Class 9 :
Kerr Black Holes

ASTR350 Black Holes (Spring 2022)

Cole Miller

THIS CLASS

- Kerr (spinning) black holes!
  - No-hair theorem
  - Twisting of spacetime (“frame-dragging”)
  - Ergosphere and “black hole machines”
  - Orbits around black holes

Muddiest points

Any astro questions?

RECAP

- ## Schwarzschild Solution
  - Describes **non-spinning**, uncharged black holes
  - Spherically Symmetric spacetime
  - External observers see infalling objects freeze/fade at the event horizon $r=2GM/c^2$
  - Time slows to zero at event horizon
  - From point of view of infalling observer, pass through the event horizon and 'hit' the spacetime singularity at the center
  - Tidal forces will stretch (Spaghettify) observer before they reach the center (effect depends on mass of BH)
  - In general, mass-energy bends spacetime (not just for Schwarzschild) and near a massive object, time as measured externally slows down
  - But, equivalence principle: to freely-falling local observer, everything seems normal!

RECAP

- Uncovered two aspects of the event horizon
  - Surface at which gravitational redshift is infinite
  - Region within horizon **cannot** causally affect outside-e.g. no information can be transmitted from inside horizon to outside world
- Orbits are not stable inside $3R_{sch}$
- Strong distortion of space-time close to a black hole

I : Roy Kerr

- Roy Kerr (1934-)
  - Discovered exact solutions of Einstein's equations describing a *spinning black hole* **in 1963** (47 years after Schwarzschild)
  - Was later shown that this solution is unique… any spinning (uncharged) black hole is described by the Kerr solution
  - Started a revolution in the theoretical understanding of *real* black holes
  - I met him! We talked about rugby as well as GR...

Spin?

- Why is spin important in GR?
  - in Newton's theory gravitational potential does not depend on spin

- But spin has energy and

energy has mass !

- But its more complex than that- the BH drags space-time
  - best *simple* explanation is an analog to gravitational analog of electromagnetic induction (production of electric field by a moving magnet)).

Real Black Holes

- Due to their origin, expect 'stellar mass' black holes to be 'spinning'
  - collapse of massive star (more later)
- Not clear if supermassive black holes should have a 'lot' of spin
  - their origin is not well understood

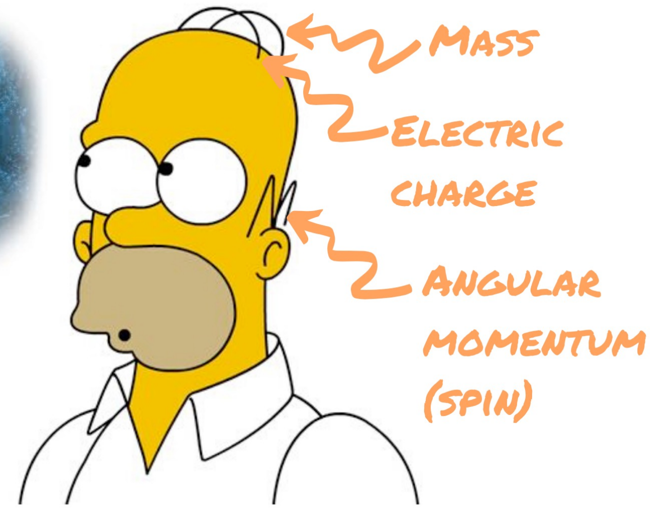
II : No-hair theorem

- Any (isolated) black hole is described by just **three** quantities…
  - Mass
  - Spin
  - Electrical Charge
- Anything not measurable by a *long range* field is effaced

- Once these quantities are specified, the properties of the black hole exterior to the horizon (e.g. spacetime curvature) are uniquely determined.
  - There can be no lumps or bumps on a BH!

Why Mass, Charge and Spin?

- The **no-hair theorem** states that all black holes can be completely characterized by only three *externally observable* classical parameters: mass, electric charge, and angular momentum.
- All other information ("hair" is a metaphor) about the matter which formed a black hole or is falling into it, "disappears" behind the black-hole event horizon and is therefore permanently inaccessible to external observers.



some hair theorem

Mass,Charge and Spin

- Suppose two black holes have the same masses, electrical charges, and angular momenta, but the first black hole was made by collapsing stars, whereas the second is made out of potatoes
  - *They will be completely indistinguishable to an observer outside the event horizon.*

- None of the special particle physics conserved quantities (baryonic number, leptonic number), all of which could be different for the material that collapsed and created the black hole) are conserved in the black hole and are they unobservable from the outside.

Mass,Charge and Spin

- These are the only quantities which can be determined from a distance by examining its gravitational and electromagnetic fields.
  - (lumps and bumps go away- perfectly spherical)

- Astrophysical black holes are expected to have non-zero angular momentum, due to their formation via collapse of rotating stellar objects and growth via accretion, but effectively zero charge, since any net charge will quickly attract the opposite charge and neutralize.

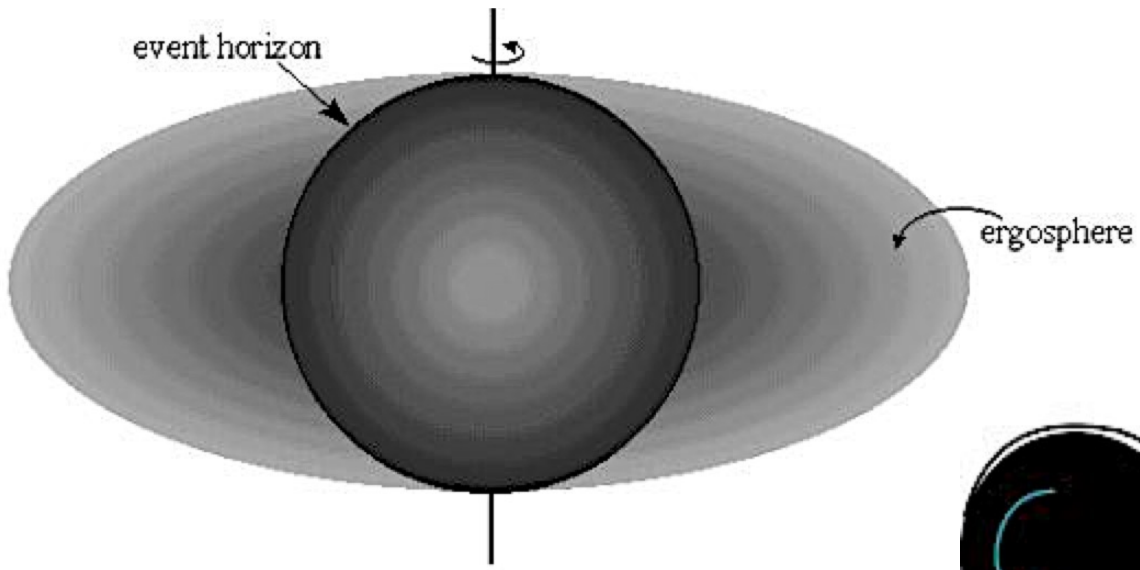
Spin Can Have a Big Effect

- Remember that energy has mass, and spin has energy
- Spinning black holes have a very different "metric"* than non-spinning ones (much more complex)

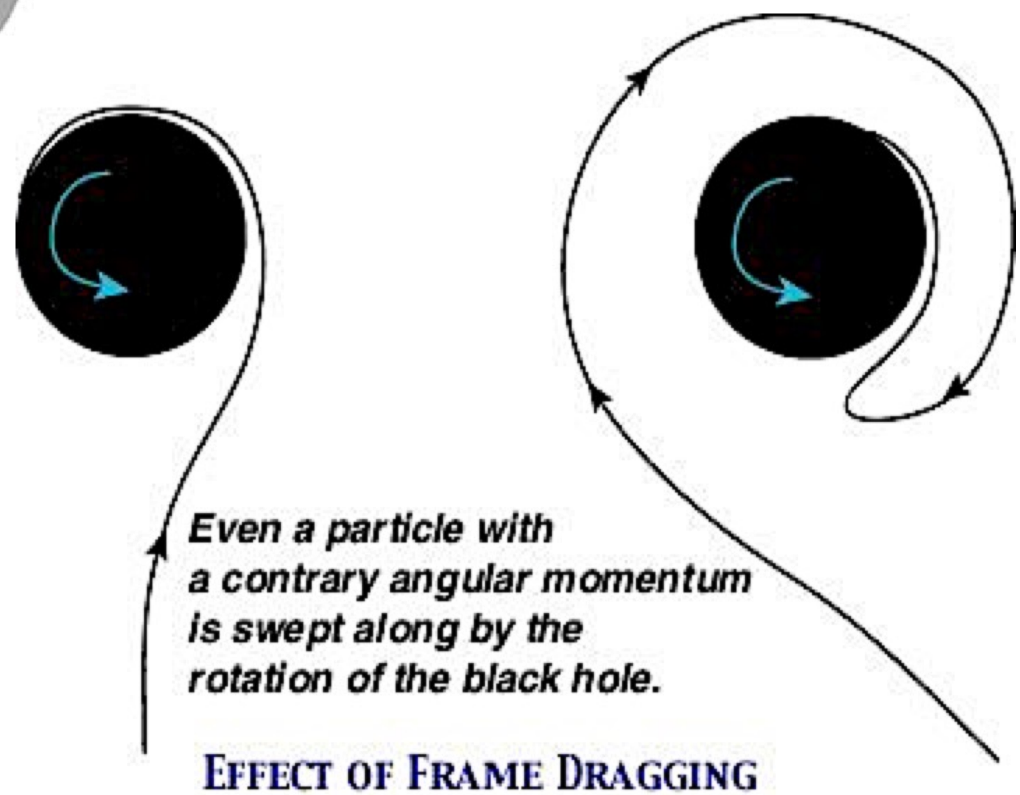
*The metric can be thought of as a generalization of the gravitational potential of Newtonian gravitation.

It captures all the geometric and causal structure of spacetime, and defines time, distance, volume, curvature, angle, and separation of the future and the past. (wikipedia)

III : Frame dragging and the ergosphere



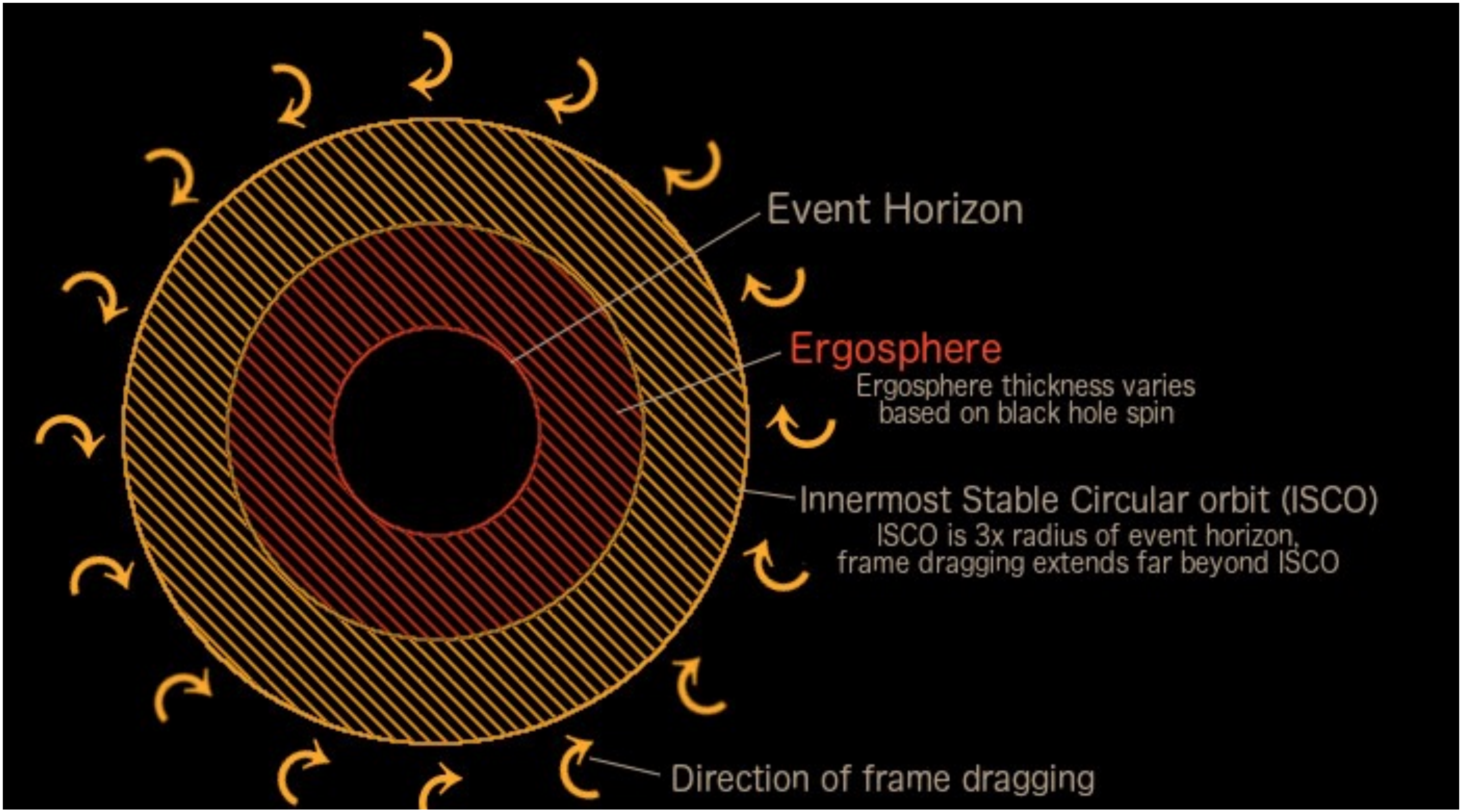
an object that falls into a black hole without any angular momentum
will still spiral into the Kerr black hole due to frame dragging



For a spinning black hole an infalling particle MUST end up rotating in the same direction as the BH.

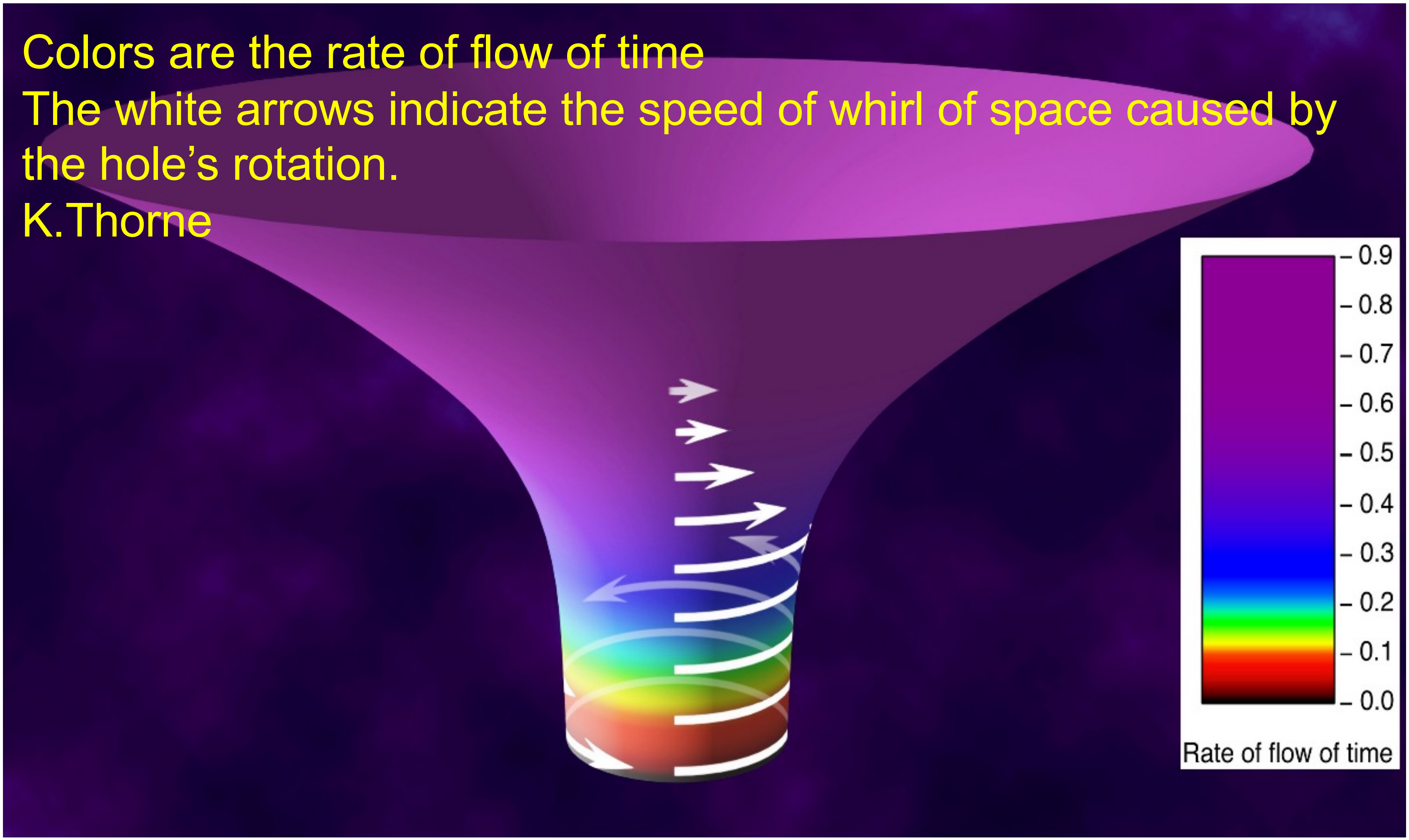
Graphics: University of Winnipeg, Physics Dept.

ErgoSphere



Event Horizon
Ergosphere
Ergosphere thickness varies based on black hole spin
Innermost Stable Circular orbit (ISCO)
ISCO is 3x radius of event horizon, frame dragging extends far beyond ISCO
Direction of frame dragging

Spacetime Around a Kerr BH

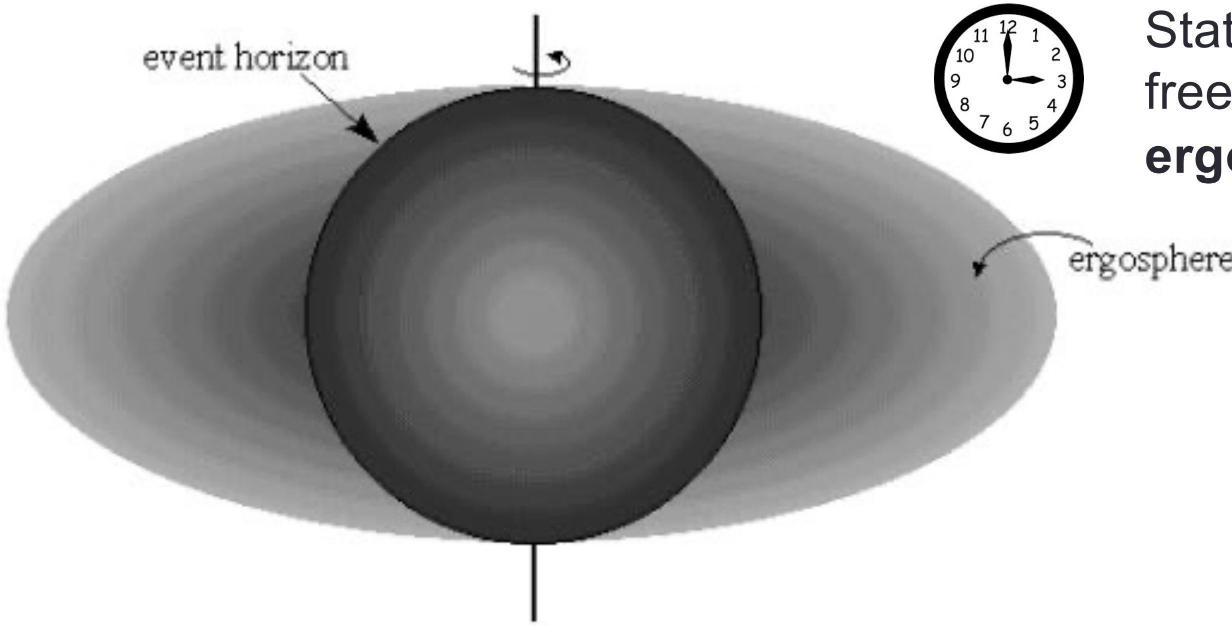


Differences Between Schwarzschild and Kerr Metric

- For a Schwarzschild BH the innermost stable radius is $3r_G=6GM/c^2$ - there are no stable circular orbits at smaller radii
  - The binding energy from this orbit is 0.0572 of the rest mass energy – the maximum that can be released
- For a Kerr the innermost stable radius depends on the spin and direction; it is $r_+=GM/c^2$ for motion in the rotation direction of a maximally rotating black hole, $r_+=9GM/c^2$ for motion opposite the rotation direction of a maximally rotating black hole, and in between for different spins and different motion
- The smaller the innermost stable orbit, the more energy can be released by infalling matter
  - For maximal Kerr BH about 40% of the energy can be released.

Ergosphere is not spherically symmetric
The faster a black hole spins the more oblate the ergosphere becomes

Stationary clock appears to freeze/stop at the edge of the **ergosphere**.



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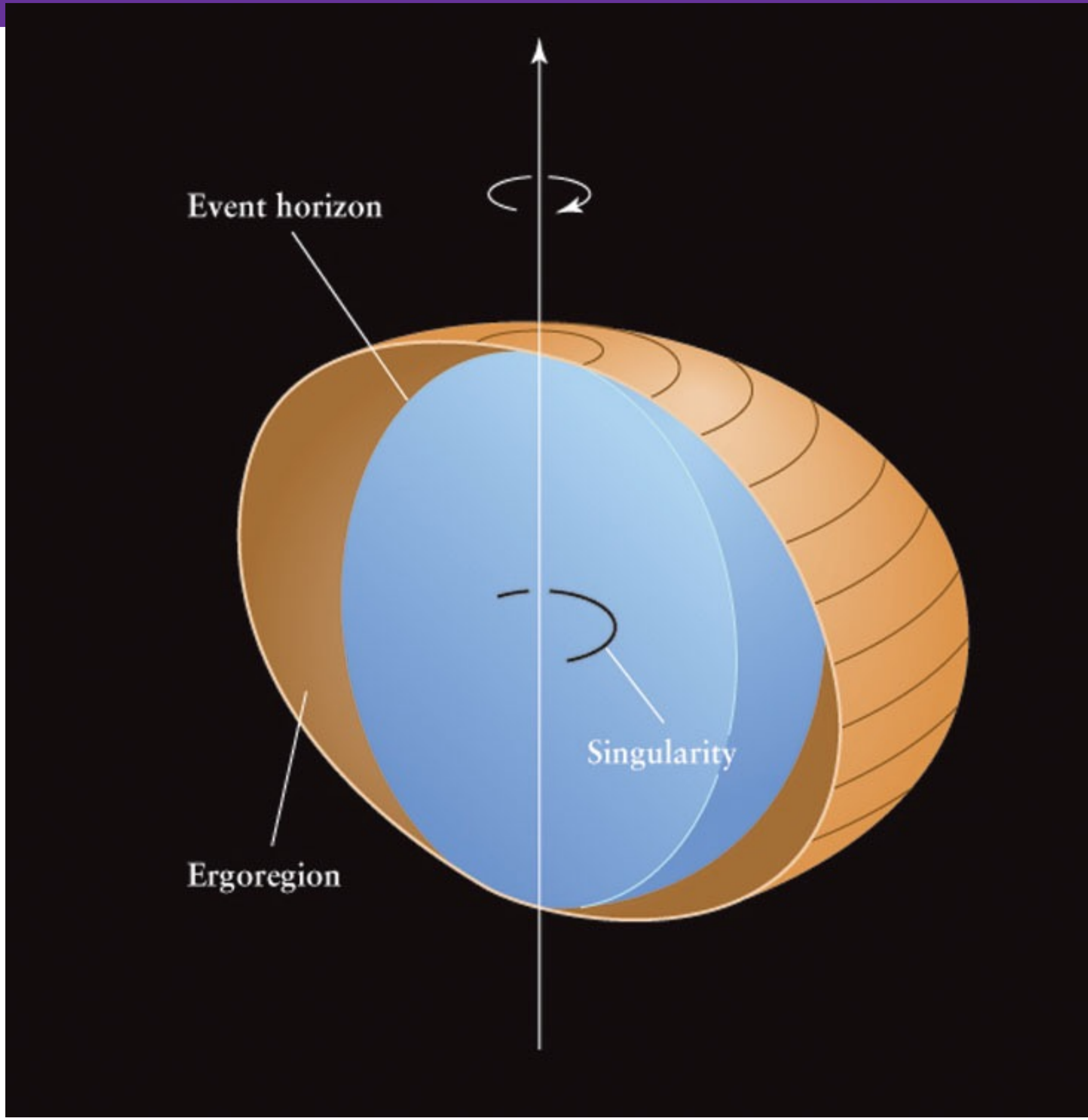
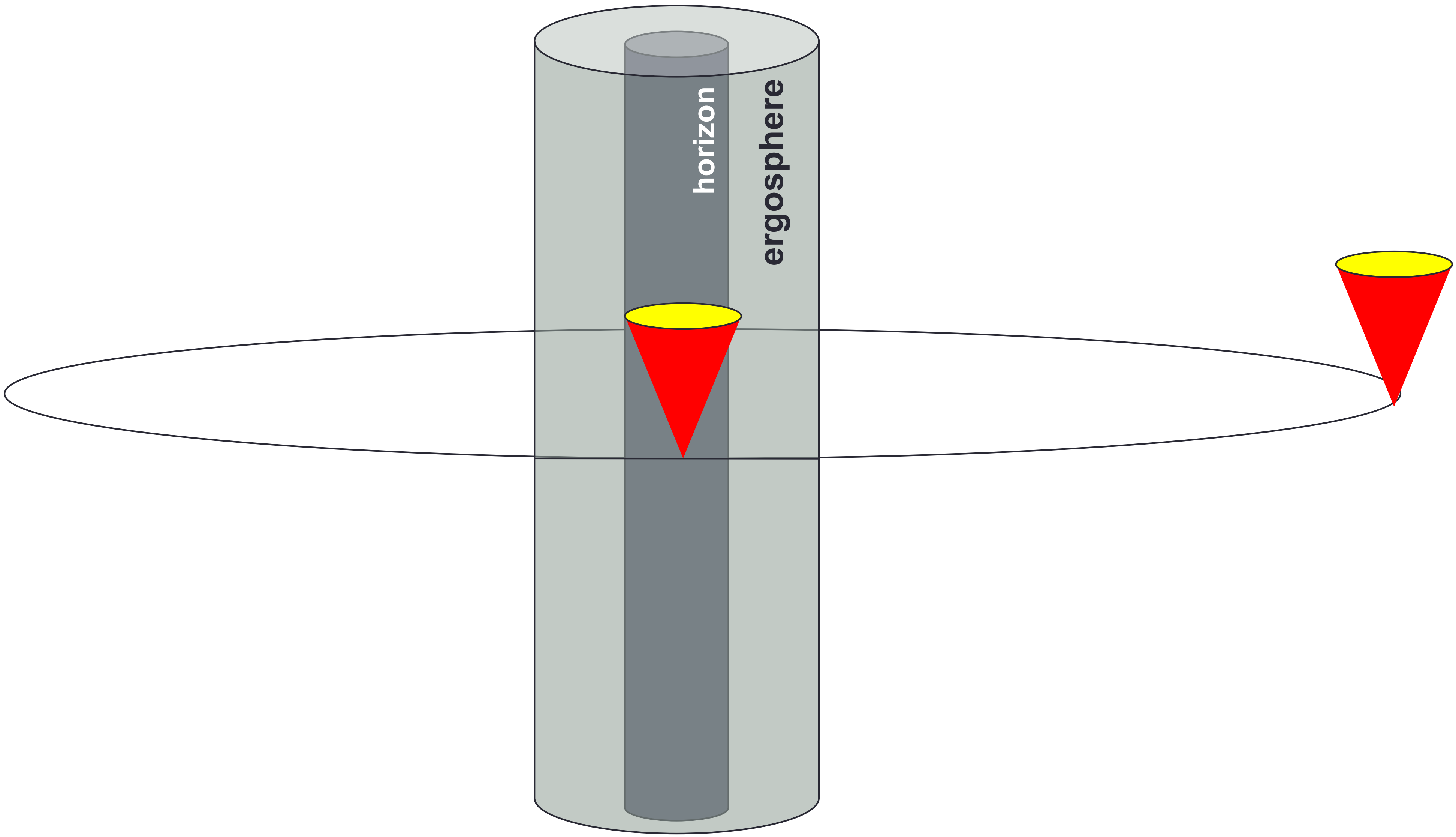
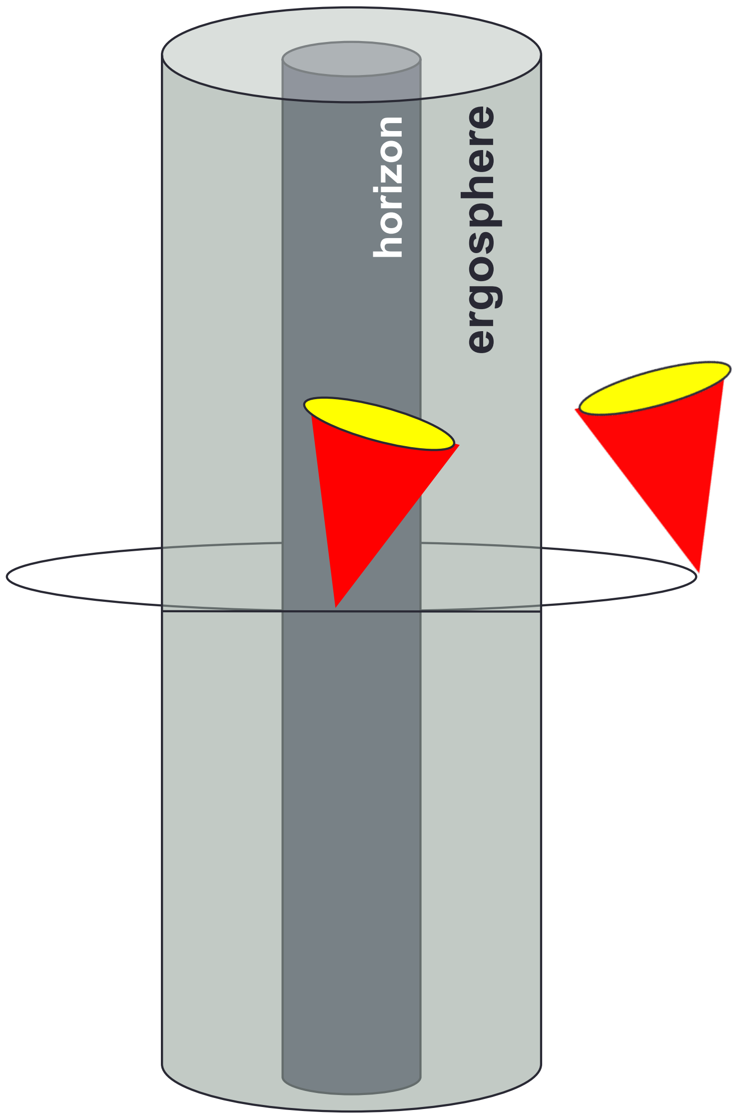

Event horizon
Singularity
Ergoregion

Space Time Diagram- Orbiting Around a BH

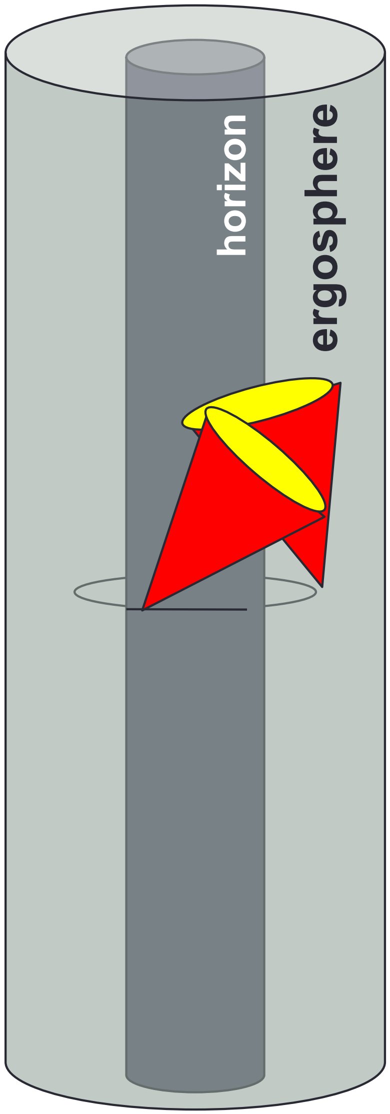


Space Time Diagram- Orbiting Around a BH - Getting Closer to BH



Frame dragging

When reaching close to the Kerr BHs, observers even with zero angular momentum will co-rotate with the BHs because of the swirling of spacetime from the rotating body.



Frame dragging effects tip over light cones in direction of rotation.

Within ergosphere, light cones tipped such that all futures rotate in sense of black hole.

In other words, within ergosphere it is impossible to stand still!

IV : The event horizon

- Ergosphere is outside of the event horizon… we can travel in and out, and can see emission from within!
- Actual event horizon has familiar properties…
  - Surface of infinite redshift, even for clocks/sources that are co-rotating with the spacetime.
  - Seals off the interior space from view
- But rotation has an effect on the location of the event horizon.
  - Define spin parameter "***a***" (proportional to the angular momentum of the BH $a=cJ/GM^2$ (where J is the Newtonian angular momentum)
  - ***a***=0 means non-spinning,
  - ***a***=1 means spinning at maximum rate

- Then the **event horizon** is at:

$$R_{evt} = \left(1 + \sqrt{1 - a^2}\right)\frac{GM}{c^2}$$

- **Smaller event horizons for spinning black holes**

Important later
for how much energy
can be extracted from
accretion

$$a = 0 \Rightarrow R_{evt} = \frac{2GM}{c^2}$$

$$a = 1 \Rightarrow R_{evt} = \frac{GM}{c^2}$$

- What happens when a>1? Called **superspinners**. Kerr solution still gives an answer, but there is no event horizon! We have a **naked singularity**!
- **Cosmic Censorship Hypothesis** asserts that nature does not allow naked singularities and hence forbids a>1 black holes. But note that "a" can be greater than 1 for objects without event horizons; e.g., for Earth, "a" is nearly 1000.

How Fast is the BH Spinning

- Angular speed of BH is
  $\sim[10^5/M_{BH}/M_{sun}]\{a/(1+sqrt(1-a))\}$ radians/sec
- If a=0.9 and M_{BH}= mass of sun, 10,000 revolutions per second (K. Thorne)
- If mass= 10^6 suns 0.01 rev/sec; for same “a”, inversely proportional to the mass of the BH

- How much energy in the spin ($0.29Mc^2$)=$5x10^{53}$ ergs for a 1 solar mass BH, if BH spins maximally

So what is the actual size ?

$R_G \sim 1.5\ (M / M_\odot)$ km

So how close are neutron stars to being black holes ?

Neutron stars are only about a factor 2—3 larger than their event horizons

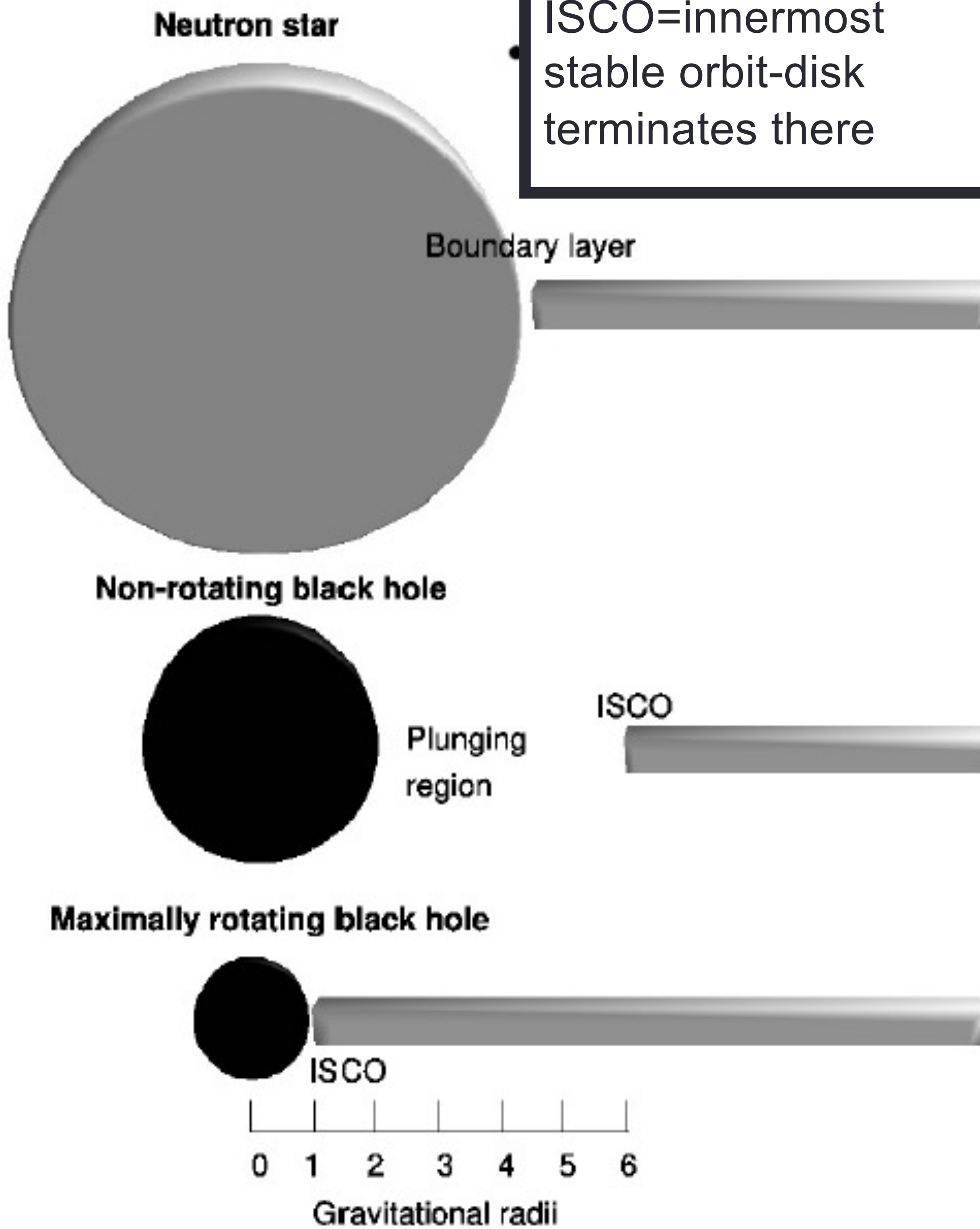
What about spin ?

A non-rotating ("Schwarzschild") black hole has its event horizon at 2 R_G and its ISCO at 6 R_G

A maximally rotating ("Maximal Kerr") black hole has both its event horizon and ISCO at R_G

→ Spinning black holes are more compact → potentially more radiatively efficient

ISCO=innermost stable orbit-disk terminates there



R. Fender 2007

V : Orbits around black holes

- Very far from black hole…
  - Gravity behaves just as Newton says!
  - Velocity of a circular orbit is

$$V = \sqrt{\frac{GM}{r}}$$

  - Orbit is **stable**… if something on a circular orbit is nudged, the orbits just becomes slightly elliptical.
- As you get closer to a black hole, gravitational force becomes more and more non-Newtonian

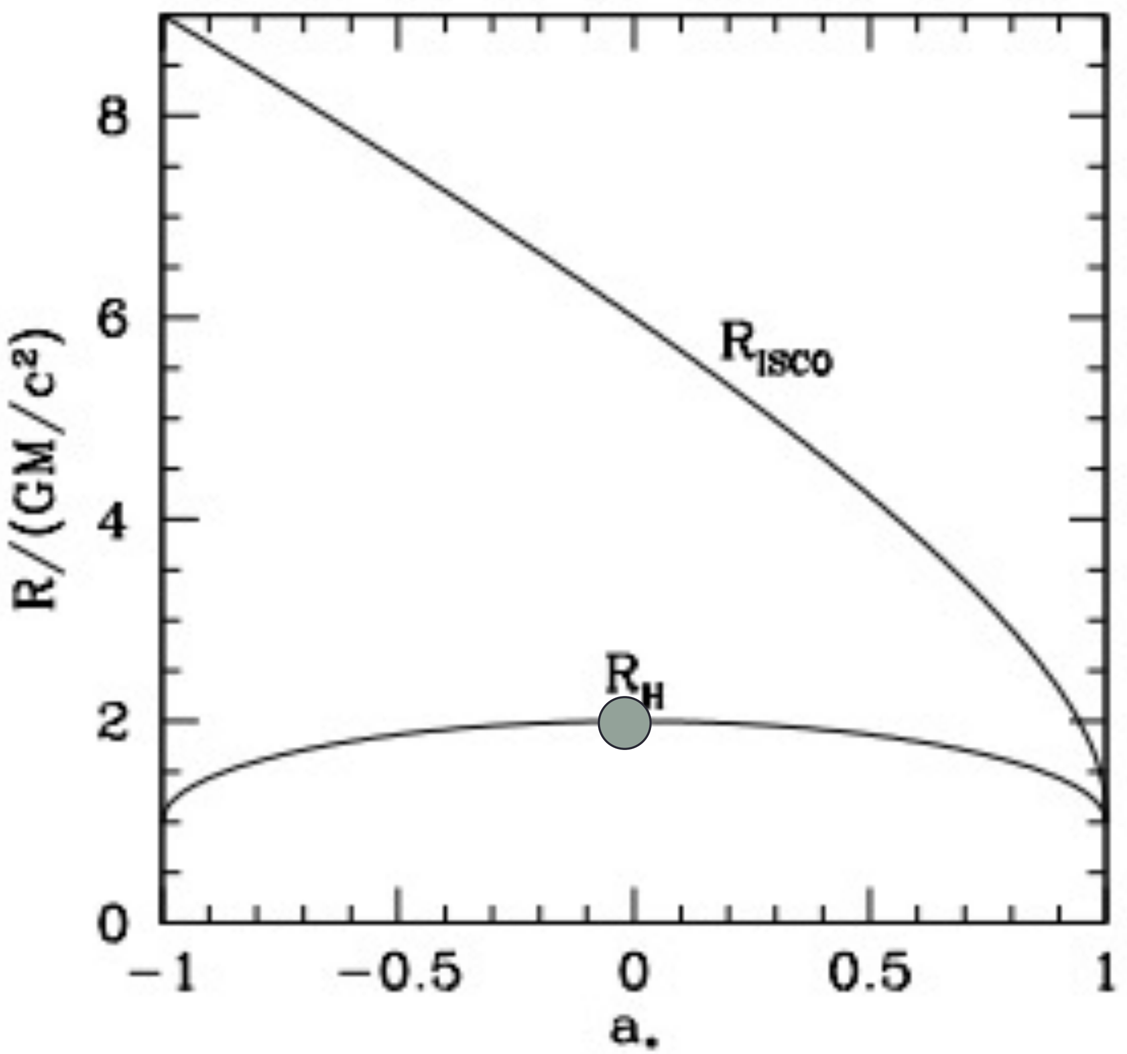
Heading inwards…two special orbits…

- Innermost stable circular orbit (ISCO)
  - $R_{ISCO}=6GM/c^2$ (**Schwarzschild**)
  - $R_{ISCO}=GM/c^2$ (**Kerr a=1**)
  - At and within this location, a particle with mass on a circular orbit is no longer stable… a small nudge and it will spiral into the black hole!
  - Very important for accretion disks (more later!)

- Photon circular orbit
  - $R_{ph}=3GM/c^2$ (Schwarzschild), $R_{ph}=GM/c^2$ (Kerr a=1)
    - -A circular orbit can only exist in the equatorial plane (Kerr)
  - Only at this one radius can photons travel in circles around the black hole But the orbit is unstable
  - **Nothing can be in orbit inside of this radius- must plunge if orbit is initially circular.** (can escape if orbit is initially outward and $R>R_{horizon}$)

How the Horizon and Innermost stable circular orbit (ISCO) Change with Spin)

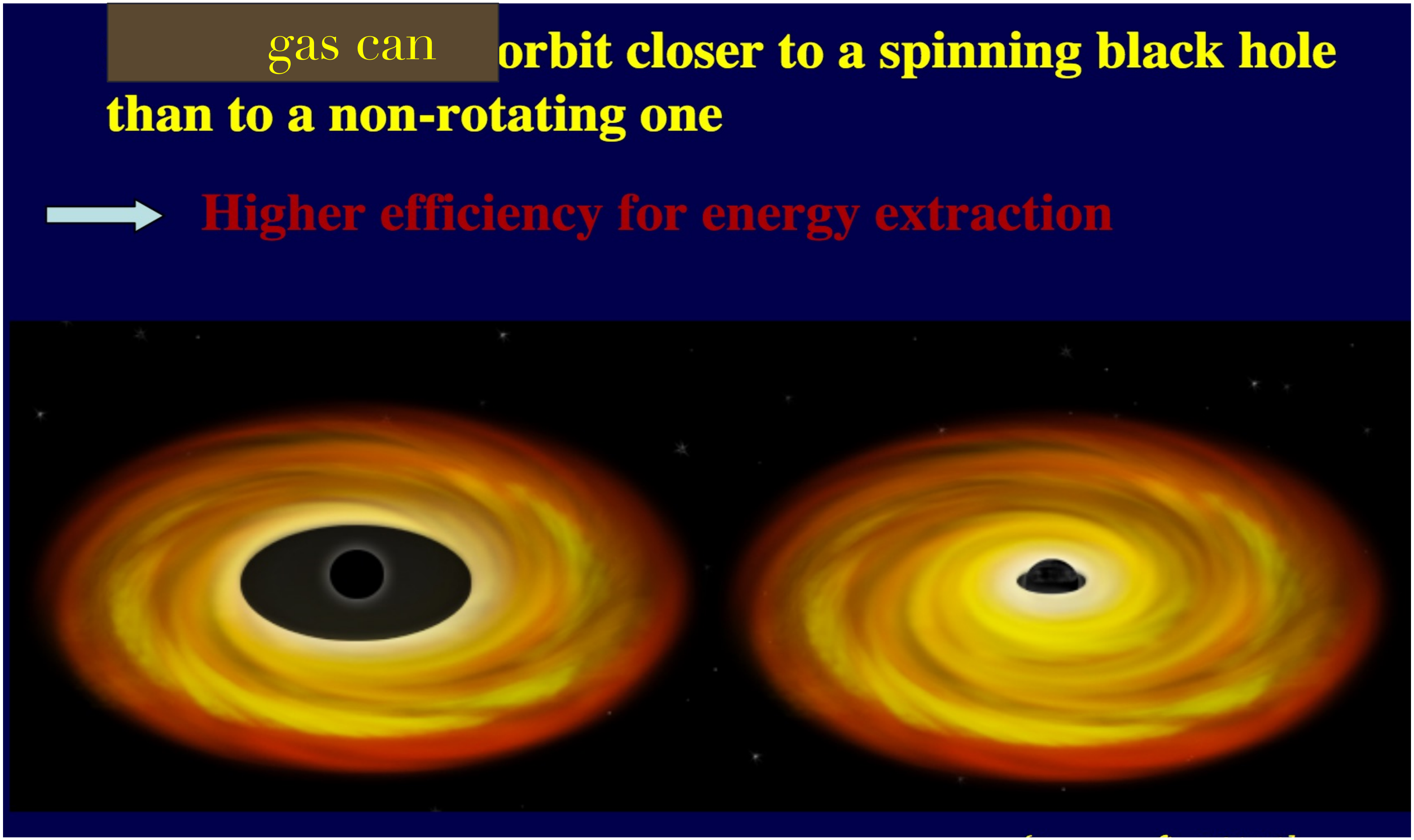
As prograde spin increases R_{ISCO} and $R_{Horizon}$ get smaller



spin

Schwarzschild solution

How the Horizon and ISCO Change with Spin



not spinning

spinning

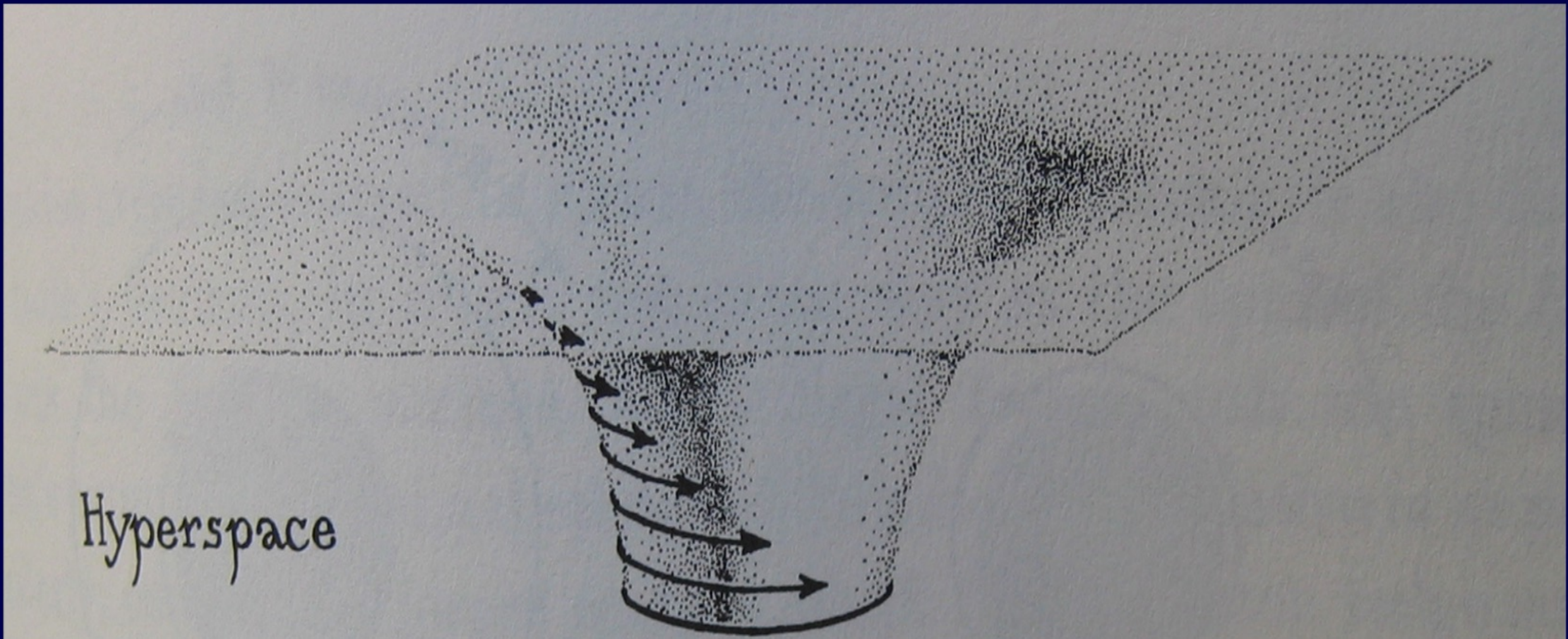
Summary: Kerr Solution

- Two “surfaces”:
  - The Horizon: region from which no signal can escape.
  - The Ergosphere: region inside which space rotates that is is impossible for a body to appear stationary to a distant observer

Summary Rotating black holes-

- Roy Kerr (1963)
  - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has "**static surface**" inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: "**frame dragging**".
  - **Ergosphere** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Space around a spinning Black Hole



[figure from Thorne's "Black holes and time warps"]

Because spacetime is "stuck" to the horizon, **space is dragged along with the spin.** This appears as a tornado-like swirl in hyperspace.

http://www.astro.sunysb.edu/rosalba/astro2030/KerrBH.pdf

Next:Real black holes

So much for theory – what about reality?

Thought to be (at least) two classes of black hole in nature

- **“Stellar mass black holes”** – left over from the collapse/implosion of a massive star ($M>8M_{\odot}$)
- **“Supermassive black holes”** – giants that currently sit at the centers of galaxies (range of mass of BH from $10^{5.5}$-10^{10} $M_{\odot}$)
  - GW results show that $M\sim120M_{sun}$ BHs exist
  - Not clear if (120-$300{,}000M_{sun}$) exist