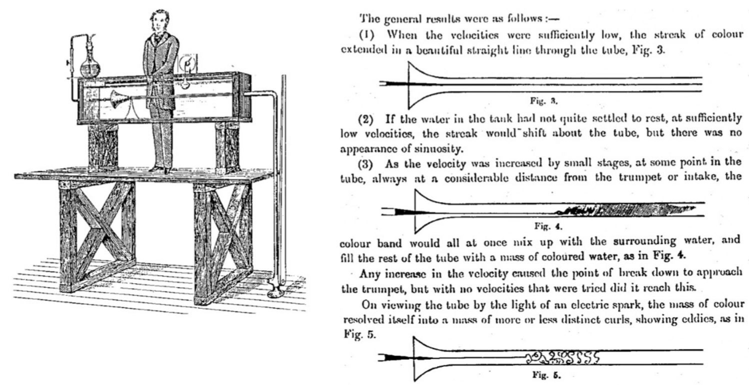
Figure 3: Comparison of simulation results (black solid line) to viscous layer scaling (dash dot orange line) and log-layer scaling (blue dashed line) at $x = 0.5$ m.

Term Paper and Presentations

- Students asked to choose a topic in turbulence, write a term paper, and present it
- Select topics chosen by students
  - Droplet breakup, Coherent Structures and Low Dimensional Models, Filtering in LES, Multiphase Turbulence, Algorithms to Identify Coherent Structures, Compressibility Effects and Closures, Modified *k*-equation Models
- NASA Writing Guide and feedback to improve communication and writing
- Grades partially assigned by peers

Historical Perspective

- Historical figures and events are presented throughout the class
- Equations and concepts, as they appear, are related to the historical concepts
    - Who created them?
    - Why did they create them?
    - What environment were they in when they did?
    - What problem were they trying to solve?
- In some cases, I reached out, and had great discussions and new original material



The general results were as follows :—

(1) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, Fig. 3.

Fig. 3.

(2) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the

Fig. 4.

colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in Fig. 4.

Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in Fig. 5.

Fig. 5.

Figure 1.1: Experiments of Reynolds.

Figure 4.4: Portrait photograph of Barrett Stone Baldwin, Jr.

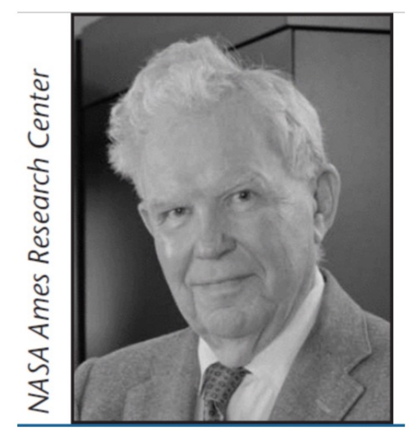


Figure 4.5: Harvard Lomax.

Outcome: Students

University course statistics and evaluations

University Core Course Evaluation Questions

	%(1)	%(2)	%(3)	%(4)	%(5)	Count	Mean	Median	SD
Course content (e.g., readings, activities, assignments) was relevant & useful.	0.0%	0.0%	0.0%	28.6%	71.4%	7	4.71	5.00	0.49
The course fostered regular interaction between student and instructor.	0.0%	0.0%	0.0%	71.4%	28.6%	7	4.29	4.00	0.49
Course activities and assignments improved my ability to analyze, solve problems, and/or think critically.	0.0%	0.0%	0.0%	57.1%	42.9%	7	4.43	4.00	0.53
Overall, this course was a valuable educational experience.	0.0%	0.0%	0.0%	28.6%	71.4%	7	4.71	5.00	0.49

Outcome – Generally positive, but we can improve through specific examples …

Select Feedback of Students

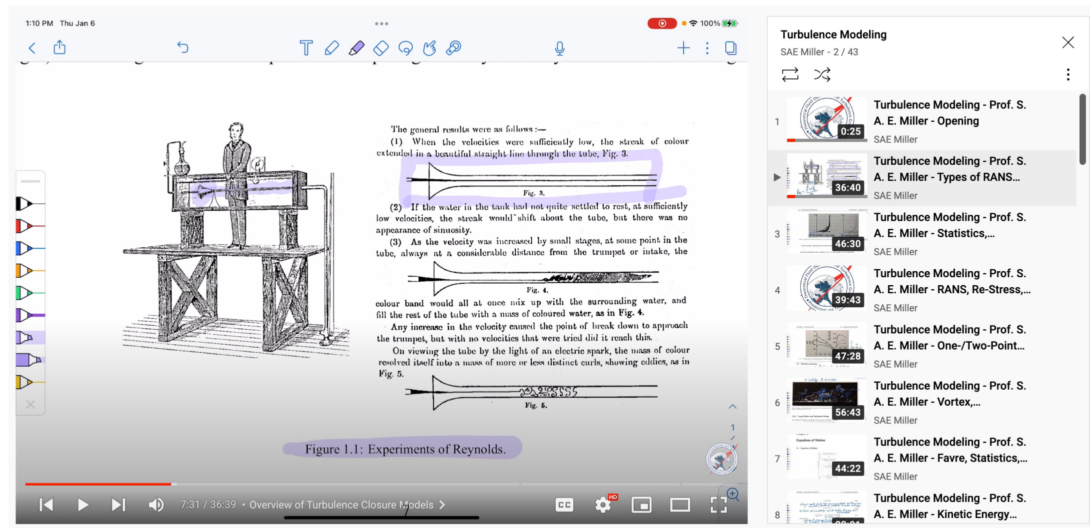
What constructive suggestion(s) do you have for improving the course materials, organization, and assignments?

Comments
Same suggestion as before.
I think some more discussion on the coherent structures generated by turbulence (for example hairpin vortices) and large eddy simulation would've been helpful.
Overall, I think the material, organization, and assignments were well thought out and developed. However, I am not sure that the first computer project was very relevant to the course material. The course focuses heavily on turbulence closure models for RANS computations, but the first project involves computation of a nonlinear dynamical model of convection. This project provides some insight into chaos, and is fun to code, but is an example of a dynamical systems approach to turbulence which is not touched upon again in the course. Perhaps the second project should be the first project in the course, and the last project could be modified to include a one equation turbulence closure model. This is just a suggestion.
We need to have access to the complete notes and voice recordings of the class perhaps after point in the semester.

Please identify the topics and/or skills you learned in the course that you believe will have the highest application for future courses or professional growth.

Comments
Learning Fortran and coding basic flow solver. Practice presentation and writing technical papers. Ability to not get overwhelmed by a lot of complex–equations and pick out what is important to my understanding. Ability to read over technical articles and summarize them.
A general understanding of turbulence is really helpful. I can read turbulence papers, listen to turbulence talks, and even write and converse about turbulence and feel like I know what is being discussed.
RANS computations, turbulence modeling, and technical writing.
Coding, Numerical Methods, Identifying which turbulence model to use for different types of flows

Outcome: Online



Youtube playlist ~ 43 lectures

Course Homepage https://saemiller.com/turbulence-modeling/

Playlist https://youtu.be/xtwRdfj00rI&list=PLbiOzt50Bx-liph4_pxAdW8Qu4QelSDvo

Thank You

Extra Slides

APS Abstract

A new graduate class is developed at the University of Florida called Modeling Inhomogeneous Turbulence with a Historical Perspective. The course covers in-depth concepts of the science and mathematics of turbulence modeling. Major topics of the class include statistics for modeling, the Russian school, law of the wall, chaos, compressible Navier-Stokes equations, mean kinetic energy, Reynolds stress transport equation, boundary layer equations, two-dimensional laminar flows, mixing length concepts, Baldwin-Lomax, Cebeci-Smith, one-half equations, one-equations, Prandtl's model, Spalart-Allmarus, k-omega, k-epsilon, Boussinesq, nonlinear relations, stress transport models, closure, Morkovin hypothesis, and studies in particular flows. These topics are related to turbulent flows that are observed in our daily lives and within various fields of engineering. Student assessment is conducted via analysis assignments, term papers, and a presentation on a topic of their choice. A four part programming project involves creating a parabolic boundary layer marching code with an algebraic closure. Feedback from students and progress on making the course publicly available are presented. Portions of the course appear on online. Course notes and assignments are available freely within a 351 page handout.