Ellipse's Versiera – Structure and Kinematics of the Generator Mechanism

Alina Duta[1], Iulian Popescu[2], Ludmila Sass[1], Mirela Cherciu[1], and Ionut-Daniel Geonea[1](✉)

[1] Faculty of Mechanics, University of Craiova, Craiova, Romania
ionut.geonea@edu.ucv.ro
[2] Academy of Technical Sciences, Craiova, Romania

Abstract. The structural and kinematic analyses of a mechanism presented in the specialty literature are performed. The mechanism is used to plot versiera of ellipses and ellipses. We wrote the equations for calculating the positions of the points necessary to realize the geometric model in the ADAMS® program using the vector contours method and the distance formula between two points. The mechanism is represented in an initial position together with successive positions. Curves have been plotted for different situations, and when the distance between two fixed lines changes beyond a certain limit, other curves are obtained: conchoides of de Sulze and curves with egg shape.

Keywords: planar mechanism · versiera of ellipse · ADAMS simulation

1 Introduction

Studied by Maria Gaetana Agnesi, also by Fermat and Guido Grandi, the name of this curve has a colorful history. *Versaria* is the name given by Grandi, meaning "turning in every direction" [5] (Fig. 1).

One of the witch of Agnesi application is related to the fact that it approximates the spectral energy distribution of spectral lines, particularly X-ray lines. Also, the cross-section of a smooth hill has a similar shape to the witch [4, 6]. Curves with this shape have been used as the generic topographic obstacle in a flow in mathematical modeling. Solitary waves in deep water can also take this shape [3]. The witch of Agnesi has applications in probability theory, too. Other ordinary plane curves of 3-rd degree are mentioned in [5]: cubic parable, folium of Descartes, cissoid of Diocles, conchoid of de Sluze, straight strofoide, demicubical parabola, serpentine curve, trident curve, trisectrix of Maclaurin.

The generated curve equation is:

$$4 \cdot b^2 \cdot x + x \cdot y^2 = 8 \cdot a \cdot b^2 \tag{1}$$

There are other curves inspired by the witch curve of Agnesi such as the versiera of ellipse [2] or Noor's curve [1]. Starting from circle's versiera we have analized an

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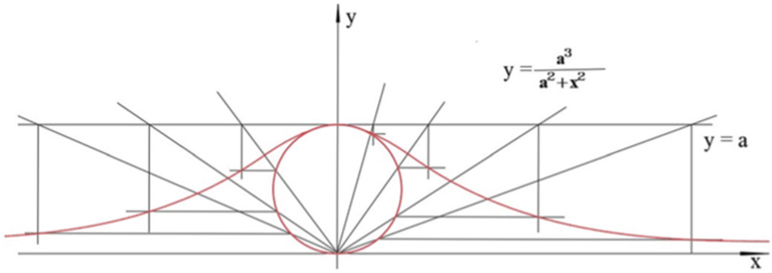

Fig. 1. The witch curve of Agnesi [5]

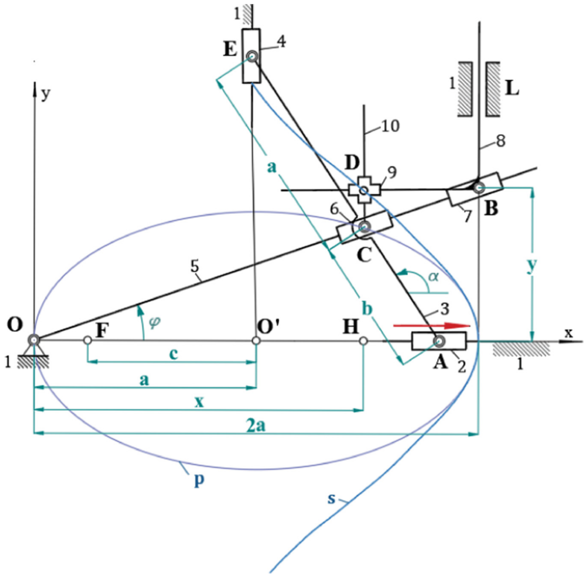

Fig. 2. The kinematic scheme of the mechanism

historical mechanism depicted by Fig. 2 [2] wich can be used to plot versiera of ellipse through the point C. From the mathematic point of view this is a planar, cubic and rational curve.

Geometric modeling and motion simulation of the designed mechanism were performed. The SolidWorks software was used for geometric modeling, and the ADAMS View software for kinematic simulation. The bar was modeled by „link" type elements, the slides by „box" type elements, an aspect that is highlighted in Fig. 3. Since the degree of mobility of the mechanism is equal to 1, it can be operated by a single motor. For this study a rotational motor positioned at joint O was chosen, case in which the vertical position of the driving element becomes the critical operating position. The corresponding kinematic joints and the law of motion for the driving element, respectively the angular velocity of 2 rad/sec have been adopted. By simulating the motion of the mechanism we aimed to obtain the trajectories of the points of interest, namely C and D. The trajectories

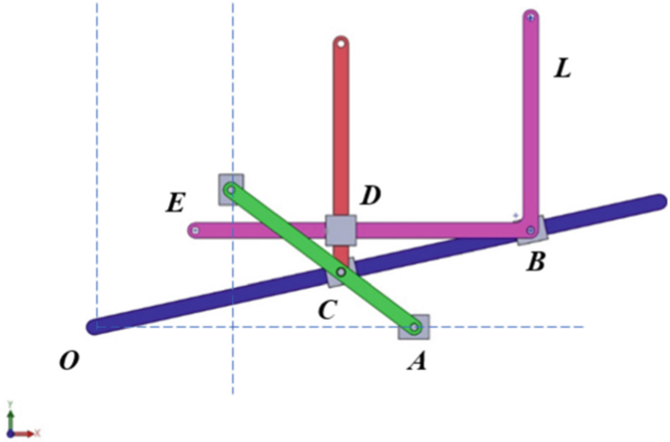

Fig. 3. The mechanism simulation model designed in the ADAMS View program

obtained by the kinematic simulation in the ADAMS program are presented in Fig. 4. It can be seen that point C describes an ellipse and point D the versiera of the ellipse.

2 The Mechanism Structure

The mechanism's structural scheme is provided in Fig. 5. One can see the leading element, denoted by A in Fig. 1, which performes a translation movement, the dyad AEEE of RRP type, the dyad CCC of type RPR, the dyad BBL of type RPP and the dyad CDD of type RPP (in brief a leading element and 4 dyads).

3 Computational Relationships

The following relationships are deduced by using the contour method:

$$x_E = x_A + AE.cos\alpha = const. \tag{2}$$

$$y_E = AE.sin\alpha \tag{3}$$

$$x_C = X_A + AC.cos\alpha = OC.cos\varphi \tag{4}$$

$$y_C = AC.sin\alpha = OC.sin\varphi \tag{5}$$

$$x_B = OB.cos\varphi = const. \tag{6}$$

$$y_B = OB.sin\varphi \tag{7}$$

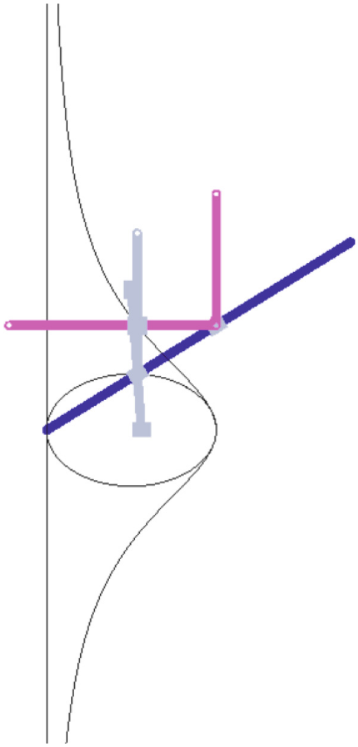

Fig. 4. The trajectories described by points C - the ellipse- and D – versiera of ellipseduring simulation.

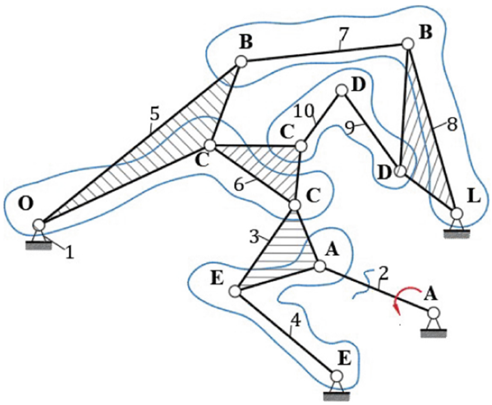

Fig. 5. The structural scheme

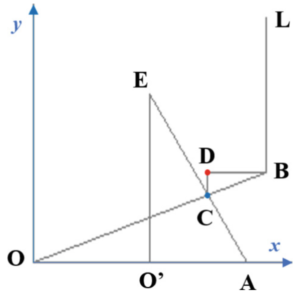

Fig. 6. Mechanism in the initial position.

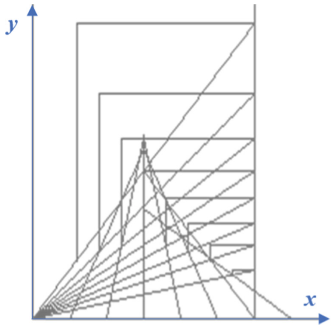

Fig. 7. The mechanism in successive positions.

$$x_D = x_C + CD.cos90^0 = x_B + BD.cos180^0 \tag{8}$$

$$y_D = y_C + CD.sin90^0 = y_B + BD.sin180^0 \tag{9}$$

The value of x_A is used through a cycling technique. Equation 2 yields α, whilst y_E is yielded by Eq. 3. Equation 5 is used to obtain φ, and OC is computed based on Eq. 4. OB is yielded by Eq. 6 whilst y_B is obtained from Eq. 7. From Eqs.8 and 9 we obtained BD, CD, x_D, y_D respectively.

4 Results

Considering as initial data AE = 50 mm, AC = 20 mm, x_E = 30 mm, x_L = 60 mm and Eqs. 2–9, the kinematic scheme of the mechanism in the GWBASIC program was obtained. Figure 6 shows the mechanism in one position and the successive positions of it in Fig. 7.

The two branches of the curve are shown as follows: in Fig. 8 the one below the Ox axis corresponds to the sign (−) used behind a square root yielded by the above equations, in Fig. 9 the one above the Ox axis when the sign (+) is used, and the whole curve as that depicted by Fig. 10. One can also notice the lines BL and O'E in Fig. 10.

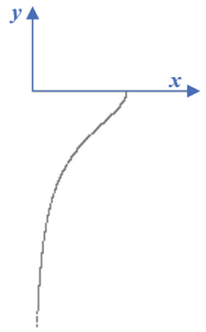

Fig. 8. The branch of the curve for (-) sign

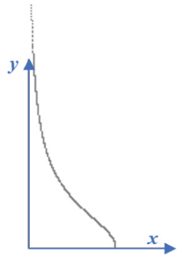

Fig. 9. Branch of the curve for (+) sign

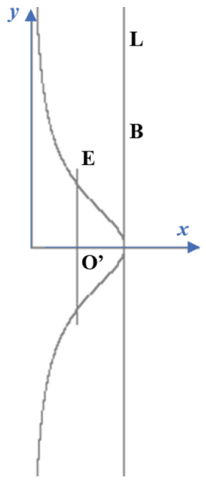

Fig. 10. Both branches of the curve

It is known that the points from the line AE which are moving with the heads on the system axes plot ellipses. Figure 11 depicts the ellipse generated by the point C and also its versiera.

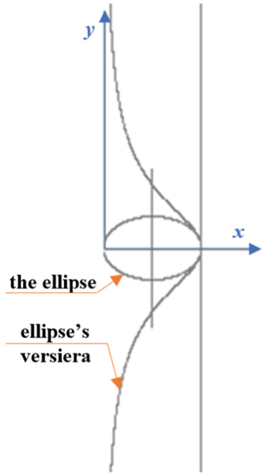

Fig. 11. The ellipse and its versiera

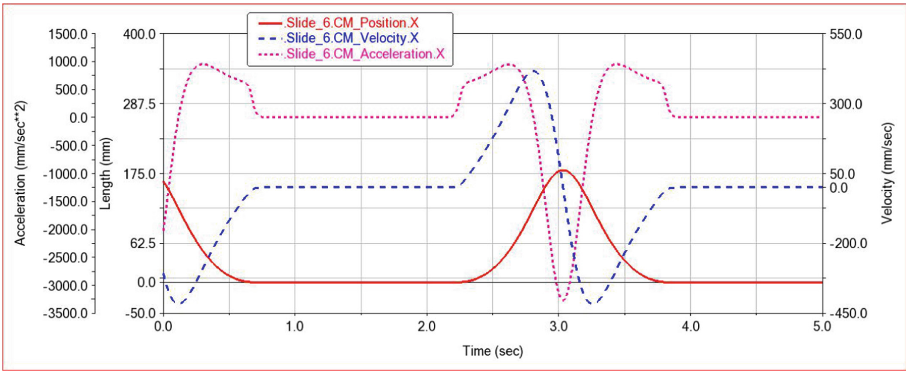

Fig. 12. Graphics for position, velocity and acceleration of the center of the mass for slide 6, in relation to the X axis of the reference system

Simultaneously, the aim was to draw the combined graphs, displacements, speeds, accelerations for points C and D. The results obtained are presented in Figs. 13, 14, 15, 16, 17, and 18. The marker attached to the center of mass of the slide 6, generates an elliptical trajectory. The position, velocity and acceleration of the center of mass of the slide 6, along the X and Y axes of the reference system, obtained by numerical simulation in ADAMS, are shown in Figs. 12 and 13. The fixed reference system is positioned in the coupling O, and has the orientation as seen in Figs. 2 and 3. The resulting values for the position, speed and acceleration of the center of mass of the slide 6 are shown in the graph in Fig. 14.

We presented the same graphical results for the slide 9. We thus presented in Figs. 15 and 16 position, speed and acceleration along the X and Y axes for the marker attached

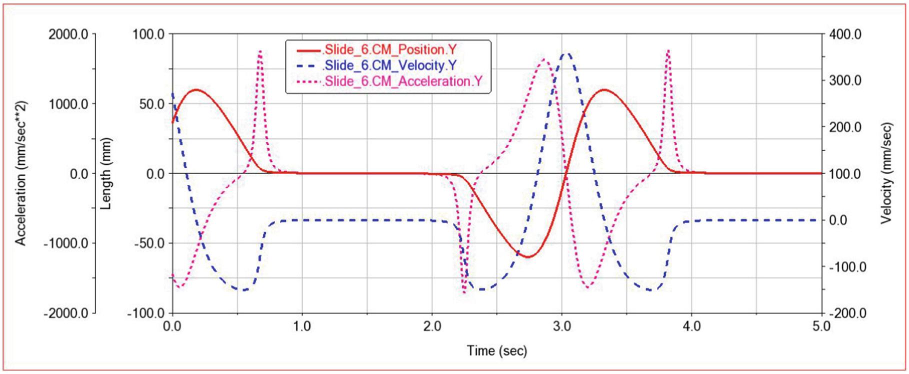

Fig. 13. Graphics for position, velocity and acceleration of the center of the mass for slide 6, in relation to the Y axis of the reference system

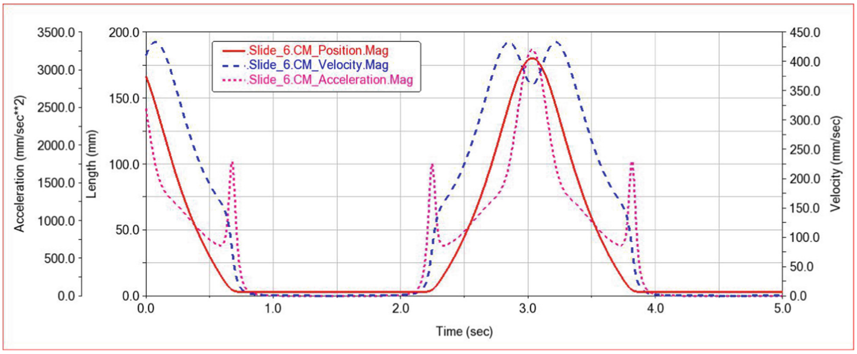

Fig. 14. Graphic results obtained for the resulting position, velocity and acceleration of the center of mass for slide 6.

to the center of mass of the slide 9. The resulting values for these kinematic parameters are shown in Fig. 17.

5 Additional Results

The position of the fixed line which crosses through E was modified, keeping the same value for $x_L = 60$ mm, yielding the curves from Figs. 18, 19, 20, 21, 22, 23, 24.

Figures 18, 19 and 23 presents some conchoids of de Sluze, whilst Figs. 20, 21, and 22 present curves with egg shapes. The modification of curves depends on the positioning of fixe lines.

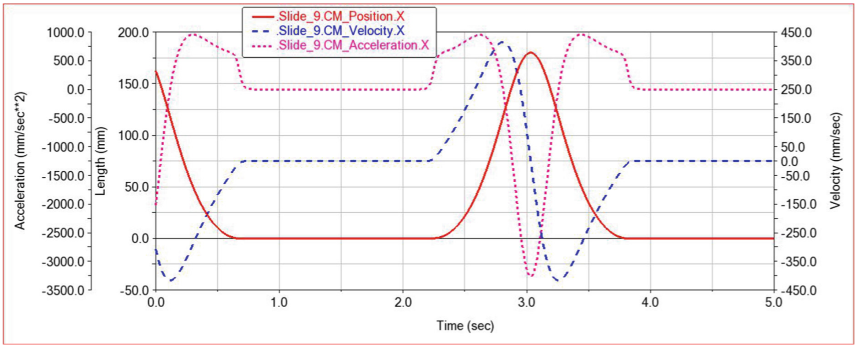

Fig. 15. Graphics for position, velocity and acceleration of the center of the mass for slide 9, in relation to the X axis of the reference system

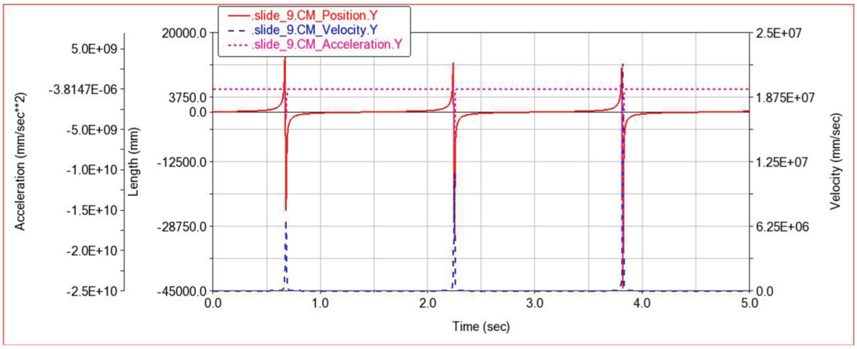

Fig. 16. Graphics for position, velocity and acceleration of the center of the mass for slide 9, in relation to the Y axis of the reference system.

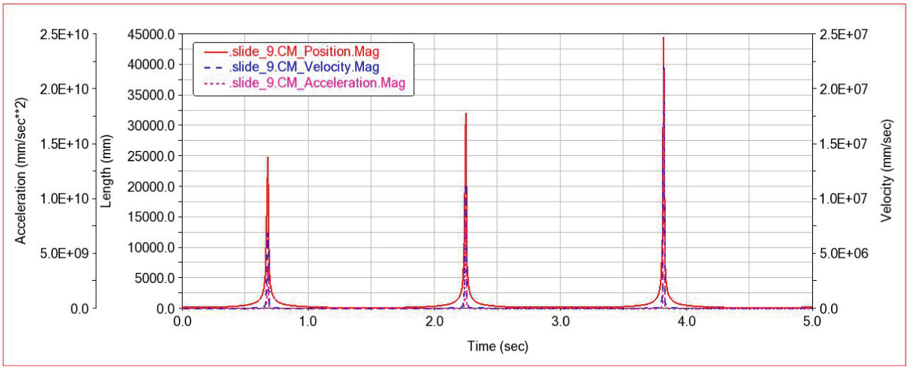

Fig. 17. Graphic results obtained for the resulting position, velocity and acceleration of the center of mass for slide 9.

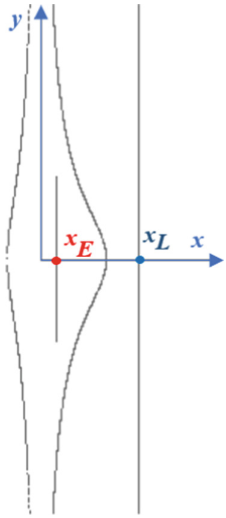

Fig. 18. The curves obtained for $x_E = 10$ mm

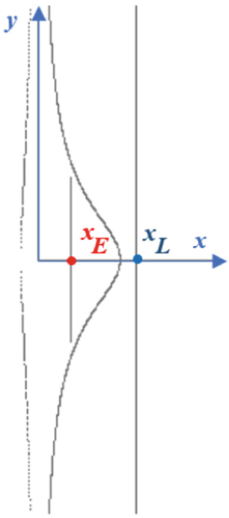

Fig. 19. The curves obtained for $x_E = 20$ mm

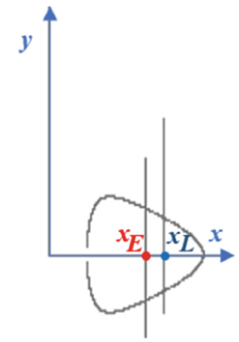

Fig. 20. The curves obtained for $x_E = 50$ mm

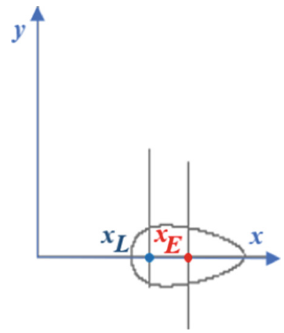

Fig. 21. The curves obtained for $x_E = 80$ mm

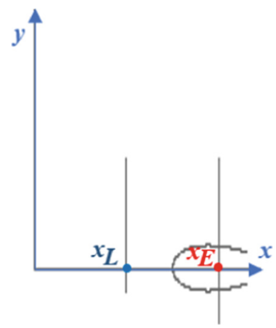

Fig. 22. The curves obtained for $x_E = 120$ mm

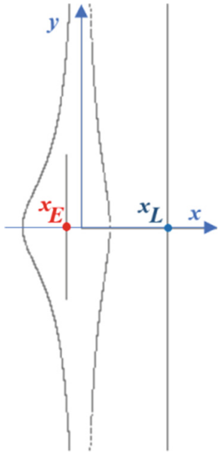

Fig. 23. The curves obtained for x_E = - 10 mm

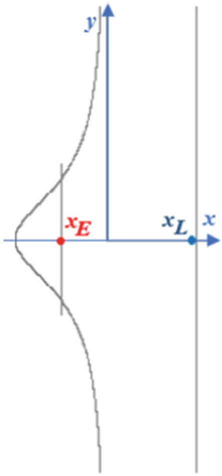

Fig. 24. The curves obtained for x_E = - 30 mm

6 Conclusions

Structural and kinematic studies were made relative to a mechanism from the specialty literature which generates the versiera of ellipse. Its abilities relative to the above-mentioned generation were proved again. The mechanism has the type P-RRP-RPR-RPP-RPP. Computational relationships were written based on the method of contours, allowing the deduction of a certain position of the mechanism.

The modelling of the mechanism and simulation of its operation in the ADAMS program was performed using different integration steps. For the non-working position of the mechanism, when two kinematic elements tend to become parallel, the program does not detect the critical position for certain values of the integration step, the simulation runs continuously, although the real mechanism cannot work in the critical position; the kinematic diagrams in that position are not interpretable, but the simulation validates the algorithm and successive positions of the mechanism.

The generated curve has two branches, one above the abscissa (for the sign $+$ in front of a square root), and the other one under the abscissa (for the sign $-$).

If the position of a fixed element is modified, versiersa of the ellipse are obtained up to a critical value and then one obtains either the conchoids of de Sluze or curves with egg shapes.

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