Maruyama, Yoshihiro

Fibred algebraic semantics for a variety of non-classical first-order logics and topological logical translation. (English) Zbl 07442024 J. Symb. Log. 86, No. 3, 1189-1213 (2021)

This paper takes first steps towards interplay between algebraic and categorical logics, connecting algebraic substructural logic [Zbl 1171.03001] with categorical predicate logic [Zbl 1035.03001, p. 39–123] via Lawvere hyperdoctrine [Zbl 0341.18002; Zbl 1114.18002]. The author employs Lawvere hyperdoctrines and their extensions as fibered algebraic models of predicated logic. A hyperdoctrine is a functor

$$P: \mathbf{C}^{\mathrm{op}} \to \mathbf{H}\mathbf{A}$$

where

- *HA* is the category of Heyting algebras and their homomorphisms with some additional conditions on *P* and *C* to express quantifers and other logical structures [http://www.lfcs.inf.ed.ac. uk/reports/92/ECS-LFCS-92-194/ECS-LFCS-92-194.pdf; https://cs.au.dk/~birke/papers/ hosl-journal.pdf; Zbl 1395.03039; Zbl 1443.14002; https://www.researchgate.net/publication/ 340322745_Higher-Order_Categorical_Substructural_Logic_Expanding_the_Horizon_of_Tripos_ Theory; https://ieeexplore.ieee.org/document/9177695; Zbl 0674.03007];
- The base category C stands intuitively for the category of types or domains of discourse, so that P(C) is the algebra of predicates on a type C.

Given a predicate logic, the corresponding hyperdoctrines can be defined as a canonical manner, serving as its fibered algebras, while there is no canonical way to define cylindric algebras [Zbl 0214.01302; Zbl 0576.03043] for a given predicate logic. Fibrations and hyperdoctrines are linked with each other via the so-called Grothendieck construction.

Fibered universal algebra studies a functor of form

 $P: \mathbf{C}^{\mathrm{op}} \to \mathbf{Alg}$

aoart from logical conditions to express quantifiers and others, where Alg is an algebraic category [Zbl 1113.18001]. There is a general principle of completeness lifting claiming that if a propositional logic L is sound and complete with respect to a variety Alg, then the corresponding fibered algebras or hyperdoctrines give sound and complete semantics for the predicate logic extending L with quantifiers. This paper demonstrates this in the fairly general context of arbitrary logics over the full Lambek calculus emcompassing a wide veriety of logical systems such as classical, intuitionistic, linear, fuzzy, substructural and relevant logics.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

- 18C10 Theories (e.g., algebraic theories), structure, and semantics
- 03G30 Categorical logic, topoi
- 03B47 Substructural logics (including relevance, entailment, linear logic, Lambek calculus, BCK and BCI logics)
- 18D30 Fibered categories

Keywords:

tripos theory; topos theory; hyperdoctrine; Lawvere-Tierney topology; categorical logic; algebraic logic; substructural logic

Full Text: DOI