CLASSIFICATION OF RED VARIABLES

Janet A. Mattei[1], Grant Foster[1], Leora A. Hurwitz[1]
Kerriann H. Malatesta[1], Lee Anne Willson[2], Marie-Odile Mennessier[3]

[1]American Association of Variable Star Observers, 25 Birch St., Cambridge, MA 02138, USA
[2]Iowa State University, Physics and Astronomy Dept., Ames, IA 50011, USA
[3]Université Montpellier II, Groupe de Recherche en Astronomie et Astrophysique du Languedoc, Unité associeé au CNRS, F-34095 Montpellier cedex 5, France

ABSTRACT

Red variables are traditionally loosely classified into Mira, semiregular (SR), and slow irregular (L) variables. The Mira variables are the best-defined subgroup, but the SR and L stars are more numerous. The SRs show significant inhomogeneities in their division into SRa (regular variability but smaller pulsation amplitudes than Miras), SRb (less regular variability), SRc (more luminous), and SRd (warmer) variables. Relationships within each group are not clear. To obtain useful $P-L$ relations, and to relate stars to distinct evolutionary states, requires clear classification criteria.

We report here on an analysis of longterm AAVSO light curves. We find that Mira-type variables are so clearly different from semiregular variables as to remove all doubt as to their distinction. M-type Miras and C-type Miras are likewise dramatically different in their light curve properties. The M-Miras show a large number of interesting relationships, and appear to form a homogeneous group. The pulsations of SR variables are *extremely* unstable. Perhaps of greatest interest, *most* SR variables are multiperiodic, showing two periods P_1 and P_2 whose ratio falls in the narrow range $1.7 \leq P_1/P_2 \leq 1.95$.

We are now poised to combine the best available astrometry data with the best available periodicity data.

Key words: stars: classification; pulsation.

1. INTRODUCTION

Red variables are asymptotic giant branch (AGB) stars and are important in our understanding of stellar evolution, enrichment of the interstellar medium via mass loss, and the kinematics of the galaxy.

Red variables are currently classified into Mira, semiregular (SR), and slow irregular (L) variables. The Mira variables are the best-defined and most homogeneous subgroup. They have periods longer than 60 days, amplitudes in the visual larger than 2.5 magnitudes, and show rather regular variability. The SRs show significant inhomogeneity in their division into SRa with periods longer than 35 days, V amplitudes less than 2.5 magnitudes, and regular variability, SRb with periods longer than 20 days, V amplitudes less than 2.5 magnitudes and less regularity in their light variation, SRc more luminous, and SRd warmer variables.

In the present classification, Miras are distinguished from SRa variables by amplitude; from SRb by regularity; from SRc by luminosity class; and from SRd by color or spectral type. The resulting classes of SR variables are not physically homogeneous, and it has not been clear whether the SR variables are all distinct physically from the Miras. Also, the classification of these red stars in standard lists is not always consistent with the above criteria when better light curve data are used: for example, there are a large number of Mira stars with amplitudes in the visual less that the supposedly defining limit $\Delta V = 2.5$ mag, and several SR stars with $\Delta V > 2.5$ mag. The 'regularity' of the light curve, an integral part of the classification, is not a well-defined parameter. Also, the IRAS colors of some SRa stars fall in the Mira region, suggesting that the SRa class may not be truly a separate class but rather a mixture of stars better classified as SRb or Mira. The work of Kerschbaum and Hron (1992) indicates that a better division of the SR variables may be into 'red' and 'blue' ones. IRAS data show that there may be two kinds of oxygen-rich SRb stars, those with and those without circumstellar dust shells. Finally, the classification of the variable carbon stars introduces some new issues; there are indications that their physical properties may be different, as well as their atmospheric structure and the dynamics associated with carbon-based dust formation.

A new physically meaningful classification of red variables can only be achieved by combining all available information. Better classification parameters are vital for current research in evolution, kinematics, pulsation modes and mass loss characteristics of red variables.

In order to correlate Hipparcos astrometry with the pulsational properties of long-period variables

(LPVs), it is necessary to assemble a precise, consistent, and homogeneous data set of the periodic behaviors of those LPVs observed by the Hipparcos satellite.

We report here the on an analysis of longterm AAVSO light curves of Hipparcos stars and other stellar properties that confirms the traditional separation of Miras from other classes, and sheds new light on the properties and classification of SR variables.

2. LIGHT CURVE SAMPLE

We have used observations from the AAVSO International Database, which contains extensive visual data for 255 Mira and semiregular variables which were observed by the Hipparcos satellite. For the vast majority we had at least 1,000 observations, and in many cases in excess of 10,000 observations, covering the time span from JD 2,437,600 (October 1961) to about JD 2,450,300 (August 1996). Spectral type and variability type were extracted from the GCVS (Kholopov et al. 1985).

Of the 255 LPVs, most (170) are Mira-type; this is clearly a selection effect of the sample, as visual observing strongly favors large-amplitude variables. Of the 85 SR variables, about a quarter (23) are classified SRa, about half (46) SRb, an eighth (11) SRc, one is SRd, and four are given no subtype.

3. ANALYSIS METHODS

Our primary goal in analysis of the light curves was to determine the periodic behaviors shown by these stars. This task is complicated by the fact that the cyclic variations of LPVs are not perfectly regular, in fact they are decidedly variable. Each cycle is likely to have a slightly different period and amplitude than all others, as well as a unique light curve shape. Sometimes the variability amplitude may increase or decrease dramatically, or the period vary significantly from one cycle to the next. Although these factors make period analysis more difficult, they also give us much information about the nature of the stars.

3.1. Period Analysis

Our chief period analysis method is the CLEANEST Fourier spectrum (Foster 1995, 1996a). It is exceptionally good at detecting and quantifying multiple periods. It also yields not just the mean period and amplitude of any periodic fluctuation, but the *time evolution* of period and amplitude. CLEANEST quantifies long-term changes in the mean period and mean amplitude, but does not respond to cycle-to-cycle jitter, giving smooth functions $P(t)$ and $A(t)$ for period and amplitude as functions of time. We computed the rms variation of these smooth functions in order to define the *instabilities* of period δ_P and amplitude δ_A for a given star. We emphasize that this is the long-term instability; if the period and amplitude only jitter, but don't *evolve*, then $P(t)$ and $A(t)$ are constant, and δ_P and δ_A are zero.

It is important to remark that the amplitude which emerges from our period analysis is the amplitude of the best-fit sinusoid, with time-dependent period and amplitude, corresponding to a given periodic fluctuation. In this way, we can deduce the amplitude for each of the many periods shown by multiperiodic stars. This is *not* the same as the usual definition of amplitude, the difference between maximum and minimum brightness. We note that the usual definition does not allow us to compute a separate amplitude for each of several separate periods, and we further note that the amplitude of the *main* period (the one with highest amplitude) is a surprisingly good estimate of the traditional, maximum-to-minimum amplitude.

For many periodic fluctuations we detected multiple harmonics, indicative that the light curve shape is not purely sinusoidal. We characterized the 'complexity' of the light curve shape by computing H, the *harmonic fraction of variance.*

Therefore for each period, for each star, we obtain the following data: mean period P, mean amplitude A, period instability δ_P, amplitude instability δ_A, and harmonic fraction of variance H.

3.2. Other Methods

So many of the stars showed evidence of multiperiodicity, that in many cases we felt compelled to investigate the data more closely, to confirm or deny the results. This was particularly true for the longer periods, over 1000 days, especially since almost all of them showed power levels in a Fourier analysis which did not conclusively prove periodicity; they may instead represent characteristic timescales rather than true periods. For some of them we studied the data from a small group of observers, to see whether or not results were consistent from one observer to another. For some stars whose periods seemed to be especially unstable, appearing and disappearing unpredictably, we applied wavelet analysis in the form of the *weighted wavelet Z-transform*, or WWZ (Foster 1996b).

Our best test of controversial results was to analyze another, completely independent data set for a given star. For this we turned to the recently digitized AAVSO data archives from 1911 to 1961. Although these data are not yet part of the AAVSO International Database, in special cases we extracted these earlier data; if the same results exist in both data sets, they may be considered conclusive.

4. CLASSIFICATION

As it turned out, three of the available quantities were most useful for classification: the mean amplitude A, amplitude instability δ_A, and period P. A plot of A versus δ_A (Figure 1) quite naturally reveals the

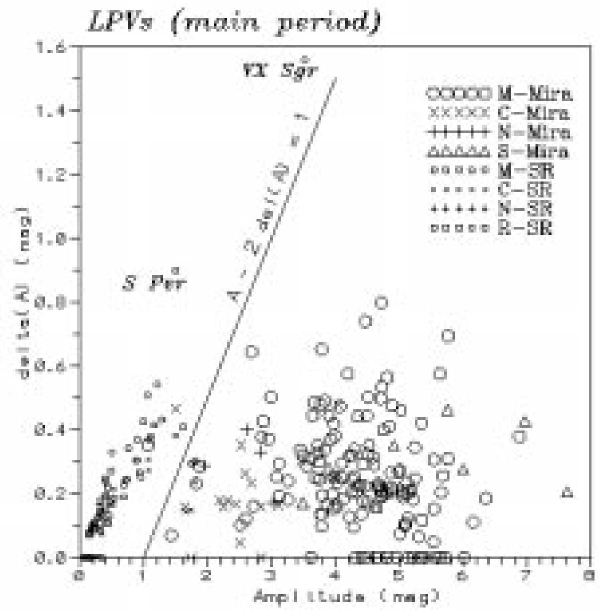

Figure 1. Amplitude A versus amplitude instability δ_A for the main period of LPVs. The line is the cutoff between Mira and SR, given by $A{-}2\delta_A = 1$.

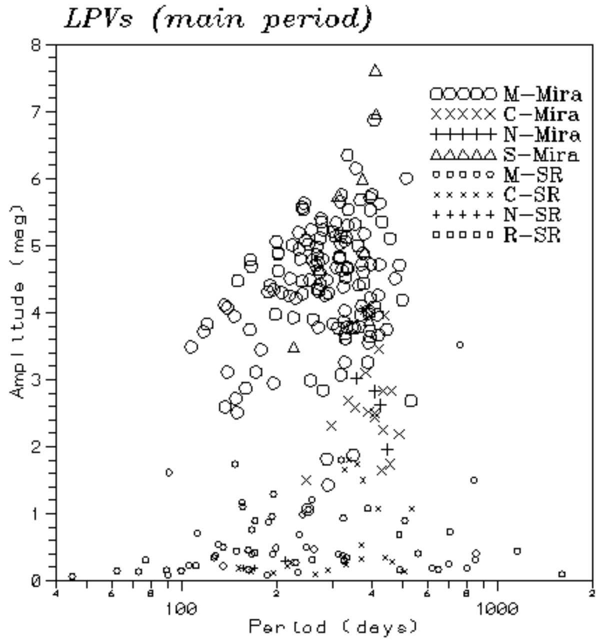

Figure 2. Period P versus amplitude A for the main period of LPVs.

presence of at least two distinct groups. LPVs separate into those with $A-2\delta_A<1$ and those >1. The low-amplitude group consistently shows *very* high δ_A compared to A; these stars are definitely *very* irregular in their light curves. In fact, most of them will, at some time or another, show dramatically reduced amplitude, dropping below our detection threshold.

Our analysis provides striking confirmation of the standard separation of Miras from SRs, based mainly but not entirely upon amplitude. It should come as no surprise that the low-amplitude, highly unstable stars are the SR variables; only two stars classified Mira fall in this group (the M-Mira S Tri and the C-Mira Y Per, both of which may be misclassified as Mira). The high-amplitude group is composed almost entirely of Mira variables.

The period-amplitude plot (Figure 2) shows that there seems to be a subgroup of the Mira variables, with higher period $P>250$ days and lower amplitude $A<4$ mag. Examination of their spectral types shows that most of this group is C-Miras. Considering M-Miras and C-Miras separately, the two groups show very different pulsation properties. The M-Miras show a ‘full’ range of periods, from as low as 100 days to as high (in this study) as 550 days. C-Miras are confined to a much smaller period range, from 250 to 500 days. They also show significantly smaller amplitudes, ranging as low as 1.5 mag, while no C-Mira shows amplitude >4.2 mag. The amplitudes of M-Miras, on the other hand, are almost never below 2.5 mag, and some of them vary as much as 7 magnitudes.

The visual amplitude of Miras is certainly the result of several factors. First, their physical pulsation has large amplitude; shorter period stars will have smaller shocks, at least. Second, the atmosphere of a pulsating star is extremely extended; as shocks propagate through, such important sources of opacity as TiO are likely to dissociate and recombine. Hence the visual magnitude can change drastically, as is characteristic of Miras.

For the carbon stars, the sources of opacity are different. Therefore, the change in visual light over the course of one cycle can be very different, even for the same physical conditions (velocity, luminosity, pulsation mode) of variation. In addition, differences in the opacity could lead to differences in the radius that helps define the pulsation period and mode.

The A versus δ_A plot for M-stars alone (Figure 3) shows why the ‘traditional’ amplitude cutoff for Mira variables is given as 2.5 mag: the vast majority of M-Miras have $A\geq 2.5$. However, a small group (five stars, T CVn, BG Cyg, Z Sco, S Aql, and T Ari) lies in between the SRs and the main body of Miras. They are clearly too stable to be SR (their amplitudes never dip below 1 mag at any time), but of significantly lower amplitude than most M-Miras; we have taken to calling them the ‘M-star oddballs.’ Whether they are more closely related to the SR or Mira stars remains to be seen, but their light curves are qualitatively more like Miras than SRs.

Examination of the amplitudes of SRs (Figure 4) shows that they separate into at least two groups, those with low amplitude $A<0.7$ mag, and high amplitude $A>0.7$ mag. There are far more low-amplitude SRs than high-amplitude, all the more pronounced when one considers that our study is strongly biased in favor of high amplitudes. The low-amplitude stars are almost all SRb, while the high-amplitude are overwhelmingly SRa. Therefore there are definitely two types of SR variable, and they can be conveniently defined by amplitude (although multiperiodicity also may play a role, see Section 6). We emphasize that the usual criterion separating SRa

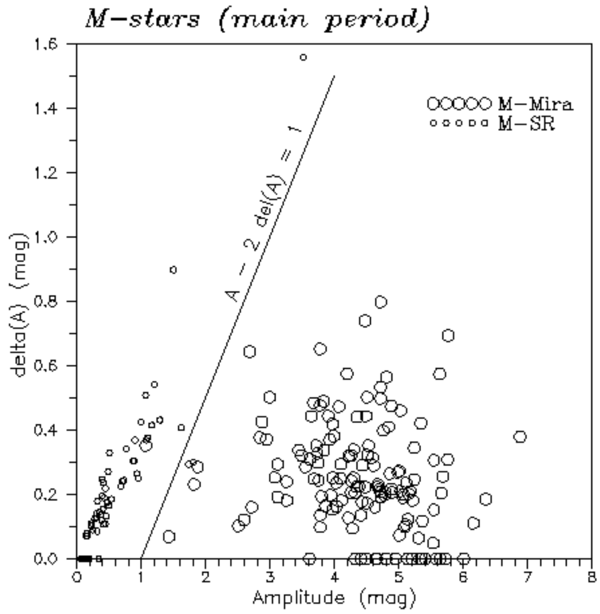

Figure 3. Amplitude A versus amplitude instability δ_A, for the main period of M-type LPVs.

from SRb, their 'regularity,' seems not to be correct; the SRa stars are every bit as irregular as the SRb stars, with just as much tendency for the amplitude, at some moment, to drop below the detection threshold.

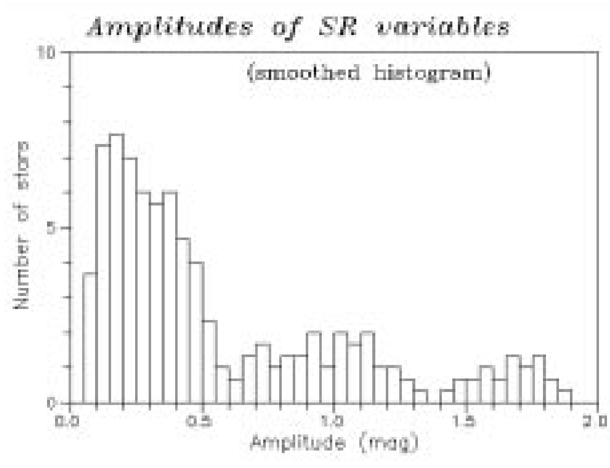

Figure 4. Smoothed histogram of the amplitudes of SR variables.

Based on these considerations, we believe that seven of the stars in our sample need to be reclassified, two called Mira which are actually SR, and five called SR which are actually Mira. They are listed in Table 1.

5. M-MIRA RELATIONSHIPS

M-Miras are by far the largest subgroup of our sample, enabling us to look for trends within the group.

Table 1. Stars which should be reclassified.

Star	Present	Proposed
S Tri	Mira	SRa
Y Per	Mira	SRa
S Aql	SR	Mira
T Ari	SR	Mira
ST And	SR	Mira
S Cam	SR	Mira
SS Vir	SR	Mira

We find several: (1) longer-period M-Miras tend to have larger amplitude, up to period 220 days; beyond that, the mean amplitude shows no correlation with period; (2) longer-period M-Miras are redder in $V-I$ and slightly more *blue* in $B-V$; (3) longer-period M-Miras tend to have less symmetric light curves, and more often show 'bumps' on the light curve. Comparison with numerical models suggests that these are produced by secondary shocks that occur when the pulsation period is roughly twice the minimum 'acoustic cutoff period' in the atmosphere (Bowen 1990).

A most intriguing result of this analysis is the clear presence of a period-dependent upper limit to visual amplitude, which is nearly a linear function of $\log P$ (Figure 5). For periods in the range 100 to 500 days, this may be expressed as:

$$A \leq 5.72 \log P - 7.85. \qquad (1)$$

The physical significance of this is not obvious, but it will likely provide a useful constraint for models of the pulsations and atmospheres of these stars.

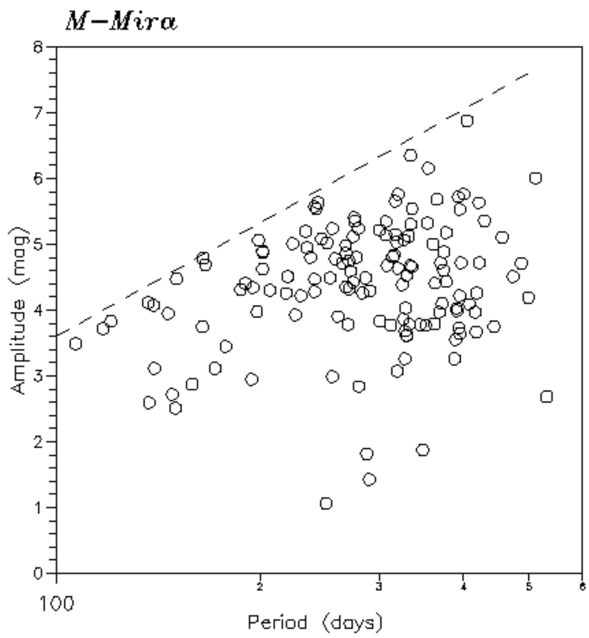

Figure 5. Period P versus amplitude A for M-Miras, showing the upper limit as a function of period.

6. MULTIPERIODIC STARS

A substantial fraction of both Miras and SRs show evidence of multiple periods. Plotting the ratios of their periods and amplitudes (Figure 6), two groups are evident: group 1 shows a large amplitude ratio $A_2/A_1 \approx 10$. These are Miras with a *possible* long period, although invariably the Fourier analysis does not show periodicity conclusively; the power is such that it may be a characteristic timescale rather than truly a period. Group 2 consists of SR variables (and a few Miras) with two periods whose ratio falls in the narrow range $1.7 \leq P_2/P_1 \leq 1.95$.

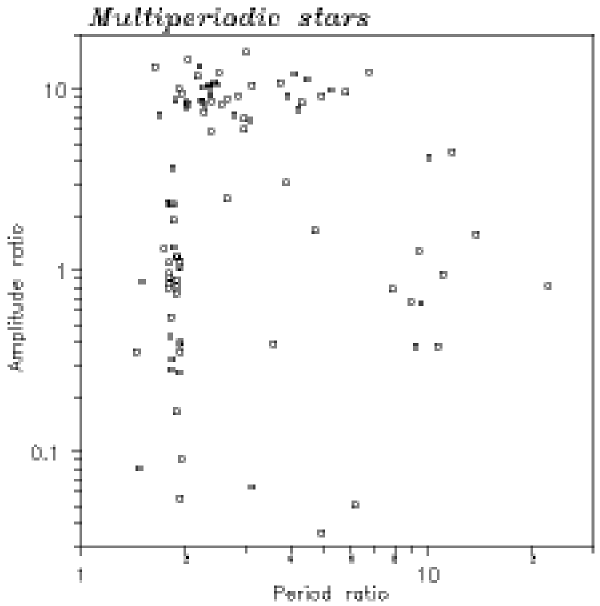

Figure 6. Period ratio $P_{\rm long}/P_{\rm short}$ versus amplitude ratio $A_{\rm long}/A_{\rm short}$ for all possibly multiperiodic stars.

Analysis of AAVSO data prior to 1961 confirmed that the secondary 'periods' of Miras in group 1 are *not* true periods at all. It is noteworthy that so many Miras show such characteristic timescales, that they so often fall near subharmonics of the main period, and that they are *so* inconsistent from one time span to another. We note that this is exactly what we would expect to find, if these Miras were either chaotic or in transition to chaos. We do *not* claim to have found chaos in Mira light curves; we merely point out that their light curves are perfectly consistent with the presence of mild chaos.

For stars with two periods in the ratio between 1.7 and 1.95 (Table 2), both periods invariably turned out to be *real.* There are 30 such stars in our sample (27 SRs and 3 Miras), and they are mostly SRb. For those SRb stars with very low amplitude, we may have missed such a secondary period due to its even lower amplitude, so we isolated all stars with amplitudes from 0.3 to 0.8 mag. For this group, 19 of 30 stars (63 percent!) show two periods in this ratio. We also note that almost all of the multiperiodics have amplitude ≤ 0.8 mag. We may therefore wish to revise the SRa/SRb amplitude boundary to 0.8 mag, and revise the definition of SRb to include multiperiodicity.

What is the meaning of these two periods? If they are modes of pulsation, then the most likely modes are fundamental and first overtone. However, standard linear pulsation analyses yield period ratios for these modes that are mostly in the range from 2.1 to 2.3, higher than the ratio observed (1.7 to 1.95).

It is also possible that these periods represent an interplay between the pulsation period and the *acoustic cutoff period*, the cutoff between periods which propagate freely into the atmosphere and those for which the wave is reflected (at approximately the photosphere) back into the stellar core. Numerical calculations by Bowen (1990) typically show that for pulsation periods longer than the acoustic cutoff period (a necessary condition for large-amplitude pulsation), the resulting atmospheric motions are complex, with secondary shocks, such that the acoustic cutoff frequency may also show up in period analyses of the resulting light curve.

A few stars show evidence of *three* periods. For three of them, we have confirmed triple periodicity, while two others are not yet confirmed (Table 3).

7. HIPPARCOS DATA

Parallax determination for Mira stars by most techniques including those with the Hipparcos satellite, are made more difficult by the physical extent of the stars - angular diameters typically at least as large as the expected parallactic displacement, if not larger. The same limitation, although perhaps to a less extreme degree, probably affects the SR distances, for they are also giants.

Only a few dozen of the stars in our sample have parallaxes which are precise enough to be used on their own, i.e., for which the ratio of the parallax error σ to the parallax itself π satisfies $\sigma_\pi/\pi \leq 0.2$ (and for which the parallax is not negative!). Under such circumstances the use of parallaxes is a statistically delicate procedure (Lutz & Kulker 1973). It is encouraging that analysis of Hipparcos parallaxes *as a data set* indicate these should be consistent and homogeneous and that the stated error estimates are accurate (Lindegren 1995). We intend to fully and carefully analyze Hipparcos data using rigorous methods and expect that they will provide useful constraints on the properties of the Mira and SR variables.

ACKNOWLEDGMENTS

We wish to express our deep and sincere gratitude to all the amateur variable star observers around the world, who for more than a century contributed so much to the science of variable stars, and who played a crucial role in assisting the Hipparcos mission with its observations of variable stars. We gratefully acknowledge the support of NASA under grant NAGW-1493 which made it possible for the AAVSO to provide data support to the Hipparcos mission. J.A.M. thanks M. Grenon and U. Bastian for fruitful discussions on how best to utilize the Hipparcos data on the stars in this study.

Table 2. Stars with two periods in the ratio 1.7 $\leq P_1/P_2 \leq$ 1.95.

Name	Type	Spectrum	P1	A1	P2	A2	P1/P2
RV And	SRA	M4e	166.430	0.761	88.800	0.126	1.8742
TV And	SRA	M4me...	112.005	0.708	62.238	0.305	1.7996
TY And	SRB	M5:e...	262.057	0.466	136.263	0.186	1.9232
UU Aur	SRB	CII...	438.005	0.340	232.828	0.277	1.8812
RX Boo	SRB	MIIIvar	304.700	0.141	162.307	0.167	1.8773
V Boo	SRA	M6e	257.632	1.203	134.443	0.466	1.9163
SV Cas	SRA	M6.5	474.268	0.901	251.395	1.066	1.8865
T Cet	SRC	M5/M6Ib/II	299.500	0.338	162.352	0.453	1.8448
RS Cnc	SRC	M6S	229.155	0.274	128.760	0.218	1.7797
T Cnc	SRB	RV...	893.000	0.189	486.959	0.692	1.8338
T Col	M	M5/M6e	225.524	3.925	116.299	0.356	1.9392
RS CrB	SRA	M7	331.600	0.341	183.000	0.096	1.8120
Y CVn	SRB	CIab:...	290.384	0.161	162.023	0.144	1.7922
AF Cyg	SRB	M5e...	165.949	0.395	92.901	0.379	1.7863
RU Cyg	SRA	M6:e...	434.219	0.297	235.145	0.688	1.8466
W Cyg	SRB	M4IIIe...	236.509	0.230	130.527	0.536	1.8120
S Dra	SRB	M7	325.023	0.304	179.700	0.167	1.8087
TX Dra	SRB	M4:e...	135.700	0.130	76.731	0.305	1.7685
UX Dra	SRA	CII...	330.365	0.231	176.464	0.203	1.8721
T Eri	M	M5/M6IIIe	253.935	4.490	131.620	0.246	1.9293
Y Gem	SRB	M6:e...	273.500	0.234	148.795	0.443	1.8381
SW Mon	SRB	M4IIIvar	193.538	0.398	103.460	0.298	1.8707
BQ Ori	SR	M5IIIvar	245.512	0.361	128.023	0.379	1.9177
RU Per	SRB	M4e	328.745	0.377	170.358	0.422	1.9297
S Sct	SRB	CII...	269.000	0.169	150.560	0.187	1.7867
S Tri	M	M2e	250.251	1.063	131.020	0.289	1.9100
RZ UMa	SRB	M5.5	258.731	0.318	144.120	0.270	1.7952
V UMa	SRB	M5.5	198.520	0.486	108.680	0.157	1.8266
V UMi	SRB	M5III:	126.500	0.104	72.966	0.137	1.7337
W Vul	SRB	M5IIIe	242.081	0.338	126.351	0.345	1.9159

Table 3. Triply periodic stars.

Name	Type	Spectrum	P1	A1	P2	A2	P3	A3	Confirmed?
RX Boo	SRb	MIIIvar	162.307	0.167	304.700	0.141	2691.837	0.349	No
S Dra	SRb	M7	325.023	0.304	179.700	0.167	1323.821	0.262	No
TX Dra	SRb	M4:e...	76.731	0.305	135.700	0.130	715.000	0.249	Yes
G Her	SR	M6III:var	89.510	0.158	62.275	0.056	888.928	0.446	Yes
V UMi	SRb	M5III:	72.966	0.137	126.500	0.104	773.000	0.100	Yes

REFERENCES

Bowen, G. H. 1990, in Numerical Modeling of Nonlinear Stellar Pulsations: Problems and Prospects, NATO Advanced Research Workshop, ed. J. R. Buchler, Kluwer Academic Publishers, pp. 155-171

Foster, G. 1995, AJ, 109, 1889

Foster, G. 1996a, AJ, 111, 541

Foster, G. 1996b, AJ, 112, 1709

Kerschbaum, F., and Hron, J. 1992, A&A, 263, 97

Kholopov, P. N., et al., General Catalogue of Variable Stars, 4th edition, Nauka Publishing House, Moscow 1985

Lindegren, L. 1995, A&A, 304, 61

Lutz, T., and Kelker, D. 1973, PASP, 85, 573