COMP 150-SEN
Software Engineering Foundations

Program Verification

Spring 2019

(Based on lecture notes by Hal Perkins, CS 211, Cornell)

Software Specifications

- A specification defines the *behavior* of an abstraction
- It is a *contract* between the user and provider
  - Provider's code must implement the specification
  - Providers can change the implementation as long as it still meets the specification
  - Users that depend on the implementation could be in trouble
    - Should only rely on the specification

It’s Hard to Write Good Specs

- Very difficult to get developers to write specs
  - Even harder to keep them up to date
- Having specs in a separate document from the code almost guarantees failure
  - Rational for javadoc and similar: extract a standalone specification from the code and embedded comments
- Hard to accurately and formally capture all properties of interest
  - Always finding important details not specified

Specifications Are Useful

- There are lots of subtle algorithms and data structures
  - Internal specs/invariants vital to correct implementation
  - Example: Binary search tree
    - All nodes reachable from left child have smaller key than current node
    - All nodes reachable from right child have larger key than current node
- In the real world, much coding effort goes into modifying previously written code
  - Often originally written by somebody else
  - Documenting and respecting specs avoids a mess

Formal vs. Informal Specifications

```
static int find(int[] d, int x) { … }
```

- Informal
  - If the array `d` is sorted, and some element of `d` is equal to `x`, then `find(d, x)` returns the index of `x` …
- Formal
  - ($\forall$i such that 0<i<d.length . d[i-1] $\leq$ d[i] and
  - $\exists$j such that 0$\leq$j<d.length . d[j] = x)
  - $\Rightarrow$ find(d,x) = j …

Pros and Cons

- Formal specs
  - Force you to be very clear
  - Might be checkable with automated tools
    - Either at compile time (static checking) or run time (dynamic checking)
- Informal specs
  - Some important properties are hard to express formally
    - Sometimes just difficult and long
    - Sometimes don’t have the necessary formal notation
  - Some people are intimidated by formal specs

Type of External Specifications

- Specifications on method
  - Preconditions: What must be true before call
  - Postconditions: What is made true after call
    - Often relates return values to argument values

```
// precondition: d is sorted
// postcondition:
//   (ret≥0 ∧ d[ret] == x) ∨ (ret=-1 ∧ x∉d)

static int find(int[] d, int x) { … }
```

  - Potentially also: side effects
    - What heap state is modified by calling the method
  - Potentially also: performance
    - Exact bounds? Maybe high-level: random access fast, insertion slow…

Types of Internal Specifications

- Specs appearing within the code itself
- Loop invariants: condition that must hold at the the beginning of each iteration of a loop

```
static int find(int[] d, int x) {
  // inv: ∀0≤j<i . d[j]≠x
  for (int i=0; i<d.length; i++) {
    if (d[i]==x) return i;
  }
}
```

  - More on these shortly!
- Object invariants (more later!)

```
class Buffer {
  char[] bytes;
  int filled;   // inv: 0≤filled<bytes.length
}
```

Specification and Subtyping

- The Liskov Substitution Principle:

> `S` is a subtype of `T` if anyone expecting a `T` can be given an `S` instead

- How do specs of original and overriden method relate?

```
// pre: d is sorted
// post: (ret≥0 ∧ d[ret] == x) ∨ (ret=-1 ∧ x∉d)
static int find(int[] d, int x) { … }
```

- If we override `find`, can the new method
  - Have true as a precondition?
  - Have `d is sorted and ∃i.d[i]=x` as a precondition?
  - Have postcond `(ret=-1 ∧ x∉d)` (only)?
  - Throw `NoSuchElementException` rather than returning -1 if `x∉d`

General Rule for Overriding

```
class A {
  // pre: preA, post: postA
  m() { … }
}
class B extends A {
  // pre: preB, post: postB
  m() { … }
}
```

- Postconditions must be related as follows
  - We want: Anyone who expects an `A` can be given a `B` (subtyping definition)
  - So, if someone is expecting `postA` to hold after the call, it's okay if `postB` holds as long as `postB ⇒ postA`

General Rule for Overriding (cont'd)

```
class A {
  // pre: preA, post: postA
  m() { … }
}
class B extends A {
  // pre: preB, post: postB
  m() { … }
}
```

- Preconditions must be related as follows
  - We want: Anyone who expects an `A` can be given a `B` (subtyping definition)
  - So, if someone makes `preA` hold before the call, it's okay to call a method expecting `preB` as long as `preA ⇒ preB`

General Rule for Overriding (cont'd)

```
class A {
  // pre: preA, post: postA
  m() { … }
}
class B extends A {
  // pre: preB, post: postB
  m() { … }
}
```

- Cumulatively, for `B < A` (`B` subtype of `A`) need
  - `postB ⇒ postA`
  - `preA ⇒ preB`
- Notice the direction flips between pre/post
  - Postcondition is *covariant*, precondition is *contravariant*

Javadoc

- Documentation as source code comments
  - Running `javadoc` on code creates separate html doc files

```
/** Javadoc Comment for this class */
public class MyClass {
  /** Javadoc Comment for field text */
  String text;

  /** Given a sorted array, returns the index into
      the array of the given element, otherwise
      returns -1.

      @param d array to search in, assumed sorted
      @param x the element to search for
      @returns i >= 0 when d[i] == x, and -1 when
               x does not occur in d */
  public static int find(int d[], int x) { }
}
```

Javadoc Output (1/2)

Class MyClass

java.lang.Object
    MyClass

```
public class MyClass
extends java.lang.Object
```

Javadoc Comment for this class

...

Method Summary

All Methods | Static Methods | Concrete Methods

Modifier and Type	Method and Description
`static int`	`find(int[] d, int x)` Given a sorted array, returns the index into the array of the given element, otherwise returns -1.

Javadoc Output (2/2)

Method Detail

find

```
public static int find(int[] d,
                       int x)
```

Given a sorted array, returns the index into the array of the given element, otherwise returns -1.

Parameters:

`d - array to search in, assumed sorted`

`x - the element to search for`

Testing vs. Verification

“Program testing can be used to show the presence of bugs, but never to show their absence!” — Edsgar Dijkstra, *Notes on Structured Programming*, 1970

- Testing only
- So then, how do we show the absence of bugs?

- Enter *program verification*
  - Given program and spec, *prove* the program meets the spec
  - Idea pioneered by
    - C.A.R. Hoare, Proof of a program: FIND. CACM 14(1), Jan. 1971
  - Difficult to make progress on idea for a long time, but there have been many recent successes
    - Warning: those successes require work far beyond what we’ll see in class

Predicates and Assertions

- We will talk about programs using *predicates*
  - A predicate is any boolean expression
  - Examples:

```
true
false
x = 5   // not assignment!
x > 5 ∨ x < 10
y ≠ null
```

- An *assertion* is a predicate written in {}'s

```
x = 4;
y = x + 2;
{ x = 4 ∧ y = 6 }
```

  - `{ p }` indicates that p is true at that program point

Hoare Triples

- Putting an assertion before and after a statement creates a *Hoare Triple*

```
{Q} S {R}
```

- This means
  - If `Q` holds before executing S and
  - if `S` is executed and
  - if `S` terminates then
  - `R` will be true in the final state
- This is a *partial correctness* spec
  - It also holds if `S` doesn't terminate
- `Q` is the precondition, `R` is the postcondition

Hoare Triple Examples

```
{x<0} x++; {x<1}
```

- If `x` is less than `0` and we increment `x`, then `x` is less than 1
  - This Hoare triple is *valid*, i.e., it is true

```
{x=4} x=5; {x=5}
```

- if `x` is `4` and we set `x` to `5`, then `x` is `5`
  - This Hoare triple is valid valid
  - But notice the precondition is *stronger* than we need
    - Here, *stronger* means *more restrictive*
  - The *weakest* or most general possible precondition is

```
{true} x=5; {x=5}
```

- Convention: use capital letters to refer to arbitrary values of variables in preconditions

```
{x=X} y=x; { x=X ∧ y=X }
```

A Longer Example

- We can interleave assertions between statements to give us several triples that combine to show a sequence of statements correct

```
{ true }
if (x ≤ y)
  { x ≤ y, hence x = min(x,y) }
  z = x;
  { z = min(x,y) }
else
  { y < x, hence y = min(x,y) }
  z = y;
  { z = min(x,y) }
{ z = min(x,y) }
```

  - `{}`’s omitted for if/else to avoid confusion with triples

Two Annotated Loops

```
{ x ≥ 0 }
while (x ≠ 0) x--;
{ x  = 0 }
```

```
{ true }
while (x ≠ 0) x--;
{ x  = 0 }
```

- Both of these are valid
  - But notice the one on the right might not terminate!
    - If `x < 0`

Total vs. Partial Correctness

- We can also define a total correctness version of Hoare triples

```
[Q] S [R]
```

- Meaning
  - If `Q` holds before executing S and
  - if `S` is executed then
  - `S` terminates and
  - `R` will be true in the final state
- Total and partial correctness are only different for loops

Two Annotated Loops, Again

```
[ x ≥ 0 ]
while (x ≠ 0) x--;
[ x  = 0 ]
```

```
[ true ]
while (x ≠ 0) x--;
[ x  = 0 ]
```

- Now the version on the left is valid, but the version on the right is invalid
  - Because the code on the right might not terminate

Proving Hoare Triples Correct

- We have examples of valid and invalid triples
  - We've been figuring this out informally so far
  - We need to establish ground rules for when triples are valid

- We need to develop a *logic*
  - In particular, we want a mechanical system of *proof rules*
    - We will stipulate that the proof rules are correct
    - Thus, we will say a triple is valid if and only if we can show that it is valid according to the proof rules

- Same rules work for partial and total correctness
  - Except for termination, which we'll discuss separately

Proof Rule: Sequencing

```
If {A} S1 {B} and {B} S2 {C} are valid
Then {A} S1; S2 {C} is valid
```

- Example
  - The following two triples are valid

```
{ true } x = 0; { x=0 }
```

```
{ x=0 } y = x + 1; { x=0 ∧ y=1}
```

  - Thus, the following is valid

```
{ true } x = 0; y = x + 1; { x=0 ∧ y=1}
```

Proof Rule: Conditional

```
If {A∧b} S1 {B} and {A∧¬b} S2 {B} are valid
Then {A} if b then S1 else S2 {B} is valid
```

- Example
  - To show the following is valid

```
{x=X} if (x<0) then y =-x; else y = x; { y=|X|}
```

  - We need to show the following two triples are valid, which they are

```
{x=X ∧ x<0} y =-x; {y=|X|}
```

```
{x=X ∧ x≥0} y = x; {y=|X|}
```

Assignment and Substitution

- The proof rule for assignment has the form

```
{"fact about e"} x=e; {"fact about x"}
```

  - For example:

```
{"2*n is even"} x=2*n; {"x is even"}
```

- To make this precise, we need to define *substitution*
  - We write `P[x↦e]` to mean predicate `P` in which variable `x` is replaced by expression `e`
    - `(x=y)[x↦w]   =   (w=y)`
    - `(x=y)[x↦x+2]   =   (x+2=y)`
    - `(y=x+x)[x↦z+x+y]   =   (y=z+x+y+z+x+y)`
    - `(z=x*y)[x↦a+b]   =   (z=x+(a*b))`   // added ()'s

Proof Rule: Assignment

```
{A[x↦e]} x=e {A} is valid
```

- Examples:

```
{ true } x = 2; { x=2 }
```

```
{ x+1≥0 } x = x+1; { x≥0 }
```

  - or, simplifying:

```
{ x≥-1 } x = x+1; { x≥0 }
```

- Notice this rule is very easy to apply “backwards”

```
{ ?? } x = x-y; { x*y + y*y = 5 }
```

  - What is `??`
  - Must be `(x-y)*y + y*y = 5`
  - Or `x*y - y*y + y*y = 5` or `x*y = 5`

Proof Rule: Consequence

- Sometimes we need to add some implications to get the assertions we really want

```
If {A} S {B} is valid and A’ ⇒ A and B ⇒ B’
Then {A’} S {B’} is valid
```

- Example: the assignment rule gives us

```
{ x>-1 } y = x + 1; { y>0 }
```

  - Since `x=0 ⇒ x>-1`, we can also conclude

```
{ x=0 } y = x + 1; { y>0 }
```

  - Also, since `y>0 ⇒ y>-5`, we can also conclude

```
{ x=0 } y = x + 1; { y>-5 }
```

Checking Validity Mechanically

- Given an assignment and a postcondition, we can use the assignment rule to compute the *weakest precondition*
  - I.e., least restrictive precondition that will cause the postcondition to hold
- This suggests a mechanical way to check whether `{A} S {B}` holds for straight line code
  - Start with `B`
  - Apply the assignment rule repeatedly to work back to the start, yielding the weakest precondition `A'`
  - Check whether `A ⇒ A'`
- We can generalize this to other constructs

Weakest Preconditions

- Define `wp(S, B)` to be the weakest precondition of `B`, as follows
  - `wp(x=e, B) = B[x↦ e]`
  - `wp(S1; S2, B) = wp(S1, wp(S2, B))`
  - `wp(if E then S1 else S2, B) =`

    `E⇒ wp(S1, B) ∧ ¬E⇒ wp(S2, B)`

Assignment and Pointers

- Warning: The assignment rule does not yield the weakest precondition in the presence of pointers

```
{ *y = 5 } *x = 5; { *x + *y = 10 }
```

  - The above is true, but the actual weakest precondition is

```
{ *y = 5 or x = y} *x = 5; { *x + *y = 10 }
```

- To extend this style or reasoning to include pointers, see work on *separation logic*

Loops

- There is no mechanical way to automatically check whether loops are correct
- Two key problems
  - Weakest preconditions might not terminate, because it needs to keep going backward through the loop repeatedly
  - We'd like to also know whether loops terminate, which weakest preconditions doesn't reason about
- Instead, we're going to develop a pen-and-paper way of reasoning about loops
  - You'll see how this is translated into practice in a future lecture

Summation in a Loop

- Goal: prove the following triple correct

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

  - Notice this is total correctness, i.e., we need to prove the loop terminates
  - (We will ignore concerns about the array length)

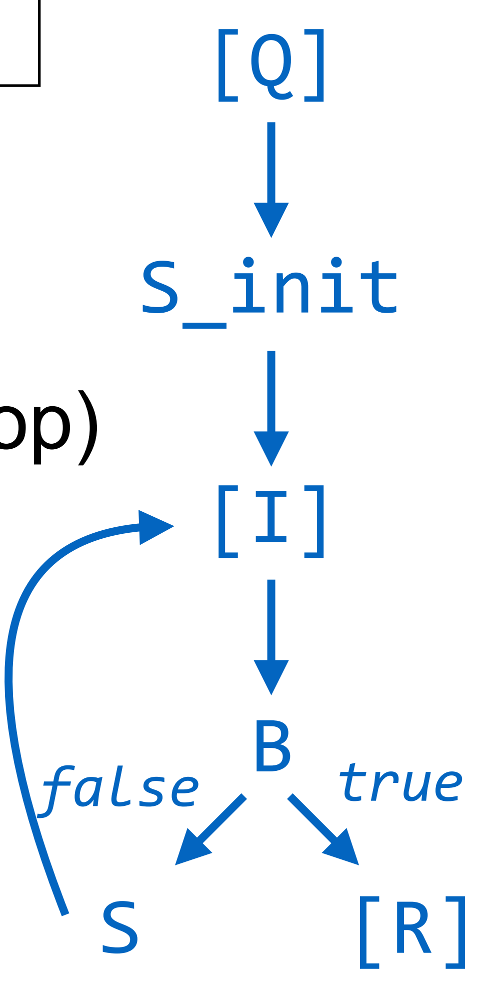
Loop Invariants

- Key idea: think about one loop iteration, generically
- A *loop invariant* is an assertion that holds at the beginning and end of each execution of the loop

```
[Q] S_init; while(B) { S } [R]
```

  - Let `I` be the loop invariant

- To prove this loop correct
  1. Show `[Q] S_init [I]` (invariant holds before loop)
  2. Show `[I∧B] S [I]` (invariant holds across loop)
  3. Show `I∧¬B⇒ R` (post holds after loop)



Loop Invariant Example

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
     0 ≤ k < n)
```

1. `[true] sum=a[0]; k=0; [sum=a[0]+…+a[k] ∧ 0≤k<n]`
   - Holds by assignment rules

Loop Invariant Example (cont'd)

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
     0 ≤ k < n)
```

2. `[sum=a[0]+…+a[k] ∧ 0≤k<n ∧ k ≠ n-1]`

   `k=k+1; sum=sum+a[k];`

   `[sum=a[0]+…+a[k] ∧ 0≤k<n]`

   - Holds by assignment rules and consequence

Loop Invariant Example (cont'd)

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
    0 ≤ k < n)
```

3. (sum=a[0]+…+a[k] ∧ 0≤k<n ∧ k = n-1) ⇒

   sum = a[0]+a[1]+…+a[n-1]

   - Holds by standard logical reasoning

Bound Function for Termination

- A *bound function* `t` is
  - An integer-valued expression defined in terms of program variables
  - `t` must strictly decrease with every loop iteration
  - `t≥0` always, so that when it reaches `0` the loop terminates

- I.e., a bound function is an upper bound on the number of remaining loop iterations

Bound Function Example

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
t:  n-1-k
```

- Check that `t` is a bound function
  - `t` strictly decreases with each step because `k` increases by `1`
  - `t` can never go below zero because the loop terminates with `k=n-1`
- Therefore, the loop terminates

Invariants are Abstractions

- An *invariant* is just an assertion at a program point
- We've seen four kinds of invariants
  - Precondition: invariant at method entry
  - Postconditions: invariant at method exit
  - Object invariant: invariant about fields that holds at beginning and end of every method within a class
  - Loop invariant: invariant at every iteration of a loop
- Invariants create an abstraction barrier
  - Invariant must be established by code before it
  - Code after it can rely on the invariant being true
- Invariants are a powerful tool for understand code!
  - Try to think about what always holds at a program point
  - Add assertions to code to confirm your understanding