Stock vs. Bond Yields, and Demographic Fluctuations

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Abstract

This paper analyzes the strong comovement between real stock and nominal bond yields at generational (low) frequencies. Life-cycle patterns in savings behavior in an overlapping generations model with cash-in-advance constraints explain this persistent comovement between financial yields. We argue that the slow-evolving time-series covariation due to changing population age structure accounts for the equilibrium relation between stock and bond markets. As a result, by exploiting the demographic information into distant future, the forecasting performance of valuation models improves. Finally, using a cross-country panel, we document the cross-sectional variation of the demographic effect and explain the cross-country differences in comovement between stock and bond markets.

KEYWORDS: demographics, cash-in-advance, stock-bond, return forecasting.

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1 Introduction

This paper shows that stock and bond yields share a common demographic component that explains the slow-evolving time-series covariation. Yields to aggregate U.S. stock and government bond markets follow surprisingly similar paths in the post-war period (e.g., Bekaert and Engstrom, 2010; Maio, 2013). This evidence incites debate on the validity of rational valuation models that rely on relative pricing of stock and bond markets (Asness, 2003; Estrada, 2009). At the same time, yields to the aggregate stock market are positively correlated with inflation (Wei, 2010). This observation is puzzling, since conventional wisdom suggests that the stock market represents real assets and hence should be a good hedge against inflation. Behavioral explanations such as the "inflation illusion" (Modigliani and Cohn, 1979; Campbell and Vuolteenaho, 2004; Feinman, 2005) or risk-based stories (Brandt and Wang, 2003; Bekaert and Engstrom, 2010) have been suggested to reconcile this evidence.[1]

Insert Figure 1 here

Information on stock-bond yield correlation provided in Figure 1 confirms prior evidence: regardless of the stock yield measure (dividend or earnings yield), the correlation is highly positive over most of the 1900-2014 period, except during the Bretton Woods and recent crisis periods. While we observe time variation in correlation due to business cycle shocks (Campbell et al., 2013; Hasseltoft, 2009, 2012)[2], our paper focuses on the persistent low-frequency correlation, at generational frequency. Based on the relation between the population age structure and financial markets, we provide a novel explanation for this strong comovement at generational frequency, while allowing for a switch in the sign of this comovement. First, by developing an overlapping generations model (OLG) with money and cash-in-advance constraints, we show that a common

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[1] Such behavioral arguments suggest that stock market investors fail to increase the expected nominal cash payouts in response to increases in expected inflation, so that one can explain the comovement of dividend yield with nominal bond yields a well as the negative relation between inflation and stock market returns. Yet, others question whether such a behavioral bias shared by only stock market participants can explain the alleged equilibrium relation between cross-asset yields. See Thomas and Zhang (2008) and Wei (2010).

[2] These papers show that risk channel at the business cycle-frequency is the determinant of time-varying stock-bond market return correlation. While the models based on the risk channel describe the temporary changes in the correlations, these changes are often not predictable.

demographic factor drives the persistent, long-run components of financial yields. We then show empirically that the link between financial yields and a demographic variable is robust in the U.S. sample over the last century. Next, we test the relevance of demographic fluctuations within a valuation model and show how demographic information about distant future can be successfully used to forecast excess stock market returns (Lander et al., 1997; Bekaert and Engstrom, 2010; Maio, 2013). Finally, we extend the analysis to a cross-country panel, and document cross-sectional variation of the demographic effect. This evidence provides out-of-sample support for the importance of a common demographic component in countries with high stock market participation, and hence explains the differences in bond-stock comovements across countries.

Insert Figure 2 here

The U.S. population age structure evolves over time and features twenty-year boom and bust cycles (see Figure 2).[3] This life-cycle pattern appears as a predictable component in financial yields. We account for the gradual change in these time series using a specific demographic variable, namely the proportion of the middle-aged to young population, MY_t. This variable affects financial markets through the demand channel at generational frequency. Figure 3 plots earnings yield to aggregate U.S. stock market and 10-year U.S. nominal bond yields together with the long-run components captured by the demographic variable MY_t.[4] Using a classification of monetary regimes that is similar to the ones proposed in earlier literature,[5] we also document the changes in the average bond yield (dotted horizontal lines) across monetary regimes (Filipova et al., 2014). The low frequency components of stock and bond data reveal surprisingly similar patterns, and starting from the second half of the Bretton Woods period, demographics-driven trends

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[3] The boom in births (baby boom generation) starts during the prosperous period that follows WWII, and comes to an end because of the historical coincidence of invention of oral contraceptives (and widespread use during the 60's) and the legalization of abortion in 1973. The wave continues with the children of baby boomers, also named as echo boomers, and the children of baby bust generation.

[4] The earnings yield time series is the cyclically adjusted earning price ratio (using a 10-year window of earnings) taken from Robert Shiller's webpage.

[5] Meltzer (1986), D'Agostino and Surico (2012), and Bordo and Haubrich (2008) identify seven regimes before the recent Quantitative Easing period: the Pre-Fed period (Gold Standard without a central bank), from 1900q1 to 1914q4; Gold Standard with a central bank, from 1915q1 to 1931q3; the Mixed Regime, from 1931q4 to 1939q3; Pegged Interest Rate, 1939q4-1951q1; Bretton Woods, 1951q2-1971q3; Great Inflation, 1971q4-1983q4; Great Moderation, 1984q1-2008q3. We use the same time frames and we add an eighth period, the QE period, starting in 2008q4.

of financial yields carry useful information beyond the drifts in the mean that are due to shifts in monetary regimes. This coincides with the period of large demographic fluctuations in U.S. population age structure.

Insert Figure 3 here

The interest in MY_t as an empirical proxy for the change in the U.S. age structure is not arbitrary; it is derived from an overlapping generations model by Geanakoplos et al. (2004, henceforth GMQ). GMQ conjecture that the life-cycle portfolio behavior (Bakshi and Chen, 1994) plays an important role in determining equilibrium asset prices; given the assumed demographic structure, consumption smoothing requires that, when MY_t is large, there is excess demand for saving by a large cohort of middle-aged population. For the market to clear, equilibrium prices of financial assets and therefore the yields should adjust. GMQ focus on the real economy and hence on real prices of financial assets,[6] so the model provides no predictions on aggregate price level or on inflation, which is an important component of nominal bond yields. The present paper extends the GMQ model by introducing money as a medium of payment. Our model shows that the age structure of the U.S. population not only affects real returns to financial assets, but also the aggregate price level and inflation. The equilibrium relation between financial yields is robust to the presence of monetary shocks that capture monetary regime shifts. On the one hand, the life-cycle portfolio behavior indicates that individuals facing a hump-shape income stream save when middle-aged, which in turn increases real asset prices. On the other hand, middle-aged workers being more productive, a large MY ratio fosters aggregate real production and aggregate real income. As economic activity grows, money demand goes up which leads to a reduction of the aggregate price level to equilibrate the money market. Therefore the price level is inversely related to the MY ratio. In other words, the population age structure not only affects real asset prices but also has a contemporaneous impact on nominal values. Therefore, isolating and quantifying the demographic effect on inflation is crucial to understand the comovement between real stock yields, e.g., dividend or earnings yield, and nominal bond yields.

The GMQ model is part of a strand of literature that aims at explaining market fluctuations with demographic factors. Bakshi and Chen (1994) develop the life-cycle

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[6] Similarly, using an OLG model, Piazzesi and Schneider (2012) analyze the relation between inflation and prices of real assets.

investment hypothesis which asserts that an investor in an early stage of her life allocates more wealth on housing and switches to financial assets at a later stage. Starting from this literature, Erb et al. (1997) and Ang and Maddaloni (2005) study the effects of demographics in an international context and document the link between demographics and risk premium. However, the evidence is not conclusive (Poterba, 2001; Goyal, 2004). While theoretical models suggest strong demographic effects on financial markets, empirical studies face difficulties documenting these effects. This is mainly because ad-hoc demographic variables are not successful in isolating the relevant low-frequency information from the noise in financial markets. Our paper contributes to this literature by analyzing jointly stock and bond markets using a demographic variable that is justified within a monetary OLG model.

Even though stock and bonds are the two main asset classes considered in long term portfolio allocation (e.g., Bali et al., 2009; Levy, 2015),[7] the literature so far mainly focuses on the business cycle comovement between stock and bond returns. While earlier empirical studies provide some evidence on stock and bond return correlation (e.g., Fama and French, 1993), this evidence is hard to reconcile within a present value model that assumes constant risk premia (e.g., Shiller and Beltratti, 1992; Campbell and Ammer, 1993). Thus two diverse strands of literature focus on predictability of stock and bond market returns separately. Despite the critical view (see, for example, Welch and Goyal, 2008), recent literature shows predictive ability of financial ratios (e.g., Ang and Bekaert, 2007; Cochrane, 2008; Ferreira and Santa-Clara, 2011). In particular, previous studies document the role of dividend yield in forecasting long term returns (e.g., Cochrane, 2008) and justify its use within the dynamic dividend growth model proposed by Campbell and Shiller (1988). This model relies on a log-linearized version of one-period returns on a stock portfolio. Yet, the derivation of the model and hence its forecasting performance crucially relies on the stationarity of dividend yields. Lettau and Van Nieuwerburgh (2008) challenge this point and find breaks in the long-run mean of dividend yield, while Favero et al. (2011) show the strong empirical link between the dividend yield persistence and demographic fluctuations.

The bond market literature, on the other hand, highlights the role of forward interest rates in forecasting future spot interest rates for longer horizons (e.g., Fama and Bliss,

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[7] A recent article ("How Much Stock Should You Own in Retirement?") published on 3 Feb 2014 in *Wall Street Journal* discusses the asset allocation problem from a long term perspective.

1987). In particular, linear combinations of forward rates are successful in predicting term premia (Cochrane and Piazzesi, 2005). An early literature attributes bond yield predictability to mean reversion of the spot rate towards a constant expected value. Yet, recent literature argues that the predictability of the spot rate captured by forward rates is either due to a moving, yet still stationary, mean (Balduzzi et al., 1998), or a time-varying, but non-stationary, long-term expected value (Fama, 2006; Duffee, 2011). Favero et al. (2015) develop a no-arbitrage affine term structure model based on the idea that the slow mean-reverting component of the spot rate is driven by demographic fluctuations, while the more rapid (business-cycle-length) mean-reverting component is captured by macroeconomic factors. However, the latter paper is agnostic about the link between demographics and inflation. We argue that stock return predictability exploiting the comovement of financial yields is tightly linked to the slow evolution of population age structure and show how a valuation model such as the Fed model can be modified to incorporate demographic information to better forecast excess stock market returns.

A growing body of literature focuses on the joint dynamics of stock and bond markets (Baele et al., 2010; Lettau and Wachter, 2011; Hasseltoft, 2012). For instance, Campbell et al. (2013) develop a model based on four state variables to explain the covariance between stock and bond returns and find that stock-bond covariance is driven by the covariance between nominal variables and the real economy. Koijen et al. (2015) propose a arbitrage-free stochastic discount factor (SDF) model where the pricing factors are motivated by a permanent/transitory decomposition of the pricing kernel and price cross-section of returns. However, none of these papers explicitly consider low frequency time-series variation in demographics as the source of a persistent, slow-moving component.

The remainder of the paper is organized as follows; Section 2 introduces the monetary overlapping generations model. Section 3 provides theoretical and empirical evidences that a demographic factor drives the long-run component of financial yields. In Section 4, we discuss the valuation models and report empirical results on return predictability. Section 5 provides cross-country evidence. Section 6 concludes.

2 A Stochastic Monetary Exchange Economy

We develop a stochastic model of a monetary exchange economy in order to show the mechanisms through which the population age structure affects both real returns, inflation, and nominal yields. The stochastic features of the model introduces a wedge between realized and expected inflation, and shows the robustness of yield correlation under different inflation regimes.

2.1 Model

2.1.1 Overview

We develop a stochastic 3-period overlapping generations model of a monetary exchange economy. We extend the stochastic model developed by GMQ (2004) to a monetary economy by introducing a Clower (1967) type cash-in-advance constraint. Each period lasts 20 years. Young and middle-aged individuals supply labor inelastically and receive labor income, while retired individuals live off their savings. The superscripts y, m and r indicate the individual's respective life stages: young, middle-aged and retired. This life-cycle portfolio behavior, as described by Bakshi and Chen (1994), plays an important role in determining equilibrium asset prices. Two types of financial instruments, bond and stock, are available and allow agents to redistribute income over time. We assume that in odd (even) periods, a large (small) cohort enters the economy, so that in every odd (even) period the demographic structure is (N,n,N) ((n,N,n)). In doing so, we focus on middle to long-run demographic fluctuations, abstracting from short-run and business cycle frequencies.

2.1.2 Stochastic Stream of Dividends and Wages

Following GMQ, we introduce random shocks to wages and dividends to circumvent the substitutability between bonds and stocks. This assumption enables us to analyze the impact of the age structure on stock prices and risk premium in a framework that incorporates the risks that individuals face when planning their life-time consumption. Labor and production plans yield real wages $w = (w^y, w^m)$ and real dividends d in each period, respectively. Income shocks are such that both wages and dividends can take low or high values: $w = \{w^L, w^H\} = \{(w^{y,L}, w^{m,L}), (w^{y,H}, w^{m,H})\}$ and $d = \{d^H, d^L\}$. Therefore, the stochastic income structure features four income states denoted by $s = \{s_1, s_2, s_3, s_4\}$ where $s_1 = (w^H, d^H)$, $s_2 = (w^H, d^L)$, $s_3 = (w^L, d^H)$, and $s_4 = (w^L, d^L)$. The stochastic wage structure $w_s = (w^y_s, w^m_s)$ reflects the higher productivity of middle-aged workers as we assume that $w^y_s < w^m_s$ in any income state s. Moreover, each individual faces a stream of wages (w^y_s, w^m_{s+1}, 0) that is concave over her life time: $w^y_s < w^m_{s+1}$ in any income states s and $s+1$.

2.1.3 The Role of Money

In our setting, the essential role of money is that of medium of payments. We build on the cash-in-advance setting proposed by Bénassy (2005) and assume that, in each period, each individual possesses an income composed of her labor income, for working individuals, and the financial returns of previous savings, if taken. Then, the bond and stock markets open, and each individual decides upon his financial investment. The rest of his income is kept in the form of money and constitutes the individual's money demand. This money holding is eventually traded against the consumption good. As a result, agents face a within-period cash-in-advance constraint that embodies the assumption that money is the only mean of purchasing the consumption good. Consequently, individuals hold money in each of their three periods of life, irrespectively of being borrower or saver. Because it does not pay interest, money is a dominated asset that is entirely consumed during each period. In other words, bonds and equities are the only instruments that are carried across periods to smooth consumption over time. Consequently, money holdings are more closely related to consumption expenditures than to savings, a feature that matches empirical regularities (Handa, 2002).[8] Such a cash-in-advance constraint, as introduced by Lucas (1982),[9] presents the following advantages. First, it isolates the money demand functions from the specific choice of utility functions, an issue that prevails in money in the utility function models. Second, differently from models that feature both money and bonds as stores of value, we obtain a monetary equilibrium without relying on additional assumptions regarding demographic change or monetary policy that affect the return of money. Finally, as argued by Heer et al. (2011), cash-in-advance constraints are useful in explaining the heterogeneity of money holdings across different age groups.

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[8] This feature is also in line with the periodicity of the model. Indeed, given that each period lasts 20 years, it is reasonable to assume that money is not carried over time to allow consumption deferral over 20 years.

[9] This cash in advance constraint also relates to the one proposed by Artus (1995) and Heer et al. (2011)

2.1.4 Monetary Regimes

Because previous literature established that, over the last century, monetary regimes differed markedly in their success to establish a credible framework to gain control over inflation,[10] we assume stochastic shocks to money supply $\bar{M}_g^S$, where the subscript $g = \{g_1, g_2, g_3, g_3\}$ represents the four states of money supply. In such a long-run setting where changes in the stock of money lead to changes in the price level, inflation is either low -g_1- (corresponding to the Mixed Regime and QE periods), or medium -g_2- (as during the Pre-Fed, Gold Standard, Bretton Woods and Great Moderation periods), or high -g_3- (Pegged Regime), or very high -g_4- (Great Inflation). Each money supply state has a probability to occur equal to the observed length of the respective period(s) that each state characterizes. With this stochastic structure of money supply, we implicitly assume that money supply did not react to demographic fluctuations. We justify this assumption of exogeneity by providing evidence that the FED did not adjust money supply in response to changes in inflation and output gap that were triggered by changes in the demographic structure (see Appendix A).

2.1.5 Individuals

The utility function features constant relative risk aversion and is intertemporally additive. Therefore, a young individual born in period j, $j = \{odd, even\}$, and income state s maximizes $U(c^y_{j,s}) + \beta U(c^m_{j+1,s+1}) + \beta^2 U(c^r_{j+2,s+2})$, where $\{c^y_{j,s}, c^m_{j+1,s+1}, c^r_{j+2,s+2}\}$ is her real consumption stream over the three life periods. Let $q_{j,s}$ and $q^e_{j,s}$ be the real bond price and real stock price in period j and income state s, respectively. $(zb^y_{j,s}, ze^y_{j,s}, zb^m_{j+1,s+1}, ze^m_{j+1,s+1})$ represent the real asset holdings of an individual born in period j and income state s. The real borrowing constraints of a young individual born in period j and income state s write:

$$\begin{aligned} c^y_{j,s} + q_{j,s} zb^y_{j,s} + q^e_{j,s} ze^y_{j,s} &= w^y_s \\ c^m_{j+1,s+1} + q_{j+1,s+1} zb^m_{j+1,s+1} + q^e_{j+1,s+1} ze^m_{j+1,s+1} &= w^m_{s+1} + zb^y_{j,s} + (q^e_{j+1,s+1} + d_{s+1}) ze^y_{j,s} \\ c^r_{j,s+2} &= zb^m_{j+1,s+1} + (q^e_{j,s+2} + d_{s+2}) zb^m_{j+1,s+1} \end{aligned}$$

where $j + 2 = j$ by the cyclicity of the demographic structure.

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[10] See, for example, Bordo and Haubrich (2008), D'Agostino and Surico (2012), Filipova et al. (2014) and Meltzer (1986).

Let $1/\sigma$ denote the intertemporal elasticity of substitution between consumption in any two periods. The maximization by young and middle-aged agents of their intertemporal utility functions leads to the following Euler equations that determine optimal consumption choices over time:

$$\begin{aligned} (c^y_{j,s})^{-\sigma} q_{j,s} &= \beta E_{j,s}(c^m_{j+1,s+1})^{-\sigma} \\ (c^m_{j,s})^{-\sigma} q_{j,s} &= \beta E_{j,s}(c^r_{j+1,s+1})^{-\sigma} \end{aligned} \tag{1}$$

and

$$\begin{aligned} (c^y_{j,s})^{-\sigma} E_{j,s} \frac{q^e_{j,s}}{q^e_{j+1,s+1} + d_{s+1}} &= \beta E_{j,s}(c^m_{j+1,s+1})^{-\sigma} \\ (c^m_{j,s})^{-\sigma} E_{j,s} \frac{q^e_{j,s}}{q^e_{j+1,s+1} + d_{s+1}} &= \beta E_{j,s}(c^r_{j+1,s+1})^{-\sigma} \end{aligned} \tag{2}$$

These equations state that individuals who are young or middle-aged in period j and income state s choose to reduce their future consumption when the real cost of deferring consumption from period j to period $j+1$, $q_{j,s}$ or $E_{j,s}\frac{q^e_{j,s}}{q^e_{j+1,s+1}+d_{s+1}}$, increases, or when the discount factor β decreases.

In each stage of life, the consumption good has to be paid for in cash. Because money is a dominated store of value, each individual's stream of nominal money demand $M_{j,s,g}$ equals the optimal consumption structure specified by the Euler equations times the price of the consumption good, $P_{j,s,g}$. Therefore, the within-period cash-in-advance constraints are as follows:

$$c^y_{j,s} = \frac{M^y_{j,s,g}}{P_{j,s,g}} \qquad c^m_{j,s} = \frac{M^m_{j,s,g}}{P_{j,s,g}} \qquad c^r_{j,s} = \frac{M^r_{j,s,g}}{P_{j,s,g}} \tag{3}$$

2.2 Equilibrium

The economy is in a decentralized equilibrium at all times; that is, all individuals choose their consumption stream optimally (Equations (1) and (2)). Moreover, the cash-in-advance constraints (Equations (3)) must be respected in equilibrium, and the following resource constraints must be satisfied in all periods:

$$\begin{aligned} Nc^y_{o,s} + nc^m_{o,s} + Nc^r_{o,s} &= Nw^y_s + nw^m_s + d_s \\ nc^y_{e,s} + Nc^m_{e,s} + nc^r_{e,s} &= nw^y_s + Nw^m_s + d_s \end{aligned} \tag{4}$$

$$\begin{aligned} N\frac{M^y_{o,s,g}}{P_{o,s,g}} + n\frac{M^m_{o,s,g}}{P_{o,s,g}} + N\frac{M^r_{o,s,g}}{P_{o,s,g}} &= \frac{\bar{M}^S_g}{P_{o,s,g}} \\ n\frac{M^y_{e,s,g}}{P_{e,s,g}} + N\frac{M^m_{e,s,g}}{P_{e,s,g}} + n\frac{M^r_{e,s,g}}{P_{e,s,g}} &= \frac{\bar{M}^S_g}{P_{e,s,g}} \end{aligned} \tag{5}$$

The first two equations represent the equilibrium on the good market, whereas the two last equations state that the money market clears in both odd and even periods.

By substituting the cash-in-advance equations into the resource constraints of the money market, the equilibrium conditions listed here can be expressed as functions of consumption levels ($c^y_{j,s}$, $c^m_{j,s}$ and $c^r_{j,s}$), asset prices ($q_{j,s}$ and $q^e_{j,s}$), saving decisions ($zb^y_{j,s}$ and $zb^m_{j,s}$) and real money supply ($\frac{\bar{M}^S_g}{P_{j,s,g}}$). It means that money is neutral, that is, increases in nominal money supply are entirely absorbed by a proportional increase in the price level and leave real activity unaffected. This explains why real variables are not indexed by the money supply state g. This feature of the model is justified in the medium to long-run.

2.3 Solving the Model

Solving for the equilibrium requires to identify the four elements that constitute the state space: the population pyramid j, the state of incomes s, the state of money supply g, and the portfolio income received by middle-aged workers which is determined by past shocks. The equilibrium is characterized by: i) young workers chose their saving and portfolio structure optimally, given their budget constraint when young and their expected budget constraint when middle-aged; ii) middle-aged workers chose their saving and portfolio structure optimally, given their budget constraint when middle-aged and their expected budget constraint when retired; iii) the bond market and the stock market clear; iv) the asset prices that individuals expect for the following period and income state, when deciding upon their portfolio, are equal to the asset prices that clear the bond and stock markets in the following period and income state, when agents receive such portfolio income; also, the savings that young workers expect to make in the following period and income state, when deciding upon their portfolio, are equal to the savings that middle-aged workers actually chose in the following period and income state, would they receive such portfolio income. The last condition assures that expectations about asset prices and saving decisions are correct.

To solve for the equilibrium, we form a grid of portfolio incomes inherited by middle-aged individuals from period $t-1$. Then we choose initial expectation functions over asset prices and saving decision that will be realized in $t+1$. We solve for the optimal portfolio decisions of young and middle-aged workers in t (retired individuals do not take any portfolio decision), for each point of the grid, given the expectation functions. Next, we solve for the optimal portfolio decisions of young and middle-aged workers, and therefore for the equilibrium asset prices and saving decisions in $t+1$, given the expectation functions and the portfolio income inherited by middle-aged workers from period t. The equilibrium asset prices and saving decisions are used to update the expectation functions. We repeat the algorithm until convergence.

2.4 Calibration

The calibration of the model is described in Table 1. For the sake of comparison, we closely follow GMQ's calibration. We interpret a period as 20 years. We take $(n, N) = (52, 79)$ as the size in millions of the Great Depression (1925-1944) and Baby Boom (1945-1964) generations so that, in the model, the middle-age to young ratio MY alternates between 0.66 in even periods and 1.52 in odd periods. In Appendix B, we provide the results obtained under the second specification $(n, N) = (69, 79)$, which characterizes the Baby Boom (1945-1964) and Baby Bust (1965-1984) generations.

Insert Table 1 here

We assume that an annual discount factor of 0.97, which translates into a discount factor of 0.5 at a 20-year frequency. The value of the intertemporal elasticity of substitution is still debated.[11] We set the value of elasticity of substitution equal to

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[11] Papers that calibrate macroeconomic models to match growth and business cycle facts usually use values around unity. After the seminal work by Kydland and Prescott (1982) who set the substitution elasticity to 0.66, most of the real business cycle literature used a value close to one. Other studies, which mainly estimate Euler equations using aggregate consumption data, support lower values. Hall (1988) stands on the opposite side of the range with a value close to zero.

1/4. Robustness checks for alternative values ($\sigma = 1; 2; 6$) show that changes in the elasticity of substitution modify only slightly the effect that the population age structure has on asset prices and does not impact the demographic effect on inflation.[12]

Concerning incomes and dividends, we set the average wage of young and middle-aged workers over income states to 2 and 3, respectively, to match the ratio of average annual real income of middle-aged to young individuals in the US. Moreover, the average ratio of dividends to wages is equal to 0.19 in the US. In the first specification characterized by an age structure of $(n, N) = (52, 79)$, total wages in odd (even) periods are, on average across income states, equal to 314 (341), so we set the average level of dividends equal to $0.19(\frac{314+341}{2})$. In the second specification, the age structure is $(n, N) = (69, 79)$. Total wages in odd (even periods) are, on average across income states, equal to 365 (375), so we set the average level of dividends equal to $0.19(\frac{365+375}{2})$. To obtain the stochastic structure of wages and dividends, the coefficient of variation of young workers' wages, middle-aged workers' wages, and dividends are set to 15%, 20% and 19%, respectively (see GMQ). As a result, the stochastic wage structure is $\{(w^{y,L}, w^{m,L}), (w^{y,H}, w^{m,H})\} = \{(1.7, 2.4), (2.3, 3.6)\}$, and the stochastic dividend structure is given by $\{d^H, d^L\} = \{74, 50\}$ under the first specification $(n, N) = (52, 79)$, and $\{d^H, d^L\} = \{83, 57\}$ under the second specification $(n, N) = (69, 79)$. We take into account the positive correlation between wages and dividends and assign the following probabilities to each of the four income states s: $(0.4, 0.1, 0.1, 0.4)$.

We normalize the initial price level in odd period to one and set money supply accordingly. The stochastic structure of money supply is set to $g = (g_1, g_2, g_3, g_4) = (0\%, 2.5\%, 4.5\%, 7\%)$ in annualized terms, so as to match the observed average annual inflation rate over the Mixed Regime and Quantitative Easing periods (state g_1), over the Pre-Fed, Gold Standard, Bretton Woods and Great Moderation periods (state g_2), over the Pegged Regime period (state g_3), and over the Great Inflation period (state g_4). We assign the following probabilities to each of the four money supply states g: $(0.15, 0.6, 0.125, 0.125)$ to roughly match the relative length of the respective monetary regime(s) over the period 1900-2014.

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[12] These robustness checks can be provided upon request.

3 Theoretical Results vs. U.S. Evidence

3.1 Data

In this section, we introduce the empirical counterparts of the variables entering the model. For the empirical counterpart of the model-implied MY_t, we use the ratio of the number of individuals aged 40-49 to the number of individuals aged 20-29.[13] Equity yield is proxied by the cyclically adjusted earnings price ratio. The long-term real bond yield is not observable, and under the Fisher hypothesis, it depends on the long-run expectations of inflation over the life-time of the bond.[14] Following D'Agostino and Surico (2012), we consider different models to generate long-run inflation expectations.[15] In order to capture the time-variation in model parameters that is due to changes in monetary regimes, we estimate a Bayesian VAR model that includes inflation and money growth, with drifting coefficients and stochastic volatility, to obtain long term inflation expectations $E_t\pi^{an}_{VAR\_tv}$ and hence real bond rate r_t. As alternative specifications, we also consider an AR(1) model, an AR(1) model with stochastic volatility, and bivariate models including M2 money growth and output growth.

To obtain long-run inflation expectations, we generate 10-year ahead forecasts by iterating forward the one-step-ahead forecasts:

$$\begin{aligned} \Pi_t &= \mu_t + A_t\Pi_{t-1} + \varepsilon_t \\ \widehat{\Pi}_{t+1|t} &= \widehat{\mu}_t + \widehat{A}_t\Pi_t \\ \widehat{\Pi}_{t+10|t} &= \sum_{j=1}^{10} \widehat{A}_t^{j-1}\widehat{\mu}_t + \widehat{A}_t^{10}\Pi_t \end{aligned}$$

where Π_t is a vector that includes endogenous variables used for inflation forecasts. For

---

[13] We use demographic projections to avoid look-ahead bias. Projected values of the demographic variable are hand-collected from various past U.S. Census reports available at http://www.census.gov/prod/www/abs/p25.html. Projected values of MY_t are obtained from the middle series of the most recent report available at the time of the forecast. For instance, the projected values for the period 1964-1969 are the forecasts from the report published in 1964. For the period before 1950, estimated values are used, since no public report was available.

[14] Long-run inflation expectations can be extracted from survey data (e.g., survey of professional forecasters) or the inflation index bond market (TIPS). However, the data is only available in the recent part of the sample, while our major focus is the long time-series relation between stock and bond markets.

[15] We thank D'Agostino and Surico for sharing their replication code.

example, in our baseline case,

$$\Pi_t \;=\; \begin{pmatrix} \pi_t \\ \Delta m_t \end{pmatrix}$$

hence we obtain real bond rate r_t by computing $E_t \pi^{an}_{VAR\_tv}$. All data sources are reported in Appendix C. In Table 2, we provide the time series properties of all the key variables over the sample period 1900-2014. MY ratio, equity yield, risk-free rate and 10-year nominal bond yield are all very persistent variables, but standard unit root tests, e.g., augmented Dickey Fuller or Phillips-Perron, agree on the existence of a unit root only for the nominal bond yield. Therefore, we construct a demeaned nominal bond yield series denoted by i_t that accounts for the differences in the long-run mean under different monetary regimes.

Insert Table 2 here

3.2 The Demographic Effect on Bond and Stock Yields

Static Model. In order to disentangle the channels through which demographic fluctuations, shocks to income, and shocks to money supply growth affect consumption and saving decisions as well as real and nominal yields, we first solve a model featuring no demographic fluctuations ($n = N = (52+79)/2$, $d = 62$), constant wages and dividends equal to their averages across income states, and no shock to money supply. We denote the output of this simulation by a tilde. Unsurprisingly, we obtain that the consumption stream $(\tilde{c}^y, \tilde{c}^m, \tilde{c}^r) = (2.00, 1.98, 1.96)$ and the annualized real interest rate on bonds and equities $r = 3.34\%$ are constant across periods.

Deterministic Model. As a second step, we introduce demographic fluctuations only, keeping wages, dividends and money supply growth constant. We observe that, in such a deterministic setting, the resource constraint on the good market would be violated, would the consumption stream be maintained at the equilibrium values of the static model $(\tilde{c}^y, \tilde{c}^m, \tilde{c}^r)$. In odd (even) periods, aggregate demand for the consumption good would exceed (be lower than) aggregate income/output:

$$
\begin{aligned}
N\tilde{c}^y + n\tilde{c}^m + N\tilde{c}^r &> Nw^y + nw^m + d \\
n\tilde{c}^y + N\tilde{c}^m + n\tilde{c}^r &< nw^y + Nw^m + d
\end{aligned}
$$

The life-cycle portfolio behavior explains these disequilibria. Individuals facing a hump-shape income stream save when middle-aged and dis-save when retired. Therefore, in odd periods, when the demographic structure is characterized by a small cohort of middle-aged individuals, aggregate saving is low and, connectedly, aggregate consumption is high. The opposite holds in even periods. The equilibrium on the good market (and consequently on the bond and stock markets) is obtained through the adjustment of the real price of financial assets. Table 3 shows that asset prices increase in even periods so as to prevent excess saving in the economy. Symmetrically, low real asset prices stimulate savings in odd period when the MY ratio is low and bring the asset and good markets to clear. This explains the decrease in the annualized real interest rate by 104% and the decrease in equity yields by 50% over 20 years, from odd to even periods.

Insert Table 3 here

This adjustment in asset prices affects the individuals' consumption pattern and distinguishes consumption profiles across cohorts. An individual born in a large cohort, that is, in odd periods, faces a high cost of borrowing when young, and a small return of savings when middle-aged. In the opposite, an individual born in a small cohort, that is, in even periods, can borrow at a low cost when young, and benefits from a high return of savings when middle-aged. Consequently, individuals born in odd periods consume less when young and retired, compared to individuals born in even periods.

Using the cash-in-advance constraints, we substitute individual consumptions into money demands in the resource constraints of the good market. Then, by embedding the resource constraints that we obtain into the resource constraints of the money market, we get

$$
P_{o,s,g} = \frac{\bar{M}_g^S}{Nw_s^y + nw_s^m + d_s} \qquad P_{e,s,g} = \frac{\bar{M}_g^S}{nw_s^y + Nw_s^m + d_s} \tag{6}
$$

The price level in the economy is determined by the money supply relative to aggregate real income/output.[16] As economic activity grows, the demand for real cash balances increases, lowering the price level and the realized inflation rate (from period $j-1$ to j). The opposite mechanism takes place when the economic activity slows down. Moreover, the level of real activity directly relates to the demographic structure. We can illustrate this relation by expressing Equations (6) as functions of $Young_j$, the number of young individuals in period j, and MY_j, the MY ratio in period j. We obtain the following equation:

$$P_{j,s,g} = \frac{\frac{\bar{M}_g^S}{Young_j}}{w_s^y + MY_j w_s^m + \frac{d_s}{Young_j}} \tag{7}$$

Middle-aged workers being more productive than young ones, a higher MY ratio implies higher aggregate productivity, and hence higher aggregate real income/output. As economic activity grows, money demand goes up which leads to a decrease in the aggregate price level to sustain money market equilibrium. Therefore the price level is inversely related to the MY ratio: prices are expected to be high (low) in odd (even) periods. Results shown in Table 3 confirm this prediction that the small proportion of middle-aged workers in odd periods pushes aggregate productivity down, leading to a low level of aggregate income/output and subsequently to a high price level. This mechanism generates a negative comovement between the MY ratio and realized inflation π_j^{an} (from period $j-1$ to j) and for the positive comovement between the MY ratio and expected inflation $E\pi_j^{an}$ (from period j to $j+1$).[17] Moreover, our theoretical analysis predicts a positive correlation between (realized) inflation and financial yields, as they both share a common factor which is the demographic structure.

Using the Fisher equation, we retrieve the annualized nominal interest rate i_j. We can therefore decompose the total effect that the MY ratio has on the nominal interest rate into its effect on the annualized real interest rate r_j and on the annualized expected inflation rate, $E\pi_j^{an}$. We observe that the 6 percentage point decrease in the nominal interest rate from odd to even period stems mainly from the 6.7 percentage point decrease in the real interest rate that is partially offset by the 0.7 percentage increase in the

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[16] Note that Equations ((6)) are special cases of the quantity theory exchange equation in which the velocity of money is constant and equal to one. Extensions to the Lucas' basic model have been provided to account for the variability of the velocity of money (see, for example, Lucas (1984), Svensson (1985) and Lucas and Stokey (1987)), but for tractability reasons we do not introduce them in our model.

[17] See also Juselius and Takats (2015) who find a stable, significant and negative correlation between inflation and the share of young and old individuals in a sample of 20 OECD countries.

expected inflation rate. Our results are therefore twofold. First, because the MY ratio is negatively correlated with the real interest rate and positively correlated with the expected inflation rate, the model predicts that nominal bond yields are less sensitive to the demographic structure than real bond yields. Second, because the demographic effect on inflation is moderate, we obtain a positive correlation between nominal bond and real stock yields, at generational frequency. Both results carry to the stochastic setting.

Stochastic Model. Next, we introduce shocks to dividends, wages and money supply. We simulate a 100,000 period model and average the results obtained in each pyramid structure j, income state s, and money growth state g. We also report averages across states. The results are presented in Table 4 for the population age structure $(n, N) = (52, 79)$ and in Appendix B for the population age structure $(n, N) = (69, 79)$. Standard deviations, shown in parenthesis, are small for almost all variables, which indicates that past shocks affect equilibrium values only marginally. Moreover, a paired sample t-test indicates that the average values are significantly different between odd and even periods.

The demographic effect on the consumption/saving decision that we obtained in the deterministic case remains valid in this stochastic environment. Moreover, long-run fluctuations in the demographic structure lead to fluctuations in asset prices and inflation that are, on average, qualitatively and quantitatively similar to the ones obtained in a deterministic setting. First, the small (large) share of middle-aged workers, who are characterized by a relatively large desire to save, in odd (even) periods pushes asset prices down (up). While this demographic effect on asset prices is observed on average, income shocks alter the results. Indeed, high wages and dividends push individuals' demand for savings up, which makes stock prices increase and real yields fall. Inversely, low stock prices and high real yields are observed when wages and dividends are low. Second, the small (large) share of middle-aged workers in odd (even) periods pushes aggregate productivity and real GDP down (up), yielding inflationary (deflationary) pressures. This demographic effect on inflation is also affected by income shocks. In good income states, wages and dividends are large, and so is real output. The price level being determined by the money supply relative to aggregate real output, high income states are associated with low price levels and low realized inflation.

Insert Table 4 here

Because of the uncertainty introduced by wage and dividend shocks, bonds and stocks are no longer perfect substitutes. The results suggest that the equity premium is roughly stable across MY ratios (around 1%),[18] indicating that the population age structure does not have a large direct effect on equity premium. However, as we show in the Section 4, the MY ratio will help to pin down ex-ante equity premium over long horizon through the equilibrium relation with stock and bond yields within a valuation framework.

Under uncertainty, the Fisher equation is also altered as it accounts for the inflation risk premium. Using the Bekaert and Engstrom (2010) decomposition, the annualized yield on the nominal bond i consists of three components:

$$i_{j,s,g} = E\pi^{an}_{j,s,g} + r_{j,s} + IRP_{j,s,g}$$

where $E\pi^{an}_{j,s,g}$ is the annualized expected inflation from period j to period $j+1$, $r_{j,s}$ is the annualized real interest rate on bonds over the same period, and the inflation risk premium $IRP_{j,s,g} = -\frac{1}{2}Var(\pi_{j+1,s+1,g+1}) + Cov(lnq^{e}_{j,s}, \pi_{j+1,s+1,g+1})$.[19] Although the stock price and inflation are both affected by the population age structure, the covariance is not significantly different from zero, and we obtain a very low inflation risk premium (below 10^4).

As in the deterministic case, the demographic effect on expected inflation is moderate, and fluctuations in nominal yields across pyramid structures mainly stem from the effect that these pyramid structures have on real yields. Moreover, the effect of income shocks s on real yields transmits into changes in nominal yields across income states. As a result, nominal and real returns correlate positively. We provide a thorough discussion on this positive correlation in the Subsection 3.3.

Empirical Evidence. In Panel A of Table 5, we report the correlation of each variable with MY_t over the sample period. The statistical significance of correlations is based on a bootstrapping exercise that accounts for the persistence of each variable and imposes the null of orthogonality between two variables.[20] The data generating process of each variable is determined based on Akaike information criterion (AIC) over the sample period (Berkowitz and Kilian, 2000).[21] The residuals are drawn from each model OLS

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[18] Our model does not include any additional channel that would generate the high equity premium that is observed in the market.

[19] See Appendix D for the derivation.

[20] We thank Alessandro Palandri for suggesting this bootstrapping exercise.

[21] For example, over the sample period, AIC suggests an ARMA(2,2) for MY_t and AR(1) for most other time series.

estimation separately and correlations are computed based on 10,000 bootstrap samples. The asterisks denoting significance at conventional levels are based on bootstrap p-values of correlations:

$$p_{bootstrap} = \begin{array}{l} \#\{\widehat{\rho}^{s} \geq \widehat{\rho}\}/10,000 \text{ if } \widehat{\rho} \geq 0 \\ \#\{\widehat{\rho}^{s} \leq \widehat{\rho}\}/10,000 \text{ if } \widehat{\rho} \leq 0 \end{array}$$

where $\#\{\widehat{\rho}^{s} \geq \widehat{\rho}\}$ denotes the number of bootstrapped correlations higher than the estimated correlation.

The signs of all the correlations are in line with the predictions of the model: both the equity yield and (demeaned) nominal bond yield are negatively correlated with MY_t. Moreover, MY_t is positively correlated with expected inflation, although the magnitude and significance of the correlation depends on the model of long-term expectations, as shown in Panel B. Finally, the correlation between the real bond yield and MY_t is also significant and slightly larger in magnitude than the correlation between the real bond yield and MY_t, regardless of the specification we use to measure long-run inflation expectations. This last result is in line with the moderate demographic effect on inflation and with the model prediction that nominal bond yields are less sensitive to the demographic structure than real bond yields.

Insert Table 5 here

Our results are confirmed by the analysis of the adjusted R^2_{adj} of a univariate regression of each variable on MY_t. The coefficients of determination suggest that, while 31 percent (15 vs. 16 percent) of equity yield (nominal vs. real bond yield) variation is explained by MY_t, the effect of demographic fluctuations on inflation expectations is more limited, varying from 2 to 14 percent depending on the model choice for inflation forecasts.

3.3 The Comovement Between Bond and Stock Yields

In this subsection, we test the model prediction that a common persistent component reflecting the time-variation in population age structure drives comovement between financial yields.

Insert Table 6 here

The first two panels of Table 6 show the empirical correlations and partial correlations (controlling for MY_t) of each variable. We note that the correlation between equity yield and nominal bond yield is positive and significant over the long sample. Importantly, the correlation coefficient drops and looses significance once we control for the demographic component in each variable. We observe the same features for the correlation between real and nominal bond yield. Panel C shows how the correlation between equity yield and nominal bond yield breaks down across monetary regimes and reveals the large time variation in the low-frequency correlation coefficient, as discussed in the introduction.

The demographic effect identified in the model is such that an increase in the MY ratio, from odd to even period, leads to a decrease in real bond returns and equity yields, and to an increase in expected inflation, as presented in the previous subsection. Because the demographic effect on expected inflation is moderate, changes in the demographic structure trigger a positive comovement between nominal and real bond yields, as well as a positive comovement between nominal bond yield and equity yields. Panel D of Table 6 reports the correlation coefficients obtained by simulating our 100,000 period model. The model-implied correlation between nominal bond yield and equity yields is 0.82, a magnitude that is comparable to the correlation coefficients observed during most of the monetary regimes, expect for the Bretton Woods and Quantitative Easing periods.[22]

However, it is important to note that, while our model predicts a positive comovement between nominal bond yields and equity yields, income and money supply shocks brings the correlation to turn negative in a few specific subperiods, when the effect of income and money supply shocks counteracts the demographic effect. To show this, we decompose the stochastic model results by demographic structure, income state, and money supply state, as shown in Table 7. First, low income states, by curbing demand for saving, push bond and stock prices down, and nominal and real yields up. As real yields are more sensitive to income shocks than nominal yields,[23] real yields will increase from odd to even period when the income shock effect dominates the demographic effect (for example from state (Odd, s_1) to state $(Even, s_4)$). In this case, an increase in the MY ratio from odd to even period will be associated with an increase in real yields and a decrease in nominal yields. We summarize the effect of income shocks on the sign of the correlation in Figure 4, Panel (a). The bottom right quarter shows an average correlation

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[22] The model implied correlation coefficient is quite robust to changes in the assumptions about the stochastic structure of money supply. We provide evidence upon request.

[23] The coefficient of variation of the real rate of return on stock across states s is equal to 4.7, whereas the coefficient of variation of the nominal rate of return on bonds is equal to 0.7.

of -0.13, indicating that, as an economy moves from a low-MY demographic structure and low income states (s_1 or s_2) to a high MY ratio and higher income states (s_3 or s_4), or vice versa, the model predicts the correlation between nominal bond yields and equity yields to be negative. This result provides a rationale for the extended period of negative correlation observed during Bretton Woods, a period characterized by a falling MY ratio and an economy heading towards relatively low income states, with four recessions from the end of Bretton Woods (1969q4-1970q4) to the Great Inflation (1973q4-1975q1, 1980q1-1980q3, and 1981q3-1982q4).

Insert Table 7 here

Insert Figure 4 here

Second, the positive correlation between nominal bond yields and equity yields is also affected by money supply shocks, and therefore by changes in individuals' expectations about future inflation. In states g_3 or g_4, when inflation is above trend, expectations that inflation will slow down bring nominal yields to decrease. The inverse occurs in state g_1 and g_2 when inflation is below trend. While both nominal and real yields decrease on average from odd to even periods, nominal yields would increase if expectations about future inflation increase simultaneously (for example from states s_2, g_4 to states s_2, g_1), as seen in Table 7. This would lead to a temporary negative comovement between nominal and real yields. We summarize the effect of money supply shocks on the sign of the correlation in Figure 4, Panel (b). In the upper left corner, we observe that, as the MY ratio increases from odd to even period, nominal bond yields and equity yields correlate negatively when the income state remains constant and the inflation rate is expected to drop sharply, from g_4 to g_1. This mechanism, when reversed, sheds light upon the negative correlation between bond and stock yields observed during the QE period, as this period is characterized by a decreasing MY ratio and increasing expectations about future inflation.[24]

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[24] The model's ability to explain the recent period is limited, since it is not designed to capture the peculiarities of each monetary regime, in particular we do not explicitly model the conduct of unconventional monetary policy given our focus on the long time series relation.

4 Valuation Models and Predictability

The strong comovement between stock yields, a real variable, and nominal bond yields - especially in the post-Bretton Woods period- perplexes both researchers and practitioners. This empirical fact, also formalized under the Fed model, is used among investor professionals to detect stock market mispricing relative to bond market (e.g., Lander et al., 1997; Maio, 2013), but reconciling this empirical "anomaly" with rational explanations of stock pricing remains disputed.[25] In this paper, our approach differs from the behavioral explanations or risk-based arguments as for the source of this comovement. The theoretical and empirical results shown in Section 3 allow us to argue that a common demographic factor drives the persistent, long-run components of financial yields and, as a next step, we show how to introduce a demographic factor shared by asset yields within a present value framework.

4.1 Present value Models

Present value models provide an ideal environment to test the conjecture whether an equilibrium relation between equity and bond yields exists. Earlier studies proposing valuation models show equity return predictability using either bond yields (Lander et al., 1997; Asness, 2003), demographic variable (Favero et al., 2011) or yield spreads (Maio, 2013). On the one hand, (Favero et al., 2011) establish the empirical link between the slowly evolving mean in the log dividend-price ratio and MY_t. Using the decomposition of log-dividend price ratio within the dynamic dividend growth model (Campbell and Shiller, 1988), they show that demographic information is useful in generating accurate forecasts for real stock market returns, but not for future changes in dividends. On the other hand, Maio (2013) build upon the Campbell and Shiller (1988) model by assuming Log Pure Expectations Hypothesis of the term structure and justify the use of the yield gap, either using (log) earnings price or dividend price ratio, as a forecasting variable for excess stock market returns. In particular, the original dividend-growth model

$$dp_t = \frac{-k}{1-\rho} + \mathrm{E}_t \sum_{j=0}^{\infty} \rho^j (ret_{t+1+j} - \Delta d_{t+1+j}) \tag{8}$$

---

[25] See Ritter and Warr (2002), Asness (2003), and Estrada (2009).

where k and ρ are log-linearization constants, dp_t is the log dividend-price ratio, and $ret_{t,t+h}$ and Δd_{t+h} are the holding-period return from the stock market and dividend growth for time t to t+h. Maio (2013) rewrites this equation in terms of excess stock market returns

$$dp_t = \frac{-k}{1-\rho} + \mathrm{E}_t \sum_{j=0}^{\infty} \rho^j (xret_{t+1+j} - \Delta d_{t+1+j}) + E_t \sum_{j=0}^{10-1} \rho^j y_{1,t+j} + E_t \sum_{j=n}^{\infty} \rho^j rf_{t+1+j} \tag{9}$$

where $y_{1,t}$ is the one-period nominal (log) bond yield and rf_t is the one-period risk-free bond.

Under Log Pure Expectations Hypothesis (EH) and assuming $\rho \approx 1$

$$\mathrm{E}_t \sum_{j=0}^{10-1} \rho^j y_{1,t+j} \approx n \times i_t \tag{10}$$

where i_t is 10-year nominal bond (log) yield. Defining the yield gap as $yg_t^d = dp_t - n \times i_t$, we can write

$$ygd_t^d \equiv yg_t^d - E_t \sum_{j=n}^{\infty} \rho^j rf_{t+1+j} = \frac{-k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j (xret_{t+1+j} - \Delta d_{t+1+j}) \tag{11}$$

To the extent that the future demographic fluctuations improves our inference on distant future level of risk-free rates, a model including yield gap and projections on MY_t should improve the forecasting accuracy on equity premium given the lack of dividend growth forecastability (e.g., Cochrane (2008)).

Equation (8) can also be written in term of earnings yield (e.g., Lamont, 1998)

$$ep_t = \frac{-k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j (ret_{t+1+j} - (1-\rho) de_{t+1+j} - \Delta e_{t+1+j}) \tag{12}$$

where de_t is the log dividend-payout ratio. Under the same assumptions (EH and $\rho \approx 1$) and defining $yg_t^e = ep_t - n \times i_t$

$$ygd_t^e \equiv yg_t^e - E_t \sum_{j=n}^{\infty} \rho^j rf_{t+1+j} = \frac{-k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j (xret_{t+1+j} - \Delta e_{t+1+j}) \tag{13}$$

In the following two subsections, we first test different stock yield specifications implied by the present value model as well as its extensions to evaluate the role of demographics in determining the low-frequency comovement between stock and bond yields. Then, we assess the implications of the equilibrium relation between stock and bond yield on stock return predictability exploiting the present-value identity.

4.2 Equilibrium Relation

Do we really understand the time-series relation between stock and bond yields, can we believe in a valuation model that relies on a joint mechanism that ties stock and bond markets? How can we reconcile the strong comovement between the stock market yield, a real variable and nominal bond yield? Bekaert and Engstrom (2010) address these questions and suggest a mechanism where expected inflation coincides with periods of high uncertainty and risk aversion, hence rationalize the strong comovement between stock and bond yields, that is, the Fed model. However, there are still some concerns; first, the Fed model works perfectly in some subsamples, but less so during the Bretton Woods monetary regime and in the recent crises period (e.g., Asness, 2003; Hasseltoft, 2009). Second, it might be conceivable to believe that short-term (e.g., one-year) inflation expectations are counter-cyclical, but it is less clear why we should expect a similar cyclical pattern for long-term inflation expectations which is the relevant metric for long term investors. Another recent paper by Maio (2013), on the other hand, focuses on the yield gap between stock and bond yields and shows strong predictability of stock returns. But, the sample is limited to the pre-crisis period, hence subject to the first criticism.

In this section, we first test the Fed model using annual data over a century. In particular, we project stock yield on the long term (10-year) nominal bond yield as the benchmark valuation model. Then we augment the model controlling for demographic fluctuations via MY_t. We also control for the relative stock-bond volatility (Asness, 2003) and compare the model with an alternative specification that includes the real bond yield instead of the nominal counterpart. In further specifications, we augment the baseline model with the demographic variable and several other controls. In particular, we consider several supply-side variables for the stock, bond and money markets. We also include time-varying habit based-risk aversion (Campbell and Cochrane, 1999). Relative stock-bond volatility is measured as the natural logarithm of the ratio of standard deviations of monthly yields (10-year window). We use the total market capitalization (NYSE, AMEX and Nasdaq) over GDP as a proxy for stock supply (Hobijn and Jovanovic, 2001), government debt over GDP for bond supply (Krishnamurthy and Vissing-Jorgensen, 2012).[26] Money supply is defined as money stock (M2) over GDP.[27] We construct the surplus ratio, $S_t = \frac{C_t - H_t}{C_t}$ to proxy for the time-varying risk aversion, where C_t is the real personal consumption, and H_t is the "habit stock", a 10 year moving average of past consumption levels. The dependent variable, stock yield, is the cyclically adjusted earnings price ratio (Panel B) collected from Robert Shiller's website.

Insert Table 8 here

The first specification (1) in Table 8 is the original univariate Fed Model. We note that the demeaned long term nominal bond yield is significant with an adjusted R^2 of 11 percent over the sample 1900-2014. However, when we augment the model with MY_t, the coefficient of bond yield is less than half and not significant, while MY_t enters significantly with a negative sign and an adjusted R^2 of 32 percent. The results are similar, though slightly less significant MY_t, when we control for the relative stock-bond volatility. The substitution of the nominal bond yield with the real counterpart (constructed using inflation expectations $E_t \pi^{an}_{VAR\_tv}$ from a Bayesian VAR, including inflation and money growth, with drifting coefficients and stochastic volatility), the real bond yield does not enter significantly, since MY_t captures the equilibrium relation between real stock and bond yield.

A simplifying assumption in our theoretical model is that the supply side in stock and bond markets do not respond to demographic fluctuations and the demographic effect prevails through the demand channel. In our empirical specifications, we explicitly control for supply side variables for stock, bond and money markets. Over the sample period, the results remain virtually untouched once we control supply-side variables supporting the idea that demographics mainly effect stock-bond yield comovement through the demand channel. Finally, in the last specification, the significance of the demographic variable still persists once we control for time-varying risk aversion. Hence the importance of demographic fluctuations in determining stock-bond yield comovement is strongly confirmed in the data. Overall, this evidence suggests that the omitted demographic

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[26] The data is available at Henning Bohn's website.

[27] We also controlled for annual growth in real M2 (deflated by CPI) as a proxy for money supply changes (Woodford, 2008), results remain similar.

component plays an important role in determining the long-run level of both stock and bond yields. Therefore, a natural question arises whether demographic information shared by stock and bond markets can be effectively used for future inference on stock market returns within the present value framework (Lander et al., 1997; Maio, 2013).

4.3 Return Predictability, Is it There?

The present value relation of Campbell and Shiller (1988) shown in Equation (9) explains the link between stock yield and future returns.[28] Moreover, it justifies the use of dividend yield as a predictor of future market returns (e.g., Ang and Bekaert, 2007). Under the Pure Expectations Hypothesis, Maio (2013) shows that the yield gap between (log) equity yield and scaled bond yield is a good predictor for equity premium. However, Maio (2013) does not explore whether the yield gap can explain the expectations on the level of distant future risk-free rates, a term included in Equation (11). We first test whether yield gap using both log dividend and earnings yield (yg_t^e and yg_t^d) as well as projections on the future level of MY_t are linked to expectations of distant future risk-free rates. As a benchmark we also use the historical level of risk-free rates. Given the Census projections we have very accurate forecasts on the expectation of the future level of MY_t, that is, $E_t(MY_t^{t+h})$, where h is the investment horizon. We consider 1-year, 5-year and 10-year investment horizon. In the last column of Table 9 we also show the out-of-sample R^2 statistics (Campbell and Thompson, 2008) which compares the forecast error of the historical mean with the forecast from predictive regressions and is computed as follows

$$\mathrm{R}^2_{OS} = 1 - \frac{\sum_{t=t_0}^{T}(\mathrm{r}_t - \hat{\mathrm{r}}_t)^2}{\sum_{t=t_0}^{T}(\mathrm{r}_t - \bar{\mathrm{r}}_t)^2}$$

where $\hat{r}_t$ is the forecast at $t-h$ and $\bar{r}_t$ is the historical average estimated until $t-h$. If R^2_{OS} is positive, it means that the predictive regression has a lower mean square error than the prevailing historical mean.

Insert Table 9 here

First of all, Table 9 shows that the historical average of risk-free rates is not informative about future risk-free rates at any horizon. Moreover, neither of the yield gap variables

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[28] A similar present-value relation can be written using earnings yield (see Appendix in Maio (2013))

is significantly related to future level of risk-free rates. On the contrary, future levels of MY_t which is known accurately at time t, is a good indicator about the future levels of risk-free rate. Moreover, as reflected R^2_{OS}, the predictive ability of MY_t is not limited to in-sample prediction. Therefore, based on Equations (11) and (13), we will argue that a modified version of yield gap variables or a bivariate forecasting model including yield gap and future level of MY_t will improve upon the forecasting models on excess market returns.

Insert Table 10 here

The long run (1-year, 5-year, 10-year) return predictability results are shown in Table 10. Yield gap variables are either marginally significant or not significant over the 1900-2014 sample. The adjusted R^2 of these variables increase with the investment horizon (except for yg^d_t at 10-year horizon),[29] but out-of-sample R^2_{OS} reveal that these variables do not outperform the historical average. Based on Equations (11) and (13), we construct two modified versions of yield gap under the assumption that future level of MY_t perfectly reflects expectations about the distant future risk-free rates. In particular we define

$$\begin{aligned} ygd^e_t &\equiv ep_t - n \times i^+_t + E_t(MY^{n+h}_{t+n}) \\ ygd^d_t &\equiv dp_t - n \times i^+_t + E_t(MY^{n+h}_{t+n}) \end{aligned}$$

where ep_t and dp_t are log of earnings-price and dividend-price ratio, i^+_t is log of 10-year nominal bond yield, hence n=10, and $E_t(MY^{n+h}_{t+n})$ reflects the average MY_t projections into distant future from time $t+n$ to $t+n+h$.

The modified yield gap variables in univariate predictive regression are highly significant with high adjusted R^2, particularly at long horizon.[30] However, the out-of-sample R^2_{OS} show that the forecasting performance does not improve upon a simple model based on historical averages. This is not surprising, since the variable we construct artificially imposes a restriction, that is, the coefficient on $E_t(MY^{n+h}_{t+n})$ should be equal to 1 in above definitions. In fact, the bivariate predictive regressions show that when we relax

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[29] Earlier literature argues that there might be a mechanical link between investment horizon and R^2 (Boudoukh et al., 2008).

[30] The significance is also confirmed by the p-values obtained from bootstrap exercises suggested in Maio (2013).

that constraint the out-of-sample R^2_{OS} are in line with in-sample adjusted R^2. Overall, long-run predictive regression results confirm the previous analysis that the addition of the demographic variable improves the forecasting performance of the Fed model, especially at long horizon.

5 Cross-Country Differences

The evidence so far is limited to the U.S. time series. Note that the OLG model which is used to motivate the empirical analysis makes strong simplifying assumptions: i) the model is designed for closed economies, ii) it is calibrated using U.S. data, iii) it assumes close to stationary population structure featuring boom and bust cycles. Given the low-frequency nature of the data and relatively short samples, it is hard to generalize the results and exclude alternative explanations of the tight link between the stock and bond yields. One way to overcome this problem is proving evidence from other countries (Estrada, 2009; Bekaert and Engstrom, 2010). In fact, Estrada (2009) criticizes the FED model, since he fails to find robust evidence on stock-bond yield comovement across countries, while Bekaert and Engstrom (2010) rationalizes the model based on stagflation incidents.

Insert Figure 5 here

Although some countries in the sample exhibit similarities in their demographic structure due to the baby boom after the WW2 and subsequent improvements in birth control, there is vast heterogeneity among countries in terms of the importance of stock markets as a channel for aggregate savings. This heterogeneity is evident in different stock market participation patterns (see Figure 5).[31] We would expect higher demographic effects on financial markets in countries where stock markets play an important role for savings. In this section, we proxy for the country-specific demographic effect by the (negative) correlation between stock yield on each country's aggregate stock market (either measured by dividend or earnings yield) and the demographic variable MY_t constructed for each country over the sample 1960-2009.[32] The sample correlation is

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[31] Data are from Giannetti and Koskinen (2010) collected from several sources around the millennium. Hence it does not take into account time-series variation in participation patterns.

[32] MY_t data for individual countries are obtained from World Bank projections and start in 1960. The

the average correlation obtained through 10,000 bootstrap samples.[33] When we project the demographic effect on cross-country stock market participation rates, we obtain a positive slope coefficient of 1.55 (t-stat=2.53) and 2.55 (t-stat=4.61) for both measures using dividend and earnings yields, indicating a stronger demographic effect on financial markets in those countries with higher stock market participation rate. This heterogeneity among countries can explain why we fail to demonstrate a robust comovement between stock and bond markets across countries.

Insert Figure 6 here

To test this claim, we proceed with a similar analysis suggested by Bekaert and Engstrom (2010): First, in Figure 6, we plot the cross-sectional stock-bond yield correlations (y-axis) and demographic effect on stock yield (x-axis). The strong relationship is evident; the upward slope across figures indicates that the higher the effect of the demographic variable on financial markets (stronger negative correlation), the higher is the link between stock and bond yields. This result is robust across different measures of stock yields.[34]

Insert Table 11 here

We test whether this explanation is robust to alternative control variables. In particular, in robust regressions reported in Table 11, we include first $\overline{\text{inf}}_i$, the full-sample country-specific mean of inflation and $\overline{\Delta GDP}_i$, the full-sample country-specific mean of real GDP growth (over the post Bretton Woods sample, 1973-2009) as country specific control variables. Then we include gradually the percentage of observations during which the country was in recession (measured by negative annual GDP growth) and $\overline{inf\_rec_i}$, the country-specific time-series mean of the interaction $inf_{i,t} \cdot rec_{i,t}$ (Bekaert and Engstrom, 2010). The role of the demographic effect in determining the comovement between financial yields remains intact despite the controls. On the whole, the effects of a time varying age-structure on financial markets vary substantially across countries

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dividend yield series are collected from Global Financial Data up to 2009.

[33] For countries with less than 30 data points, rank transformed variables are used to measure Spearman correlation, otherwise standard Pearson correlation is calculated for each bootstrap sample.

[34] Because the variables included in the regression analysis are correlations and thus limited to the interval [-1,1], as a robustness check, we also transformed the variables applying $ln(1+corr)/ln(2-corr)$, which maps the variables to the [$-\infty, \infty$] interval. The t-stats and R^2 are lower, but still highly significant.

covered in the panel. Yet this demographic effect provides a consistent explanation for the joint path (and the lack thereof) of stock and bond yields.

6 Concluding Remarks

This paper documents the role of changing population age structure on stock and bond yields. This is not the first study that stresses the importance of demographic changes for financial markets. The net demand for financial assets by certain age groups does provide important information on the aggregate demand for financial assets as the population structure changes. Thus this paper suggests a channel through which demography shapes the puzzling time series behavior of both key financial variables and provides an economic rationale for the co-movement of stock yields and nominal bond yields by introducing money in an OLG model. Clearly, demographics cannot explain all the time-variation in these variables, neither it should, but the first-order effects of the population age structure on financial markets are too important to be dismissed.

Our results have important implications for long term investors with stylized portfolio choice. Time series patterns of financial yields not only drive the predictability of returns, but are also crucial for portfolio allocations (Schotman and Schweitzer, 2000). In this paper, we argue that persistent changes in the population age structure is a common source of variation both for stock and bond markets. This evidence suggests that keeping a substantial portion of a retirement portfolio in local stock and bond markets might not be a good idea for diversification purposes. Finally, it implies that excluding a country's population age structure from the information set may harm an investor who considers international markets for long term investment.

Appendix A Money Supply Rule

In Section 2.1.4, we introduce the assumption that the central bank does not adjust money supply in response to changes in inflation and in the output gap that are triggered by changes in the demographic structure. To justify this assumption, we first estimate the following money supply rule which mirrors the Taylor rule:

$$\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \epsilon_t$$

as introduced by Chowdhury and Schabert (2008). μ_t represents the growth rate of non-borrowed reserves, $E_t\pi_{t+n}$ the expected inflation rate in $t+n$, y_t the real output, and y_t^* the time-varying potential output. The data are quarterly time series taken from the St. Louis FRED database. The growth rate of non-borrowed reserves is constructed as the annual log difference in non-borrowed reserves. The inflation rate is the compounded annual rate of change in the CPI index from time t to $t+n$.[35] Output gap is the percentage gap between actual and potential output.

The results presented in Table A.1 show that, over the entire period and the pre-crisis period, money supply did not significantly react to inflation, a result which is in line with the existing literature (Chowdhury and Schabert, 2008; Sargent and Surico, 2011) and with the history of the FED's monetary policy strategy (Meulendyke, 1998). We also split the sample into two sub-periods: the pre-Volker period (1961Q1-1979Q2) and the post-Volker period (1982Q4-2013Q1). The results suggest the absence of a consistent money supply's feedback to inflation over these two sub-periods. Looking at the entire period, the pre-crisis period, and the two subperiods, the results also indicate that the FED targeted money supply to stabilize output.

Next, we test for the reaction of money supply to changes in the MY ratio, directly or indirectly through inflation and the output gap. We add the MY ratio as a control variable in our money supply rule:

$$\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \mu_{MY}MY_t + \epsilon_t$$

The estimates of the regression coefficients of inflation and the output gap are affected only slightly, and the estimated coefficient of the MY ratio does not significantly differ from zero. This result indicates that the central bank does not systematically adjust money supply to offset inflationary and expansionary effects of the MY ratio. For this reason, we assume that money supply growth is exogenous.

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[35] The use of the GDP deflator instead of the CPI does not alter the results significantly. Also, the results are shown for $n = 1$ and robustness checks indicate that the results are not affected by a change in the horizon ($n = 4$). These robustness checks are provided by the authors upon request.

Table A.1: GMM-Estimation of the Money Supply Rule

Baseline model: $\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \epsilon_t$
Baseline model + control: $\mu_t = \rho\mu_{t-1} + \mu_\pi E_t\pi_{t+n} + \mu_y(y_t - y_t^*) + \mu_{MY} MY_t + \epsilon_t$

	Whole sample 1961q1-2013q1		Pre-crisis period 1961q1-2007q4		Pre-Volker period 1961q1-1979q2		Post-Volker period 1982q4-2013q1	
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
ρ	0.770^{**}	0.861^{**}	0.846^{**}	0.985^{**}	0.766^{**}	0.724^{**}	0.833^{**}	0.668^{**}
	(14.89)	(20.31)	(16.73)	(19.61)	(11.71)	(9.98)	(13.81)	(11.57)
μ_π	0.087^{*}	0.040	0.056	0.100	0.066	0.067	-0.061	0.285
	(1.78)	(0.60)	(1.21)	(1.62)	(1.14)	(1.28)	(-0.35)	(0.81)
μ_y	-0.521^{**}	-0.398^{**}	-0.368^{**}	-0.241^{*}	-0.204	-0.262^{*}	-0.650^{*}	-1.259^{**}
	(-3.39)	(-3.27)	(-2.87)	(-2.07)	(-1.26)	(-2.63)	(-2.61)	(-3.82)
μ_{MY}		0.347		-0.612		0.189		-0.973
		(0.74)		(-1.52)		(0.45)		(-0.92)
$Adj.R^2$	0.69	0.68	0.67	0.64	0.52	0.52	0.70	0.69
J	0.419	0.194	0.308	0.168	0.553	0.739	0.398	0.594

The set of instruments includes four lags of money supply growth, inflation and output gap, as well as four lags of the MY ratio in specifications that include the MY ratio. Standard errors are in parenthesis. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). We test the overidentifying restrictions of our model specification and report the p-value of the Hansen's J-statistics. In columns (a), (c), (e) and (g), we estimate our baseline model. In columns (b), (d), (f) and (h), we add the MY ratio as a control variable.

Appendix B Alternative Demographic Structure

In Tables B.1 and B.2, we show the results obtained by simulating the stochastic monetary exchange economy model with the alternative demographic structure (n,N)=(69,79).

Table B.1: Stochastic Model - Results

Panel A: Consumption and Savings Decisions

Odd	$c^y_{o,s}$	$c^m_{o,s}$	$c^r_{o,s}$	$zb^y_{o,s}$	$ze^y_{o,s}$	$zb^m_{o,s}$	$ze^m_{o,s}$
s_1	2.17 (0.01)	2.37 (0.02)	2.25 (0.02)	-1.10 (0.00)	0.01 (0.00)	1.26 (0.00)	0.00 (0.00)
s_2	2.14 (0.01)	2.28 (0.02)	2.04 (0.02)	-1.09 (0.00)	0.01 (0.00)	1.25 (0.00)	0.00 (0.00)
s_3	1.64 (0.00)	1.74 (0.02)	1.69 (0.02)	-1.09 (0.00)	0.01 (0.00)	1.25 (0.00)	0.00 (0.00)
s_4	1.61 (0.00)	1.60 (0.01)	1.51 (0.01)	-1.07 (0.00)	0.01 (0.00)	1.22 (0.00)	0.00 (0.00)
Average across s states	1.89 (0.00)	1.99 (0.01)	1.88 (0.01)	-1.08 (0.00)	0.01 (0.00)	1.24 (0.00)	0.00 (0.00)

Even	$c^y_{e,s}$	$c^m_{e,s}$	$c^r_{e,s}$	$zb^y_{e,s}$	$ze^y_{e,s}$	$zb^m_{e,s}$	$ze^m_{e,s}$
s_1	2.48 (0.03)	2.34 (0.04)	2.47 (0.07)	-1.09 (0.00)	0.01 (0.00)	0.95 (0.00)	0.01 (0.00)
s_2	2.42 (0.02)	2.25 (0.03)	2.25 (0.06)	-1.09 (0.00)	0.01 (0.00)	0.95 (0.00)	0.01 (0.00)
s_3	1.79 (0.01)	1.76 (0.03)	1.85 (0.05)	-1.10 (0.00)	0.01 (0.00)	0.96 (0.00)	0.01 (0.00)
s_4	1.73 (0.01)	1.62 (0.03)	1.69 (0.04)	-1.09 (0.00)	0.01 (0.00)	0.95 (0.00)	0.01 (0.00)
Average across s states	2.10 (0.01)	1.99 (0.02)	2.07 (0.03)	-1.09 (0.00)	0.01 (0.00)	0.95 (0.00)	0.01 (0.00)

Panel B: Stock and Bond Yields

Odd	$r_{o,s}$	$q^e_{o,s}$	$ey_{o,s}$	$rp_{o,s}$	$i_{o,s,g}$ *Av. across g states*	$\pi^{an}_{o,s,g}$ *Av. across g states*	$E\pi^{an}_{o,s,g}$ *Av. across g states*
s_1	-0.15% (0.00)	145.36 (2.62)	0.06 (0.00)	1.03% (0.00)	4.11% (0.02)	2.25% (0.02)	4.30% (0.02)
s_2	0.35% (0.00)	133.42 (2.06)	0.04 (0.00)	1.04% (0.00)	4.31% (0.02)	2.54% (0.02)	4.00% (0.02)
s_3	5.85% (0.00)	46.06 (0.74)	0.18 (0.00)	1.10% (0.00)	8.65% (0.02)	3.69% (0.02)	2.84% (0.02)
s_4	6.77% (0.00)	40.36 (0.50)	0.14 (0.00)	1.11% (0.00)	9.20% (0.02)	4.08% (0.02)	2.47% (0.02)
Average across s states	3.27% (0.00)	92.24 (1.09)	0.10 (0.00)	1.07% (0.00)	6.62% (0.01)	3.15% (0.01)	3.40% (0.01)

Even	$r_{e,s}$	$q^e_{e,s}$	$ey_{e,s}$	$rp_{e,s}$	$i_{e,s,g}$ *Av. across g states*	$\pi^{an}_{e,s,g}$ *Av. across g states*	$E\pi^{an}_{e,s,g}$ *Av. across g states*
s_1	-2.53% (0.00)	206.84 (10.47)	0.04 (0.00)	0.98% (0.00)	1.98% (0.02)	2.00% (0.02)	4.55% (0.02)
s_2	-1.91% (0.00)	183.31 (8.41)	0.04 (0.00)	0.99% (0.00)	2.31% (0.02)	2.26% (0.02)	4.26% (0.02)
s_3	3.63% (0.00)	59.74 (2.83)	0.14 (0.00)	1.02% (0.00)	6.58% (0.02)	3.59% (0.02)	3.00% (0.02)
s_4	4.81% (0.00)	48.75 (1.95)	0.12 (0.00)	1.05% (0.00)	7.43% (0.02)	3.88% (0.02)	2.66% (0.02)
Average across s states	1.08% (0.00)	126.54 (4.35)	0.08 (0.00)	1.01% (0.00)	4.65% (0.01)	2.93% (0.02)	3.61% (0.01)

This table presents the simulation results of the stochastic model calibrated to the population age structure of (n,N)=(69,79). $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bond from period j to period $j+1$, respectively. $ey_{j,s}$ refers to the annualized earnings yield on stocks and is defined as $ey_{j,s} = 2 * (d_s/20)/q^e_{j,s}$. $rp_{j,s}$ is the annualized risk premium defined as $rp_{j,s} = average((\frac{q^e_{j+1,s+1}+d_{s+1}}{q^e_{j,s}})^{\frac{1}{20}} - 1 - r_{j,s})$. $\pi^{an}_{j,s,g}$ is the annualized inflation rate from period $j-1$ to period j. $E\pi^{an}_{j,s,g}$ is annualized expected inflation, i.e., the inflation rate from period j to period $j+1$. Because the inflation risk premium is very small (below 10^4 in absolute value), we do not report it.

Table B.2: Stochastic Model, Stock and Bond Yields across States

Odd	g	$r_{o,s}$	$ey_{o,s}$	$i_{o,s,g}$	$\pi^{an}_{o,s,g}$	$E\pi^{an}_{o,s,g}$
s_1	g_1	-0.15%	0.06	7.13%	-0.65%	7.33%
	g_2			4.51%	1.82%	4.71%
	g_3			2.51%	3.80%	2.70%
	g_4			0.11%	6.27%	0.31%
s_2	g_1	0.35%	0.04	7.37%	-0.45%	7.06%
	g_2			4.75%	2.08%	4.44%
	g_3			2.75%	4.13%	2.44%
	g_4			0.36%	6.49%	0.05%
s_3	g_1	5.85%	0.18	11.63%	0.73%	5.81%
	g_2			9.04%	3.24%	3.23%
	g_3			7.07%	5.27%	1.25%
	g_4			4.70%	7.87%	-1.11%
s_4	g_1	6.77%	0.14	12.18%	1.14%	5.45%
	g_2			9.61%	3.63%	2.88%
	g_3			7.64%	5.62%	0.91%
	g_4			5.28%	8.21%	-1.44%
Average		3.27%	0.10	6.62%	3.15%	3.40%
Even	g	$r_{e,s}$	$ey_{e,s}$	$i_{e,s,g}$	$\pi^{an}_{e,s,g}$	$E\pi^{an}_{e,s,g}$
s_1	g_1	-2.53%	0.04	4.99%	-0.89%	7.57%
	g_2			2.37%	1.58%	4.94%
	g_3			0.36%	3.57%	2.93%
	g_4			-2.05%	6.03%	0.53%
s_2	g_1	-1.91%	0.04	5.36%	-0.64%	7.30%
	g_2			2.73%	1.81%	4.68%
	g_3			0.73%	3.79%	2.68%
	g_4			-1.67%	6.24%	0.28%
s_3	g_1	3.63%	0.14	9.61%	0.66%	6.01%
	g_2			7.01%	3.11%	3.42%
	g_3			5.03%	5.15%	1.44%
	g_4			2.66%	7.62%	-0.92%
s_4	g_1	4.81%	0.12	10.42%	0.92%	5.66%
	g_2			7.84%	3.43%	3.08%
	g_3			5.87%	5.48%	1.11%
	g_4			3.51%	7.89%	1.11%
Average		1.08%	0.08	4.65%	2.93%	3.61%

This table presents the stock and bond yield simulation results of the stochastic model calibrated to the population age structure of (n,N)=(69,79). $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bond from period j to period $j+1$, respectively. $ey_{j,s}$ refers to the annualized earnings yield on stocks and is defined as $ey_{j,s} = 2 * (d_s/20)/q^e_{j,s}$. $\pi^{an}_{j,s,g}$ is the annualized inflation rate from period $j-1$ to period j. $E\pi^{an}_{j,s,g}$ is annualized expected inflation, i.e., the inflation rate from period j to period $j+1$.

Appendix C Description of Time-series and Data Sources

Equity market data: S&P 500 index yearly prices from 1900 to 2014 from are from Welch and Goyal (2008) and Robert Shiller's website (December observations). Dividends (Earnings) are twelve-month moving sums of dividends (earnings) paid on the S&P 500 index. Equity yields are defined as the ratio of one-year trailing dividends (earnings) to equity market index, i.e., S&P500.

Cyclically adjusted earnings yield: The ratio of ten-year moving average of earnings to equity market index, i.e. S&P500 collected from Robert Shiller's website for the period 1900-2014.

Stock market returns: For S&P 500 index, to construct the continuously compounded return r_t, we take the ex-dividend-price P_t add dividend D_t over P_{t-1} and take the natural logarithm of the ratio.

Risk-free rate: 3-Month Treasury Bill rate is taken from Welch and Goyal (2008) extended collecting data from St. Louis FRED database.

Inflation: Inflation is defined as annual log difference of GDP deflator. Pre-war sample data is taken D'Agostino and Surico (2012) and extended using data collected from St. Louis FRED database.

Bond yields: Long-term government bond yields are both from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook taken from Welch and Goyal (2008) and from Robert Shiller's website (annual one-year treasury bond-yield series).

Demographic Variables: The U.S. annual population estimates series are collected from U.S. Census Bureau and the sample covers estimates from 1900-2050. Middle-aged to young ratio, MY_t, is calculated as the ratio of the age group 40-49 to age group 20-29. Past MY_t projections for the period 1950-2013 are hand-collected from various past Census reports available at http://www.census.gov/prod/www/abs/p25.html.

Supply-side variables: We use the total market capitalization (NYSE, AMEX and NASDAQ from CRSP dataset) over GDP as a proxy for equity supply (1925-2014), government debt over GDP (available at Henning Bohn's website, 1900-2012) for bond supply and M2 over GDP (from FRED St. Louis) for money supply.

International database: Cross-country stock and bond yields are collected from Global Financial Data up to 2009. Stock yield is the dividend yield to the benchmark index and bond yield is the 10-year constant maturity government bond yields. For Finland and Japan, shorter maturity bonds, 5 year and 7-year, respectively, are used, since a longer time-series is available. International MY_t estimates for the period 1960-2008 are from World Bank Population estimates and projections from 2009-2050 are collected from International database (U.S. Census Bureau).

Appendix D Stochastic Fisher Equation

Qt and qt are respectively the nominal and the real bond prices, with $Q_t = \frac{P_t}{P_{t+1}} q_t$, $E(Q_t) = \frac{1}{1+i_t}$, and $E(q_t) = \frac{1}{1+r_t}$, where i_t and r_t are respectively the nominal and real interest rates.

We assume that $\frac{P_t}{P_{t+1}}$ and q_t are jointly log-normal distributed, which implies that

$$\begin{aligned} E(\frac{P_t}{P_{t+1}}) &= exp(E(ln(\frac{P_t}{P_{t+1}}))) + \frac{1}{2} Var(ln(\frac{P_t}{P_{t+1}})), \text{ and} \\ E(\frac{P_t}{P_{t+1}} q_t) &= E(q_t) E(\frac{P_t}{P_{t+1}}) exp(Cov(ln(\frac{P_t}{P_{t+1}}), ln(q_t))) \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} \frac{1}{1+i_t} &= E(Q_t) = E(\frac{P_t}{P_{t+1}} q_t) = E(q_t) E(\frac{P_t}{P_{t+1}}) exp(Cov(ln(\frac{P_t}{P_{t+1}}), ln(q_t))) \\ &= E(q_t) E(\frac{P_t}{P_{t+1}}) exp(Cov(ln(-\pi_{t+1}), ln(q_t))) \end{aligned}$$

By taking the log of both sides, we have

$$\begin{aligned} ln(\frac{1}{1+i_t}) &= ln(E(Q_t)) \\ -ln(1+i_t) &= ln(E(q_t)) + ln(E(\frac{P_t}{P_{t+1}})) + Cov(ln(-\pi_{t+1}), ln(q_t)) \\ &= ln(E(q_t)) + ln(exp(E(ln(\frac{P_t}{P_{t+1}}))) + \frac{1}{2} Var(ln(\frac{P_t}{P_{t+1}}))) - Cov(ln(\pi_{t+1}), ln(q_t)) \\ &= ln(E(q_t)) - E(\pi_{t+1}) + \frac{1}{2} Var(\pi_{t+1}) - Cov(ln(\pi_{t+1}), ln(q_t)) \\ &= -ln(1+r_t) - E(\pi_{t+1}) + \frac{1}{2} Var(\pi_{t+1}) - Cov(ln(\pi_{t+1}), ln(q_t)) \end{aligned}$$

The linear approximation leads to

$$\begin{aligned} -i_t &\approx -r_t - E(\pi_{t+1}) + \frac{1}{2} Var(\pi_{t+1}) - Cov(ln(\pi_{t+1}), ln(q_t)) \\ i_t &\approx r_t + E(\pi_{t+1}) - \frac{1}{2} Var(\pi_{t+1}) + Cov(ln(\pi_{t+1}), ln(q_t)) \end{aligned}$$

where $-\frac{1}{2} Var(\pi_{t+1}) + Cov(ln(\pi_{t+1}), ln(q_t))$ is the inflation risk premium.

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Table 1: Calibration: Exchange Monetary Economy

Description	Parameter	Value
First specification		
Large cohort	N	79
Small cohort	n	52
Real dividends	d	62
Nominal money supply	$\bar{M}^S$	376
Second specification		
Large cohort	N	79
Small cohort	n	69
Real dividends	d	70
Nominal money supply	$\bar{M}^S$	435
Common to both specifications		
Discount factor	β	0.5
Elasticity of substitution	$1/\sigma$	1/4
Av. real wages of young workers	w^y	2
Av. real wages of middle-aged workers	w^m	3

This table shows the parameter values used in the calibration of the exchange monetary economy over 20-year periods. In the first specification, the age structure is $(n, N) = (52, 79)$, whereas in the second specification, the age structure is $(n, N) = (69, 79)$.

Table 2: Data Summary Statistics

	mean	stdev	skew	kurt	min	max	AC(1)
ey_t	0.07	0.03	1.19	4.60	0.02	0.19	0.875
i_t^+	4.8%	0.02	1.56	5.40	1.9%	14.6%	0.934
i_t	0.0%	0.01	1.17	6.07	−3.0%	5.6%	0.732
r_t	2.7%	0.03	−1.04	7.02	−8.2%	10.8%	0.790
rf_t	3.6%	0.03	1.10	5.42	0.0%	15.5%	0.879
$\pi_{t,cum}^{an}$	0.0%	0.02	−0.38	4.79	−5.8%	5.3%	0.846
$E_t\pi_{VAR\_tv}^{an}$	2.1%	0.03	0.84	3.99	−2.8%	11.8%	0.861
MY_t	0.79	0.18	0.43	1.99	0.56	1.16	0.981

This table presents descriptive statistics of the empirical measures of stock yield, ey_t, proxied by the cyclically adjusted earnings price ratio obtained from Robert Shiller's website; 10-year U.S. nominal bond yield (p.a.), i_t^+, where $^+$ indicates the existence of a unit root; nominal bond yield demeaned across monetary regimes, i_t; real interest rate obtained using inflation expectations ($E_t\pi_{VAR\_tv}^{an}$) from a VAR model with drifting coefficients and stochastic volatility including money growth, r_t, Treasury-bill rate, rf_t, demeaned (annualized) ten-year cumulative inflation, $\widetilde{\pi}_{t,cum}^{an}$; and middle aged-young ratio MY_t. We also report first order autocorrelation. Annual data. Sample 1900-2014.

Table 3: Deterministic Model - Results

	$(n, N) = (52, 79)$		$(n, N) = (69, 79)$	
	Odd	Even	Odd	Even
MY_j	0.66	1.52	0.87	1.14
y_j	376.00	403.00	435.00	445.00
c_j^y	1.77	2.39	1.91	2.10
c_j^m	1.98	2.03	1.97	2.00
c_j^r	1.69	2.28	1.87	2.06
zb_j^y	0.81	-0.37	0.21	-0.16
zb_j^m	2.28	1.69	2.06	1.87
q_j^e	52.23	120.22	66.73	88.56
π_j^{an}	2.35%	1.65%	2.11%	1.89%
$E\pi_j^{an}$	1.65%	2.35%	1.89%	2.11%
ey_j	0.12	0.06	0.10	0.08
r_j	6.45%	-0.26%	4.42%	2.20%
i_j	8.08%	2.09%	6.30%	4.31%

This table presents the solution to the deterministic model. π_j^{an} is the annualized inflation rate from period $j-1$ to period j. $E\pi_j^{an}$ is annualized expected inflation, that is, the inflation rate from period j to period $j+1$. ey_j refers to the annualized earnings yield on stocks and is defined as $ey_j = 2 * (d/20)/q_j^e$. r_j and i_j are annualized real and nominal interest rates from period j to period $j+1$, respectively.

Table 4: Stochastic Model - Results

Panel A: Consumption and Savings Decisions

Odd	$c^y_{o,s}$	$c^m_{o,s}$	$c^r_{o,s}$	$zb^y_{o,s}$	$ze^y_{o,s}$	$zb^m_{o,s}$	$ze^m_{o,s}$
s_1	1.95 (0.01)	2.44 (0.06)	2.05 (0.05)	−1.06 (0.01)	0.01 (0.00)	1.60 (0.01)	0.00 (0.00)
s_2	1.94 (0.01)	2.35 (0.05)	1.83 (0.05)	−1.04 (0.01)	0.01 (0.00)	1.58 (0.01)	0.00 (0.00)
s_3	1.53 (0.01)	1.73 (0.05)	1.56 (0.04)	−1.01 (0.01)	0.01 (0.00)	1.53 (0.02)	0.00 (0.00)
s_4	1.52 (0.00)	1.60 (0.04)	1.34 (0.03)	−0.97 (0.01)	0.01 (0.00)	1.48 (0.02)	0.00 (0.00)
Average across s states	1.73 (0.00)	2.02 (0.03)	1.69 (0.02)	−1.02 (0.01)	0.01 (0.00)	1.54 (0.01)	0.00 (0.00)

Even	$c^y_{e,s}$	$c^m_{e,s}$	$c^r_{e,s}$	$zb^y_{e,s}$	$ze^y_{e,s}$	$zb^m_{e,s}$	$ze^m_{e,s}$
s_1	2.90 (0.05)	2.34 (0.05)	2.74 (0.13)	−1.05 (0.00)	0.01 (0.00)	0.69 (0.00)	0.01 (0.00)
s_2	2.80 (0.05)	2.24 (0.04)	2.53 (0.12)	−1.05 (0.00)	0.01 (0.00)	0.69 (0.00)	0.01 (0.00)
s_3	2.00 (0.03)	1.79 (0.04)	2.05 (0.10)	−1.05 (0.00)	0.01 (0.00)	0.69 (0.00)	0.01 (0.00)
s_4	1.90 (0.02)	1.64 (0.04)	1.91 (0.08)	−1.05 (0.00)	0.01 (0.00)	0.69 (0.00)	0.01 (0.00)
Average across s states	2.40 (0.02)	2.00 (0.03)	2.32 (0.06)	−1.05 (0.00)	0.01 (0.00)	0.69 (0.00)	0.01 (0.00)

Panel B: Stock and Bond Yields

Odd	$r_{o,s}$	$q^e_{o,s}$	$ey_{o,s}$	$rp_{o,s}$	$i_{o,s,g}$ *Av. across g states*	$\pi^{an}_{o,s,g}$ *Av. across g states*	$E\pi^{an}_{o,s,g}$ *Av. across g states*
s_1	2.04% (0.00)	102.70 (3.52)	0.08 (0.00)	1.08% (0.00)	6.14% (0.02)	2.48% (0.01)	4.07% (0.01)
s_2	2.54% (0.00)	97.23 (2.77)	0.06 (0.00)	1.08% (0.00)	6.22% (0.02)	2.79% (0.01)	3.75% (0.01)
s_3	7.94% (0.00)	36.71 (1.06)	0.20 (0.00)	1.13% (0.00)	10.52% (0.02)	3.89% (0.01)	2.64% (0.01)
s_4	8.52% (0.00)	34.16 (0.72)	0.14 (0.00)	1.13% (0.00)	10.81% (0.02)	4.30% (0.01)	2.26% (0.01)
Average across s states	5.27% (0.00)	68.14 (1.47)	0.12 (0.00)	1.11% (0.01)	8.46% (0.00)	3.38% (0.01)	3.17% (0.00)

Even	$r_{e,s}$	$q^e_{e,s}$	$ey_{e,s}$	$rp_{e,s}$	$i_{e,s,g}$ *Av. across g states*	$\pi^{an}_{e,s,g}$ *Av. across g states*	$E\pi^{an}_{e,s,g}$ *Av. across g states*
s_1	−5.00% (0.00)	289.37 (23.13)	0.02 (0.00)	0.91% (0.00)	−0.26% (0.02)	1.75% (0.01)	4.80% (0.01)
s_2	−4.19% (0.00)	247.76 (18.29)	0.02 (0.00)	0.92% (0.00)	0.18% (0.02)	2.02% (0.01)	4.50% (0.01)
s_3	1.24% (0.00)	79.30 (6.25)	0.10 (0.00)	0.95% (0.00)	4.36% (0.02)	3.37% (0.01)	3.21% (0.01)
s_4	2.59% (0.00)	60.61 (4.39)	0.08 (0.00)	0.97% (0.00)	5.43% (0.02)	3.67% (0.01)	2.87% (0.01)
Average across s states	−1.26% (0.00)	172.70 (9.62)	0.06 (0.00)	0.94% (0.01)	2.52% (0.00)	2.70% (0.01)	3.84% (0.01)

This table presents the simulation results of the stochastic model calibrated to the population age structure of (n,N)=(52,79). $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bond from period j to period $j+1$, respectively. $ey_{j,s}$ refers to the annualized earnings yield on stocks and is defined as $ey_{j,s} = 2*(d_s/20)/q^e_{j,s}$. $rp_{j,s}$ is the annualized risk premium defined as $rp_{j,s} = average((\frac{q^e_{j+1,s+1}+d_{s+1}}{q^e_{j,s}})^{\frac{1}{20}} - 1 - r_{j,s})$. $\pi^{an}_{j,s,g}$ is the annualized inflation rate from period $j-1$ to period j. $E\pi^{an}_{j,s,g}$ is annualized expected inflation, that is, the inflation rate from period j to period $j+1$. Because the inflation risk premium is very small (below 10^4 in absolute value), we do not report it.

Table 5: Empirical Correlations with the MY Ratio

Panel A. Correlations	$corr(x_t, MY_t)$	R^2_{adj}
ey_t	-0.57^{**} (-7.18)	0.31
i_t	-0.39^{**} (-4.48)	0.15
r_t	-0.41^{*} (-4.82)	0.16
$E_t\pi^{an}_{VAR\_tv}$	0.17 (2.01)	0.03
Panel B. Sensitivity to Inflation Forecasts	$corr(x_t, MY_t)$	R^2_{adj}
$r_{t,AR(1)}$	-0.42^{*} (-4.91)	0.17
$r_{t,AR(1)\_tv}$	-0.62^{***} (-8.46)	0.38
$r_{t,VAR}$	-0.40^{*} (-4.72)	0.16
r_{t,VAR_GDP_tv}	-0.49^{**} (-5.94)	0.23
$E_t\pi^{an}_{AR(1)}$	0.32^{*} (3.76)	0.10
$E_t\pi^{an}_{AR(1)\_tv}$	0.39^{*} (4.49)	0.14
$E_t\pi^{an}_{VAR}$	0.31 (3.50)	0.09
$E_t\pi^{an}_{VAR\_GDP\_tv}$	0.18 (1.91)	0.02

Panel A shows the correlation coefficients between the MY ratio and the following variables: earnings yield, ey_t, proxied by cyclically adjusted earnings price ratio; 10-year U.S. nominal bond yield (p.a.) demeaned across monetary regimes, i_t; real interest rate obtained using long-term inflation expectations ($E_t\pi^{an}_{VAR\_tv}$) from a VAR model with drifting coefficients and stochastic volatility including money growth, r_t. Panel B shows the correlation with MY_t with different specifications of real interest rates and long term inflation expectations. Different specifications include an $AR(1)$ model, an $AR(1)$ model with stochastic volatility, a bivariate VAR model including inflation and money growth, and a bivariate VAR model with drifting coefficients and stochastic volatility including GDP growth. The reported t-statistics in parentheses for Pearson's correlation are calculated using a Student's t-distribution. Statistical significance of correlation is based on a bootstrapping exercise which accounts for the persistence of each variable and imposes the null hypothesis of no correlation between two variables. Asterisks *,** and *** denote significance at 10, 5 and 1 percent, respectively. The last column reports the adjusted R^2_{adj}. Annual data. Sample 1900-2014.

Table 6: Correlation and Partial Correlation Matrices

Panel A. Correlations	ey_t	i_t	r_t	$E\pi^{an}_{t,VAR\_tv}$
ey_t	1	0.35^{**} (3.92)	0.33^{*} (3.74)	-0.12 (-1.30)
i_t		1	0.30^{**} (3.24)	0.25 (2.79)
r_t			1	-0.58^{***} (-7.58)
$E\pi^{an}_{t,VAR\_tv}$				1

Panel B. Partial Correlations	ey_t	i_t	r_t	$E\pi^{an}_{t,VAR\_tv}$
ey_t	1	0.17 (1.80)	0.13 (1.42)	-0.02 (-0.22)
i_t		1	0.16 (1.67)	0.36^{**} (4.09)
r_t			1	-0.56^{***} (-7.21)
$E\pi^{an}_{t,VAR\_tv}$				1

Panel C. Correlations Over Time	$corr(ey_t, i_t)$	
	corr	dyn.corr
Pre-Fed (1900m1-1914m12)	0.88	0.39
Gold (1915m1-1931m9)	0.91	0.54
Mixed (1931m10-1939m9)	0.73	0.23
Pegged (1939m10-1951m3)	0.62	0.50
Bretton Woods (1951m4-1971m9)	-0.43	-0.02
Great Inflation (1971m10-1983m12)	0.80	0.55
Great Moderation (1984m1-2008m10)	0.85	0.17
QE (2008m11-2014m12)	0.23	-0.36
Panel D. Model-Implied Correlation	0.82	-

Panel A shows full-sample correlations, Panel B shows partial correlations controlling for middle aged-young ratio MY_t, and Panel C shows stock-bond correlations across monetary regimes. The variables are stock yield ey_t, proxied by cyclically adjusted earnings price ratio; demeaned (monetary regimes) 10-year nominal bond yield, i_t; real interest rate obtained using long-term inflation expectations ($E_t\pi^{an}_{VAR\_tv}$) from a VAR model with drifting coefficients and stochastic volatility including money growth, r_t. The reported t-statistics in parentheses for Pearson's correlation are calculated using a Student's t-distribution. Statistical significance of correlation is based on a bootstrapping exercise which accounts for the persistence of each variable and imposes the null hypothesis of no correlation between two variables. Asterisks *,** and *** denote significance at 10, 5 and 1 percent, respectively. In Panel C, *dyn.corr* measures the average dynamic correlation using a fixed window of 36 months. Panels A and B: annual data. Panel C: monthly data. Sample 1900-2014.

Table 7: Stochastic Model: Stock and Bond Yields across States

Odd	g	$r_{o,s}$	$ey_{o,s}$	$i_{o,s,g}$	$\pi^{an}_{o,s,g}$	$E\pi^{an}_{o,s,g}$
s_1	g_1	2.04%	0.08	9.16%	−0.42%	7.09%
	g_2			6.54%	2.05%	4.47%
	g_3			4.54%	4.04%	2.47%
	g_4			2.15%	6.52%	0.08%
s_2	g_1	2.54%	0.06	9.27%	−0.20%	6.80%
	g_2			6.66%	2.33%	4.19%
	g_3			4.66%	4.39%	2.19%
	g_4			2.29%	6.75%	−0.19%
s_3	g_1	7.94%	0.20	13.49%	0.93%	5.61%
	g_2			10.91%	3.44%	3.03%
	g_3			8.94%	5.47%	1.06%
	g_4			6.58%	8.08%	−1.29%
s_4	g_1	8.52%	0.14	13.78%	1.36%	5.23%
	g_2			11.21%	3.85%	2.66%
	g_3			9.25%	5.85%	0.70%
	g_4			6.90%	8.45%	−1.65%
Average		5.27%	0.12	8.46%	3.38%	3.17%
Even	g	$r_{e,s}$	$ey_{e,s}$	$i_{e,s,g}$	$\pi^{an}_{e,s,g}$	$E\pi^{an}_{e,s,g}$
s_1	g_1	−5.00%	0.02	2.76%	−1.12%	7.82%
	g_2			0.13%	1.34%	5.19%
	g_3			−1.87%	3.32%	3.18%
	g_4			−4.30%	5.78%	0.77%
s_2	g_1	−4.19%	0.02	3.24%	−0.87%	7.55%
	g_2			0.60%	1.57%	4.93%
	g_3			−1.41%	3.54%	2.92%
	g_4			−3.80%	5.99%	0.52%
s_3	g_1	1.24%	0.10	7.40%	0.45%	6.23%
	g_2			4.79%	2.89%	3.64%
	g_3			2.79%	4.93%	1.66%
	g_4			0.42%	7.40%	−0.72%
s_4	g_1	2.59%	0.08	8.43%	0.71%	5.87%
	g_2			5.84%	3.22%	3.28%
	g_3			3.87%	5.27%	1.31%
	g_4			1.50%	7.67%	−1.06%
Average		−1.26%	0.06	2.52%	2.70%	3.84%

This table presents the stock and bond yield simulation results of the stochastic model calibrated to the population age structure of (n,N)=(52,79). $r_{j,s}$ and $i_{j,s,g}$ are the annualized real and nominal rates of return on bond from period j to period $j+1$, respectively. $ey_{j,s}$ refers to the annualized earnings yield on stocks and is defined as $ey_{j,s} = 2 * (d_s/20)/q^e_{j,s}$. $\pi^{an}_{j,s,g}$ is the annualized inflation rate from period $j-1$ to period j. $E\pi^{an}_{j,s,g}$ is annualized expected inflation, that is, the inflation rate from period j to period $j+1$.

Table 8: Fed Model

ey_t	$const$	i_t	MY_t	σ_t^{EB}	$ctrl_t$	R^2_{adj}
(1) Fed Model	0.072*** (10.96)	0.739*** (2.74)				0.11
(2) Fed Model + MY_t	0.140*** (5.45)	0.323 (1.59)	-0.086*** (-3.11)			0.32
(3) Fed Model + σ_t^{EB}	0.067*** (12.46)	0.906*** (3.06)		0.011*** (2.77)		0.29
(4) Model (3) + MY_t	0.120*** (4.26)	0.540* (1.91)	-0.064** (-2.09)	0.008** (2.07)		0.38
(4') (r_t) + MY_t	0.120*** (6.12)	0.209 (1.63)	-0.072*** (-3.19)	0.006* (1.81)		0.36
(4) + $ctrl_t = es_t$	0.146*** (9.01)	0.207 (1.37)	-0.089*** (-4.09)	0.004 (1.65)	-0.007 (-0.97)	0.63
(4) + $ctrl_t = bs_t$	0.129*** (6.09)	0.418 (1.60)	-0.089*** (-3.61)	0.006 (1.24)	0.028 (1.35)	0.40
(4) + $ctrl_t = ms_t$	0.110 (1.88)	0.527* (1.89)	-0.065** (-2.37)	0.007 (1.23)	0.019 (0.26)	0.38
(4) + $ctrl_t = ra_t$	0.135*** (5.30)	0.463* (1.79)	-0.068** (-2.38)	0.006 (1.48)	-0.119** (-2.19)	0.43

This table reports the estimates of valuation models such as the FED model and its extensions. The baseline model (1) posits a long run relation between stock yields ey_t and demeaned long term nominal bond yields i_t. The second specification augments the model by including MY_t. Third specification controls for relative stock-bond volatility σ_t^{EB} in the original Fed Model and fourth model includes MY_t. Next specification substitutes the nominal bond yield with real interest rate obtained using inflation expectations from a VAR model with drifting coefficients and stochastic volatility including money growth r_t. Further controls include es_t, stock supply (1925-2014) measured by total market cap (NYSE+AMEX+NASDAQ) over nominal GDP, bs_t, bond supply measured by government debt over GDP (1900-2012), ms_t, money supply (M2) over GDP, and time-varying habit based-risk aversion ra_t. Relative stock-bond volatility is logarithm of the ratio of the realized volatilities (10-year window of monthly observations). The dependent variable, earnings yield, is measured by cyclically adjusted earnings price ratio. The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). Asterisks *, ** and ** indicate significance at 10, 5 and 1 percent, respectively. The last column reports adjusted R^2_{adj}. Annual data. Sample 1900-2014.

Table 9: Future Risk-free Rates

Short-term interest rates: $rf_{t,t+h} = \alpha_0 + \alpha_1 x_t + \varepsilon_{t,t+h}$

Panel A. h=1 year	$\overline{rf}_t$	yg_t^e	yg_t^d	$E_t(MY_t^{t+h})$	R^2_{adj}	R^2_{OS}
$rf_{t,t+1}$	0.000 (0.06)				0.00	–
$rf_{t,t+1}$		0.009 (1.54)			0.03	−0.01
$rf_{t,t+1}$			0.004 (0.87)		0.00	−0.05
$rf_{t,t+1}$				-0.024^{**} (−2.01)	0.06	0.04
Panel B. h=5 years	$\overline{rf}_t$	yg_t^e	yg_t^d	$E_t(MY_t^{t+h})$	R^2_{adj}	R^2_{OS}
$rf_{t,t+5}$	0.001 (0.29)				0.00	–
$rf_{t,t+5}$		0.008 (1.56)			0.08	0.09
$rf_{t,t+5}$			0.007 (1.47)		0.06	0.11
$rf_{t,t+5}$				-0.026^{**} (−2.34)	0.15	0.21
Panel C. h=10 years	$\overline{rf}_t$	yg_t^e	yg_t^d	$E_t(MY_t^{t+h})$	R^2_{adj}	R^2_{OS}
$rf_{t,t+10}$	0.002 (0.66)				0.00	–
$rf_{t,t+10}$		0.002 (0.53)			0.00	0.08
$rf_{t,t+10}$			−0.004 (−0.99)		0.04	0.09
$rf_{t,t+10}$				-0.024^{**} (−3.92)	0.28	0.29

The table shows the slope estimates of a univariate predictive regressions of cumulative risk-free rates, that is, the average of the (demeaned) Treasury-bill rates over the investment horizon, on a constant ($\overline{rf}_t$), two yield gap proxies yg_t^e and yg_t^d and expectation of future middle-aged to young ratio $E_t(MY_t^{t+h})$. Each panel shows the results for different horizon h, 1-year, 5-years and 10-years. The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the lag length is equal to the investment horizon. Asterisks *,** and *** indicate significance at 10, 5 and 1 percent levels, respectively. The last two columns report adjusted R^2_{adj} and out-of-sample coefficient of determination R^2_{OS}. Annual data. Sample 1900-2014.

Table 10: Stock Return Predictability

Excess Stock Returns: $xret_{t,t+h} = \alpha_0 + \alpha_1 x_t + \varepsilon_{t,t+h}$							
Panel A. h=1 year	yg_t^e	ygd_t^e	yg_t^d	ygd_t^d	$E_t(MY_{n+1}^{n+h})$	R^2_{adj}	R^2_{OS}
$xret_{t,t+1}$	0.038 (1.11)[0.16]					0.00	−0.03
$xret_{t,t+1}$		0.066** (2.03)[0.05]				0.02	0.01
$xret_{t,t+1}$			0.035 (1.03)[0.16]			0.00	−0.10
$xret_{t,t+1}$				0.060* (1.74)[0.03]		0.02	−0.06
$xret_{t,t+1}$	0.059* (1.83)				0.211*** (2.81)	0.03	0.03
$xret_{t,t+1}$			0.062 (1.80)		0.239*** (2.83)	0.04	0.00
Panel B. h=5 years	yg_t^e	ygd_t^e	yg_t^d	ygd_t^d	$E_t(MY_{n+1}^{n+h})$	R^2_{adj}	R^2_{OS}
$xret_{t,t+5}$	0.026 (1.21)[0.05]					0.02	−0.20
$xret_{t,t+5}$		0.050*** (2.74)[0.00]				0.09	−0.09
$xret_{t,t+5}$			0.031* (1.68)[0.00]			0.06	−0.53
$xret_{t,t+5}$				0.053*** (3.55)[0.00]		0.13	−0.36
$xret_{t,t+5}$	0.044*** (3.52)				0.199*** (6.32)	0.20	0.20
$xret_{t,t+5}$			0.054*** (4.36)		0.224*** (6.18)	0.27	0.10
Panel C. h=10 years	yg_t^e	ygd_t^e	yg_t^d	ygd_t^d	$E_t(MY_{n+1}^{n+h})$	R^2_{adj}	R^2_{OS}
$xret_{t,t+10}$	0.036* (1.83)[0.00]					0.12	−0.24
$xret_{t,t+10}$		0.059** (5.36)[0.00]				0.32	0.03
$xret_{t,t+10}$			0.019 (0.99)[0.00]			0.04	−0.28
$xret_{t,t+10}$				0.037** (2.34)[0.00]		0.16	−0.20
$xret_{t,t+10}$	0.053*** (5.39)				0.201*** (6.56)	0.58	0.51
$xret_{t,t+10}$			0.038*** (2.87)		0.207*** (6.51)	0.50	0.48

This table reports the results of long run (1, 5, 10 years) stock return predictability regressions based on univariate and bivariate models. Univariate models are based on yield gap proxies yg_t^e and yg_t^d and their modified versions accounting for expectations of distant future risk-free rates captured by expectations of distant future middle-aged to young ratio $E_t(MY_{n+1}^{n+h})$. In the univariate model the coefficient of $E_t(MY_{n+1}^{n+h})$ is restricted to be one. The unrestricted bivariate model includes both yield gap proxies and $E_t(MY_{n+1}^{n+h})$. The dependent variable is the cumulative excess stock market (S&P500) returns. The reported t-statistics (in parenthesis) are based on HAC covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the lag length equals to the investment horizon. In the univariate models we also report in square brackets the p-values obtained from a bootstrap exercise which accounts for the persistence of predictor variable and imposes the joint null hypothesis of no predictability of returns. Asterisks *,** and *** indicate significance at 10, 5 and 1 percent levels, respectively. The last two columns report adjusted R^2_{adj} and out-of-sample coefficient of determination R^2_{OS}. Annual data. Sample 1900-2014.

Table 11: Cross-Country Results

Panel A. Model: $corr_i(dy_t, by_t) = \alpha_0 + \alpha_1 corr_i(dy_t, MY_t) + \alpha_2 ctrl_i + \varepsilon_i$, $n = 19$

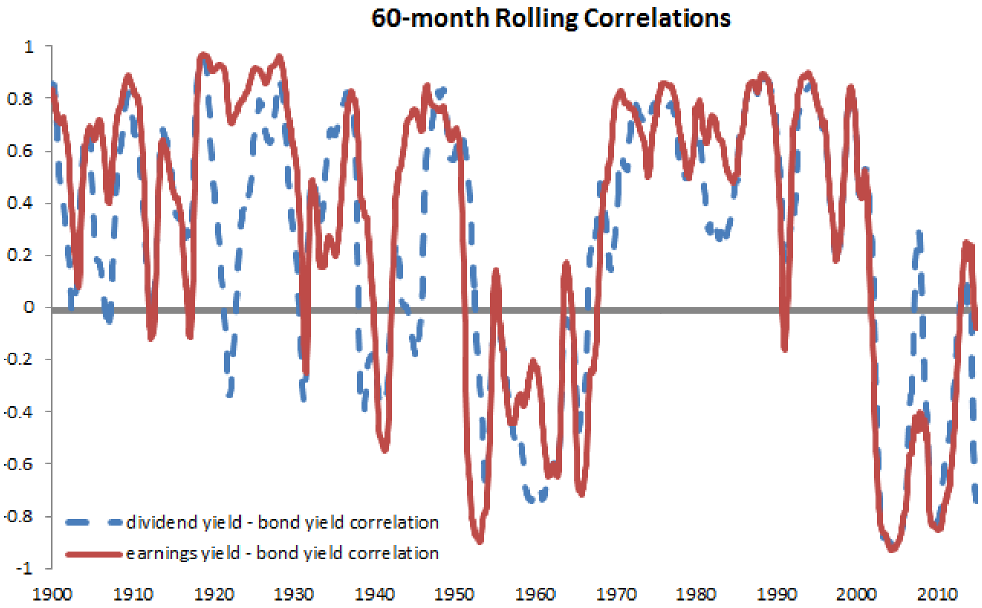
$corr_i(dy_t, by_t)$	$corr_i(dy_t, MY_t)$	$\overline{\text{inf}}_i$	$\overline{\Delta GDP}_i$	$rec_i^{percent}$	$\overline{inf\_rec_i}$	R^2_{adj}
(1)	-0.610^{***} (-3.37)					0.41
(2)	-0.535^{***} (-3.23)	-0.146^{*} (-1.89)				0.54
(3)	-0.454^{**} (-2.66)	-0.160^{**} (-2.14)	-0.076 (-1.36)			0.56
(4)	-0.375^{*} (-2.08)	-0.265^{**} (-2.28)	-0.100 (-1.57)	-0.022 (-1.16)	0.607 (1.38)	0.57

Panel B. Model: $corr_i(ey_t, by_t) = \alpha_0 + \alpha_1 corr_i(ey_t, MY_t) + \alpha_2 ctrl_i + \varepsilon_i$, $n = 17$

$corr_i(ey_t, by_t)$	$corr_i(ey_t, MY_t)$	$\overline{\text{inf}}_i$	$\overline{\Delta GDP}_i$	$rec_i^{percent}$	$\overline{inf\_rec_i}$	R^2_{adj}
(1)	-0.785^{**} (-4.94)					0.67
(2)	-0.535^{***} (-3.16)	-0.080^{**} (2.30)				0.74
(3)	-0.406^{**} (-2.63)	-0.104^{***} (-3.40)	0.134^{**} (2.66)			0.76
(4)	-0.669^{***} (-3.75)	-0.053 (-1.29)	-0.239^{*} (-2.08)	-0.029^{*} (-1.93)	0.772^{*} (1.85)	0.80

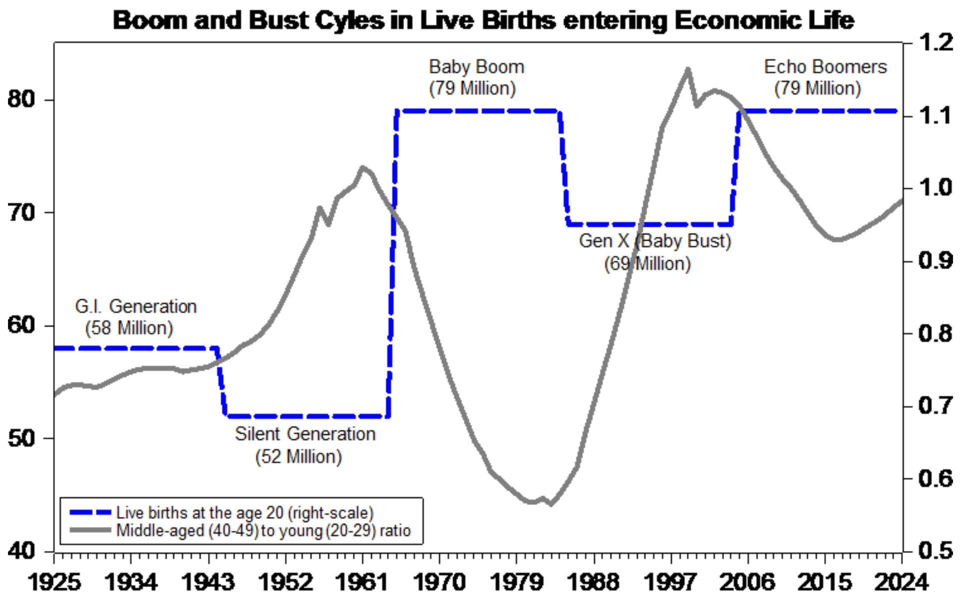
The table reports robust regression results. $corr_i(dy_t, MY_t)$ (resp. $corr_i(ey_t, MY_t)$) is the bootstrap correlation in country i, averaged across 10,000 bootstrap samples, between the stock yield measured by dividend yield dy_t (resp. measured by earnings yield ey_t) and the MY ratio. In both specifications, the dependent variable is the correlation between the stock yield (dividend yield, dy_t, or earnings yield, ey_t) and the long term nominal bond yield, by_t, for each country i over the period 1973-2009. $ctrl_i$ includes the following control variables: the full-sample country-specific mean of inflation, $\overline{\text{inf}}_i$; the full-sample country-specific mean of real GDP growth, $\overline{\Delta GDP_i}$; the percentage of observations during which the country was in recession (measured by negative annual GDP growth), $rec_i^{percent}$; and the country-specific time-series mean of the interaction $inf_{i,t} \cdot rec_{i,t}$, $\overline{inf\_rec_i}$. The reported t-statistics are based on the robust regression using bisquare weighting function. Asterisks *, ** and *** indicate significance at 10, 5 and 1 percent levels, respectively. n is the number of countries in each specification. The last column reports the OLS adjusted R^2.

Figure 1: Correlation Between Stock and Bond Yields



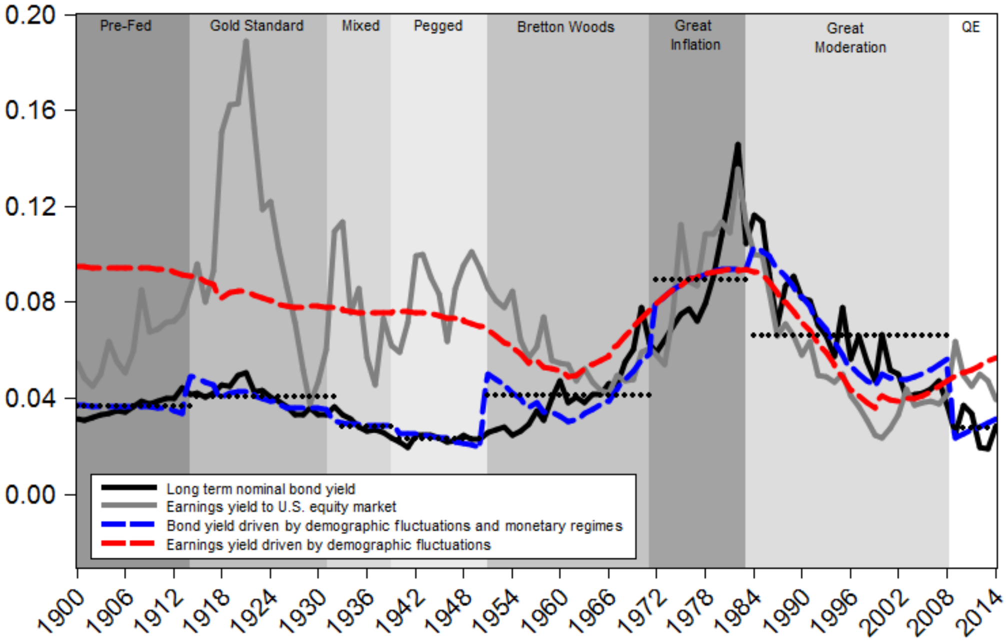
This figure plots the 60 month rolling correlation between dividend yield and bond yield, and the 60 month rolling correlation between earnings yield and bond yield. Sample 1900-2014. Monthly data.

Figure 2: Boom Bust Cycles in Live Births



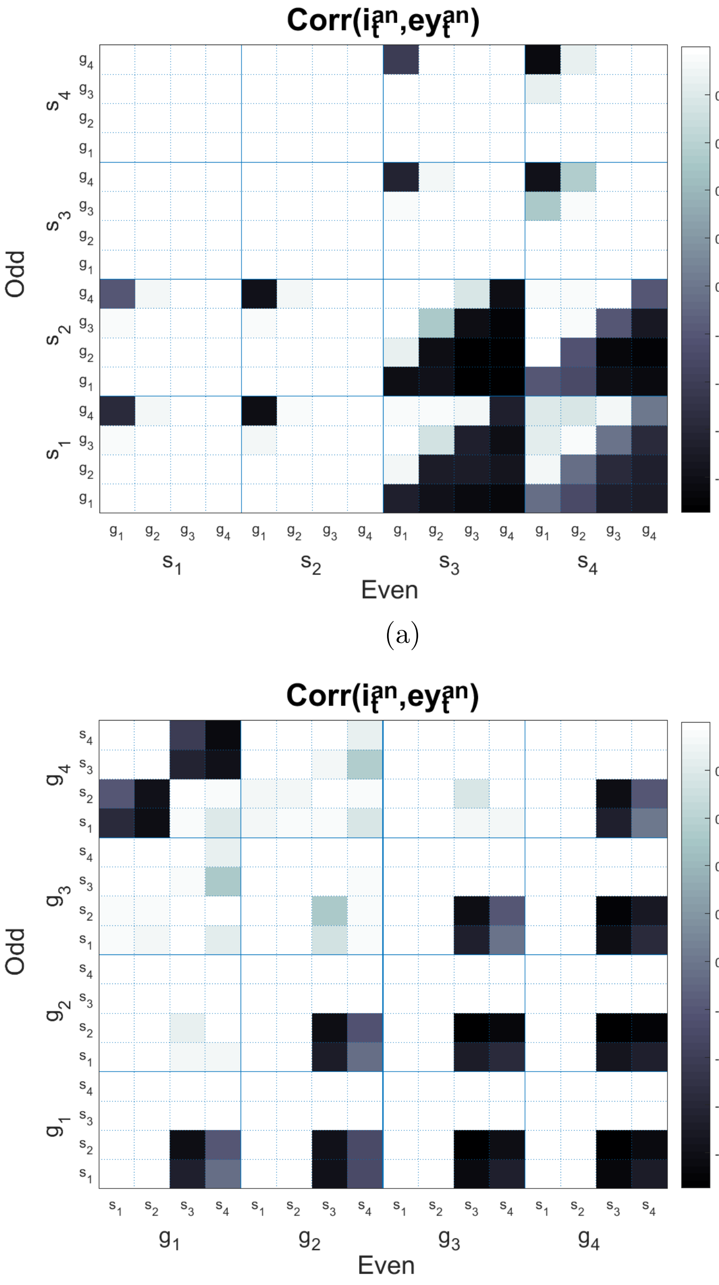
This figure plots the total number of life births (bar graph with dashed-line) at age 20 (the start of economic life) and the demographic variable, MY_t (solid line) measured as the proportion of middle-aged (40-49) to young (20-29) population. Sample 1925-2024. Annual data.

Figure 3: Stock vs. Bond Yields and Demographic Fluctuations



This figure plots the cyclically adjusted earnings yield (computed using a 10-year window of earnings) and 10-year nominal bond yields, together with the generational frequency components obtained by fitting each variable on the demographic variable MY_t, the proportion of middle-aged to young population, and controlling for monetary regimes (shaded areas). Dotted horizontal lines indicate the average bond yield in each monetary regime. Sample 1900-2014. Annual data.

Figure 4: Correlation between Bond and Stock Yields across States

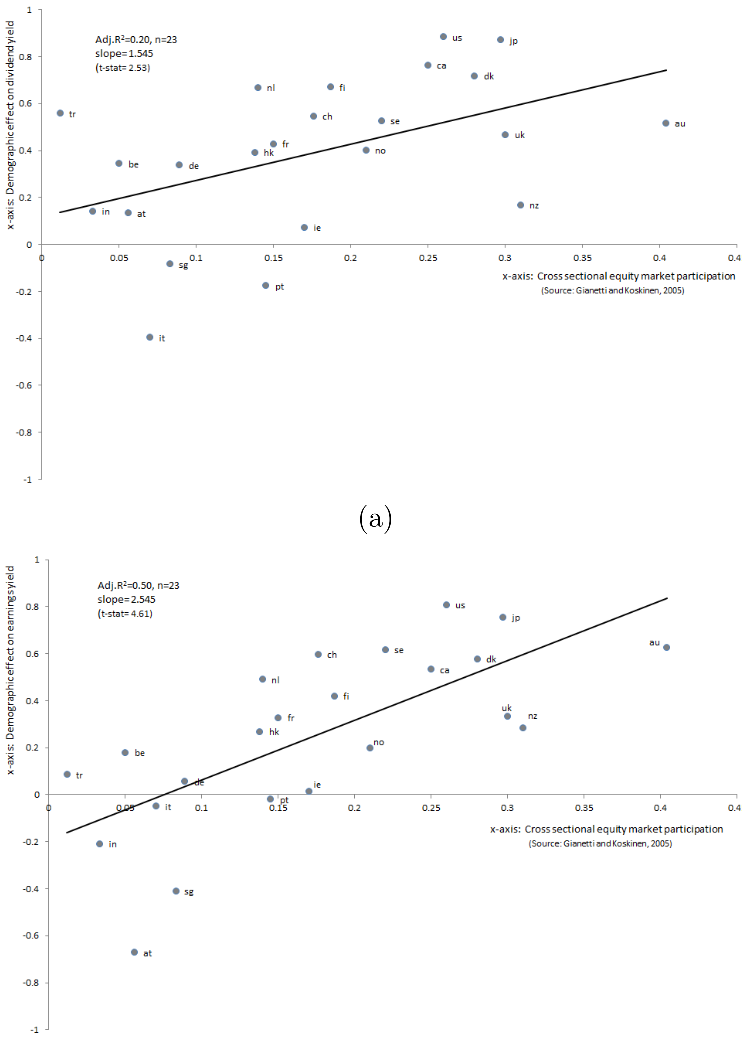


(a)

(b)

Panels (a) and (b) report the correlation between nominal bond yields and equity yields. In panel (a), the correlation shows the comovement between yields from state (j,s,g) to state (j+1,s+1,g+1), where $j = \{odd, even\}$, $s = \{s_1, s_2, s_3, s_4\}$ and $g = \{g_1, g_2, g_3, g_4\}$. In panel (b), the correlation shows the comovement between yields from state (j,g,s) to state (j+1,g+1,s+1).

Figure 5: Stock Market Participation and Demographic Effect

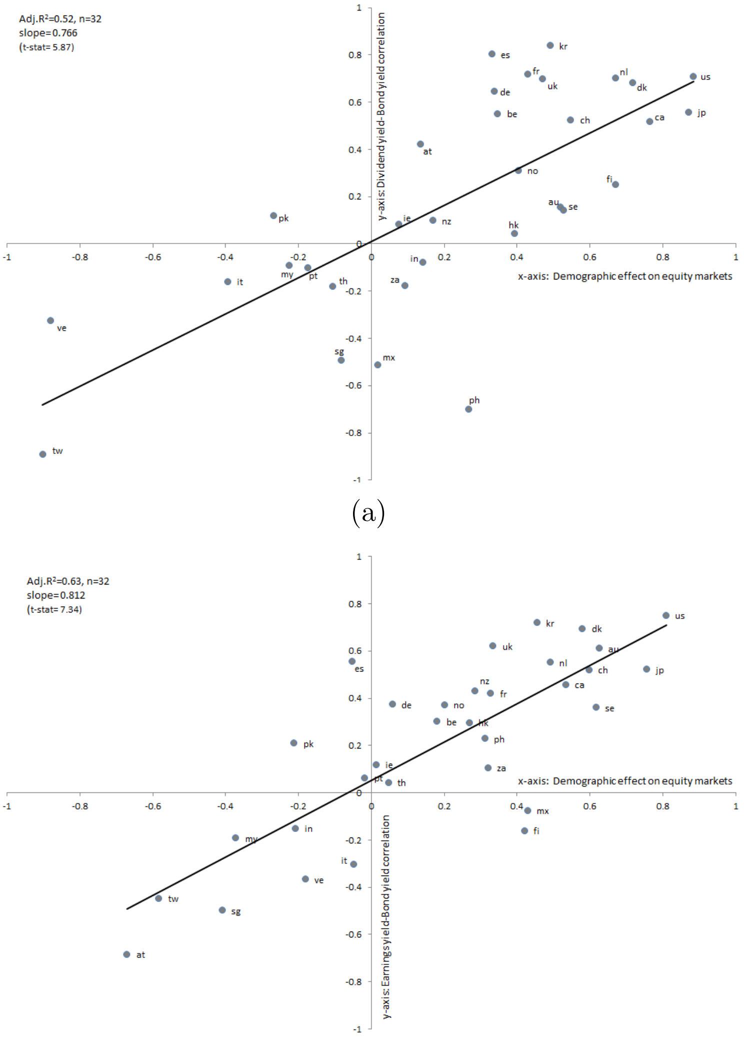


(a)

(b)

The upper (lower) panel provides a scatterplot of the demographic effect on dividend (earnings) yield (y-axis, measured by the negative correlation between dividend yield and middle age to young population ratio of each country available in the panel) and the cross sectional (around the millennium) stock market participation rates (x-axis, obtained by Giannetti and Koskinen, 2005). The reported correlation is the average correlation obtained by 10,000 bootstrap samples. For countries with less than 30 data, rank transformed variables are used to measure Spearman correlation, otherwise standard Pearson correlation is calculated for each bootstrap sample. The panels report the sample size (n), the adjusted R^2, the slope coefficient and the associated OLS t-statistics for the regression of the y-axis variable on the x-axis variable.

Figure 6: Cross-country Evidence on Demographic Effect



(a)

(b)

The upper (lower) panel provides a scatterplot of the demographic effect on dividend (earnings) yield (y-axis) and stock-bond yield correlation. The demographic effect on stock yield is proxied by the negative correlation between dividend (earnings) yield and middle age to young population ratio of each country. The reported correlation is the average correlation obtained by 10,000 bootstrap samples. For countries with less than 30 data points, rank transformed variables are used to measure Spearman correlation, otherwise standard Pearson correlation is calculated for each bootstrap sample. The panels report the sample size (n), the adjusted R^2, the slope coefficient and the associated OLS t-statistics for the regression of the y-axis variable on x-axis variable.