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THE *ENTASIS* OF GREEK DORIC COLUMNS AND CURVE FITTING A CASE STUDY ON THE PERISTYLE COLUMN OF THE TEMPLE OF ATHENA ALEA AT TEGEA

1. Description of the project

This paper introduces a small part of the research I am currently doing for my doctoral dissertation. The whole project is concerned with how to produce directly from measurement data a computer generated reconstruction of the late Classical temple of Athena Alea at Tegea, Arcadia (the two most important publications on the architecture of the Classical temple designed by Skopas are Dugas *et al.* 1924 and Norman 1984). The new field work for this project has been done as part of the excavations at the sanctuary; they have been conducted by the Norwegian Institute at Athens as an international cooperation under the direction of Professor Erik Østby[1]. Much of my work during the summer 1994 was concentrated on column drums scattered around the site and after that on analyzing the measurement data and drawings in order to determine the height of the peristyle column. The field work was continued during a five week study season in summer 1995[2], and another five week session is scheduled for 1996.

While working with the reconstruction of columns it became obvious that means to cope with *entasis*, the slightly convex tapering of columns, had to be found. In previous research literature this feature at Tegea has not been studied: it has either been noted that the *entasis* is slightly perceivable or non-existent (Dugas *et al.* 1924, 19; Cooper 1978, 104). The study has to be

[1] On the excavation see Østby *et al.* 1994 and Østby 1994. My involvement with Tegea started in 1993, the fourth year of the five year excavation project; my research has evolved directly from the documentation project of the building blocks. I owe my greatest gratitude to Professor Erik Østby, the director of the Norwegian Institute at Athens, for his continuous support and guidance. The Greek collaborator of the excavation project are Dr. Th. G. Spyropoulos, the ephor of antiquities of Arcadia and Laconia, and Dr. A. Delivorrias, the director of the Benaki Museum at Athens. In the documentation of building blocks I have periodically been assisted by the following persons: Anne-Claire Chauveau, Øystein Ekroll, Christina M. Joslin, Marianne Knutsen, Petra Pakkanen, Tom Pfauth, and Heather Russel. Without their help this study would not have been possible. I also wish to thank the three successive directors and the staff of the Finnish Institute at Athens: Professor Henrik Lilius, Gunnar af Hällström, and Kirsti Simonsuuri, Dr. Petra Pakkanen and Mrs. Maria Martzoukou. Professor Seppo Mustonen from the Department of Statistics at the University of Helsinki has commented upon my computer programs and also provided the possibility to use the computer program Survo, the main tool I have used in this study.

[2] The main new results obtained during the restudy of the building blocks are the following: 1) The peristyle columns were standing vertical instead of being inclined toward the interior as the reconstruction by Dugas and Clemmensen shows. 2) Both the crepidoma and the entablature were curving. 3) The height of the column cannot be precisely defined with the presently preserved material, but a range for the height can be defined; for the last point see Chapter 4 of this paper.

based on measurements of individual drums since none of them are *in situ*.

The programs used in the analyses have all been written especially for the purposes of this study: they have been implemented on top of MS-DOS program Survo 84C. Survo was a natural selection for me because – besides providing very good tools for graphics, report generating, statistical analysis, and database management – it also supports programs made by the user. Both sucros (Survo or super macros) and additional modules written in C language have been used (on sucros see MUSTONEN 1992, 399-443, and on programming Survo in C see MUSTONEN 1989). Sucro programs function also with the reduced public domain version of Survo, Survos, but the new C modules require the standard full version.

The results of the programs are stored in Survo data files (MUSTONEN 1992, 75-130) or printed out on paper. Some programs produce a DXF (Drawing Interchange Format) file; it is an image which can be imported to CAD and graphics applications[3]. The programs for entasis analysis consist of a new C module, ready Survo C modules and new sucros integrated to function as a single program.

2. CURVE FITTING METHODS

The main objective of this paper is to introduce how is it possible to find a function which describes the *entasis* curve accurately enough to generate computer reconstruction images of Greek Doric columns. The first tested method is third degree interpolation which is used to draw a third degree curve through the determined data points. The second method is least squares approximation and the fitted curves are second and third degree polynomials ($y = a + bx + cx^2$ and $y = d + ex + fx^2 + gx^3$) (on third degree interpolation with Survo see MUSTONEN 1992, 277-279; on the general least square equations see e.g. RESEARCH AND EDUCATION ASSOCIATION 1983, 256).

F.C. Penrose's measurements of the peristyle column of the Parthenon can be used to demonstrate the properties and differences of the two approaches (PENROSE 1851, pl. 14). Fig. 1 presents the interpolated third degree curve. In the figure the x and y axes are drawn at different scales in order to make the *entasis* more perceptible: the scale for x is ten times greater than for y. This distorts the angle between the maximum *entasis* line and the straight dashed line connecting the bottom and the top of the shaft; the lines are perpendicular even though they do not appear to be so. Fig. 1 clearly demon-

[3] DXF is a file format for exchanging computer-aided design (CAD) three-dimensional data; it is a vector file format designed by designed by Autodesk Inc. for AutoCAD and it is supported by most CAD applications, many graphics applications, and desktop publishing programs. On DXF see e.g. KAY, LEVINE 1992, 157-174. The drums and the capital of the column reconstruction in Fig. 8 are two-dimensional projections of the three-dimensional DXF files produced directly from measurement data. The final image with texts and additional drawing was done with CorelDraw.

strates the handicaps of the third degree interpolation method: it gives no curve between the first and the last two points and it cannot tolerate outliers. The curve is drawn through the points and due to outliers it becomes slightly wavy (compare Figs. 1 and 2). Also, due to an outlier close to the maximum *entasis* point the amount of *entasis* is given slightly larger than it actually is and the line for it is drawn too low.

In Fig. 2 the actual measurements are plotted as dots and the fitted curve is $y = 0.05 + 68.0x - 183.7x^2 + 381.2x^3$. The fit of the third degree polynomial is good and much better than the fit of the second degree polynomial (not illustrated). The curve could be forced to go closer to the first and the last points by introducing double data points at these positions: now the curve intersects the y axis at $y = 0.05$ m (with doubled data points $y = 0.03$ m). The third degree polynomial tolerates outliers and gives the position and amount of the maximum *entasis* accurately. It can also easily be applied in the programs that generate the reconstruction images.

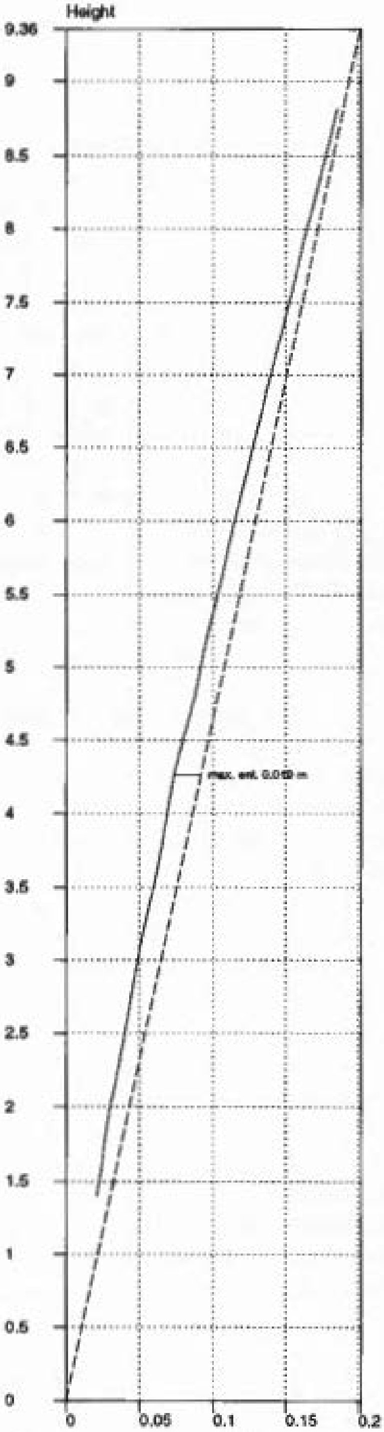
3. Column drums at Tegea

Scattered around the temple and lifted back on the foundations there are fifty column drums preserving the important dimensions: they have the full height and both the lower and upper diameters[4]. The blocks were documented on individual zone sheets: they contain the taken measurements and schematic drawings of the bottom and the top surfaces with the empolion cutting and two dowel holes. Also an error margin particular to each measurement was determined.

The documentation confirmed that each column was made of six drums (column drum measurements taken by Clemmensen are published in Dugas *et al.* 1924, 131-133). The height of the drums in the first two levels (*A* and *B*) is almost constant, but from third to sixth level (from *C* to *F*) there is considerable variation. Taking the error margins of the measurements into consideration, the drum heights are the following: level *A*, 1.46–1.48 m; level *B*, 1.46–1.49 m; level *C*, 1.32–1.67 m; level *D*, 1.41-1.71 m; level *E*, 1.34–1.66 m; level *F*, 1.32–1.64 m. The height of the peristyle column cannot be simply calculated from the average heights of the drums.

I used the diameter measurements and drum surface drawings to search for possibly matching column drums: three pairs of drums were discovered to be matching according to the 1:25 drawings. When the three matching

[4] The peristyle consisted of 36 columns of 6 drums each, so the building had originally 216 drums. If all the 50 drums are from the peristyle (all the top drums can be shown to be from the peristyle order, but the question remains open for the second drums from the top; any one of the shortest drums in this group could be from the porch orders), 23% of the original material is well or fairly well preserved. In addition to these 50 drums there are 29 blocks at the site which have at least one missing critical dimension.



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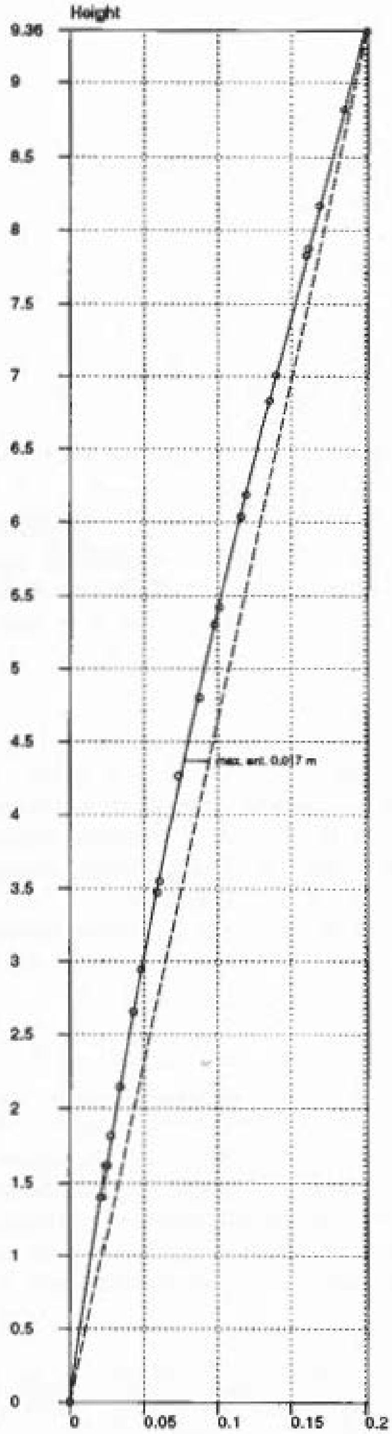

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Fig. 1 – Parthenon: *entasis* of peristyle column. Third degree interpolation.
Fig. 2 – Parthenon: *entasis* of peristyle column. Third degree polynomial and measurements.

drum pairs were rechecked and drawn at a scale of 1:10 in 1995, only one of them passed this test.

This pair consisting of a *D* and an *E* drum was then used to carry the study further. In order to see the possible ways of combining the column drums I had already earlier written a small computer program: it takes as input data the number of the block, upper and lower diameters of the drum at the bottom of two opposite flutes, the height of the drum, and the error margin for each measurement. The arrises of most of the blocks are largely broken, thus matching the drums on the basis of diameter at the arrises and flute width was not possible – only combining the drums according to the diameters at the bottom of the flutes was tested.

Fig. 3 gives the column shaft heights of the possible combinations as a histogram: with the matching pair of drums there are 1096 ways to combine the other column drums with this pair. The minimum height of the column is 8.73 m and the maximum 9.18 m. The mean, 8.91 m, is fairly close to the shaft height established by Dugas and Clemmensen, 8.885 m. But when we examine the histogram, the center classes of height are surprisingly vacant, and there are two clear clusters which do not coincide with the average height: the first one is at 8.78–8.87 m, and the second at 8.98–9.02 m. The histogram by itself cannot give any certain indication what was the column height at Tegea.

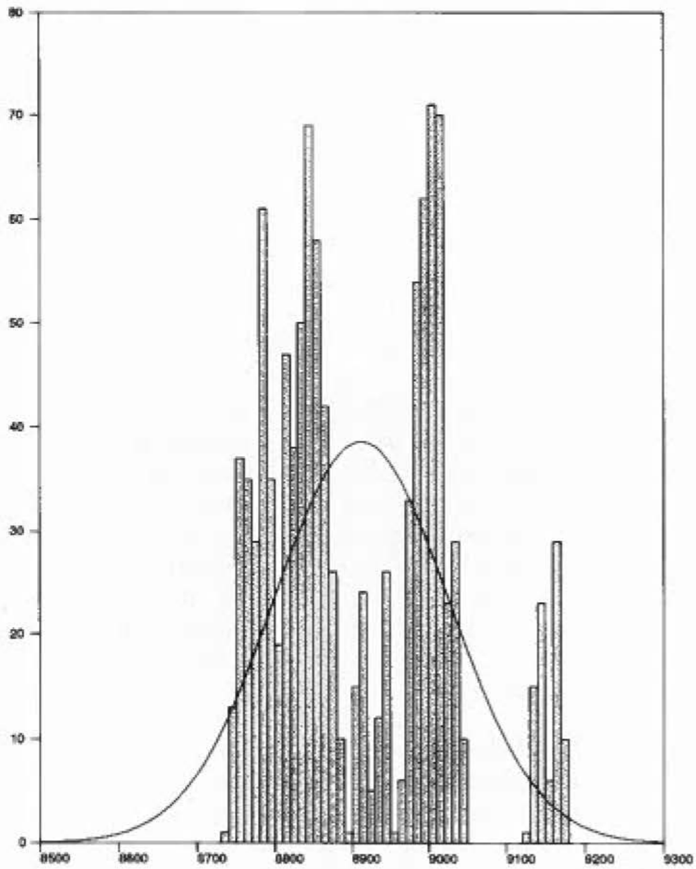
4. *ENTASIS* AND COLUMN HEIGHT

Comparative material on *entasis* in fourth century B.C. Doric buildings is quite rare: I have come across two buildings which are well enough preserved and published so that the data can be used, namely the tholos at Delphi (Fig. 4) and the pronaos column of the temple of Zeus at Nemea (Fig. 5)[5]. The *entasis* at Delphi is very slight and the maximum *entasis* is just above the center of the column shaft height. At Nemea the *entasis* is proportionally twice as great as at Delphi and exactly at the center of the shaft. These examples give some guidelines for the evaluation of the *entasis* at Tegea as well: the curve should be smooth and fit to the data points, and the *entasis* should not be too pronounced.

As previously mentioned, there is at Tegea one matching pair of column drums, a *D* and an *E* drum. The possibly fitting *C* and *F* drums were grouped into classes with approximately the same height. The average measurements of the drum classes were then calculated and used to draw the profile as a third degree polynomial[6]. Since *A* and *B* drums are almost equal

[5] On the tholos at Delphi see AMANDRY, BOUSQUET 1940-41, 121-124; on the temple of Zeus see HILL 1966, 22. The reconstruction of a third possible building, the tholos at Epidauros, is hypothetical for the top three drums of the column and the upper diameter of the shaft is not certain; ROUX 1961, 138-140.

[6] The third degree curve fitted by least squares approximation follows very closely the data points and statistical analysis of the curve fit can only give little new information. Therefore, the following conclusions are made on the basis of visual observation of the different curves.

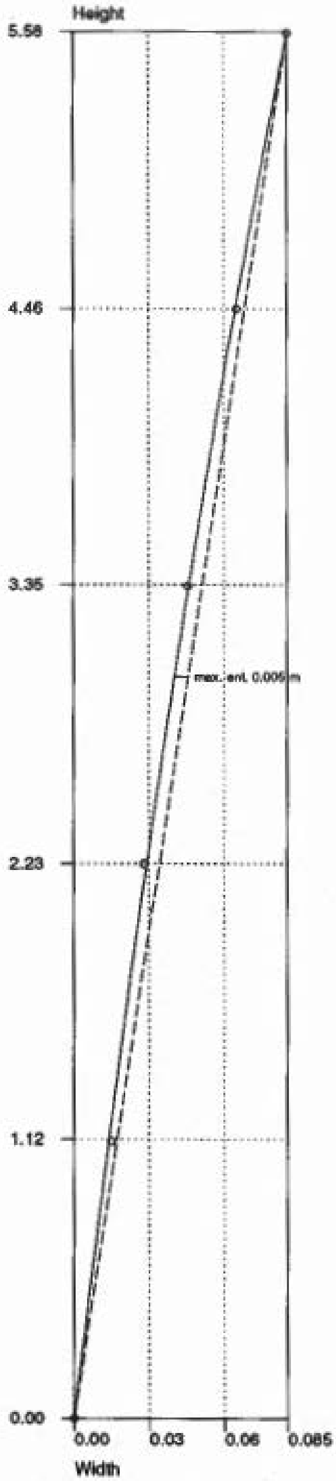


Histogram of possible column height combinations: N = 1096
New measurements, tolerance particular to each block
Matching drums: 35 (D) and 115 (E)

Fig. 3

in height, they can be left without any special attention at this stage of the study of the column height. Three out of the seven height groups were immediately ruled out from further study: in them the top of the curve almost reaches the dashed line at the joint between *E* and *F* drums creating an unthinkable turning point in the curve, and the maximum *entasis* is well below the center of the shaft (Fig. 6 gives an example of such a group). The four other groups were discovered more promising: the *entasis* curve is smooth and the position of the maximum entasis is closer to the center of the shaft, as can be expected also from the comparative material at Delphi and Nemea.

In the next stage the groups were broken down to single C and *F* drums and the curves drawn from this data: this made it possible to study how well the individual drums fit the pair of matching drums. The most coherent group



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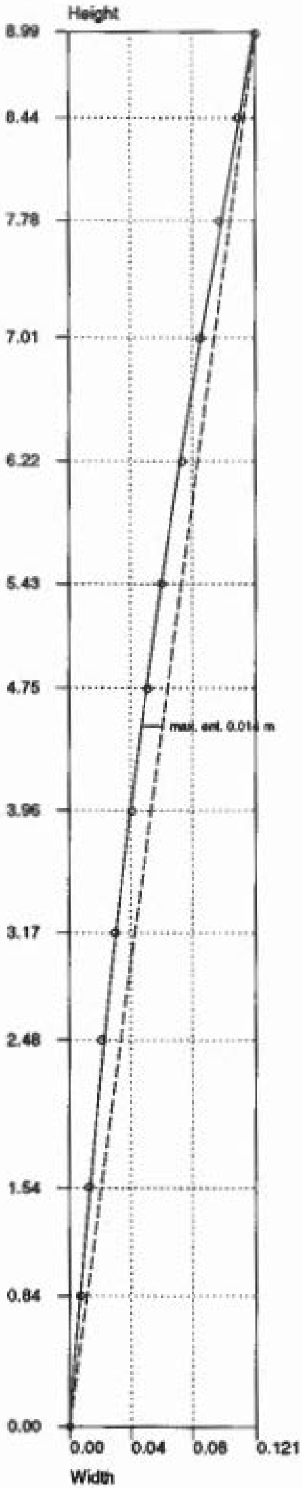

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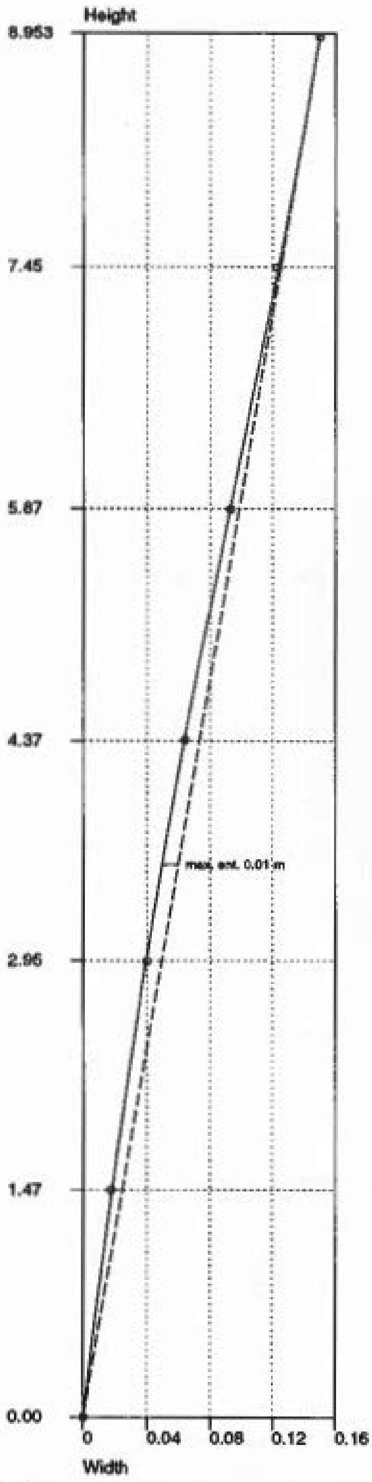
Fig. 4 – Column shaft profile of the tholos at Delphi.
Fig. 5 – Pronaos shaft profile of the temple of Zeus at Nemea.



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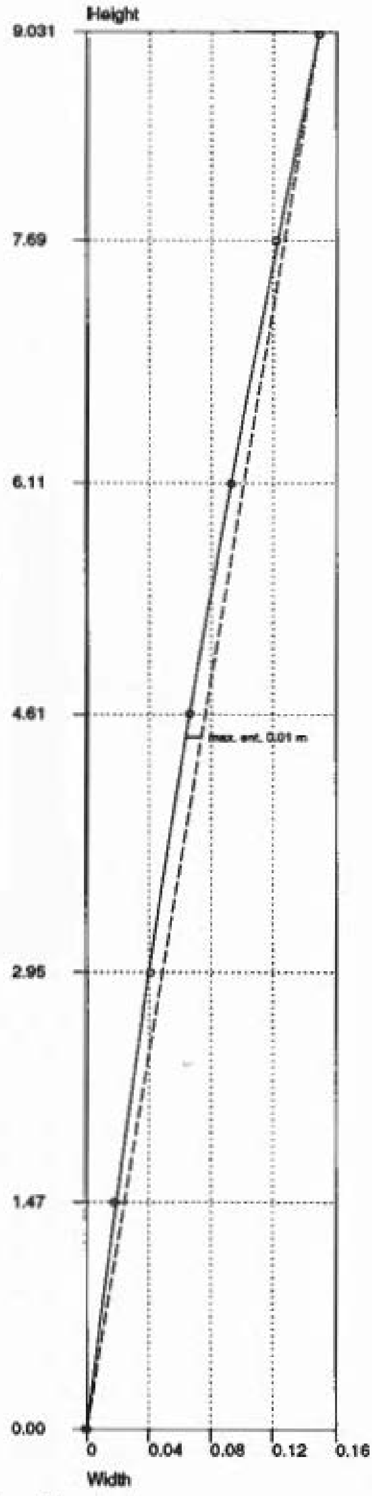

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Fig. 6 – Drum combination creating unlikely "s-shaped" *entasis*.
Fig. 7 – Possible drum combination creating smooth *entasis*.

has a shaft height range of 8.96–9.06 m. In this group four of the six curves have an *entasis* curve which fits almost perfectly to the data points; they also have a consistent length and position of the maximum *entasis*. Fig. 7 presents one of the curves of the group. This height for the column shaft is also supported by the fact that it is possible to find other *D* and *E* drums which

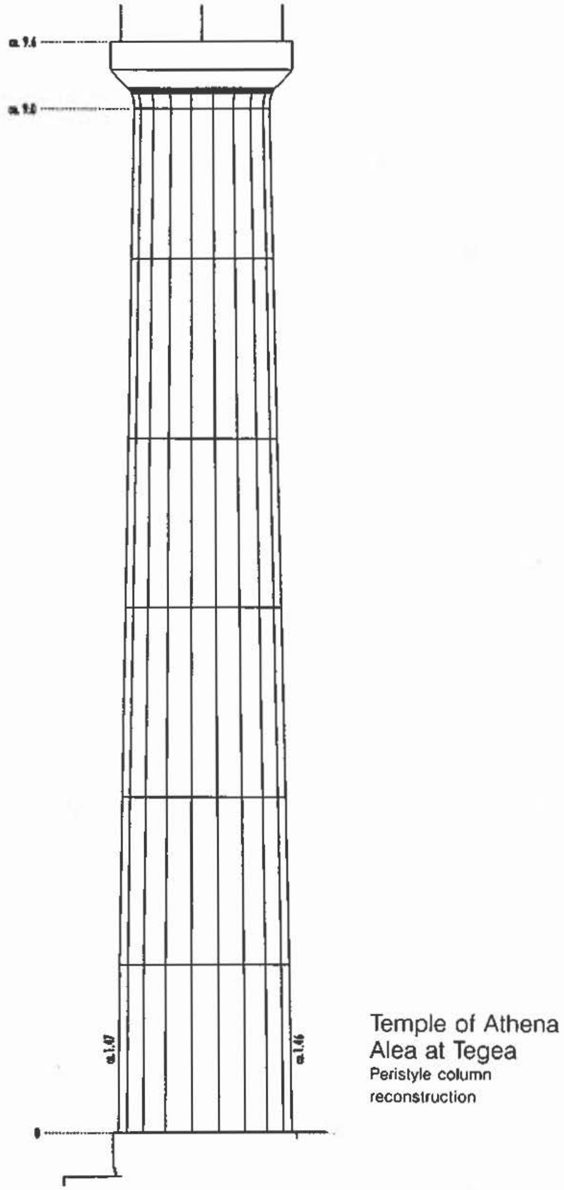

Fig. 8

produce a column shaft ca. 9.0 m high and a smooth *entasis*. The total height range of the column with the capital is 9.55–9.67 m. This is ca. 0.1–0.2 m higher than the previous reconstruction of 9.474 m, but perhaps even more significant than defining the new height is the observation that millimeter exact reconstruction of the peristyle column at Tegea cannot be reached with the presently preserved material. In Fig. 8 is presented one possible drum combination creating a ca. 9.6 m high column.

JARI PAKKANEN
Finnish Institute at Athens

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ABSTRACT

In this paper it is suggested that least squares approximation can be used to analyse and to produce computer images of the *entasis* of Greek Doric columns. The curve fitting method is then applied in a case study on the peristyle Column of the temple of Athena Alea at Tegea. By analysing the shaft profile of the different drum combinations it is possible to give the most likely range for the column height.