Stellar Atmospheres
Lecture 7

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Stellar Opacity

BOUND-BOUND (LINE) ABSORPTION

BOUND-FREE AND FREE-FREE (CONTINUOUS) ABSORPTION

Opacity

- The removal of energy from a beam of photons as it passes through matter is governed by line absorption (**bound-bound**), photoelectric absorption (**bound-free**), inverse bremsstrahlung (**free-free**) and photon scattering.
- Stimulated emission acts as negative opacity by creating photons that add to the beam.
- Stellar atmospheres are predominantly hydrogen (90% by number), whilst helium makes up almost all the rest. These two elements provide most of the opacity over most wavelengths for most (hot) stars.

Chemical composition (Pop I)

- Stellar atmosphere = mixture, composed of many chemical elements, present as atoms, ions, or molecules
- Abundances, e.g., given as mass fractions β_k
- **Solar abundances**

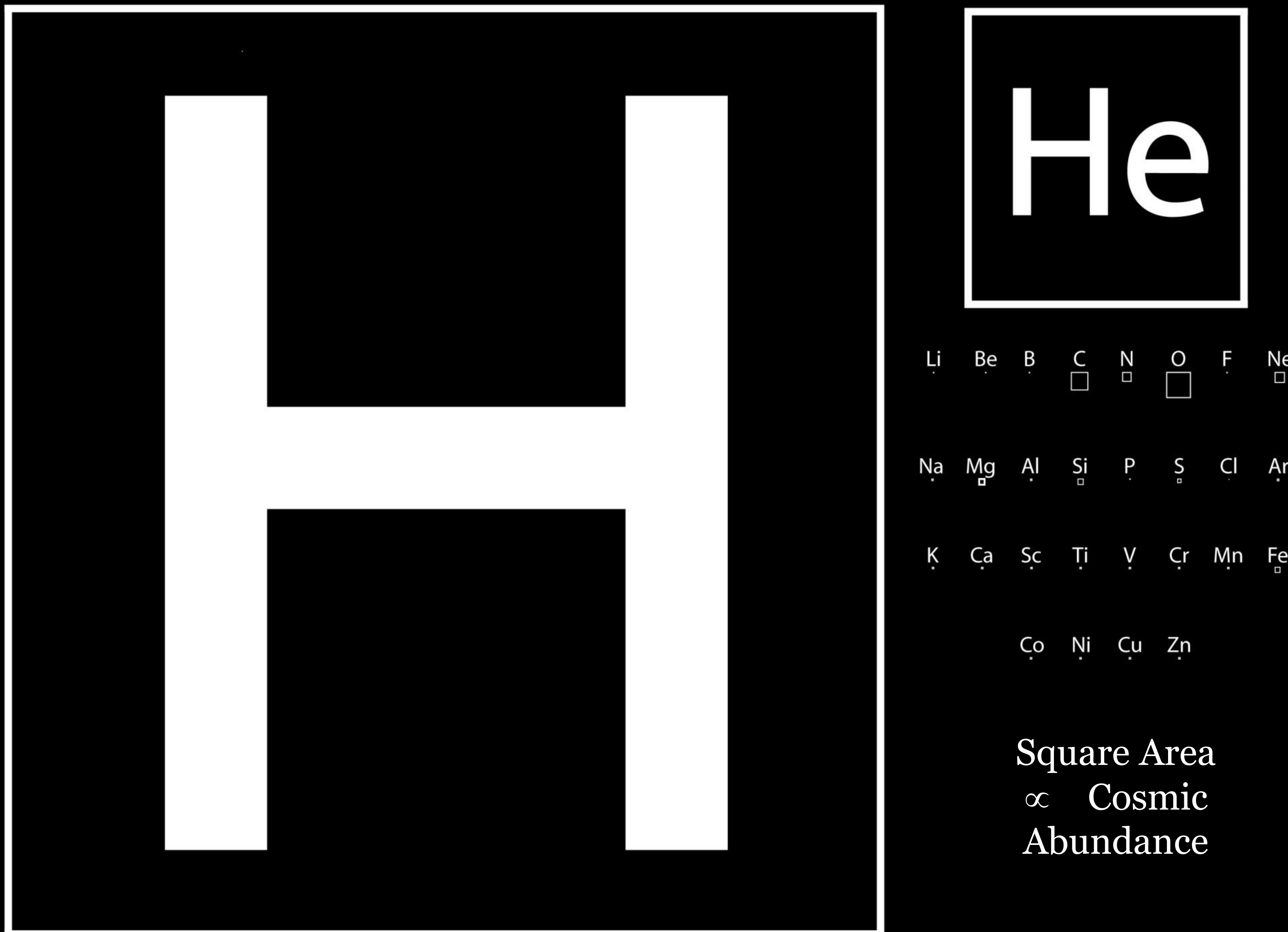
$$\beta_H = 0.71 \longrightarrow X$$
$$\beta_{He} = 0.28 \longrightarrow Y$$

"Metals" (**Z**):

$$\beta_C = 0.004$$
$$\beta_N = 0.001$$
$$\beta_O = 0.009$$
$$\vdots$$
$$\beta_{Fe} = 0.001$$
$$\vdots$$

Universal abundance for Population I stars

X+Y+Z=1



Square Area $\propto$ Cosmic Abundance

Chemical composition (Pop II)

- Population II stars

$$\beta_H = \beta_H^{\odot}$$
$$\beta_{He} = \beta_{He}^{\odot}$$
$$\beta_Z = 0.1 \cdots 0.00001\ \beta_Z^{\odot}$$

- Chemically peculiar stars, e.g., helium stars

$$\beta_H \le 0.002 << \beta_H^{\odot}$$
$$\beta_{He} = 0.964 >> \beta_{He}^{\odot}$$
$$\beta_C = 0.029 >> \beta_C^{\odot}$$
$$\beta_N = 0.003 \approx \beta_N^{\odot}$$
$$\beta_O = 0.002 < \beta_O^{\odot}$$

- Chemically peculiar stars, e.g., PG1159 stars

$$\beta_H \le 0.05 << \beta_H^{\odot}$$
$$\beta_{He} = 0.25 >> \beta_{He}^{\odot}$$
$$\beta_C = 0.55 >> \beta_C^{\odot}$$
$$\beta_N < 0.02$$
$$\beta_O = 0.15 >> \beta_O^{\odot}$$

Other definitions

- Particle number density N_k = number of atoms/ions of element k per unit volume. Relation to mass density:

$$\beta_k \rho = A_k m_H N_k$$

with A_k = mean mass of element k in atomic mass units (AMU)
m_H = mass of hydrogen atom

- Particle number fraction $$\frac{N_k}{\sum N_{k'}}$$
- Logarithmic $$\varepsilon_k = \log(N_k / N_H) + 12.00$$

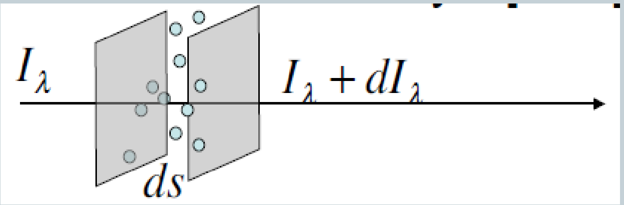
- Iron(Fe)-to-Hydrogen(H) ratio, for the Sun: $\log\left(\frac{N_{Fe}}{N_H}\right) \cong -4.3$

For other stars: $[\mathrm{Fe/H}] = \log\frac{(\mathrm{Fe/H})_*}{(\mathrm{Fe/H})_\odot} = \log(\mathrm{Fe/H})_\odot$- $\log(\mathrm{Fe/H})_*$

$$[\mathrm{Fe/H}]_\odot \equiv 0$$

Absorption coefficient

- The monochromatic absorption coefficient specifies the energy fraction taken from a light beam. It may be defined per particle, per gram, or in terms of a geometrical cross-section in cm^2:



- Per gram: $\boldsymbol{dI_\lambda \equiv -\kappa_\lambda \rho I_\lambda ds}$, where κ_λ is the mass absorption coefficient [$cm^2\ g^{-1}$], ρ is the density [$g\ cm^{-3}$].

- Per cm path length: $\boldsymbol{dI_\lambda \equiv -\alpha_\lambda I_\lambda ds}$, where $\boldsymbol{\alpha_\lambda}$ is the absorption coefficient [cm^{-1}]

$$\alpha_\lambda = \kappa_\lambda \rho$$

- Per particle: $\boldsymbol{dI_\lambda \equiv -\sigma_\lambda n\, I_\lambda ds}$, where $\boldsymbol{\sigma_\lambda}$ is the absorption cross-section per particle for individual transitions and $\boldsymbol{n}$ is the number density [particles cm^{-3}]

$$\boldsymbol{\alpha_\lambda = \sigma_\lambda n = \kappa_\lambda \rho}$$

Dominant sources of opacity

- The most important transitions for the continuous absorption are those which ionise atoms (with a continuum of final states). For H and He the line spectra do not greatly affect radiative transport. Some metals, with very complex line spectra do contribute to the continuum.
- New stellar opacities have been recalculated in the past 15-20 years by two groups – OPAL (Iglesias et al.) and The Opacity Project/OP (Seaton et al.) which have led to a factor of 3 increase in opacity under some temperature-density conditions via improved treatment of atomic data.

Line absorption

- A bound-bound transition absorbs or emits at $h\nu = hc/\lambda = \chi_u - \chi_l$ where χ is the excitation of the upper and lower levels above the ground state. Such transitions contribute to the **line** absorption.

- For H, the excitation energy for a level n of H is $\chi_n = \chi_{\text{ion}}\left(1 - \frac{1}{n^2}\right)$, where χ_{ion} =13.6eV.

- A bound-bound transition between n=low and n=high occurs at a wavelength of

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right), \quad h\nu = \chi_{ion}\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

where R_H=109677.5 cm^{-1} is the Rydberg constant for H.

- The line Lyman α at 1215A is the transition between the ground-state (n=1) and first excited state (n=2).

Oscillator strength

- The bound-bound cross-section is given by

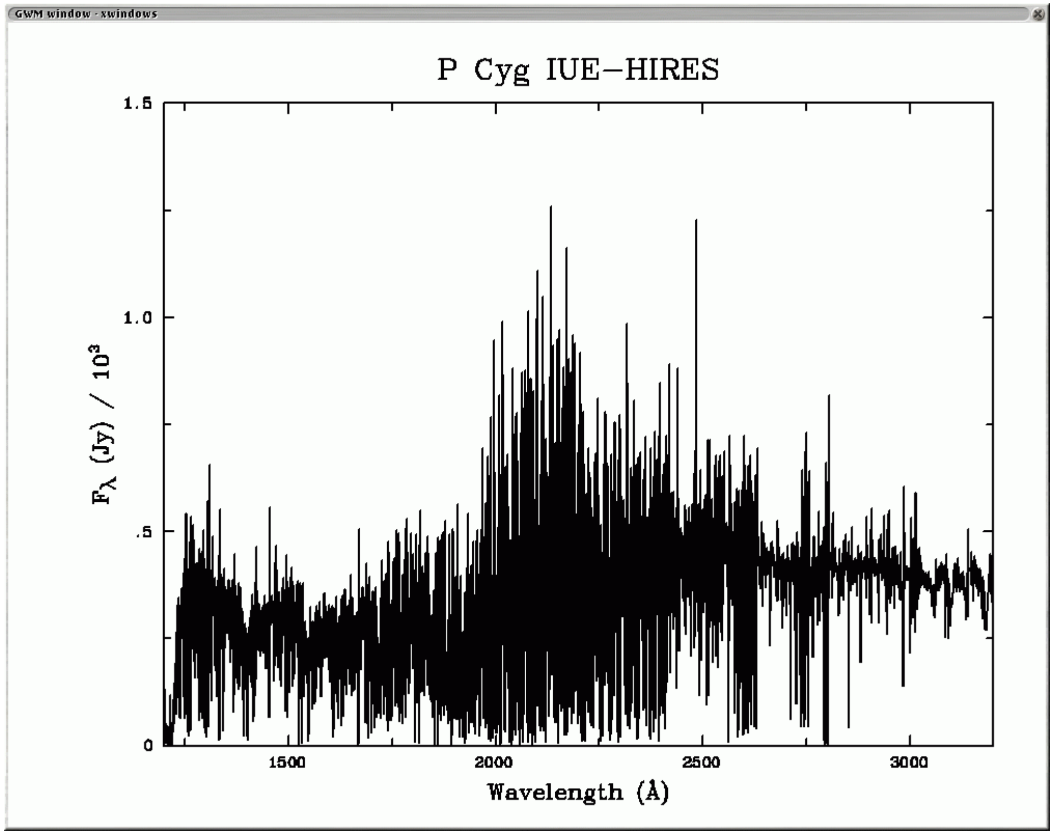
$$\sigma_\nu^{bb} = \frac{\pi e^2}{m_e c} f_{ij} \phi_\nu$$

where f is the transition oscillator strength, and ϕ is the line profile function (these quantities will be introduced later).

- The cumulative effect of many lines can behave much as continuous opacity in the upper photosphere. Problems associated with line opacity are due to the large numbers of lines involved.
- Data for millions of atomic line transitions have been calculated by Kurucz and more recently by the OP (Opacity Project).

Observations

Here is the effect of many lines (Fe II-III) on the emergent UV continuum of the extreme supergiant P Cygni.



Continuous absorption

For **continuous** sources of absorption, there has to be a continuum of energy levels, i.e. at least one end of the transition involving a free state of the electron (at an energy above χ_{ion}). Two possibilities…

1. A transition from a bound state (level n) to a free state with velocity v. The energy of the absorbed bound-free photon is given by

$$h\nu = hc/\lambda = (\chi_{\text{ion}} - \chi_n) + mV^2/2$$

   Each bound-free transition corresponds to an ionization process (since the electron is free afterwards). The emission of a photon by a free-bound transition corresponds to a recombination process.

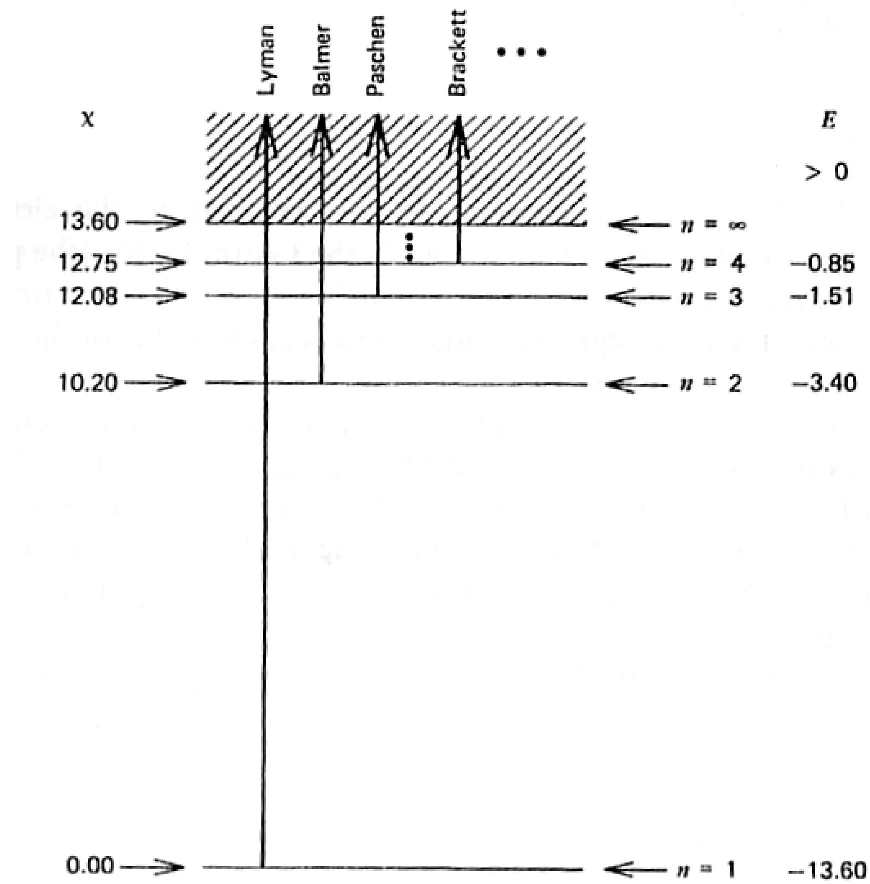
2. Finally, one can get a continuum of transitions if the electron goes from one free-state (with velocity V_1) to another free-state (with velocity V_2). The energy of the free-free transition is

$$h\nu = \frac{hc}{\lambda} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2}$$

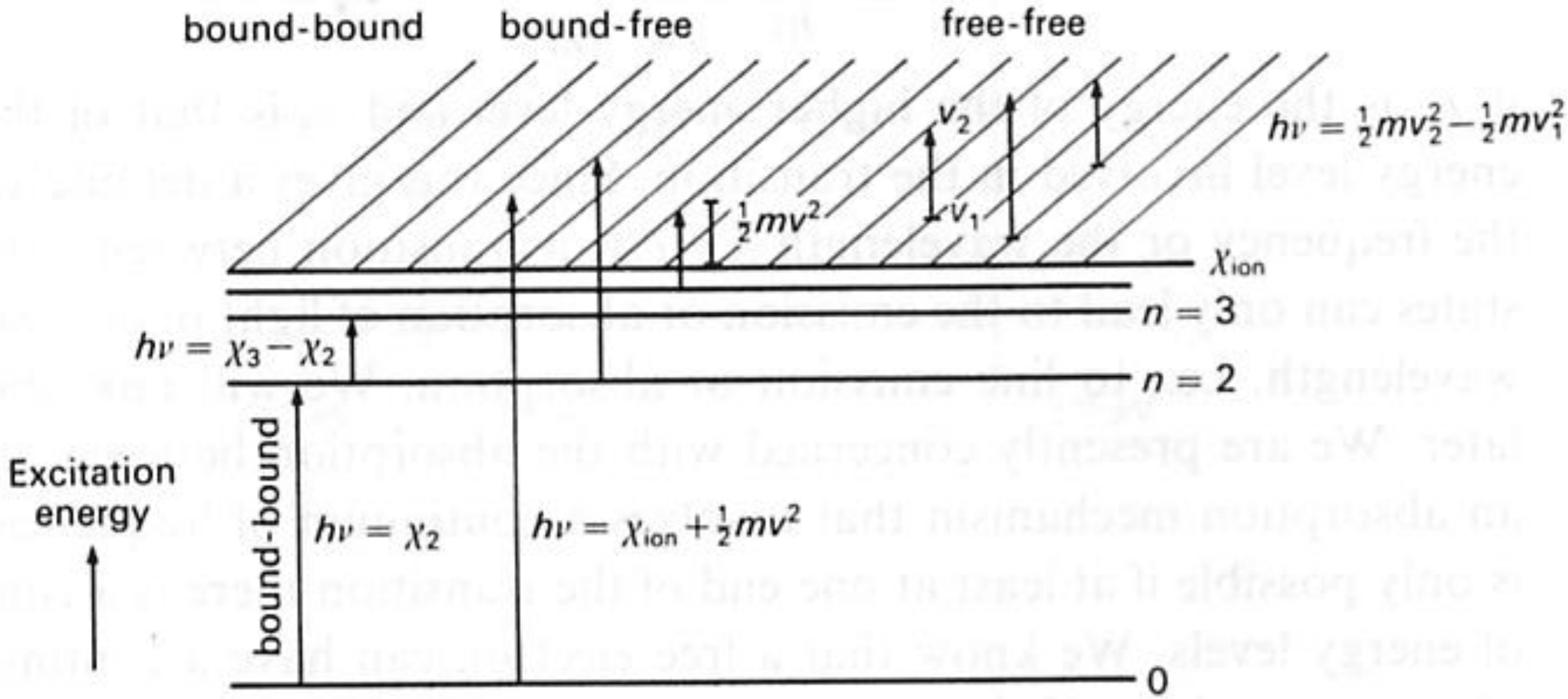
Lyman, Balmer, Paschen continua

- For hydrogen, transitions occurring between n=1 and another bound state n=2, 3, 4, etc. are known as the Lyman series (observed in the UV), between n=2 and higher members are the Balmer series (seen in the optical), with higher series observed in the IR: Paschen (n=3), Brackett (n=4), Pfund (n=5), etc.

- The Lyman continuum etc, refer to a bound-free transition between n=1 and the H+ continuum



bb, bf, ff-processes



1 eV=11600 K =1.602 10^{-12} erg

χ_{ion}=13.6 eV, χ_{ion}/k=158000 K

Continuous absorption

Which states contribute at a given wavelength?

- Photons need an energy great enough to overcome the ionization energy i.e. $h\nu > \chi_{ion} - \chi_n$ or $\lambda < hc/(\chi_{ion} - \chi_n)$. At long wavelengths only energy levels with very large χ_n can contribute to α, so most continuous opacity is from mainly free-free transitions.
- The contribution of level n will start at $\lambda_n = hc/(\chi_{ion} - \chi_n)$ and continue for shorter λ. There is a **discontinuity** at λ_n because of a sudden change in the number of absorbing atoms, e.g.

  Lyman jump (912A) due to the contribution of **n=1**.
  Balmer jump (3647A) due to the contribution of **n=2**.

- To derive α_λ, the total absorption at wavelength λ, we have to multiply σ_n by the number of atoms in this state and sum up all states n that contribute at this wavelength. For this we need to use the Boltzmann formula.

$$\alpha_\lambda = \sigma_\lambda n$$

Bound-free absorption coefficient

Kramers approximation for continuous cross-section for level *n* for H-like nucleus of charge Z:

Gaunt factor ≈ 1

$$\boldsymbol{\sigma_{bf}(\mathrm{H}) = \frac{32\pi^2}{3\sqrt{3}} \frac{e^6}{c^3\, h^3} R \frac{\lambda^3}{n^5} G_{bf} = a_0 \frac{\lambda^3}{n^5} G_{bf}}\ \textbf{cm}^2\ \textbf{per neutral H atom}$$

Rydberg constant

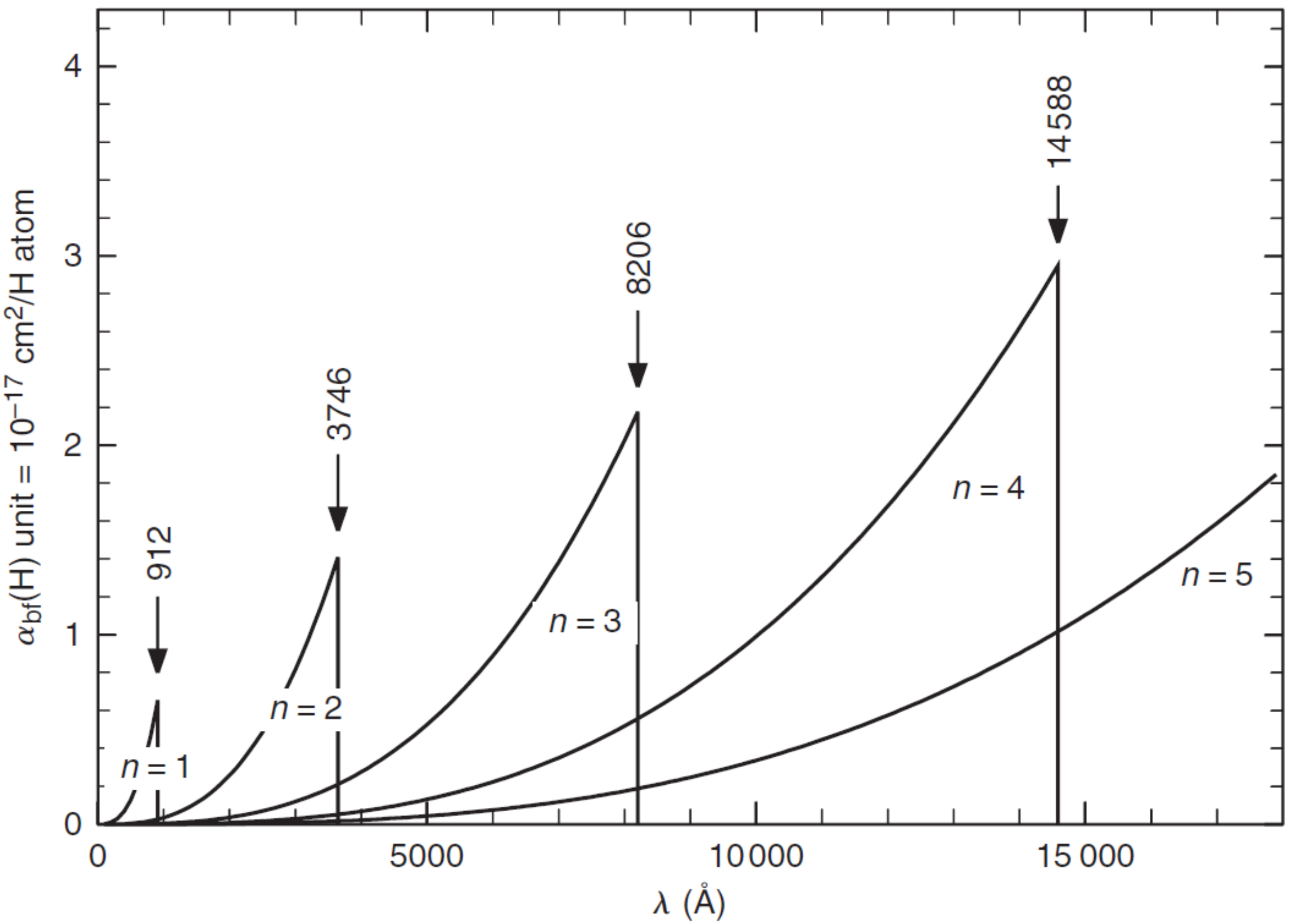
$R=2\pi^2 me^4/h^3c$

$a_0=1.0449\times10^{26}$ for λ in angstroms

The photoionization threshold is $E_n=h\nu_{nc}$, so σ_n decreases with ν (increases with λ). For H, at the threshold $\sigma_{1c}=6.3\times10^{-18}$ cm²

$$\nu_{nc} = \chi_{ion} - \chi_n$$

The total absorption coefficient for H is: $\alpha_{bf}^H = \sum_1^\infty \sigma_{nc} N_n$



The bound–free absorption coefficient for hydrogen increases with *n*.

Free-free absorption coefficient (1)

- The free-free continuous absorption coefficient for H is much smaller than the bound-free coefficient.
- When a free electron collides with a proton, its orbit (**unbound**) is altered. A photon may be absorbed during such a collision, the orbital energy of the electron being increased by the photon energy.
- The strength of the absorption depends on the electron velocity (slower electrons are more likely to absorb a photon because a slow encounter increases the probability of a photon passing by during the collision.
- We adopt a Maxwellian distribution.
- Kramers (1923):

Rydberg constant

$$d\sigma_{ff}(\mathrm{H}) = \frac{2}{3\sqrt{3}} \frac{h^2 e^2 R}{\pi m_e^3} \frac{1}{\nu^3\, v} dv$$

Cross section for the fraction of electrons in the velocity interval

Free-free absorption coefficient (2)

- Integrate over velocity:

$$\sigma_{ff}(\mathrm{H}) = \frac{2}{3\sqrt{3}} \frac{h^2 e^2 R}{\pi m_e^3} \frac{1}{\nu^3} \left(\frac{2m_e}{\pi \kappa T}\right)^{1/2}$$

- The total absorption coefficient for H is: $\kappa_{ff}^{H} = \frac{\sigma_{ff} G_{ff} N_i N_e}{N}$

  where the number density of electrons, ions and neutral Hydrogen are N_e, N_i and N, respectively.

- $N_i N_e / N$ can be substituted:

$$\kappa_{ff}^{H} = \sigma_{ff} G_{ff} \lambda^3 \frac{\log e}{2\Theta \mathrm{I}} 10^{-\Theta \mathrm{I}}$$

  where I=hc$R$, R=2π²me⁴/h³c

- This absorption process is the inverse of Bremsstrahlung emission.

Wavelength dependence of α(H)

- Consider the H absorption coefficient $\boldsymbol{\alpha}$ (per atom) for T=5040K (Θ=5040/T=1). Let us compare the value of $\boldsymbol{\alpha}$ in the Balmer (n=2) to Lyman (n=1) continua at 912A:

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{\sigma_{i2}}{\sigma_{i1}}\frac{N_2}{N_1} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}e^{-(10.2eV/kT)} = \frac{\sigma_{i2}g_2}{\sigma_{i1}g_1}10^{-10.2\times 5040\,\mathrm{K}/T)}$$

- From above, $\sigma_n \propto n^{-5}$ and $g_n = 2n^2$ so $\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}6.3\times 10^{-11} = 8\times 10^{-12}$

- There is a huge difference in hydrogen absorption coefficient at 912Å (Lyman edge) at T=5040K.

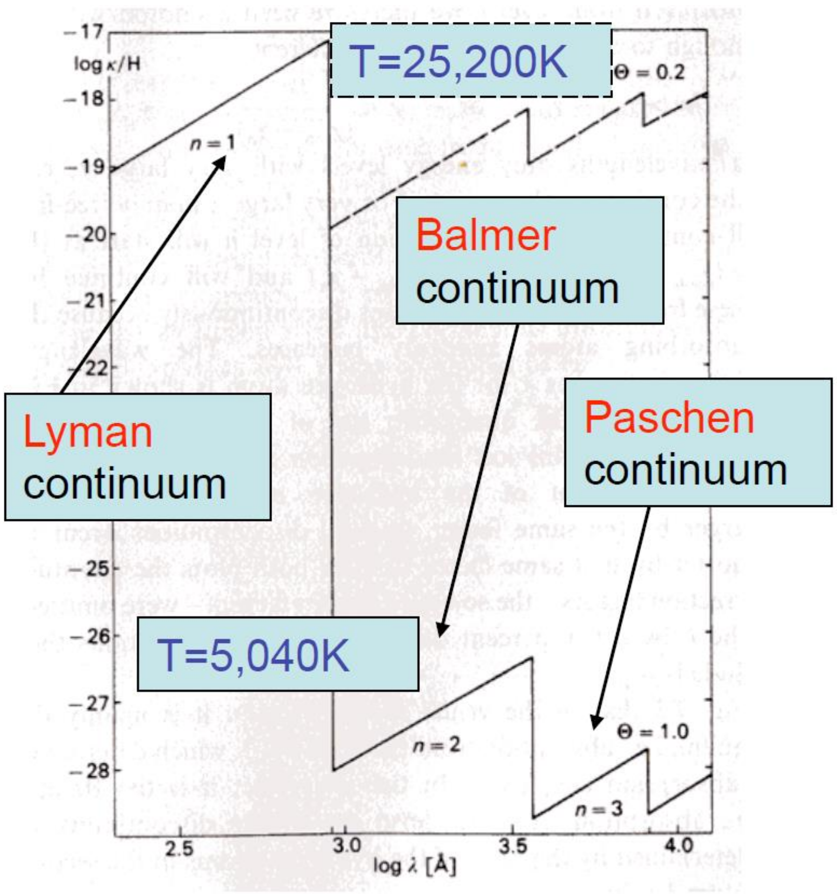
- Similar calculations at T=25 200K (Θ=5040/T=0.2) give

$$\frac{\alpha(Balmer)}{\alpha(Lyman)} = \frac{2^{-5}\times 8}{1\times 2}0.009 = 0.001$$

- Hydrogen absorption coefficient is very T sensitive!

Wavelength dependence of α(H)

- Primarily, the Paschen continuum (absorption from n=3) determines the H absorption coefficient in the visual (3647Å<λ<8205Å).
- For He^+, the ionization energy is larger by a factor of $Z^2=4$ than that of the H atom. All discontinuities occur at wavelengths shorter by a factor of 4, i.e. 228Å instead of 912Å for the He^+ Lyman continuum.



Negative hydrogen ion H^-

- The H atom is capable of holding a second electron in a bound state (binding energy 0.754eV). All photons with $\lambda < 1.64\mu m$ have sufficient energy to ionize the H^- ion back to neutral H atom plus a free electron. The extra electrons needed to form H^- come from ionized metals (such as Ca^+).
- For Solar-like stars, it turns out that H^- is the dominant continuum opacity source at optical wavelengths. In early-type stars H^- is too highly ionized to play a role, whilst in late-type stars there are too few free electrons (since no ionized metals).

Importance of H^- in the Sun

We can use the Saha equation to derive the relative population of $N(H^-)$ in the Sun (u^-=1, T=5777K, χ_{ion}=0.754 eV),

$$\log\frac{N^+}{N^0} = \log\frac{u^+}{u^0} + \log 2 + \frac{5}{2}\log T - \chi_{ion}\Theta - \log P_e - 0.48$$

$$\log\frac{N(H^0)}{N(H^-)} = \log\frac{2}{1} + \log 2 + 9.40 - 0.66 - 1.18 - 0.48 = +7.68$$

So, only **2 out of 10^8** hydrogen atoms is in the form of H^-

Why then the H^- absorption coefficient so important?
Recall, only H atoms in the 3rd quantum level (n=3, Paschen continuum) can contribute to the visual continuous opacity. From the Boltzmann formula

$$\log N(H_{n=3})/N(H_{n=1}) = \log 2(3)^2/2(1)^2 - 5040/5777 \times 12.1 = -9.6$$

i.e. $N_H(n=3)/N_H(n=1)=2.4\times10^{-10}$ for the Sun. We can now compare the number of H- ions and H atoms in the Paschen continuum:

$$\log N(H_{n=3})/N(H^-) = 2.4\times10^{-10}/2.1\times10^{-8} = 0.01$$

Importance of H^- in the Sun

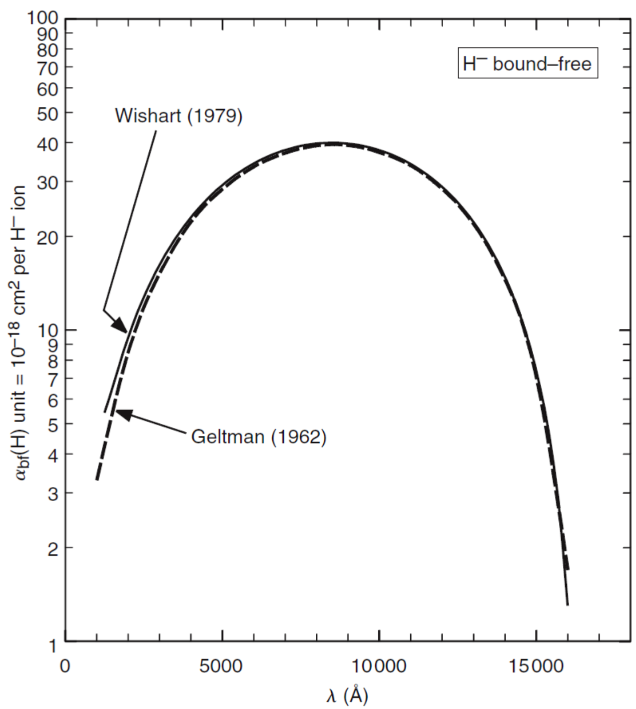
The atomic absorption coefficients per absorbing atom are comparable, so we expect H^- b-f absorption to be 100 times more important than the H Paschen continuum for the Sun.

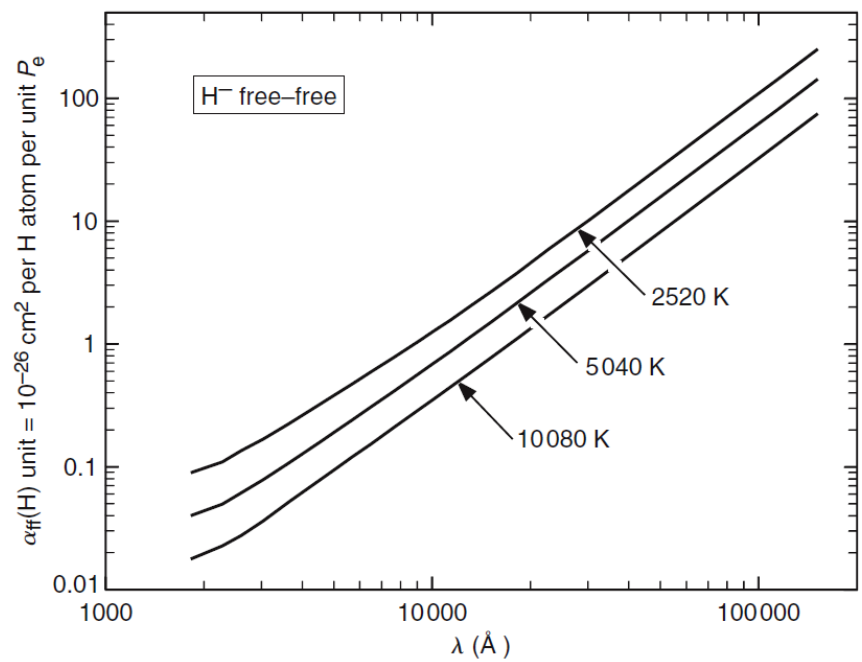
The Balmer continuum (n=2) cannot so easily be neglected, and does contribute to the opacity at shorter wavelengths.

Note: For early type stars (A and earlier) we find $N_H(n=3)/N(H^-) \gg 1$ so absorption of neutral H is much more important than H^-. This is why such stars have very strong discontinuities in the Balmer & Paschen limits. We will discuss the importance of the Balmer jump shortly.

H^- continuous opacity

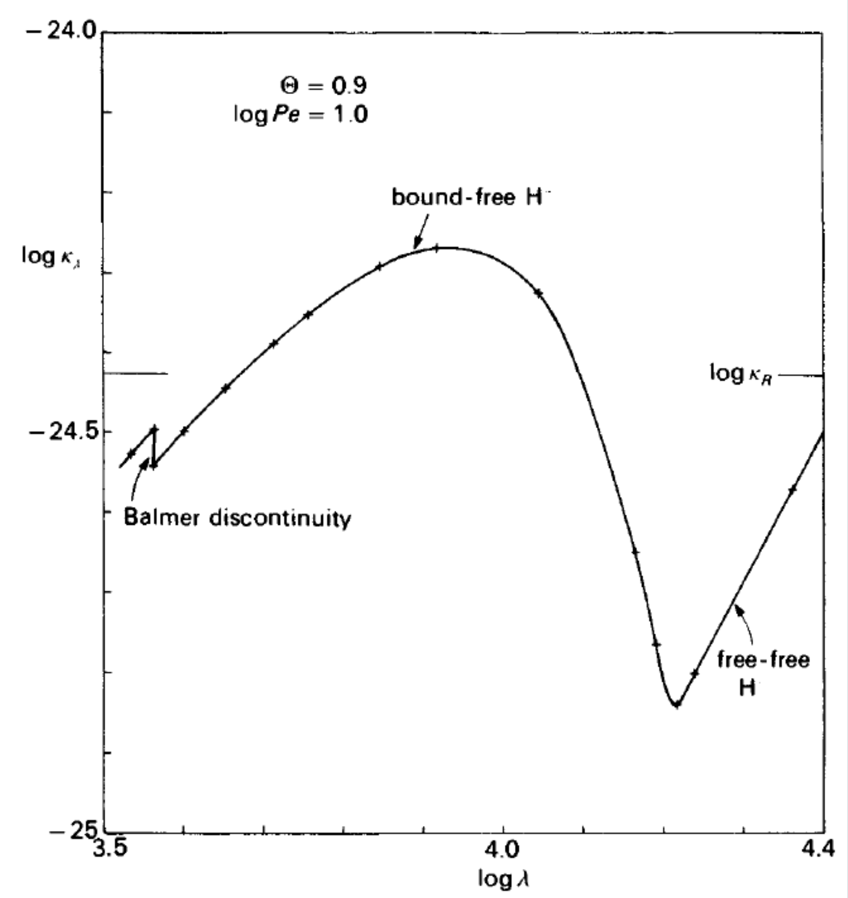
The bound-free H^- absorption can occur for $\lambda<16500$Å, with a different behaviour from H, reaching a maximum at 8000Å, and decreasing towards the ultraviolet. At longer wavelengths, there is only free-free H^- absorption (with a $\nu^{-3} \propto \lambda^3$ dependence).





Hydrogen continuous opacity

- We have identified H^- (bound-free) in the visual and H^- (free-free) in the IR as principal sources of opacity in the Sun.
- The H Balmer continuum shortward of the 3647Å Balmer jump is an additional contributor.
- What observational evidence is there that this is true for the Sun, and what other forms of opacity play a role in other stars?



Summary

- Bound-bound transitions contribute to the line absorption. Bound-free and free-free transitions (plus scattering) contribute to the continuous absorption, mostly by H & He.
- Atomic H absorption coefficient highly T sensitive. For late-type stars in the optical and IR, bound-free and free-free transitions of the H^- ion dominate the continuous opacity, since the population of atomic H in n=3 (Paschen series) is so low.
- For early-type stars, atomic H dominates, producing strong jumps in the opacity at the Lyman, Balmer & Paschen edges.