# *Problem Set #5*

due Friday, September 29

**PHYSICS 851, FALL 2000**

1. (a) Derive and solve the equations of motion for the Heisenberg operators $a(t)$ and $a^\dagger(t)$ for the harmonic oscillator.

   (b) Calculate $[a(t), a^\dagger(t')]$.

2. (a) Show that $[r_{op}, f(p_{op})] = i\hbar\nabla_p f(p_{op})$, where $f(p)$ is an arbitrary function of the momentum operator.

   (b) Using this result show that

$$e^{ip_{op}\lambda/\hbar} r_{op} e^{-ip_{op}\lambda/\hbar} = r_{op} + \lambda$$

3. (a) Calculate the correlation function $\langle 0|x(t)x(t')|0\rangle$ where $|0\rangle$ is the harmonic oscillator ground state, and $x(t)$ is the position operator in the Heisenberg representation.

   (b) Suppose that a time dependent force $F(t)$ is applied to a particle in the oscillator potential. Show that $x(t)$ obeys the equation of motion,

$$m\left(\frac{d^2}{dt^2} + \omega^2\right) x(t) = F(t)$$

   where $\omega$ is the oscillator frequency.

4. What are the matrix elements of the operator $1/|\mathbf{p}|$ in the position representation? That is, find $\langle \mathbf{r}|1/|\mathbf{p}||\mathbf{r}'\rangle$. Work the problem in three dimensions.