3.1 - Reciprocal of a Linear Function

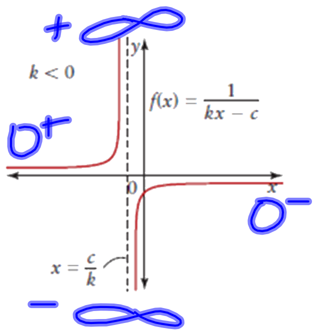
The reciprocal of a linear function has the form:

$$f(x) = \frac{1}{kx - c}.$$

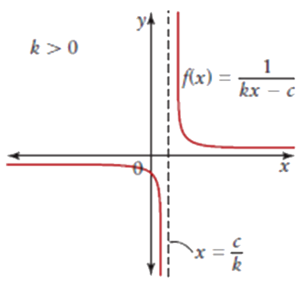
The restriction on the domain as well as the equation of the vertical asymptote ARE THE SAME THING and can be found by setting the denominator equal to zero and isolating for x.

The horizontal asymptote of a reciprocal linear function has the equation y = 0.

If the reciprocal function is negative, the graph is in quadrant 2 and 4.



If the reciprocal function is positive, the graph is in quadrant 1 and 3.



Example One

Consider the function $f(x) = \frac{1}{2x-1}$.

a) State the domain.

$2x-1 \neq 0$, $2x \neq 1$, $x \neq \frac{1}{2}$

$\{x \in \mathbb{R} \mid x \neq \frac{1}{2}\}$

b) Describe the behaviour of the function near the vertical asymptote.

CONNECTIONS

$x \to a^+$ means as x approaches a from the right.

$x \to a^-$ means as x approaches a from the left.

x	f(x)
~~0~~	
~~0.4~~	
~~0.45~~	
0.49	−500 (−∞)

$\frac{1}{2(0.499)-1} = -500$ $(-\infty)$

$\therefore x \to^{-} 0.5$, $y \to -\infty$

x	f(x)
~~1~~	
~~0.6~~	
~~0.55~~	
~~0.51~~ 0.501	500 (+∞)

$\therefore x \to^{+} 0.5$, $y \to +\infty$

$\frac{1}{2(0.501)-1} = +500$ $(+\infty)$

c) Describe the end behaviours.

x	f(x)
~~-10~~	
~~-100~~	
~~-1 000~~	
-10 000 (−∞)	−0

f(x) ⟶ 0^- means as f(x) approaches 0 from below.

$f(-10000) = \frac{1}{2(-10000)-1} = \frac{1}{-20001} = -0.0000497$ (0^-)

$\therefore x \to -\infty$, $y \to 0^-$

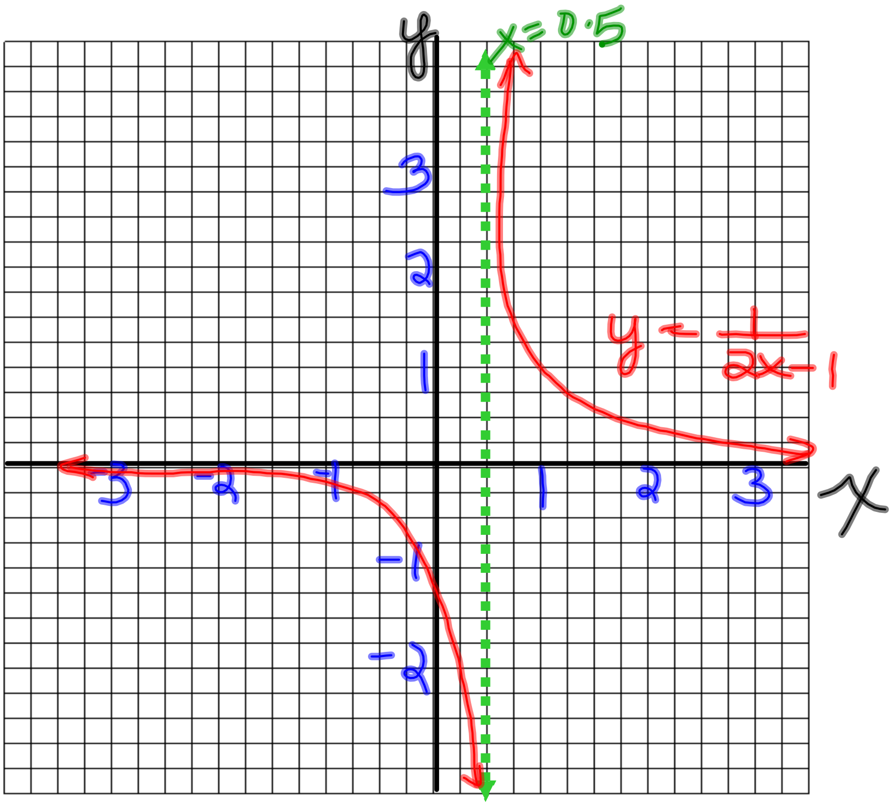
x	f(x)
~~10~~	
~~100~~	
~~1 000~~	
10 000 (+∞)	0^+

f(x) ⟶ 0^+ means as f(x) approaches 0 from above.

$f(10000) = \frac{1}{2(10000)-1} = \frac{1}{19999} = 0.000052$ (0^+)

$x \to +\infty$, $y \to 0^+$

d) Sketch a graph of the function.



e) State the range.

$\{y \in \mathbb{R} \mid y \neq 0\}$

horizontal asymptote

f) Write equations to represent the horizontal and vertical asymptotes.

Vertical Asymptote: $x = \frac{1}{2}$

Horizontal Asymptote: $y = 0$

Example Two

Determine the x- and y-intercepts of the function $g(x) = \frac{2}{x+5}$.

X- intercept

Let $y=0$, solve for x.

$\frac{0}{1} = \frac{2}{x+5}$, $2(1) = 0(x+5)$

$2 = 0$

There is no value of x that will make the fraction equal to 0.

$\therefore$ no x- intercept.

y- intercept

Let $x=0$ & solve for y.

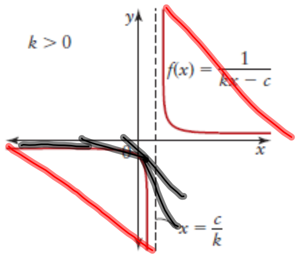
$y = \frac{2}{0+5}$

$y = \frac{2}{5}$

$\therefore$ y- intercept is @ $(0, \frac{2}{5})$

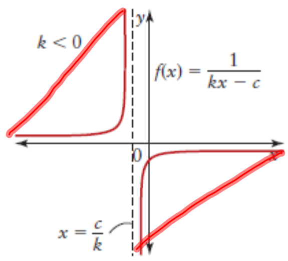
Note

- If $k > 0$, the left branch of a reciprocal linear function has a negative, decreasing slope, and the right branch has a negative, increasing slope.



decreasing means "more negatively sloped"

- If $k < 0$, the left branch of a reciprocal function has positive, increasing slope, and the right branch has positive, decreasing slope.



increasing means "more positively sloped"

Steps for Discussing the Slope of the Branches

1. Calculate the vertical asymptote.

2. Based on the sign of the reciprocal function, sketch the branches in the appropriate quadrants.

3. Discuss the slope of each branch, from left to right along the x-axis.

Example Three

Describe the intervals where the slope is increasing and the intervals where it is decreasing in the two branches of the rational function $h(x) = -\frac{1}{5x+2}$

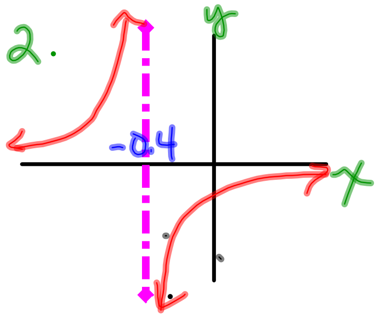
1. Vertical Asymptote

$5x+2=0$

$5x=-2$

$x=\frac{-2}{5}=-0.4$

2.



3.

$x<-0.4$	$x>-0.4$
positive & increasing	positive & decreasing

Complete: p. 154-155 # 1, 2, <u>3,5,7 ac</u>, 8.