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Hausdorff on Ordered Sets

J. M. Plotkin
Editor

American Mathematical Society
London Mathematical Society

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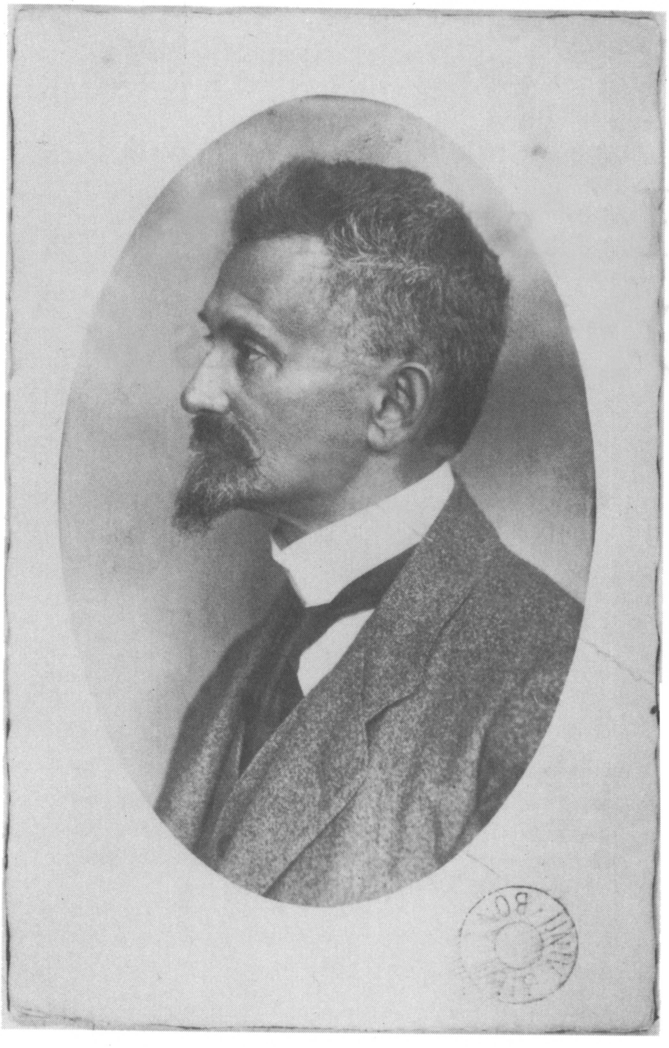

Photograph by Max Kempe, Greifswald

Felix Hausdorff in Greifswald, 1914–1921
Universitäts-und Landesbibliothek Bonn, Hss.-Abt.
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Translated by **J. M. Plotkin**

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Original German text translated by J. M. Plotkin

The article *Sum of* $\aleph_1$ *sets* by Felix Hausdorff, originally published as "Summen von $\aleph_1$-Mengen", Fundamenta Mathematicae XXVI (1936) is reprinted by permission of Instytut Matematyczny Polskiej Akademii Nauk.

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To the memory of Felix Hausdorff

Photograph by Ludwig Hogrefe, Godesburg

Felix Hausdorff in his study at home, Bonn, June 8–14, 1924
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Preface

Introduction

As the nineteenth century was ending, Georg Cantor (1845–1918) published his own *Summa Theologica* for set theory. This two-part work, entitled *Beiträge zur Begründung der transfiniten Mengenlehre*, appeared in 1895 and 1897 in volumes 46 and 49, respectively, of Mathematische Annalen. The second installment of *Beiträge* turned out to be Cantor's last published work on set theory. Though *Beiträge* could well serve as a primer on basic set theory, it also seemed unfinished. Fundamental questions remained. Are all cardinals comparable? Can all sets be well-ordered? Are there just two possibilities for the cardinalities of infinite sets of reals? Yet in some cosmically just way this work marked a fitting ending for a journey that began with the bread and butter question of uniqueness of representations of functions by trigonometric series and ended in the realm of pure set theory.

The year 1897 also saw Cantor's contributions gain official and public recognition from his peers. The First International Congress of Mathematicians took place in Zurich in August of 1897, and Cantor was in attendance. In a major address, the Zurich mathematician Adolf Hurwitz (1859–1919), who was one of the congress's organizers, devoted considerable space to an exegesis of Cantor's ordinal numbers and their application to the analysis of closed point sets. Hurwitz propounded Cantor's work as the basis for an investigation of analytic functions in terms of their sets of singularities.[1] At this same congress, Jacques Hadamard (1865-1963) gave a short talk entitled *Sur certaines applications possibles de la théorie des ensembles.*

The next international congress was held in Paris in August 1900. There, David Hilbert (1862–1943) gave the famous address in which, after a provocative meditation on the esthetics of mathematical problems and on the criteria for admissible solutions, he listed twenty-three problems whose solutions would advance the entire enterprise of mathematics.[2] The first three

[1] *Über die Entwickelung der allgemeinen Theorie der analytischen Funktionen in neurer Zeit*, Verhandlungen des ersten internationalen Mathematiker-Kongresses in Zürich vom 9. bis 11. August 1897 (Leipzig: Teubner), 1898, 91–112. Hurwitz's discussion of Cantor's work appears on pp. 94–97.

[2] Because of time constraints, Hilbert mentioned only ten problems in his actual address: three from foundations, four from arithmetic and algebra, and three from function theory. All twenty-three problems are in the printed version: *Mathematische Probleme*,

problems dealt with foundations, and problem No. 1 was labelled *Cantors Problem von der Mächtigkeit des Continuums.* As its title indicates, the first problem calls for a proof of what is now termed Cantor's Weak Continuum Hypothesis: there are only two possible cardinalities for the infinite subsets of the continuum, that of the set of natural numbers and that of the entire set of reals. In addition, the first problem also calls for a proof of the well-orderability of the continuum, again as conjectured by Cantor.[3]

Thus as the twentieth century began, Cantor, whose work in set theory was finally receiving its due, had begun a de facto retirement, and David Hilbert, one of the two mathematical greats of the day—the other being Henri Poincaré (1854–1912)—had made the positive resolution of two of Cantor's most important conjectures his own number one problem.[4] It was time for the appearance of the second generation of Cantorians.

The first decade of the new century witnessed a burst of activity in set theory with many of the participants belonging to a group twenty to thirty years younger than Cantor; among them were Felix Bernstein (1878–1956), Gerhard Hessenberg (1874–1925), Phillip Jourdain (1879–1919), Ernst Zermelo (1871–1953), and Felix Hausdorff (1868–1942). Although he was not of this younger generation, the veteran mathematician Arthur Schoenflies (1853–1928) also worked in set theory during this period. He was an ardent disseminator of Cantorian ideas, writing an encyclopedia article and book-length reports on set theory and its application to the study of point sets, i.e., subsets of $\mathbb{R}$ and $\mathbb{R}^n$ ([Sch 1898; 1900; 1908]).

However, in the years 1900–1910, it was Zermelo and Hausdorff who came to dominate the discourse on set theory and who represented the two trends that were to characterize set theory's future: its abstraction and its pursuit as a mathematical theory. Zermelo, in seeking to clarify the existence assumptions in his proof of well-ordering, axiomatized the concept of set based solely on the membership relation and a few simple operations; he became the father of abstract set theory.[5] Hausdorff carried on in Cantor's footsteps, developing set theory as a branch of mathematics worthy of study in its own right and capable of undergirding both general topology and

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Göttinger Nachrichten (1900), 253–297. His entire text, translated into English by Dr. Mary Winston Newton, appears in the *Bulletin of the American Mathematical Society*, **8** (1901–1902), 437–479.

[3]Hilbert felt that the well-orderability of the continuum had a close connection with the continuum problem and that perhaps it was the key to its solution. He asked that the solver provide a specific (i.e., definable) well-ordering: "Es erscheint mir höchst wünschenswert, *einen direkten Beweis dieser merkwürdigen Behauptung von Cantor zu gewinnen* [his italics], etwa durch wirkliche Angabe einer solchen Ordnung der Zahlen ..." ⟦"It appears highly desirable to me *to produce a direct proof of Cantor's remarkable assertion*, perhaps by actually indicating one such ordering of the [real] numbers ..."⟧

[4]Actually, their position on Hilbert's list is an artifact of his organizational scheme, but their placement did have a psychological effect.

[5]See [Ka 1996, 9–12] and [Ka 2004]. The latter article is a penetrating examination of Zermelo's place in the history of set theory.

measure theory; he was a founder of the first and a significant contributor to the second.

At least superficially the education and early professional careers of both men seem quite similar: dissertations and habilitations in applied mathematics, an interest in Cantor's set theory as manifested in 1901 by the offering of the first courses devoted entirely to set theory and by the appearance of their own first publications in set theory. We pause to present brief biographical sketches for both (up through the mid-teens of the early nineteen hundreds), asking the reader's indulgence as we maintain the conceit of parallel development.

Ernst Zermelo was born in Berlin in 1871. After studying at its university and then at Halle and Freiburg, he returned to Berlin where he received his doctorate in 1894; his dissertation on the calculus of variations was directed by H. A. Schwarz (1843–1921), Weierstrass's successor. He then became an assistant to Max Planck at Berlin's Institute for Theoretical Physics, staying there from 1894 to 1897. He moved to Göttingen in 1897, where he completed his *Habilitation* in 1899 with a work on hydrodynamics. As a *Privatdozent* at Göttingen, Zermelo was influenced by Hilbert to turn his attention to questions in set theory and foundations.[6]

At Göttingen in the winter semester of 1900–1901, Zermelo offered a course devoted entirely to set theory (in the spirit of Cantor's *Beiträge*). This was the first such course offered anywhere. In 1901, he published his first set theory paper, *Über die Addition transfiniter Cardinalzahlen*, in the Göttinger Nachrichten. It was a modest contribution to the arithmetic of cardinal numbers. However, his second publication in set theory ([Ze 1904]) ensured his immortality: he proved that every set could be well-ordered on the basis of a newly articulated principle, the Axiom of Choice.[7] Initial reactions were less than laudatory; criticism came swiftly and was not *sotto voce*. Zermelo enjoyed the private support of Hilbert, but in public he was largely left to defend himself. And defend himself he did. In [Ze 1908a], he offered a new proof of the Well-Ordering Theorem together with a brilliantly written reply to his critics. In a second, related, paper [Ze 1908b], Zermelo presented his axiomatization for the concept of set, which, with important later contributions by Fraenkel and Skolem, is the widely recognized ZF (ZFC) set theory of today. This first phase of Zermelo's foundational

[6] In [Pe 1990], Volker Peckhaus gives a detailed picture of Zermelo's years at Göttingen. Though plagued by financial and health problems, Zermelo enjoyed increasing status with Hilbert and the members of his school. From 1905, he was *Titularprofessor*, a position financed by "soft money." Hilbert's manuevers to regularize his position failed.

[7] At the Third International Congress of Mathematicians, held in Heidelberg in August 1904, Julius König presented a proof—later withdrawn—that the continuum could not be well-ordered. (See the *Introduction to "The Concept of Power in Set Theory,"* p. 24.) A little over a month after this unsettling (for set theorists) meeting, Zermelo discovered his proof that the continuum and any other set could be well-ordered. Zermelo's proof is contained in a brief letter to Hilbert dated September 18, 1904.

research ended in 1910 when he left Göttingen for a Professorship at Zurich. In 1916, Zermelo retired from this position for health reasons.[8]

Felix Hausdorff was born in Breslau (now Wrocław) in 1868 and moved with his parents to Leipzig at an early age. After studying at Leipzig, Freiburg, and Berlin, he returned to Leipzig where he received his doctorate in 1891; his dissertation on the refraction of light in the atmosphere was supervised by Heinrich Bruns (1848–1919), a mathematical astronomer and, from 1882, director of Leipzig's observatory. In 1893–1895, Hausdorff worked at the observatory doing scientific calculations. As a *Privatdozent* for astronomy and mathematics, he completed his *Habilitation* at Leipzig in 1895 with a work on the absorption of light in the atmosphere. By the late 1890s, Hausdorff began to move away from applied mathematics and towards pure mathematics with publications in probability theory and non-Euclidean geometry.

At Leipzig, in the summer semester of 1901, he taught a *Beiträge*-influenced course on set theory, historically the second such course—after Zermelo. In 1901, he also published his first paper on set theory, *Über eine gewisse Art geordneter Mengen*, in the Reports of Leipzig's Königlich Sächsischen Gesellschaft der Wissenschaften. In 1902, he became *aussordentlicher Professor* at Leipzig; shortly thereafter, in what may have been an unwise career move, he turned down the offer of a similar position at Göttingen.[9] Hausdorff was appointed *Extraordinarius* at Bonn in 1910. He became *ordentlicher Professor* at Greifswald in 1913.[10] During the period 1900–1910, he published thirteen articles on mathematics. Seven of them—some 281 journal pages—were in set theory, mostly about ordered sets. In 1914, he published the book *Grundzüge der Mengenlehre*.[11] The first six chapters, approximately half of this classic and highly praised textbook, are devoted to Hausdorff's version of Cantorian set theory as developed through

[8]Zermelo's move to Zurich terminated what Peckhaus calls his first *pro*-Hilbert phase [Pe 2002]. After his retirement, he moved back to Germany in 1921 and took up residence in the Black Forest near Freiburg. In 1926, he was appointed to an honorary Professorship at Freiburg, which was an unsalaried position. He returned to foundational research in the period 1927–1935; Peckhaus labels this Zermelo's *contra*-Hilbert phase, because he totally opposed Hilbert's finitistic proof theory. Zermelo lost his position at Freiburg in 1935 for failing to give the Nazi salute properly; he was reinstated in 1946. He died in Freiburg on May 21, 1953.

[9]See [Di 1967, 52] on this episode.

[10]Hausdorff returned to Bonn as *Ordinarius* in 1921. He was forcibly retired by the Nazis in 1935 because he was a Jew. He died in Bonn on January 26, 1942. Hausdorff, his wife, and his wife's sister committed suicide just before they were to be interned at the former cloister *Zur Ewigen Anbetung* in Endenich. Endenich was the first stop before deportation to the East and almost certain death. A photocopy and text transcription of Hausdorff's stoic and poignant farewell letter, dated January 25, 1942, appears in [Br 1996, 263–267]; an English translation of this letter appears in [Ei 1992, 101–102].

[11]Hausdorff's dedication in *Grundzüge* reads: "Dem Schöpfer der Mengenlehre Herrn Georg Cantor in dankbarer verehrung gewidmet" ⟦"Dedicated to the creator of set theory Herr Georg Cantor in grateful admiration"⟧.

a decade of his own research; the last four chapters form an important founding text for general topology.

This ends our brief attempt at comparative biography. An intriguing question remains unanswered: for Zermelo, it was Hilbert's influence that brought him to set theory and foundations, but what led Hausdorff to set theory? Both Cantor and Hausdorff attended the 1897 Zurich congress, and the mathematicians at Halle and Leipzig met frequently. These gatherings, known as *Kränzchen*, alternated between the two locations, and some at Halle were even held at Cantor's home ([Kow 1950, 106]). But social interaction alone does not account for intellectual affinity. For some important clues, we need to consider the works of the writer Paul Mongré.

Mongré's first book, which showed the influence of both Nietzsche and Schopenhauer, was the aphoristic *Sant' Ilario–Gedanken aus der Landschaft Zarathustras*; it appeared in 1897. A second book followed in 1898. Entitled *Das Chaos in kosmischer Auslese–Ein erkenntniskritischer Versuch*, it was critical of metaphysics and disparaged the ability of the empirical world as recognized by our consciousness to reveal anything about the transcendental world.[12] In 1900, Mongré published *Ekstasen*, a book of poetry, and in 1904 his one act play *Der Arzt seiner Ehre*, a comedy, was performed in Berlin and Hamburg. In the period 1897–1910, Paul Mongré, the friend of avant garde artists and writers and the quasi-disciple of Nietzsche, also published essays and reviews in various cultural periodicals. The variety and quantity of Mongré's output alone might pique our interest, but for us his most remarkable attribute is his true identity—he was Felix Hausdorff.[13]

It seems that it was Hausdorff's philosophical-literary alter ego, Paul Mongré, who inspired his interest in set theory. Walter Purkert, in the first section of his very informative introduction to the second volume of Hausdorff's Collected Works [Pu 2002, 2–7], presents strong internal evidence that Hausdorff became acquainted with Cantorian set theory somewhere in the period 1897–1898. In particular, Purkert establishes that between the writing of *Sant' Ilario* and *Das Chaos* Hausdorff-Mongré learned of Cantor's

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[12] We refer to these books as *Sant' Ilario* and *Das Chaos* for short. The English translations of their titles are: *Saint Ilario–Thoughts from Zarathustra's Countryside* and *Chaos in Cosmic Selection–A Critical Epistemological Essay*, respectively.

[13] A complete Hausdorff-Mongré bibliography is available from

http://www.aic.uni-wuppertal.de/fb7/hausdorff/schriver.htm.

All our references to Hausdorff's publications follow the designations assigned by this bibliography.

A reprint of *Das Chaos* appeared in 1976 with an altered title: *Zwischen Chaos und Kosmos oder Vom Ende der Metaphysik*, with an introduction by Max Bense (Baden-Baden: Agis Verlag). Both *Sant' Ilario* and *Das Chaos* are reprinted in *Felix Hausdorff, Gesammelte Werke, Band VII, Philosophisches Werk*, W. Stegmaier (ed.) (Berlin-Heidelberg-New York: Springer-Verlag), 2004. This volume also includes line-by-line commentaries for *Sant' Ilario* and *Das Chaos*. Werner Stegmaier's introduction describes Hausdorff's philosophical antecedents and the main ideas in his philosophical works. In addition, three short articles by Hausdorff on Nietzche's theory of *return* ⟦*Wiederkunft des Gleichen*⟧ and his *Will to Power* ⟦*Der Wille zur Macht*⟧ are reprinted.

1878 result that all the $\mathbb{R}^n$ have the same cardinality. He also cites several places in *Das Chaos* where connections between philosophical argumentation and set theoretic concepts and theorems are made.[14]

Philosophical interests, as expressed in his writings as Paul Mongré, may have brought Hausdorff to Cantor, but several commentators have seen the solution of Cantor's continuum problem as the driving force behind Hausdorff's work on ordered sets in the years 1901–1909.[15] It is not hard to imagine that any mathematician who was worth his or her salt and who had more than a passing acquaintance with Cantor's work would be tantalized by the challenge set by Hilbert and would take up the continuum problem. With respect to this challenge, starting in 1904, Hausdorff, the skilled mathematician, extended Cantor's concept of cardinal exponentiation to create the tool of *ordered powers* of ordered sets; he discovered the fundamental relation of *cofinality*, which led him to isolate an important class of invariants, the regular cardinals and their inverses. He then used ordered products and his invariants to develop a representation theory, *ab initio*, for ordered sets in general and the important subfamily of densely ordered sets in particular. His representation theory often bumped into the continuum problem, but he seemed to use such occurrences to direct his attention to more realistic goals or to measure the difficulty of a specific problem.

For the interested scholar, there is no better place to begin to seek answers to meta-questions, such as the role played by the continuum problem in Hausdorff's research—motivational goal or cautionary symbol of intractability—than in the articles themselves. These same articles are also a treasure trove for those wanting to find the sources of concepts and methods that have hitherto been seen as originating in *Grundzüge der Mengenlehre.* These range from the perhaps mundane identification of sets with functions via characteristic functions to the famous back-and-forth construction of isomorphisms, from the extremely fruitful concept of η-set to what we now call Hausdorff's maximal principle. Of course, we also find things that did not make it into *Grundzüge*: worthwhile ideas, such as the concept of *type ring* and important results, such as the calculation of a sharp lower bound for the cardinality of irreducible, continuous $c_{\sigma\sigma}$-sets and the proof that Hausdorff gaps exist in certain partially ordered sets. Then there are the *firsts*: for example, the first elimination of CH from a proof ([H 1907a]), the first statement of GCH ([H 1908]), and the first use of a maximal principle in algebra ([H 1909a]). Finally, through these articles we are afforded glimpses of Hausdorff's opinions on foundations as he tries to do battle against the ineffability of the uncountable.

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[14] *Das Chaos* is discussed in connection with Hausdorff's mathematical work in [Ei 1992, 85–91; 1994, 3–10]; Erhard Scholz echoes this title in his article *Logischen Ordnungen im Chaos* ([Scho 1996]) and uses it thematically in commenting on Hausdorff's work up to 1919.

[15] In particular, see [Mo 1989, 109; 1996, 127, 130], [Br 1996, 5], [Koe 1996, 86], and [Scho 1996, 109, 114].

Our main aim in translating Hausdorff's papers on ordered sets is to make them available to a larger audience for whatever purpose. At the same time, we hope to expose more people to the early work of a creative and productive mathematician of the first rank. The period covered is truly an amazing one in the history of set theory: "a hundred flowers bloomed," and many wilted in the intense light. However, under the aegis of Hilbert's school, Zermelo created the new branch of abstract set theory, while separately the inner-directed Hausdorff exploded upon the scene and usurped the position of the era's number one Cantorian.[16]

The Translations and Introductory Essays

The articles translated in this volume are: *Über eine gewisse Art geordneter Mengen* ⟦*About a Certain Kind of Ordered Sets*⟧ [H 1901b]; *Der Potenzbegriff in der Mengenlehre* ⟦*The Concept of Power in Set Theory*⟧ [H 1904a]; *Untersuchungen über Ordnungstypen I, II, III* ⟦*Investigations into Order Types I, II, III*⟧ [H 1906b]; *Untersuchungen über Ordnungstypen IV, V* ⟦*Investigations into Order Types IV, V*⟧ [H 1907a]; *Über dichte Ordnungstypen* ⟦*On Dense Order Types*⟧ [H 1907b]; *Grundzüge einer Theorie der geordneten Mengen* ⟦*The Fundamentals of a Theory of Ordered Sets*⟧ [H 1908]; *Die Graduierung nach dem Endverlauf* ⟦*Graduation by Final Behavior*⟧ [H 1909a]; and in the Appendix, *Summen von* $\aleph_1$ *Mengen* ⟦*Sums of* $\aleph_1$ *Sets*⟧ [H 1936b]. Further bibliographical data for these articles appears in our *Selected Hausdorff Bibliography.*

The provenance of these translations requires some explanation. I was led to Hausdorff's *Grundzüge der Mengenlehre* and then to his earlier papers on ordered sets in the early 1990s as I tried to discover the origins of the *back-and-forth* construction. This resulted in [Pl 1993] and in my realization of the wealth of material in [H 1906b; 1907a]. I was determined to produce translations of both. In the summer of 1995, I finished a first version of [H 1906b]. The project lay dormant until 1997, when at the AMS winter meetings in San Diego, I spoke about Hausdorff's work on order types. Afterwards, Marion Scheepers introduced himself, and I learned of his interest in these same papers. Marion had done some partial translations of Hausdorff while pursuing his study of gap theorems (see [Sc 1993]).

Shortly after our meeting in San Diego, we began an extensive e-mail correspondence in which we decided to work together on translations of the remaining papers on ordered sets. As amateur translators we were both feeling our way. We agreed that the translations were to be fairly literal and that anachronistic terminology was to be avoided. Each of us had primary responsibility for certain papers: I took [H 1901b; 1907b; 1908; 1909a §1] and Marion took [H 1904a; 1907a; 1909a §§2-5]. Translations were passed

[16]The creative outpouring from Hausdorff-Mongré in the span 1897–1910 is truly remarkable, as is its intellectual scope. The Hausdorff-Mongré bibliography lists twenty-two items for Hausdorff and twenty-one items for Mongré during this era; for Mongré, this includes three books and for Hausdorff, three memoir-length articles.

back and forth electronically, and the task of smoothing rough spots and maintaining uniformity across various iterations was my responsibility. By September 1, 1998, the first versions of all these papers had been completed. In the spring of 1999, Marion decided to add a translation of [H 1936b] because of its connection with [H 1909a] and its relevance to the gap literature; a first version was finished by June. The joint phase of our translating work ended in the fall of 1999. Since then, I have gone back to the originals and reworked the translations. Of course, having the versions from our joint efforts made the resulting iterations much less burdensome, and it allowed me to concentrate on passages that had earlier caused us difficulty. Multiple passes of proofreading helped to increase the accuracy of the translations. This reworking took place from the winter of 2000 through the fall of 2002.

In January of 2003, I began writing introductory essays for the papers. Each translation, except that of [H 1936b] in the Appendix, is preceded by an essay, prosaically entitled *Introduction to —*. These essays, each equipped with its own end notes, are "programs" that take the reader through the highlights of each article, make connections among articles, and offer commentary on relationships to the work of others. (All translations from German sources appearing in the introductions were done by me.) The introductions can be read before, during, or after the relevant article—or they can be ignored. If *before*, I hope that they inspire a possibly wavering reader to take up the actual work.

I wish to offer further comments on the particulars of the translations and introductions. The numbers appearing in the margins of the pages of the translations refer to the corresponding pages in the originals. All page references to Hausdorff's articles that appear in the introductions are to the relevant page or pages in the original article, which can be easily found in the translation by using the numbers in the margins. Usually the first occurrence of a translated technical term in a given article is accompanied by the original German term in double brackets, for example: initial segment ⟦Abschnitt⟧. This is not done for certain ubiquitous terms. The "set" words always receive the same translation: *Menge* = *set*, *Inbegriff* = *aggregate*, and *Gesamtheit* = *totality*; the term *Mächtigkeit*, which Cantor appropriated from the geometer Jacob Steiner, is always translated as *cardinality.* Double brackets are also used to enclose clarifying information. In the introductions, I have freely used standard set theory abbreviations: CH = continuum hypothesis; GCH = generalized continuum hypothesis; AC = axiom of choice; ZF (ZFC) = Zermelo-Fraenkel set theory (Zermelo-Fraenkel set theory plus the Axiom of Choice).

Through the miracle of LaTeX 2_ε, I have attempted to maintain the look of the originals, even making the font sizes in proper names a little larger or adding space between their letters as was the custom of the times. I have duplicated some of Hausdorff's one of a kind notations and have continued his practice of eliding the letter "i" or the letter "j" (but not both) when itemizing a list by letters. The organization into paragraphs is exactly as in the originals; however, sentences have been punctuated to reflect English usage. Minor and obvious misprints, which were amazingly very few in number, are corrected without comment. I have kept the original indexing style for footnotes (numerals or asterisks) and, where possible, the same values for indices; any deviations from this are noted by placing the original designation in double brackets within the current footnote. Notes from the translator are rare, but when they occur, they are indexed by small roman letters.

Acknowledgements

In preparing my essays, I have benefited from scholarly works emanating from Germany, in particular the books edited by Eugen Eichorn and Ernst-Jochen Thiele ([EiT 1994]) and by Egbert Brieskorn ([Br 1996]).[17] These volumes are still both important and useful. But currently the greatest boon to Hausdorff scholarship is (or will be) the Hausdorff Edition, a project undertaken by a consortium of scholars to publish the collected works of Hausdorff-Mongré. This nine-volume edition is also to include excerpts from his *Nachlaß*, along with editorial essays and commentary. Three volumes have already appeared. Of particular importance to me during the preparation of this book was the publication of the Hausdorff Edition's volume containing *Grundzüge der Mengenlehre.* It was the first volume published (designated officially as volume II), and I profited greatly from reading Walter Purkert's introductory essay; his essay is available on-line ([Pu 2002]). It was also very helpful to have the catalog (Findbuch) of Hausdorff's *Nachlaß* prepared by Walter Purkert; it too is available on-line ([Pu1995]).[18]

Sources in English that mention Hausdorff, other than in passing, are rather sparse. An exception is [Mo 1982], in which the works in the period 1904–1909 that we have translated are covered. I found Gregory Moore's book very helpful, not only for his commentary on specific articles, but also for the rich portrait that he paints of an important era. Moore has also written a short article on Hausdorff highlighting the years 1901–1908 ([Mo 1996]).

In a more personal vein, I want to thank Marion Scheepers whose participation came at a crucial time. His enthusiasm for this project provided the will to pursue what seemed like a daunting task. I also want to thank

[17]The first appraisals of Hausdorff's life and work appeared in Germany twenty-five years after his death with the traditional, but belated, essays of remembrance in the *Jahresbericht* of the **DMV** [Di 1967], [Lo 1967]. No doubt the necessity to acknowledge the circumstances that forced Hausdorff from his professorship and eventually led him to choose suicide had been a difficult psychological barrier to societal recognition of this great German mathematician.

[18]The miraculous survival of the Second World War by Hausdorff's scientific *Nachlaß* is recounted in [Bo 1967] and [Ber 1967]; also see Bergmann's article in [Br 1996, 271–281]. The forward to [Pu 1995] provides a complete history of the *Nachlaß*.

my colleagues at Michigan State University: Susan Schuur, Christel Rotthaus and Martin Fuchs in the Department of Mathematics; and Elizabeth Mittman in the German Department. Professor Schuur read my essays and made helpful comments and suggestions. Professors Rotthaus and Fuchs provided emergency help when I was overcome by a German language conundrum, and Professor Mittman allowed me to attend her classes and be a student of German once more.

As with any undertaking of long duration, there were peaks and valleys, and I gratefully acknowledge the debt I owe to friends who offered their encouragement and provided a sympathetic ear when I needed it. Foremost among these is my wife and colleague Susan Schuur.

It goes without saying that any of this volume's shortcomings are my responsibility and no one else's.

J.M. Plotkin
East Lansing, 2004

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