



# Lecture No. 3

## Moment of Inertia

### Introduction

The second moment of area, also known as area moment of inertia, is a geometrical property of an area, which reflects how its points are distributed with regard to an arbitrary axis. The unit of dimension of the second moment of area is length to fourth power,  $L^4$ , and should not be confused with the mass moment of inertia. If the piece is thin, however, the mass moment of inertia equals the area density times the area moment of inertia.

### Moment Of Inertia of a Plane Area

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let

$a_1, a_2, a_3 \dots$  = Areas of small elements, and

$r_1, r_2, r_3, \dots$  = corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 * r_1^2 + a_2 * r_2^2 + a_3 * r_3^2 + \dots$$

$$= \sum a * r^2$$

### Units of Moment of Inertia

In fact, the units of moment of inertia of a plane area depend upon the units of

The area and the length. *e.g.*

- If area is in  $m^2$  and the length is in m, the moment of inertia is expressed in  $m^4$ .
- If area in  $mm^2$  and the length is also in mm, then moment of inertia is expressed in  $mm^4$ .

## Moment Of Inertia by Integration

The moment of inertia of an area may also be found out by the method of integration as discussed below: Consider a plane figure, whose moment of inertia is required to be found out about  $X-X$  axis and  $Y-Y$  axis as shown in Figure (3.1). Let us divide the whole area into a no. of strips. Consider one of these strips.

Let

- $dA$  = Area of the strip
- $x$  = Distance of the center of gravity of the strip on  $X-X$  axis and
- $y$  = Distance of the center of gravity of the strip on  $Y-Y$  axis.

We know that the moment of inertia of the strip about  $Y-Y$  axis

$$= dA \cdot x^2$$

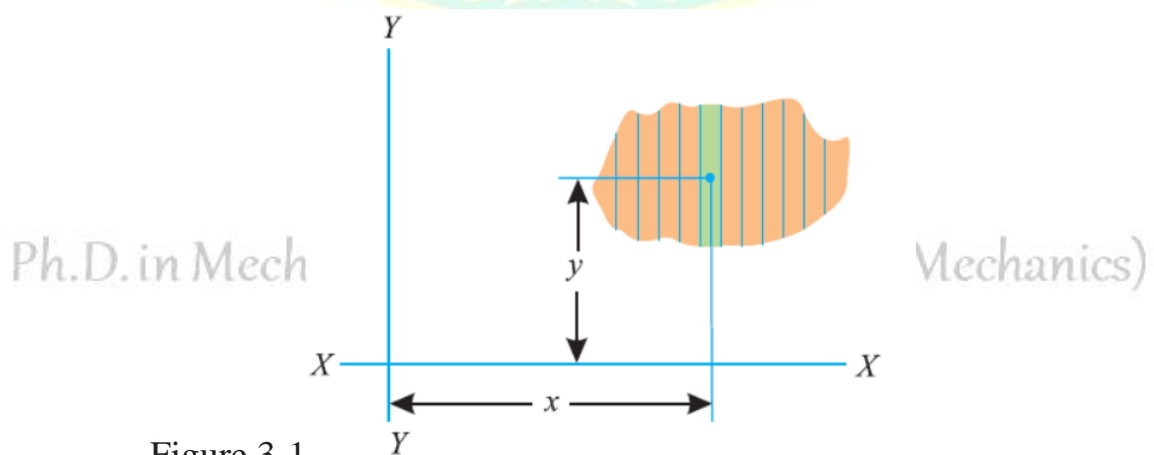
Now the moment of inertia of the whole area may be found out by integrating above Equation.  
*i.e.*

$$I_{yy} = \sum dA \cdot x^2$$

Similarly

$$I_{xx} = \sum dA \cdot y^2$$

In the following pages, we shall discuss the applications of this method for finding out the Moment of inertia of various cross-sections.



## Moment Of Inertia of a Rectangular Section

Consider a rectangular section  $ABCD$  as shown in Figure 3-2 whose moment of inertia is required to be found out.

Let

- $b$  = Width of the section and
- $d$  = Depth of the section.

Now consider a strip  $PQ$  of thickness  $dy$  parallel to  $X-X$  axis

And at a distance  $y$  from it as shown in the figure

$$\therefore \text{Area of the strip} = b \cdot dy$$

We know that moment of inertia of the strip about  $X-X$  axis,

$$= \text{Area} \times y^2 = (b \cdot dy) y^2 = b \cdot y^2 \cdot dy$$

Now \*moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina i.e. from  $-\frac{d}{2}$  to  $\frac{d}{2}$

$$I_{xx} = \int_{-\frac{d}{2}}^{\frac{d}{2}} b \cdot y^2 \cdot dy = b \int_{-\frac{d}{2}}^{\frac{d}{2}} y^2 \cdot dy$$

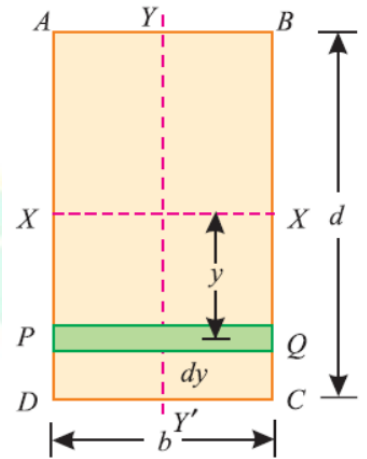
$$= b * \left[ \frac{y^3}{3} \right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$I_{xx} = \frac{b * d^3}{12}$$

Similarly

$$I_{yy} = \frac{d * b^3}{12}$$

**Note.** Cube is to be taken of the side, which is at right angles to the line of reference.



## Moment Of Inertia of a Hollow Rectangular Section

Consider a hollow rectangular section, in which  $ABCD$  is the main section and  $EFGH$  is the cut out section as shown in Fig 7.3

Let

- $b$  = Breadth of the outer rectangle,
- $d$  = Depth of the outer rectangle and
- $b_1, d_1$  = Corresponding values for the cut out rectangle.

We know that the moment of inertia, of the outer rectangle  $ABCD$  about  $X-X$  axis

$$I_{xx} = \frac{B * D^3}{12}$$

And moment of inertia of the cut out rectangle  $EFGH$  about  $X-X$  axis

$$I_{xx} = \frac{b * d^3}{12}$$

Moment of inertia of the hollow rectangular section about  $x-x$  axis,

$$I_{xx} = \text{M.I. of rectangle } ABCD - \text{M.I. of rectangle } EFGH$$

$$I_{xx} = \frac{B * D^3}{12} - \frac{b * d^3}{12}$$

Similarly

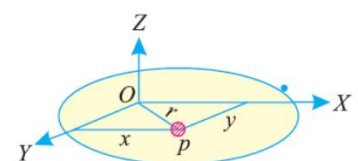
$$I_{yy} = \frac{D * B^3}{12} - \frac{d * b^3}{12}$$

## Theorem of Perpendicular Axis

It states, If  $I_{xx}$  and  $I_{yy}$  be the moments of inertia of a plane section about two perpendicular axis meeting at  $O$ , the moment of inertia  $I_{zz}$  about the axis  $Z-Z$ , perpendicular to the plane and passing through the intersection of  $X-X$  and  $Y-Y$  is given by:

$$I_{zz} = I_{xx} + I_{yy}$$

Proof:





Consider a small lamina ( $P$ ) of area  $da$  having co-ordinates as  $x$  and  $y$  along  $OX$  and  $OY$  two mutually perpendicular axes on a plane section as shown in Figure.

Now consider a plane  $OZ$  perpendicular to  $OX$  and  $OY$ .

Let

( $r$ ) Be the distance of the lamina ( $P$ ) from  $Z-Z$  axis such that

$$OP = r.$$

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina  $P$  about  $X-X$  axis,

$$I_{xx} = da * y^2$$

$$[I = \text{Area} \times (\text{Distance})^2]$$

Similarly,

$$I_{yy} = da * x^2$$

$$I_{zz} = da * r^2 = da * (x^2 + y^2).$$

$$= da.x^2 + da.y^2$$

$$I_{zz} = I_{xx} + I_{yy}$$

## Moment of Inertia of a Circular Section

Consider a circle  $ABCD$  of radius ( $r$ ) with centre  $O$  and  $XX'$  and  $Y-Y'$  be two axes of reference through  $O$  as shown in Fig. 7.5. Now consider an elementary ring of radius  $x$  and thickness  $dx$ .

Therefore area of the ring,

$$da = 2 \pi x . dx$$

And moment of inertia of ring, about  $X-X$  axis or  $Y-Y$  axis

$$= \text{Area} \times (\text{Distance})^2$$

$$2 \pi x . dx * x^2$$

$$2 \pi x^3 . dx$$

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to  $r$ .



$$I_{xx} = \int_0^r 2 \pi x^3 \cdot dx = 2 \pi \int_0^r x^3 \cdot dx$$

$$= 2 \pi * \left[ \frac{x^4}{4} \right]_0^r$$

$$I_{xx} = \frac{\pi}{2} * r^4 = \frac{\pi}{32} * d^4$$

We know from the Theorem of Perpendicular Axis that

$$I_{zz} = I_{xx} + I_{yy}$$

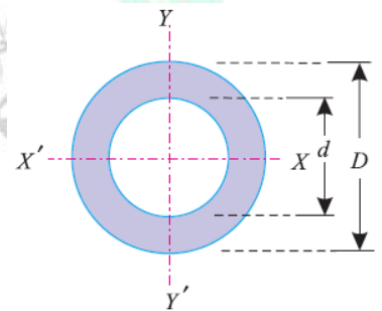
$$I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi}{64} * d^4$$

### Moment of Inertia of a Hollow Rectangular Section

Consider a hollow rectangular section, in which  $ABCD$  is the main section and  $EFGH$  is the cut out section as shown in Fig 7.3

Let

- $D$  = Diameter of the main circle,
- $d$  = Diameter of the cut out circle.



We know that the moment of inertia, of the outer rectangle  $ABCD$  about  $X-X$  axis

$$I_{xx} = \frac{\pi}{64} * D^4$$

And moment of inertia of the cut out rectangle  $EFGH$  about  $X-X$  axis

$$I_{xx} = \frac{\pi}{64} * d^4$$

Moment of inertia of the hollow rectangular section about  $x-x$  axis,

$$I_{xx} = M.I. \text{ of rectangle } ABCD - M.I. \text{ of rectangle } EFGH$$

$$I_{xx} = \frac{\pi}{64} * D^4 - \frac{\pi}{64} * d^4 =$$

$$I_{xx} = \frac{\pi}{64} * (D^4 - d^4)$$

Similarly

$$I_{yy} = \frac{\pi}{64} * (D^4 - d^4)$$

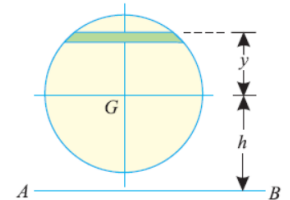
### Theorem of Parallel Axis

It states, if  $I_G$ , then moment of inertia of the area about any other axis  $AB$ , parallel to the first, and at a distance  $h$  from the center of gravity is given by denote the moment of inertia of a plane area about an axis through its center of gravity:

$$I_{AB} = I_G + a * h^2$$

Where

- $I_{AB}$  = Moment of inertia of the area about an axis  $AB$ ,
- $I_G$  = Moment of Inertia of the area about its center of gravity
- $a$  = Area of the section, and
- $h$  = Distance between center of gravity of the section and axis  $AB$ .



### Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line  $AB$  as shown in Figure.

Let

- $\delta a$  = Area of the strip
- $y$  = Distance of the strip from the center of gravity the section and
- $h$  = Distance between center of gravity of the section and the axis  $AB$ .

We know that moment of inertia of the whole section about an axis passing through the center of gravity of the section

$$= \delta a . y^2$$

And moment of inertia of the whole section about an axis passing through its center of gravity,

$$I_G = \sum \delta a . y^2$$

∴ Moment of inertia of the section about the axis  $AB$ ,

$$\begin{aligned} I_{AB} &= \sum \delta a (h + y)^2 = \sum \delta a (h^2 + y^2 + 2 h y) \\ &= (\sum \delta a * h^2) + (\sum \delta a * y^2) + (\sum \delta a * 2 h y) \\ &= a h^2 + I_G + 0 \end{aligned}$$

It may be noted that  $\sum \delta a * h^2 = a h^2$  and  $\sum \delta a * y^2 = I_G$  and  $\sum \delta a * y$  is the algebraic sum of moments of all the areas, about an axis through center of gravity of the section and is equal to  $a * y$ , where  $y$  is the distance between the section and the axis passing through the center of gravity, which obviously is zero.

## Moment Of Inertia of a Triangular Section

Consider a triangular section  $ABC$  whose moment of inertia is required to be found out.

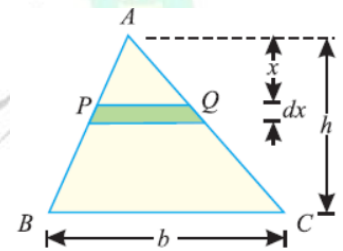
Let

- $b$  = Base of the triangular section and
- $h$  = Height of the triangular section.

Now consider a small strip  $PQ$  of thickness  $dx$  at a distance of  $x$  from the vertex  $A$  as shown in Figure from the geometry of the figure, we find that the two triangles  $APQ$  and  $ABC$  are similar. Therefore

$$\frac{PQ}{BC} = \frac{x}{h}$$

$$PQ = \frac{BC * x}{h} = \frac{b * x}{h}$$



We know that area of the strip  $PQ$

$$= \frac{b * x}{h} * dx$$

And moment of inertia of the strip about the base  $BC$

$$= Area \times (Distance)^2 = \frac{b * x}{h} * dx(h - x)^2 = \frac{b * x}{h} (h - x)^2 * dx$$

Now moment of inertia of the completely triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to  $h$ .

$$I_{BC} = \int_0^h \frac{b * x}{h} (h - x)^2 * dx$$

$$= \frac{b}{h} \int_{-\frac{d}{2}}^{\frac{d}{2}} (h^2 + x^2 + 2 * h * x) * x dy$$



$$= \frac{b}{h} \int_{-\frac{d}{2}}^{\frac{d}{2}} (h^2x + x^3 + 2 * h * x^2) dy$$

$$= \frac{b}{h} * \left[ \frac{h * x^2}{2} + \frac{x^4}{4} + \frac{2 * h * x^3}{3} \right]_0^h$$

$$I_{BC} = \frac{b * h^3}{12}$$

We know that distance between center of gravity of the triangular section and base  $BC$ ,

$$d = \frac{h}{3}$$

Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to  $X-X$  axis,

$$I_G = I_{AB} - a * h^2$$

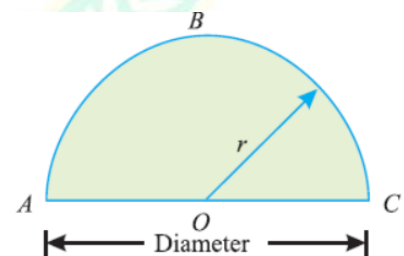
$$I_G = \frac{b * h^3}{12} * \frac{b * h}{2} * \left(\frac{h}{3}\right)^2 = \frac{b * h^3}{36}$$

## MOMENT OF INERTIA OF A SEMICIRCULAR SECTION

Consider a semicircular section  $ABC$  whose moment of inertia is required to be found out as shown in Fig. 7.10.

Let  $r$  = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base  $AC$  is equal to half the moment of inertia of the circular section about  $AC$ . Therefore moment of inertia of the semicircular section  $ABC$  about the base  $AC$



$$I_{AC} = \frac{1}{2} * \frac{\pi}{64} * D^4$$

We also know that area of semicircular section,

$$a = \frac{1}{2} * \frac{\pi}{4} * D^2$$

In addition, distance between center of gravity of the section and the base  $AC$ ,

$$h = \frac{4 * r}{3 * \pi}$$

Moment of inertia of the section through its center of gravity and parallel to x-x axis,

$$I_G = I_{AB} - a * h^2$$

$$I_G = \frac{1}{2} * \frac{\pi}{64} * D^4 - \frac{1}{2} * \frac{\pi}{4} * D^2 * \frac{4 * r}{3 * \pi}$$

$$I_G = 0.11 * r^2$$

### Example 1

Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.

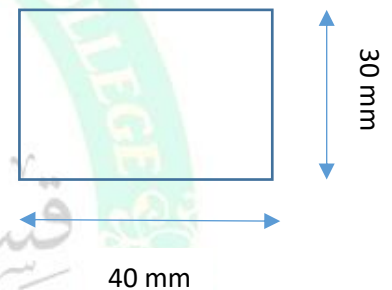
**Solution:**

$$I_{xx} = \frac{b * d^3}{12}$$

$$I_{xx} = \frac{30 * 40^3}{12} = 160 e3 mm^4$$

$$I_{xx} = \frac{d * b^3}{12}$$

$$I_{xx} = \frac{40 * 30^3}{12} = 90 e3 mm^4$$



### Example 2

Find the moment of inertia of a hollow rectangular section about its center of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.

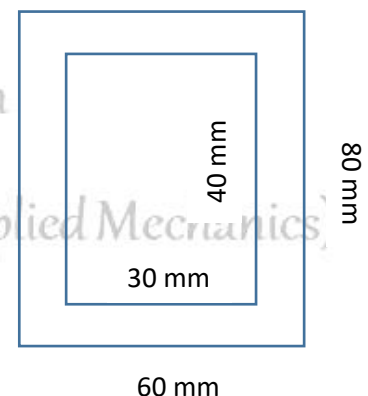
**Solution:**

$$I_{xx} = \frac{B * D^3}{12} - \frac{b * d^3}{12}$$

$$I_{xx} = \frac{60 * 80^3}{12} - \frac{30 * 40^3}{12} = 2400 e3 mm^4$$

$$I_{yy} = \frac{D * B^3}{12} - \frac{d * b^3}{12}$$

$$I_{yy} = \frac{80 * 60^3}{12} - \frac{40 * 30^3}{12} = 1350 e3 mm^4$$



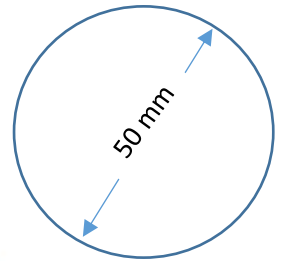
### Example 3

Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its center.

**Solution:**

$$I_{xx} = \frac{\pi}{64} * d^4$$

$$I_{xx} = \frac{\pi}{64} * (50)^4 = 307 \text{ e3 } mm^4$$



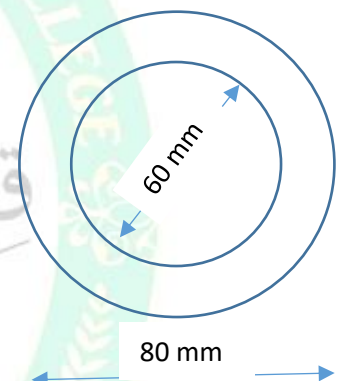
### Example 4

A hollow circular section has an external diameter of 80 mm and internal diameter of 60 mm. Find its moment of inertia about the horizontal axis passing through its center.

**Solution:**

$$I_{xx} = \frac{\pi}{64} * (D^4 - d^4)$$

$$I_{xx} = \frac{\pi}{64} * (80^4 - 60^4) = 1374 \text{ e3 } mm^4$$



### Example 5

An isosceles triangular section ABC has base width 80 mm and height 60 mm. Determine the moment of inertia of the section about the center of gravity of the section and the base BC.

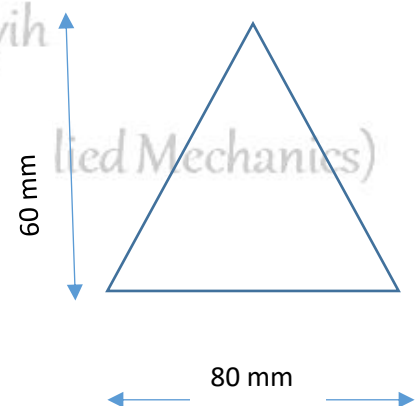
**Solution:**

$$I_{BC} = \frac{b * h^3}{12}$$

$$I_{BC} = \frac{80 * 60^3}{12} = 1440 \text{ e3 } mm^4$$

$$I_G = \frac{b * h^3}{36}$$

$$I_G = \frac{80 * 60^3}{36} = 480 \text{ e3 } mm^4$$



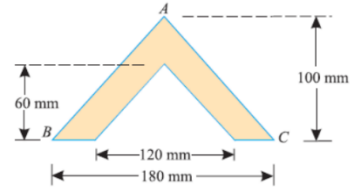
### Example 6

A hollow triangular section shown in Figure is symmetrical about its vertical axis. Find the moment of inertia of the section about the base BC.

**Solution:**

$$I_{BC} = \frac{B * H^3}{12} - \frac{b * h^3}{12}$$

$$I_{BC} = \frac{180 * 100^3}{12} - \frac{120 * 60^3}{12} = 12.84 e3 mm^4$$



### Example 7

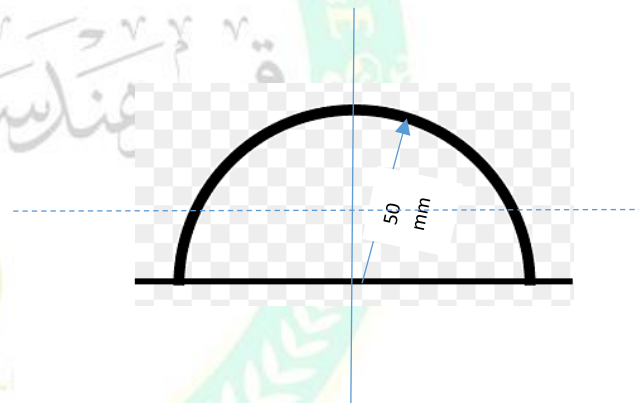
Determine the moment of inertia of a semicircular section of 100 mm diameter about its center of gravity and parallel to X-X and Y-Y axes.

**Solution:**

$$I_G = 0.11 * r^2$$

$$I_G = 0.11 * 50^2 = 687.5 e3 mm^4$$

$$I_{yy} = \frac{1}{2} * \frac{\pi}{64} * 100^4 = 2456 e3 mm^4$$



Dr. Mujtaba A. Flayyih

Ph.D. in Mechanical Engineering (Applied Mechanics)



## Moment of Inertia of a Composite Section

The moment of inertia of a composite section may be found out by the following steps:

- First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the center of gravity of the section).
- Find the moments of inertia of these areas about their respective centers of gravity.
- Now transfer these moment of inertia about the required axis (AB) by the Theorem of Parallel Axis, *i.e.*,

$$I_{AB} = I_G + a h^2$$

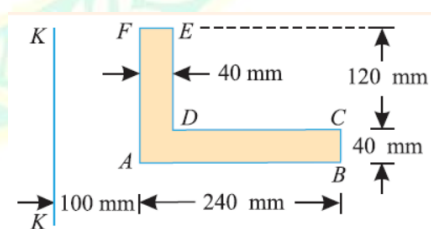
where

- $I_G$  Moment of inertia of a section about its center of gravity and parallel to the axis.
- $a$  Area of the section,
- $h$  Distance between the required axis and center of gravity of the section.

- The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

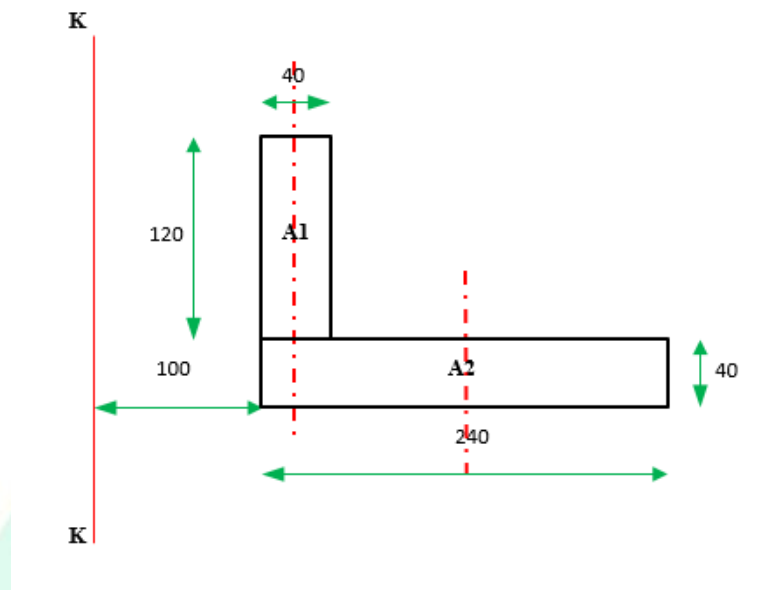
### Example 7

Figure below shows an area ABCDEF. Compute the moment of inertia of the above area about axis K-K.



Dr. Mujtaba A. Flayyih

Ph.D. in Mechanical Engineering (Applied Mechanics)



Split up the given section into plane areas (*i.e.*, rectangular A1 and A2)

Moment of inertia of section (A1) about its center of gravity

$$I_{G1} = \frac{b * h^3}{12} = \frac{120 * 40^3}{12} = 640 \text{ e3 } mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_1 = 100 + \frac{40}{2} = 120 \text{ mm}$$

Moment of inertia of section (A1) about axis K-K

$$\begin{aligned} I_{K-K(1)} &= I_{G1} + A_1 h_1^2 \\ &= 640 \text{ e3} + (120 * 40) * 120^2 \\ &= 69.76 \text{ e6 } mm^4 \end{aligned}$$

Moment of inertia of section (A2) about its center of gravity

$$I_{G2} = \frac{b * h^3}{12} = \frac{40 * 240^3}{12} = 46.08 \text{ e6 } mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

Moment of inertia of section (A1) about axis K-K

$$I_{K-K(2)} = I_{G2} + A_2 h_2^2$$

$$= 46.08 e6 + (240 * 40) * 220^2$$

$$= 510.72 e6 mm^4$$

Now moment of inertia of the whole area about axis  $K-K$ ,

$$I_{K-K} = I_{K-K(1)} + I_{K-K(2)}$$

$$= 69.76 e6 + 510.72 e6$$

$$= 580.48 e6 mm^4$$

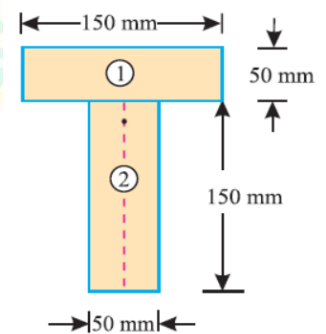
### Example 8

Find the moment of inertia of a T-section with flange as  $150 \text{ mm} \times 50 \text{ mm}$  and web as  $150 \text{ mm} \times 50 \text{ mm}$  about X-X and Y-Y axes through the center of gravity of the section.

### Solution

First of all, let us find out center of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its center of gravity will lie on this axis. Split up the whole section into two rectangles viz., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

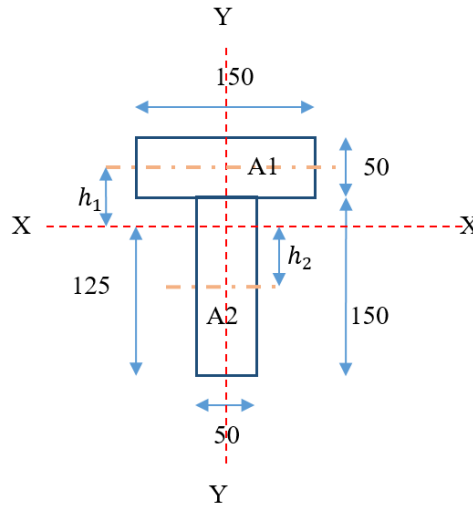
$$\bar{y} = \frac{1,875,000}{15000} = 125$$



Dr. Mujtaba A. Flayyih

Ph. D. in Mechanical Engineering (Applied Mechanics)

| No.          | Area ( $mm^2$ )       | $\bar{y}$                  | $a * \bar{y}$                     |
|--------------|-----------------------|----------------------------|-----------------------------------|
| 1            | $50 * 150 = 7500$     | $150 + \frac{50}{2} = 175$ | $7500 * 175 = 1,312,500$          |
| 2            | $150 * 50 = 7500$     | $\frac{150}{2} = 75$       | $7500 * 75 = 562,500$             |
| <b>Total</b> | $3000 + 3600 = 15000$ |                            | $1,312,500 + 562,000 = 1,875,000$ |



Moment of inertia about X-X axis

Moment of inertia of section (A1) about its center of gravity

$$I_{G1} = \frac{b * h^3}{12} = \frac{150 * 50^3}{12} = 1.56 e6 mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_1 = 175 + 125 = 25 mm$$

Moment of inertia of section (A1) about axis X-X

$$\begin{aligned} I_{X-X(1)} &= I_{G1} + A_1 h_1^2 \\ &= 1.56 e6 + 7500 * 25 \\ &= 20.3125 e6 mm^4 \end{aligned}$$

Moment of inertia of section (A2) about its center of gravity

$$I_{G2} = \frac{b * h^3}{12} = \frac{50 * 150^3}{12} = 14.0625 e6 mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_2 = 125 + 75 = 50 mm$$

Moment of inertia of section (A2) about axis X-X

$$\begin{aligned} I_{X-X(2)} &= I_{G2} + A_2 h_2^2 \\ &= 14.0625 e6 + 7500 * 50 \\ &= 32.8125 e6 mm^4 \end{aligned}$$

Now moment of inertia of the whole area about axis X-X



$$\begin{aligned} I_{X-X} &= I_{X-X(1)} + I_{X-X(2)} \\ &= 20.3125 e6 + 20.3125 e6 \\ &= 53.125 e6 mm^4 \end{aligned}$$

Moment of inertia about Y-Y axis

Moment of inertia of section (A1) about its center of gravity

$$I_{G1} = \frac{b * h^3}{12} = \frac{50 * 150^3}{12} = 14.0625 e6 mm^4$$

Moment of inertia of section (A2) about its center of gravity

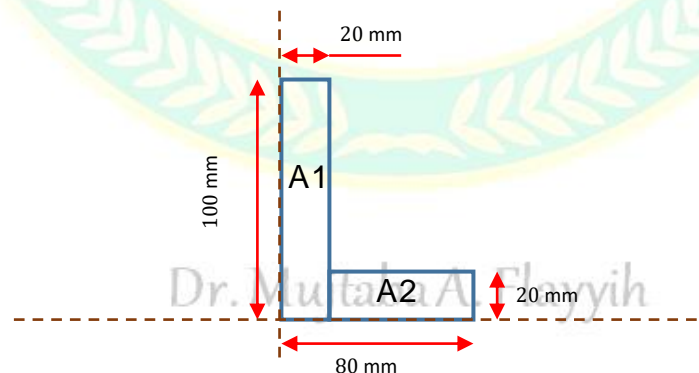
$$I_{G2} = \frac{b * h^3}{12} = \frac{150 * 50^3}{12} = 1.5625 e6 mm^4$$

Now moment of inertia of the whole area about axis Y-Y

$$\begin{aligned} I_{Y-Y} &= I_{Y-Y(1)} + I_{Y-Y(2)} \\ &= 20.3125 e6 + 20.3125 e6 \\ &= 15.625 mm^4 \end{aligned}$$

### Example 9

Find the moment of inertia about the centroid X-X and Y-Y axes of the angle section shown in Figure below



Ph.D. in Mechanical Engineering (Applied Mechanics)

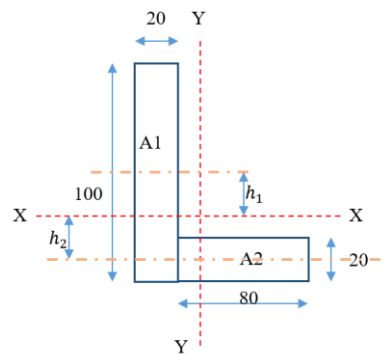
$$\bar{x} = \frac{68000}{3200} = 25 \text{ mm}$$

$$\bar{y} = \frac{112000}{3200} = 35 \text{ mm}$$



| No.          | Area ( $mm^2$ )      | $\bar{x}$                | $\bar{y}$            | $a * \bar{x}$           | $a * \bar{y}$             |
|--------------|----------------------|--------------------------|----------------------|-------------------------|---------------------------|
| 1            | $100 * 20 = 2000$    | $\frac{20}{2} = 10$      | $\frac{100}{2} = 50$ | $2000 * 10 = 20000$     | $2000 * 50 = 100000$      |
| 2            | $20 * 60 = 1200$     | $\frac{60}{2} + 20 = 40$ | $\frac{20}{2} = 10$  | $1200 * 50 = 60000$     | $1200 * 10 = 12000$       |
| <b>Total</b> | $1200 + 2000 = 3200$ |                          |                      | $20000 + 60000 = 80000$ | $100000 + 12000 = 112000$ |

Moment of inertia about X-X axis



Moment of inertia of section (A1) about its center of gravity

$$I_{G1} = \frac{b * h^3}{12} = \frac{20 * 100^3}{12} = 1.66 e6 mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_1 = 50 + 35 = 15 mm$$

Moment of inertia of section (A1) about axis X-X

$$\begin{aligned} I_{X-X(1)} &= I_{G1} + A_1 h_1^2 \\ &= 1.66 e6 + 2000 * 15^2 \\ &= 2.117 e6 mm^4 \end{aligned}$$

Moment of inertia of section (A2) about its center of gravity

$$I_{G2} = \frac{b * h^3}{12} = \frac{60 * 20^3}{12} = 0.04 e6 mm^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_2 = 35 + 10 = 25 mm$$

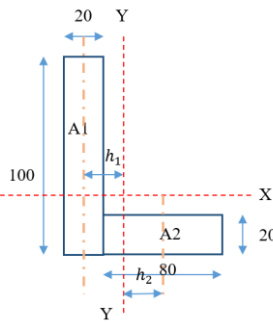
Moment of inertia of section (A1) about axis X-X

$$\begin{aligned}
 I_{X-X(2)} &= I_{G1} + A_1 h_1^2 \\
 &= 0.04 \text{ e6} + 1200 * 25^2 \\
 &= 0.79 \text{ e6} \text{ mm}^4
 \end{aligned}$$

Now moment of inertia of the whole area about axis X-X

$$\begin{aligned}
 I_{X-X} &= I_{X-X(1)} + I_{X-X(2)} \\
 &= 2.117 \text{ e6} + 0.79 \text{ e6} \\
 &= 2.907 \text{ e6} \text{ mm}^4
 \end{aligned}$$

Moment of inertia about Y-Y axis



Moment of inertia of section (A1) about its center of gravity

$$I_{G1} = \frac{b * h^3}{12} = \frac{100 * 20^3}{12} = 0.067 \text{ e6} \text{ mm}^4$$

Distance between center of gravity of section (1) and axis K-K,

$$h_1 = 25 + 10 = 15 \text{ mm}$$

Moment of inertia of section (A1) about axis X-X

$$\begin{aligned}
 I_{Y-Y(1)} &= I_{G1} + A_1 h_1^2 \\
 &= 0.067 \text{ e6} + 2000 * 15^2 \\
 &= 0.517 \text{ e6} \text{ mm}^4
 \end{aligned}$$

Moment of inertia of section (A2) about its center of gravity

$$I_{G2} = \frac{b * h^3}{12} = \frac{20 * 60^3}{12} = 0.36 \text{ e6} \text{ mm}^4$$

Distance between center of gravity of section (1) and axis K-K,

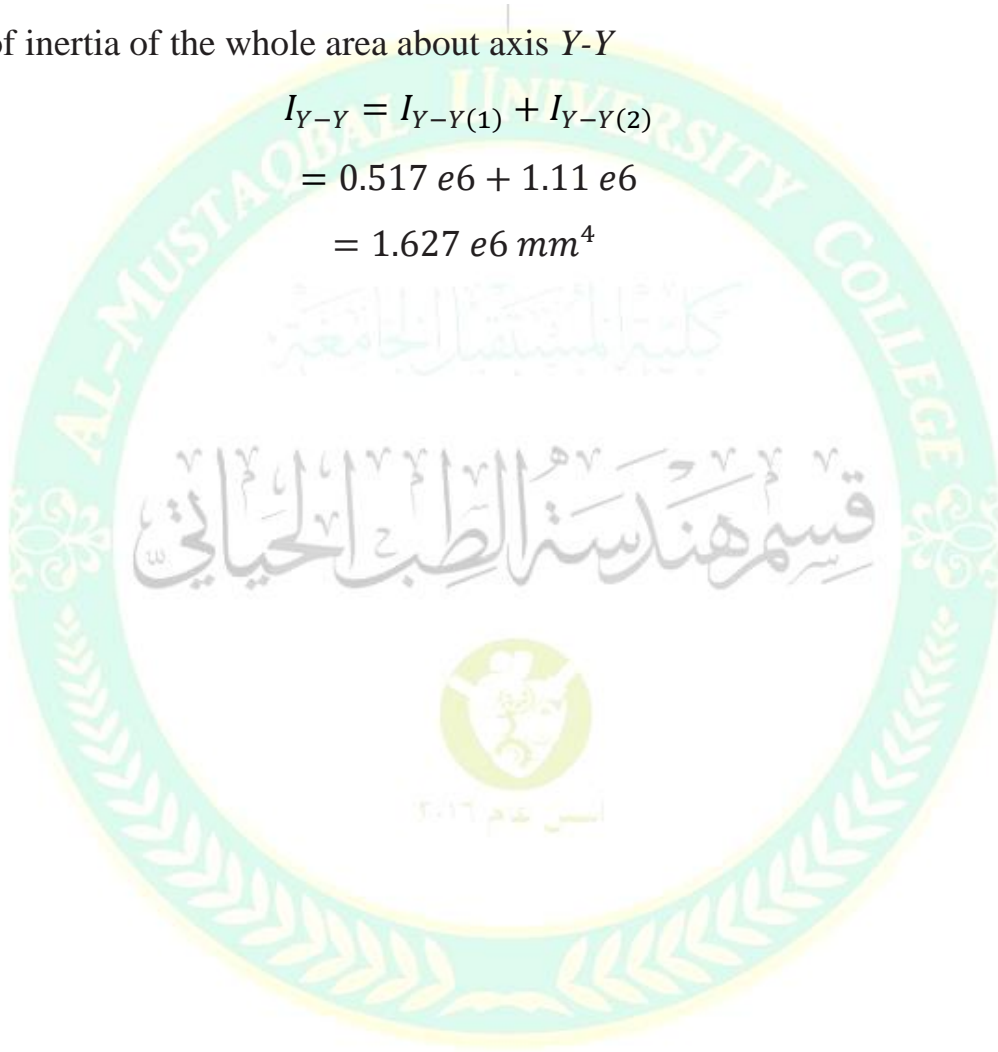
$$h_2 = 50 + 25 = 25 \text{ mm}$$

Moment of inertia of section (A1) about axis X-X

$$\begin{aligned}I_{X-X(2)} &= I_{G1} + A_1 h_1^2 \\ &= 0.36 \text{ e6} + 1200 * 25^2 \\ &= 1.11 \text{ e6 mm}^4\end{aligned}$$

Now moment of inertia of the whole area about axis Y-Y

$$\begin{aligned}I_{Y-Y} &= I_{Y-Y(1)} + I_{Y-Y(2)} \\ &= 0.517 \text{ e6} + 1.11 \text{ e6} \\ &= 1.627 \text{ e6 mm}^4\end{aligned}$$



Dr. Mujtaba A. Flayyih

Ph.D. in Mechanical Engineering (Applied Mechanics)