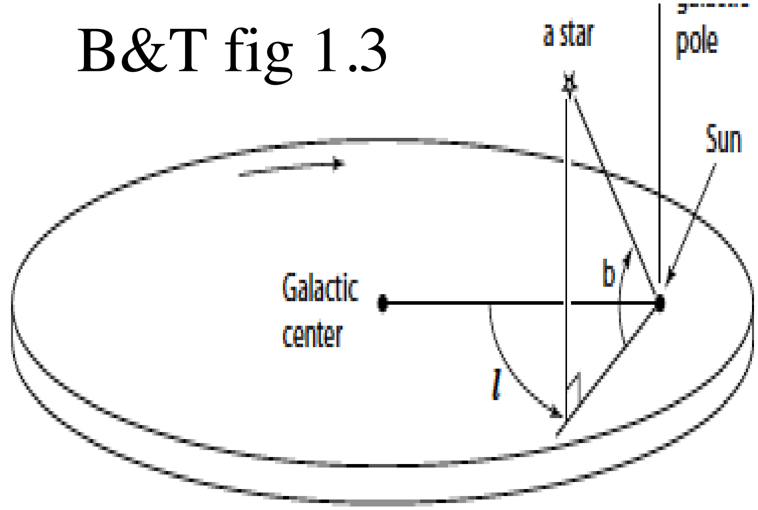
Galactic Rotation

- See SG 2.3, BM ch 3 B&T ch 2.6,2.7 and ch 6
- Coordinate system: define velocity vector by π, θ, z
  - π radial velocity wrt galactic center
  - θ motion tangential to GC with positive values in direct of galactic rotation
  - z motion perpendicular to the plane, positive values toward North galactic pole
- origin is the galactic center (center or mass/rotation)
- Local standard of rest (BM pg 536)
- velocity of a test particle moving in the plane of the MW on a closed orbit that passes thru the present position of the sun

B&T fig 1.3



If the galaxy is axisymmetric and in steady state then each pt in the plane has a velocity corresponding to a circular velocity around center of mass of MW

$(\pi,\theta,z)_{LSR}=(0,\theta_0,0)$ with

$\theta_0 = \theta_0(R)$

Coordinate Systems

The stellar velocity vectors are

z:velocity component perpendicular to plane

θ: motion tangential to GC with positive velocity in the direction of rotation

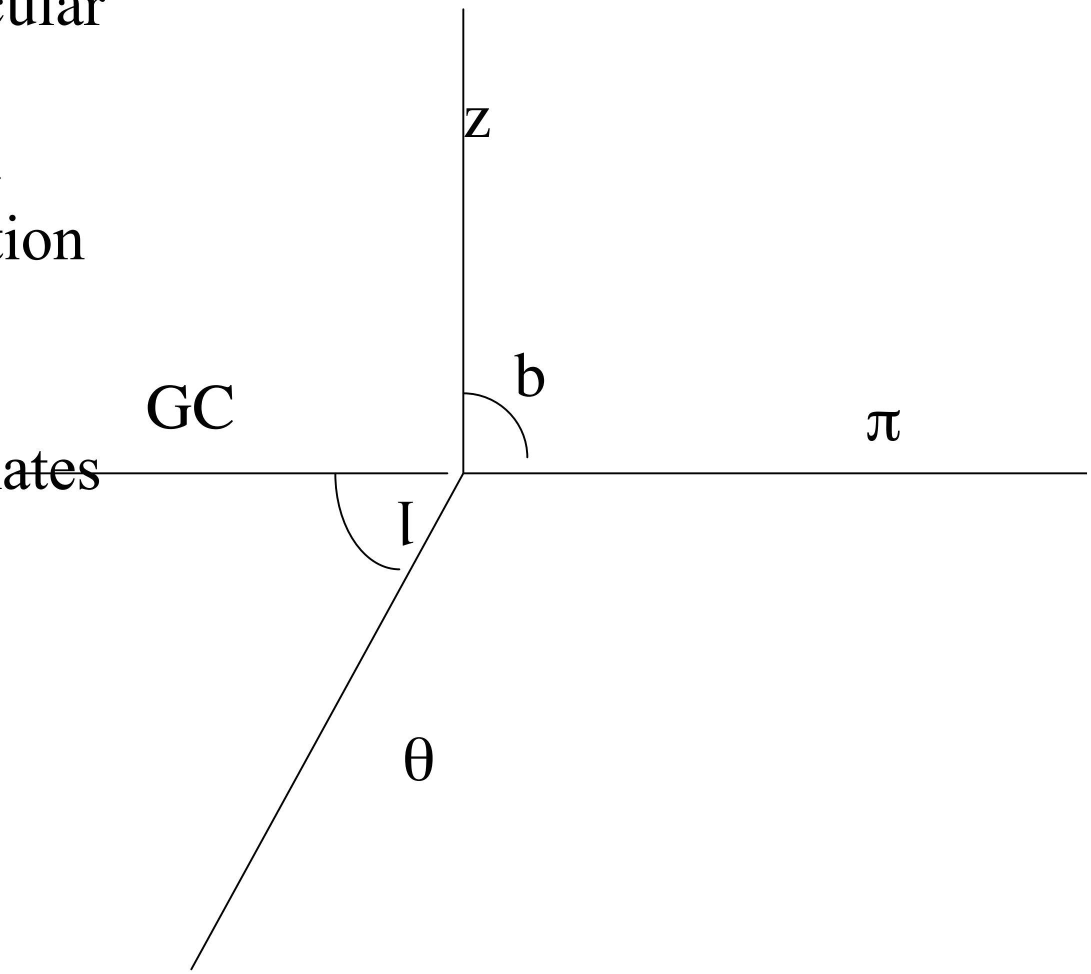
π: radial velocity wrt to GC

With respect to galactic coordinates

+π= (l=180,b=0)

+θ= (l=30,b=0)

+z= (b=90)



Local standard of rest: assume MW is axisymmetric and in steady state

If this each true each point in the pane has a 'model' velocity corresponding to the circular velocity around of the center of mass.

An imaginery point moving with that velocity at the position of the sun is defined to be the LSR

$(\pi,\theta,z)_{LSR}=(0,\theta_0,0)$;where$\theta_0 = \theta_0(R_0)$

Description of Galactic Rotation (S&G 2.3)

- For circular motion: relative angles and velocities observing a distant point
- T is the tangent point

$V_r = R_0 \sin l (V/R - V_0/R_0)$

Because V/R drops with R (rotation curve is ~flat); for value $0<l<90$ or $270<l<360$ reaches a maximum at T

So the process is to find V_r^{max} for each l

and deduce $V(R) = V_r + R_0 \sin l$

For R>R0 : rotation curve from HI or CO is degenerate ; use masers, young stars with known distances

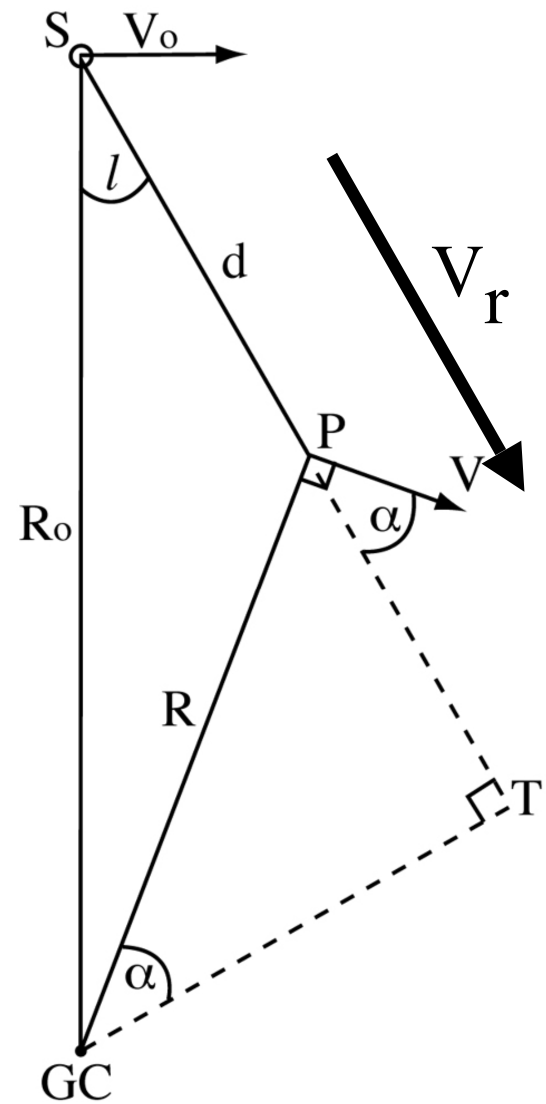


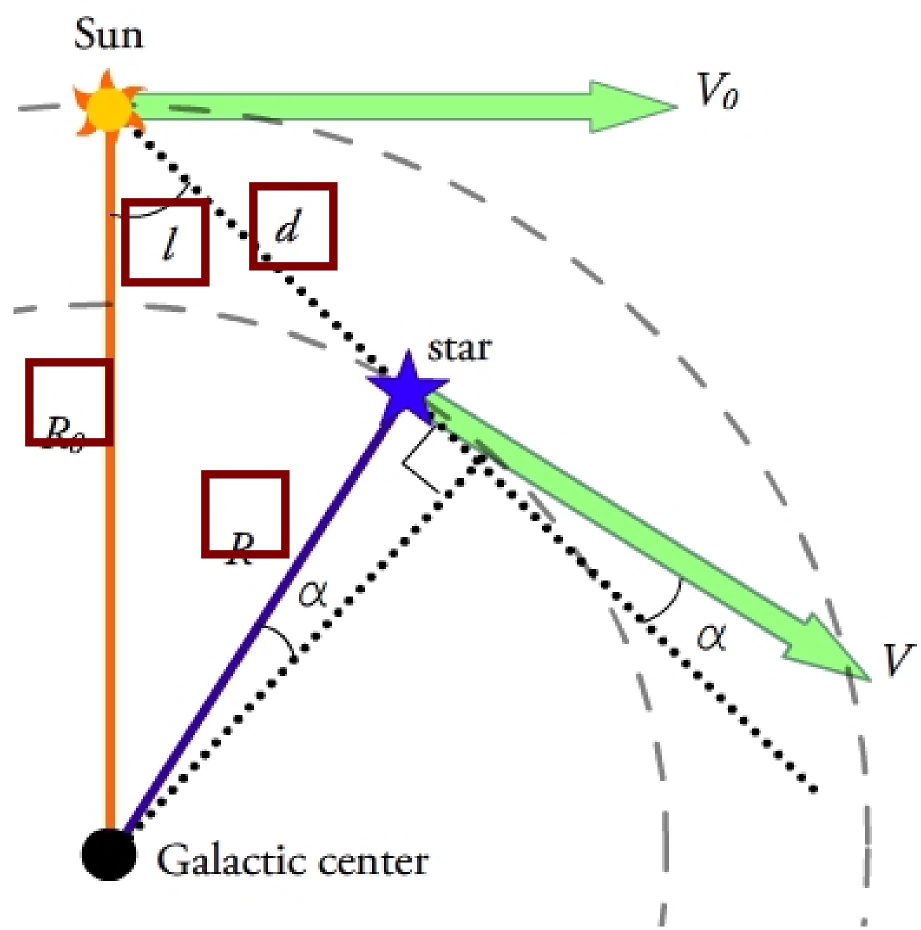
Fig 2.19 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Galactic Rotation- S+G sec 2.3, B&T sec 3.2

- Consider a star in the midplane of the Galactic disk with Galactic longitude, l, at a distance d, from the Sun. Assume circular orbits radii of R and R_0 from the galactic center and rotational velocities of V and , V_0

- The 2 components of velocity- radial and transverse are then for circular motion
- $V_{observered, radial}=V_0(\cos\alpha)-V_0\sin(l)$
- $V_{observered ,tang}=V_0(\sin\alpha)-V_0\cos(l)$
- using the law of sines
- $\sin l/R=\sin(90+\alpha)/R_0=\cos\alpha/R_0$

  $\sin l/R\sim\cos\alpha/R_0$



wikipedia

Galactic Rotation

- Then using a bit of trig

$R(\cos\alpha) = R_0 \sin(l)$

$R(\sin\alpha) = R_0 \cos(l) - d$

so

$V_r = R_0 \sin l (V/R - V_0/R_0)$

and defining $\omega = V/R$ (rotation rate)

$V_{observered,radial} = (\omega - \omega_0) R_0 \sin(l)$

Similarly

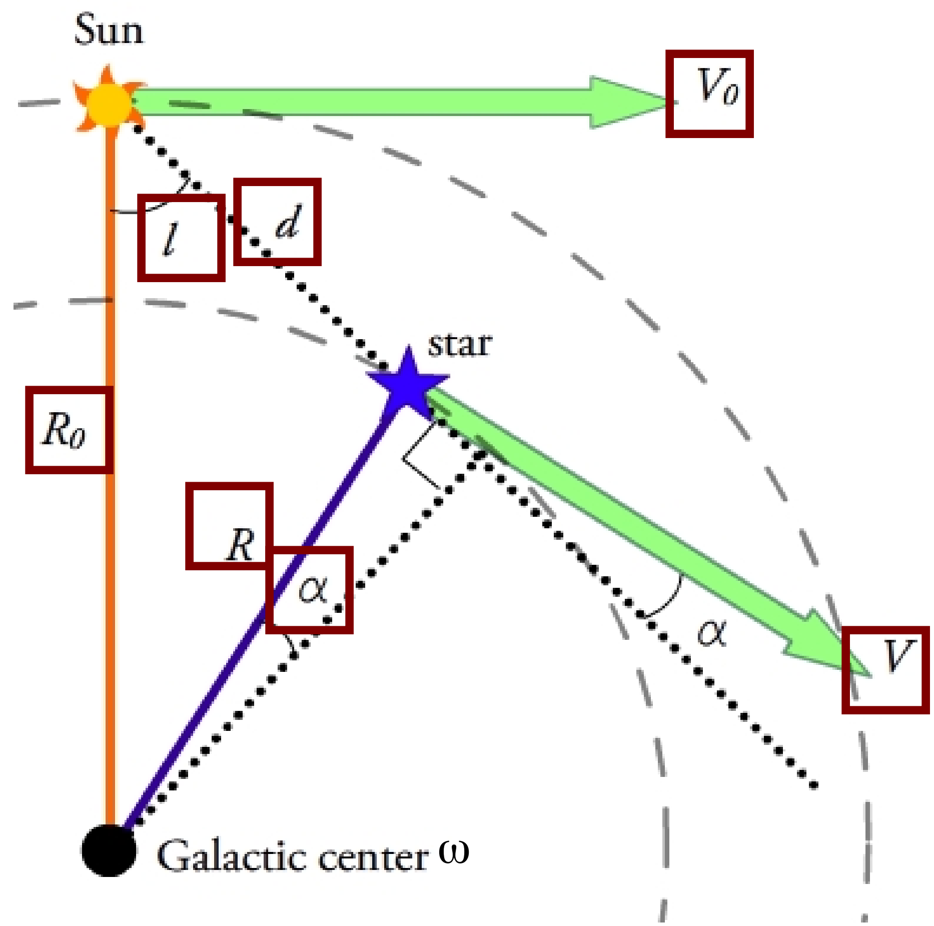
$V_{observered,tang} = V\sin\alpha - V_0 \cos l$

but since $R\sin\alpha = R_0 \cos l - d$

$V_{observered,tang} = V/R(R_0 \cos l - d) - V_0 \cos l$

then

$V_{observered,tang} = (\omega - \omega_0) R_0 \cos(l) - \omega d$



Galactic Rotation

- Then using a bit of trig

$R(\cos\alpha) = R_0 \sin(l)$

$R(\sin\alpha) = R_0 \cos(l) - d$

so

$V_{observered,radial} = (\omega - \omega_0)\, R_0 \sin(l)$

$V_{observered,tang} = (\omega - \omega_0)\, R_0 \cos(l) - \omega d$

then following for small distances from the sun expand $(\omega - \omega_0)$ around R_0 and using the fact that most of the velocities are local e.g. $R-R_0$ is small and d is smaller than R or R_0 (not TRUE for HI) and some more trig

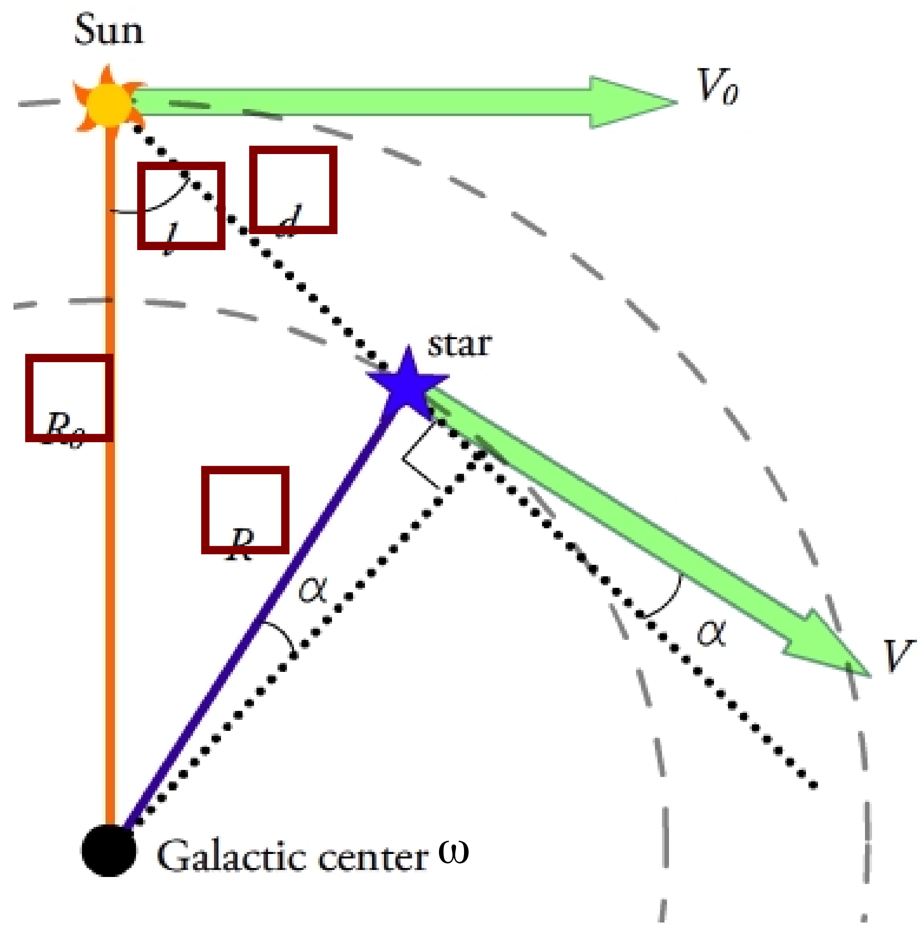
get

$V_{observered,radial} = Ad\sin(2l); V_{obs,tang} = Ad\cos(2l) + Bd$

Where

$A = -1/2\, R_0 d\omega/dr$ at R_0

$B = -1/2\, R_0 d\omega/dr - \omega$



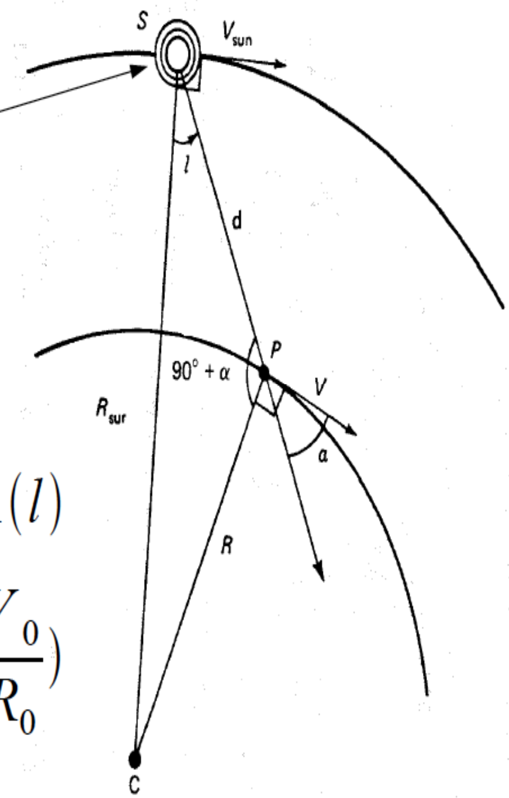
Galactic Rotation Curve- sec 2.3.1 S+G

Assume gas/star has
a perfectly circular orbit

At a radius R_0 orbit with velocity V_0; another star/parcel of gas at radius R has a orbital speed V(R)

since the angular speed V/R drops with radius V(R) is positive for nearby objects with galactic longitude 1<l<90 etc etc (pg 91 bottom)

- Galactic Rotation Curve
  - At R_{sun} the lsr has a velocity of V_0
  - A star at P has an apparent velocity of



1) $$V_r = V\cos(\alpha) - V_0\sin(l)$$

2) $$V_r = R_0\sin(l)\left(\frac{V}{R} - \frac{V_0}{R_0}\right)$$

- Convert to angular velocity ω
- $V_{observered,radial} = \omega R(\cos\alpha) - \omega_0 R_0 \sin(l)$
- $V_{observered,tang} = \omega R(\sin\alpha) - \omega_0 R_0 \cos(l)$

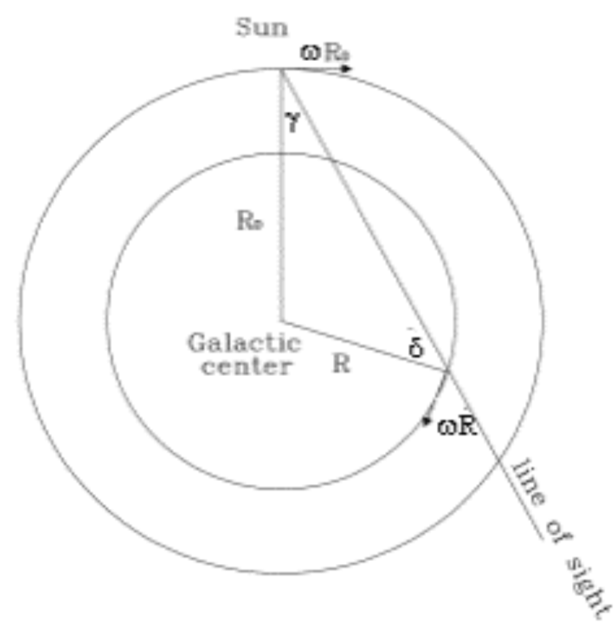
In terms of Angular Velocity

- model Galactic motion as circular motion with monotonically decreasing angular rate with distance from center.
- Simplest physics:if the mass of the Galaxy is all at center angular velocity ω at R is $\omega = M^{1/2} G^{1/2} R^{-3/2}$
- If looking through the Galaxy at an angle l from the center, velocity at radius R projected along the line of site minus the velocity of the sun projected on the same line is
- $V = \omega R \sin d - \omega_o R_o \sin l$
- ω = angular velocity at distance R

  ω_o = angular velocity at a distance R_o

  R_o = distance to the Galactic center

  l = Galactic longitude
- Using trigonometric identity $\sin d = R_o \sin l / R$ and substituting into equation (1)
- $V = (\omega - w\,\omega_o) R_o \sin l$

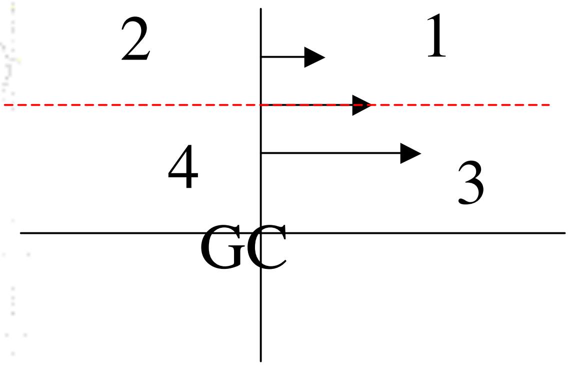


http://www.haystack.mit.edu/edu/undergrad/srt/SRT Projects/rotation.html

Continued

- The tangential velocity $v_T = V_o \sin\alpha - V_o \cos l$
- and $R \sin\alpha = R_o \cos l - d$
- a little algebra then gives
- $V_T = V/R(R_o \cos l - d) - V_o \cos l$
- re-writing this in terms of angular velocity
- $V_T = (\omega - \omega_o) R_o \cos l - \omega d$

- For a reasonable galactic mass distribution we expect that the angular speed $\omega = V/R$ is monotonically decreasing at large R (most galaxies have flat rotation curves (const V) at large R) then get a set of radial velocities as a function of where you are in the galaxy
- V_T is positive for $0<l<90$ and nearby objects- if $R>R_0$ it is negative
- For $90<l<180$ V_T is always negative
- For $180<l<270$ V_T is always positive (S+G sec 2.3.1)



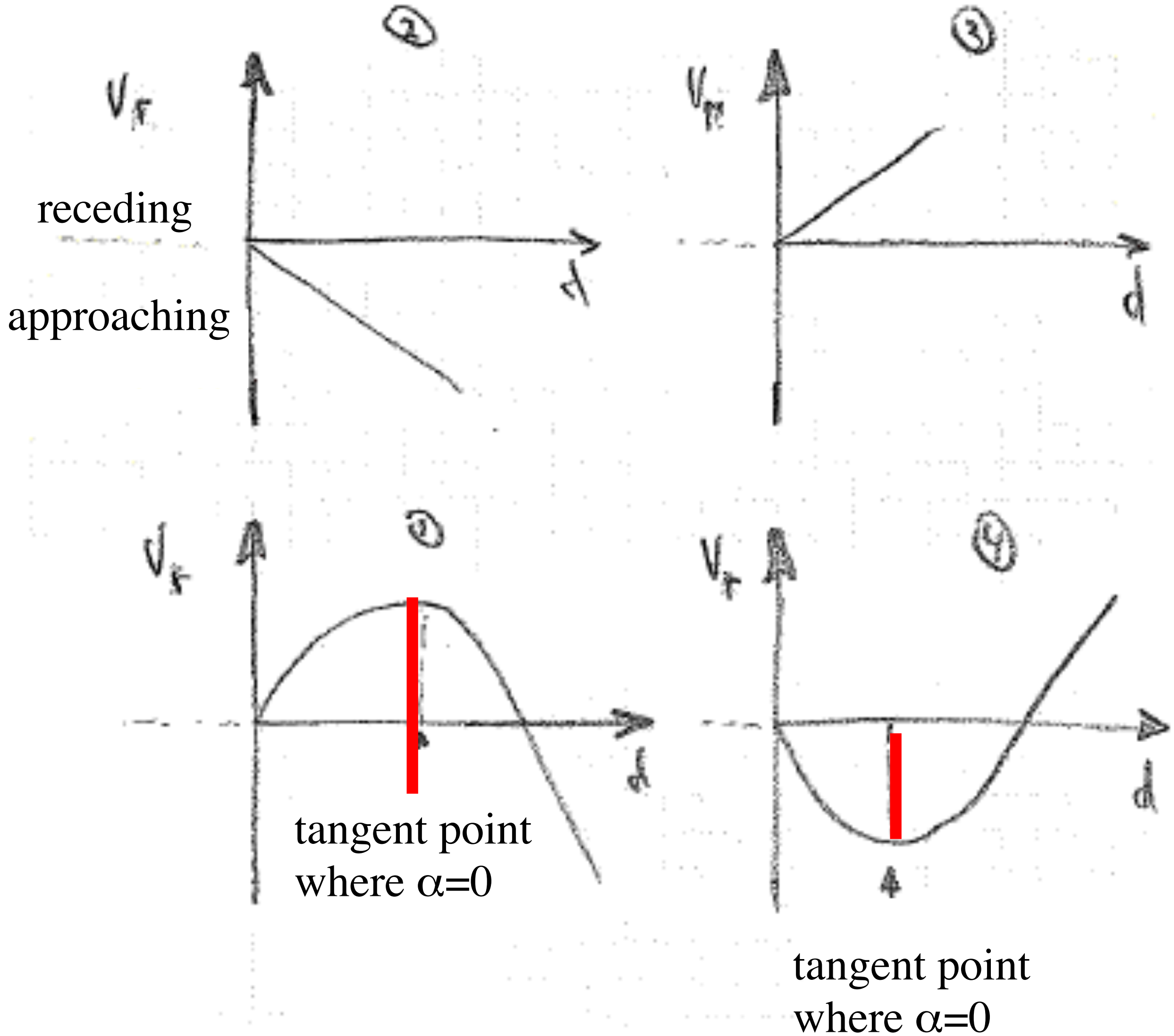
(1) $0<l<90$

(2) $90<l<180$

(3) $180<l<270$

(4) $270<l<360$

Veilleux 2010



Oort Constants

- Derivation:
- for objects near to sun, use a Taylor series expansion of $\omega-\omega_o$

$\omega-\omega_o = d\omega/dR\ (R-R_o)$

$\omega = V/R;\ d\omega/dR = d/dr(V/R) = (1/R)dV/dr - V/R^2$

then to first order $V_r = (\omega-\omega_o)R_o \sin l = [dV/dr - V/R](R-R_o) \sin l$; when $d \ll R_o$

$R-R_o = d\cos l$ which gives

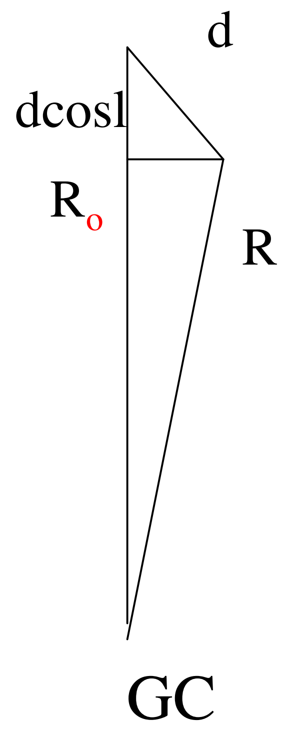
$V_r = (V_o/R_o - dV/dr)d \sin l \cos l$

using trig identity $\sin l \cos l = 1/2 \sin 2l$

one gets the Oort forumla

$V_r = Ad\sin 2l$ where

$$A = \frac{1}{2}\left[\frac{V_\circ}{R_\circ} - \left(\frac{dV}{dR}\right)_{R_\circ}\right]$$



One can do the same sort of thing for V_T

Oort Constants

- For nearby objects (d<<R) then (l is the galactic longitude)
  - $V(R) \sim R_0 \sin l\,(d(V/R)/dr)(R-R_0)$

  ~dsin(2l)[-R/2(d(V/R)/dr)~ d**A**sin(2l)
- A is one of 'Oorts constants'
- The other (pg 93 S+G) is related to the tangential velocity of a object near the sun V_t=d[Acos(2l)+B]
- So, stars at the same distance r will show a systematic pattern in the magnitude of their radial velocities across the sky with Galactic longitude.
- A is the Oort constant describing the shearing motion and B describes the rotation of the Galaxy
- A=14.8km/s/kpc
- B=-12.4 km/s/kpc

$$A = \frac{1}{2}\left[\frac{V_\circ}{R_\circ} - \left(\frac{dV}{dR}\right)_{R_\circ}\right]$$

$$B = -\frac{1}{2}\left[\frac{V_\circ}{R_\circ} + \left(\frac{dV}{dR}\right)_{R_\circ}\right]$$

$$A + B = -\left(\frac{dV}{dR}\right)_{R_\circ} \quad ; \quad A - B = \frac{V_\circ}{R_\circ}$$

A=-1/2[Rdω/dr]

Useful since if know A get kinematic estimate of d

Radial velocity $v_r \sim 2AR_0(1-\sin l)$ only valid near l~90 measure $AR_0 \sim 115$km/$_s$

Oort 'B'

- B measures 'vorticity' B=-(ω+=-1/2[Rdω/dr])=-1/2(V/R+dV/dR)
  ω=A-B=V/R; angular speed of Local standard of rest (sun's motion)

  Oort constants are local description of differential rotation

  Vaues

  A=14.8 km/s/kpc

  B=-12.4

  Velocity of sun $V_0=R_0(A-B)$

  I will not cover epicycles: stars not on perfect circular orbits:see sec 3.2.3 in B&T

  important point σ^2_y/σ^2_x= -B/A-B

Best Estimate of Oort's Constants

- $V_0 = R_0(A-B) = 218 +/- 8\ (R_0/8kpc)$
- using Hipparchos proper motions of nearby stars

- Can also use the proper motion of SgrA* (assumed to be the center of mass of the MW)

Best Estimate of Oort's Constants

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Rotation Curve

- For R<R0 The inner rotation curve is measured by the terminal (tangential)-velocity method applied to radio line observations such as the HI and CO lines.

Bovy and Tremaine continued- modern example of using Jeans and Poisson eq

- The radial Jeans equation for the disk is

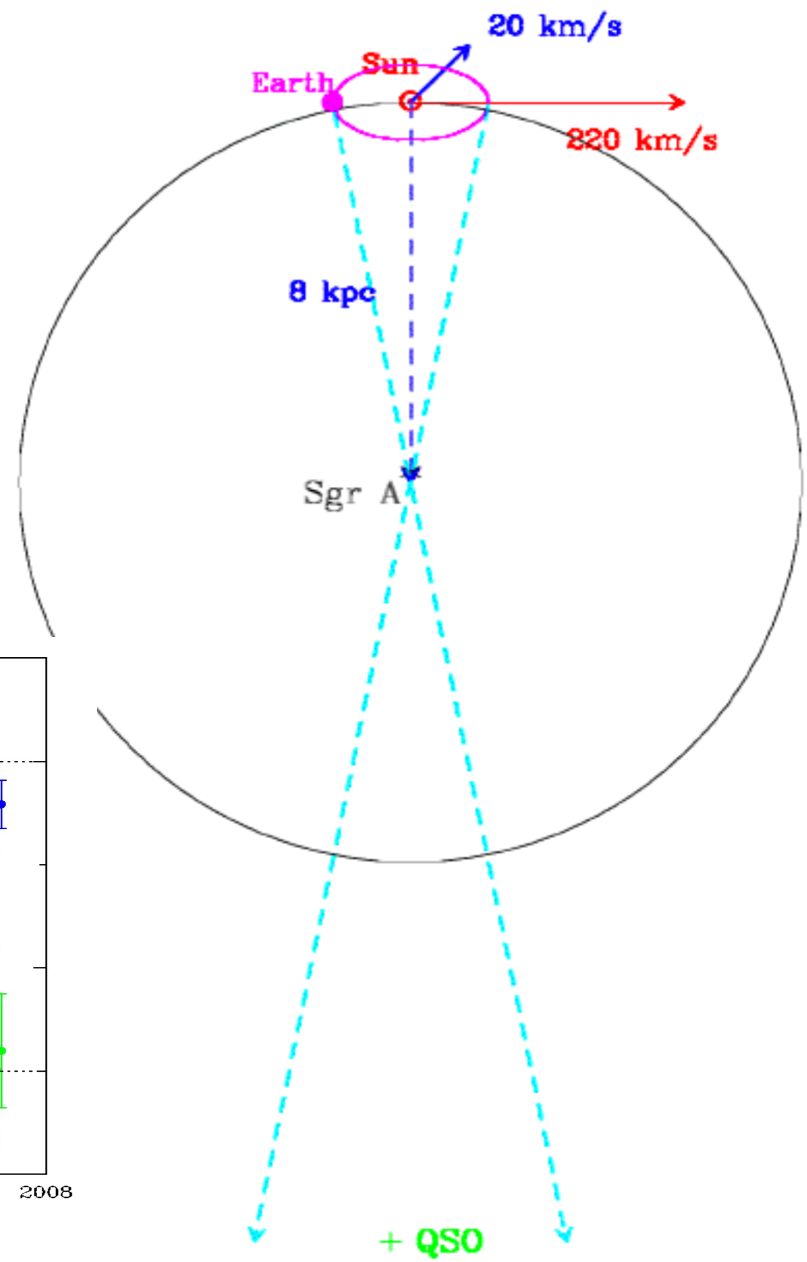
- $FR(R, Z) = \partial\Phi(R, Z)/\partial R = (1/\nu)\ (\partial\ (\nu\sigma^2_U\ )/\partial R) + (1/\nu)\ (\partial\ (\nu\sigma^2_{UW}\ )/\partial z) + (\sigma^2_U - \sigma^2_v - V^2\ )/R$
- where Φ is the gravitational potential, ν is the tracer-density profile, σ^2_U and σ^2_v are the radial and azimuthal velocity dispersions squared, σ^2_{UW} is the off-diagonal radial–vertical entry of the dispersion-squared matrix, and $\bar{V}$ is the mean azimuthal velocity;
- all of these quantities are functions of R and Z.

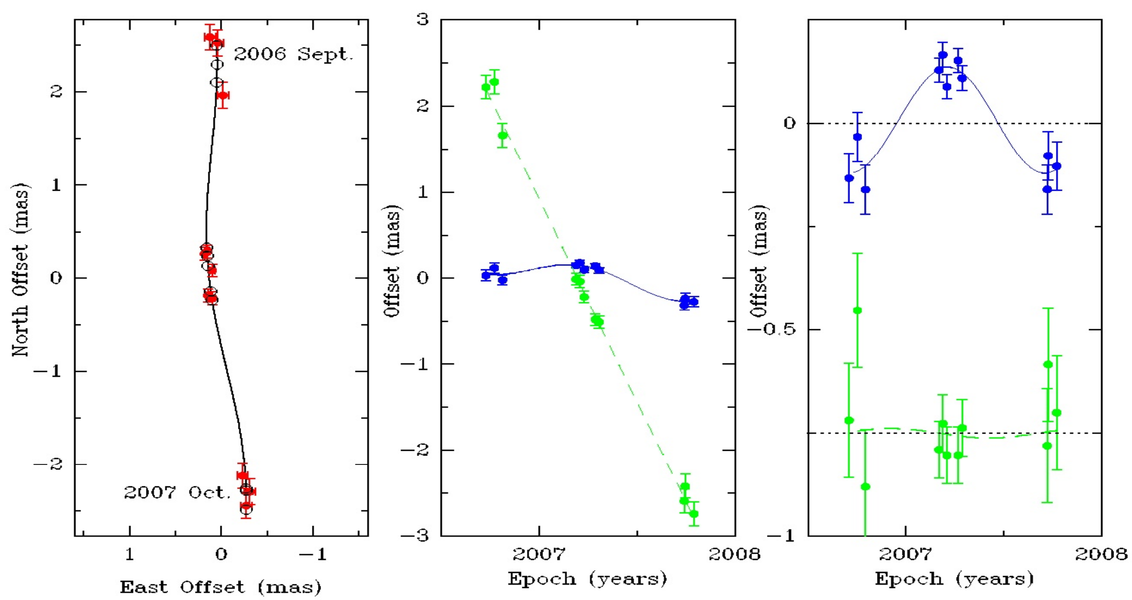
- The mean azimuthal velocity of a population of stars differs from the circular velocity due to the asymmetric drift. This offset arises because both the density of stars and the velocity dispersion typically decline with radius. This means that more stars with guiding centers at $R < R_0$ are passing through the solar neighborhood than stars with guiding centers $R > R_0$; the former are on the outer parts of their orbits, where their azimuthal velocity is less than the circular velocity

Distances From Motions

- measure the proper motion+parallax of SgrA* caused by the velocity of the sun
- East in blue, north in green -right panel has proper motion removed. left panel motion on sky (Reid et al 2009)
- However they find $R_0 = 8.4 \pm 0.6$ kpc

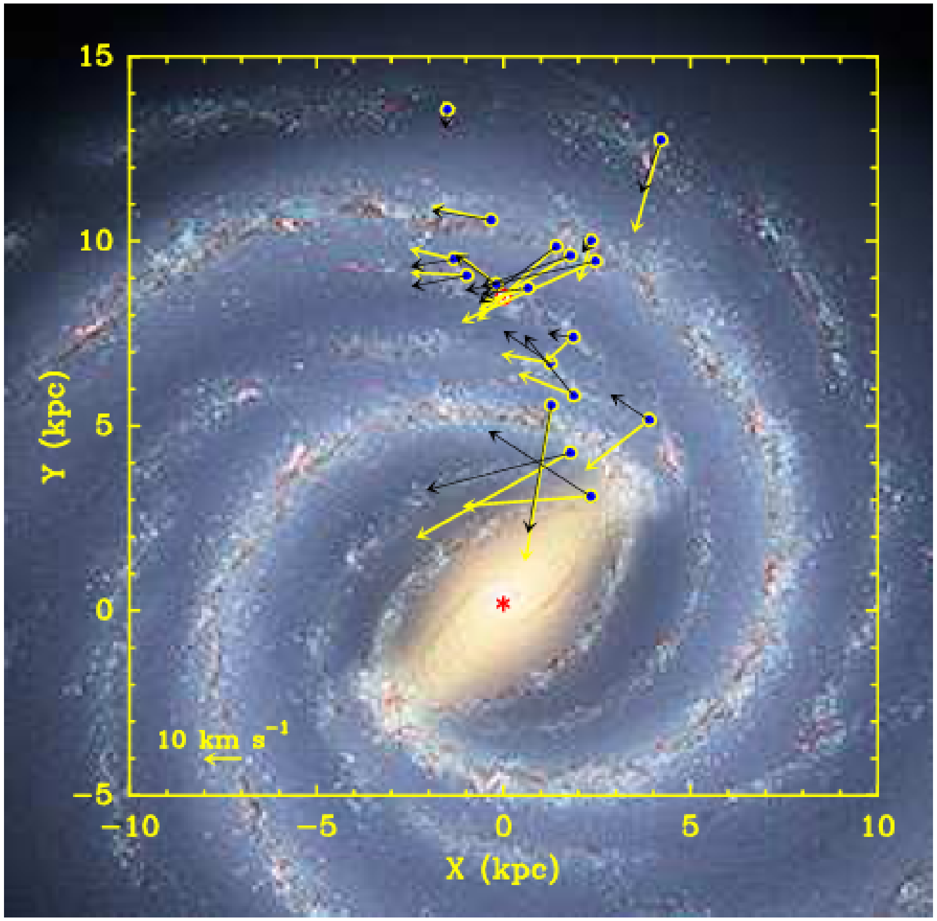
and a circular rotation speed $\Theta_0 = 254 \pm 16$ km/s.. oops





Peculiar Velocities

- Peculiar motion vectors of high mass star forming regions projected on the Galactic plane after transforming to a reference frame rotating with
- the Galaxy.
- A 10 km/s motion scale is in the lower left.
- High mass star forming regions rotate slower than galactic model indicates (Reid et al 2010)



HI Observables

- Observed intensity $T_B(l, b, v)$ observed in Galactic coordinates longitude l and latitude b need to be converted into volume densities $n(R, z)$ (Burton & te Lintel Hekkert 1986, Diplas & Savage 1991).
- Assuming that most of the gas follows an axisymmetric circular rotation yields a relation for the differential rotation velocity (e.g., Burton1988)

  $v(R, z)=[(R_{\odot}/R)\ \Theta(R, z)-\Theta_{\odot}]\sin l\ -\cos b$ where v is the radial velocity along a line of sight(directly measurable; and Θ is the tangential velocity

- for $R < R_{\odot}$, distances are ambiguous,
- for $R > R_{\odot}$, one needs to know the Galactic constants $R_{\odot}$ and $\Theta_{\odot}$ and the form of $\Theta(R, z)$.

$R_{\odot}$ is the distance of the sun from the galactic center and $\Theta_{\odot}$ is the velocity of rotation at the sun . (a lot more later)

Rotation Curve of MW

- Rotation curve o: MW (Reid et al 2010)