# ME 417

## Design of Alternative Energy Systems

### Fuel Cell Problems Solutions

1. Determine the ideal voltage, current, and required mass flow rate of hydrogen for an ideal air/hydrogen fuel cell operating at 101 kPa and 350 K that is producing 25 kW.

**Solution:**

Our general energy equation is

$$\bar{w}_{elec} = \sum_{reactants} \nu_i \bar{h}_i - \sum_{products} \nu_j \bar{h}_j - T_{FC}\left\{\sum_{reactants} \nu_i \bar{s}_i - \sum_{products} \nu_j \bar{s}_j\right\}$$

For a hydrogen/air fuel cell the balanced chemical reaction equation is given by

$$H_2 + ½O_2 + ½(3.76)N_2 \rightarrow H_2O + ½(3.76)N_2$$

which allows use to write our energy equation as

$$\bar{w}_{elec} = \bar{h}_{H_2} + (0.5)\bar{h}_{O_2} + (1.88)\bar{h}_{N_2,r} - \bar{h}_{H_2O} - (1.88)\bar{h}_{N_2,p}$$
$$- T_{FC}\left\{\bar{s}_{H_2} + (0.5)\bar{s}_{O_2} + (1.88)\bar{s}_{N_2,r} - \bar{s}_{H_2O} - (1.88)\bar{s}_{N_2,p}\right\}$$

We assume that we have an isothermal fuel cell at 350 K, so that

$$T_{FC} = T_p = T_r = 350 \text{ K}$$

So we can now evaluate our enthalpies

$$\bar{h}_{H_2} = \bar{h}_{f,H_2} + \Delta\bar{h}_{H_2} = 0 + (9971 - 8468) = 1503 \text{ kJ/kmole}$$

$$\bar{h}_{O_2} = \bar{h}_{f,O_2} + \Delta\bar{h}_{O_2} = 0 + (10{,}213 - 8682) = 1531 \text{ kJ/kmole}$$

$$\bar{h}_{H_2O} = \bar{h}_{f,H_2O} + \Delta\bar{h}_{H_2O} = -241{,}820 + (11{,}652 - 9904) = -240{,}070 \text{ kJ/kmole}$$

We have not included the enthalpies for the $N_2$, since we know that they will cancel out. To evaluate the entropies we need to have our partial pressures. Calculating them we have

$$y_{H_2} = \frac{1}{1 + 0.5 + 1.88} = 0.30$$

$$y_{O_2} = \frac{0.5}{1 + 0.5 + 1.88} = 0.15$$

$$y_{N_2,r} = \frac{1.88}{1 + 0.5 + 1.88} = 0.55$$

$$y_{H_2O} = \frac{1}{1 + 1.88} = 0.35$$

$$y_{N_2,p} = \frac{1.88}{1+1.88} = 0.65$$

Going to the ideal gas tables at 350 K we find

$$\bar{s}^o_{H_2} = 135.21\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{O_2} = 209.765\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{N_2,r} = 196.173\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{H_2O} = 194.125\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{N_2,p} = 199.173\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

Now substituting

$$\bar{w}_{elec} = 1503 + (0.5)(1531) - (-240{,}070)$$

$$-(298)\left\{\begin{array}{l}(135.21-(8.314)\ln\{0.3\}) + (0.5)(209.768-(8.314)\ln\{0.15\}) \\ +(1.88)(196.173-(8.314)\ln\{0.55\}) - (194.125-(8.314)\ln\{0.35\}) \\ -(1.88)(196.173-(8.314)\ln\{0.65\})\end{array}\right\}$$

$$\bar{w}_{elec} = 242{,}340 - (350)(34.22) = 230{,}360\ \text{kJ/kmole}$$

The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{FC}\left\{\sum\limits_{reactants}\nu_i\bar{s}_i - \sum\limits_{products}\nu_j\bar{s}_j\right\}}{\sum\limits_{reactants}\nu_i\bar{h}_i - \sum\limits_{products}\nu_j\bar{h}_j} = 1 - \frac{(350)(34.22)}{(242{,}340)} = 0.95$$

or 95%. Next we find

$$\dot{m}_{fuel} = \frac{MW_{fuel}\dot{W}_{elec}}{\bar{w}_{elec}} = \frac{(2)(25)}{230{,}360} = 2.17 \text{ x } 10^{-4}\ \text{kg/s}$$

Our ideal voltage is then given by

$$V = \frac{\bar{w}_{elec}}{(96{,}487)N_e} = \frac{(230{,}360)}{(96{,}487)(2)} = 1.19\ \text{V}$$

The current is

$$I = \frac{\dot{W}_{elec}}{V} = \frac{25{,}000}{1.19} = 20{,}943\ \text{amps}$$

2. Determine the ideal voltage, current, and required mass flow rate of ethanol ($C_2H_6O$) for an ideal air/ethanol–water fuel cell operating at 101 kPa and 280 K that is producing 500 kW. You may take the $c_P$ of ethanol to be 2.77 kJ/(kg·K).

**Solution:**

Our general energy equation is

$$\overline{w}_{elec} = \sum_{reactants} \nu_i \overline{h}_i - \sum_{products} \nu_j \overline{h}_j - T_{FC}\left\{\sum_{reactants} \nu_i \overline{s}_i - \sum_{products} \nu_j \overline{s}_j\right\}$$

For a ethanol-water/air fuel cell the balanced chemical reaction equation is given by

$$C_2H_5OH(\ell) + 3O_2 + 3(3.76)N_2 \rightarrow 2CO_2 + 3H_2O(\ell) + 3(3.76)N_2$$

which allows use to write our energy equation as

$$\overline{w}_{elec} = \overline{h}_{C_2H_6O(\ell)} + 3\overline{h}_{O_2} + (11.28)\overline{h}_{N_2,r} - 2\overline{h}_{CO_2} - 3\overline{h}_{H_2O(\ell)} - (11.28)\overline{h}_{N_2,p}$$
$$- T_{FC}\left\{\overline{s}_{C_2H_6O(\ell)} + 3\overline{s}_{O_2} + (11.28)\overline{s}_{N_2,r} - 2\overline{s}_{CO_2} - 3\overline{s}_{H_2O(\ell)} - (11.28)\overline{s}_{N_2,p}\right\}$$

We assume that we have an isothermal fuel cell at 280 K, so that

$$T_{FC} = T_p = T_r = 280 \text{ K}$$

So we can now evaluate our enthalpies

$$\overline{h}_{C_2H_6O(\ell)} = \overline{h}_{f,C_2H_6O(\ell)} + \Delta\overline{h}_{C_2H_6O(\ell)} = -277{,}690 + (2.44)(46)(280-298)$$
$$= -279{,}110 \text{ kJ/kmole}$$
$$\overline{h}_{O_2} = \overline{h}_{f,O_2} + \Delta\overline{h}_{O_2} = 0 + (8150 - 8682) = -532 \text{ kJ/kmole}$$
$$\overline{h}_{H_2O(\ell)} = \overline{h}_{f,H_2O(\ell)} + \Delta\overline{h}_{H_2O(\ell)} = -285{,}830 + (4.2)(18)(280-298)$$
$$= -286{,}230 \text{ kJ/kmole}$$
$$\overline{h}_{CO_2} = \overline{h}_{f,CO_2} + \Delta\overline{h}_{CO_2} = -393{,}520 + (8697 - 9364) = -394{,}190 \text{ kJ/kmole}$$

To evaluate the entropies we need to have our partial pressures. Calculating them we have

$$y_{O_2} = \frac{3}{3+11.28} = 0.21$$
$$y_{N_2,r} = \frac{11.28}{3+11.28} = 0.79$$
$$y_{CO_2} = \frac{2}{2+11.28} = 0.15$$
$$y_{N_2,p} = \frac{11.28}{2+11.28} = 0.85$$

Determining the entropies at 280 K we find

$$\overline{s}^o_{C_2H_6O(\ell)} = 160.70 + (2.44)(46)\ln(280/298) = 153.71 \text{ kJ/(kmole·K)}$$

$$\bar{s}^o_{O_2} = 203.191\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{N_2,r} = 189.673\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{H_2O} = 69.29 + (4.2)(18)\ln(280/298) = 64.58\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{CO_2} = 213.685\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

$$\bar{s}^o_{N_2,p} = 189.673\,\text{kJ}/(\text{kmole}\cdot\text{K})$$

Now substituting

$$\bar{w}_{elec} = -279{,}110 + (3)(-532) - (2)(-394{,}190) - (3)(-286{,}230)$$

$$-(280)\left\{\begin{array}{l}(153.71) + (3)(203.191 - (8.314)\ln\{0.21\}) \\ +(11.28)(189.673 - (8.314)\ln\{0.79\}) - (2)(213.685 - (8.314)\ln\{0.15\}) \\ -(3)(64.58) - (11.28)(189.673 - (8.314)\ln\{0.85\})\end{array}\right\}$$

$$\bar{w}_{elec} = 1{,}366{,}400 - (280)(156.42) = 1{,}322{,}600\ \text{kJ/kmole}$$

The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{FC}\left\{\sum\limits_{reactants}\nu_i\bar{s}_i - \sum\limits_{products}\nu_j\bar{s}_j\right\}}{\sum\limits_{reactants}\nu_i\bar{h}_i - \sum\limits_{products}\nu_j\bar{h}_j} = 1 - \frac{(280)(156.42)}{(1{,}366{,}400)} = 0.97$$

or 97%. Next we find

$$\dot{m}_{fuel} = \frac{MW_{fuel}\dot{W}_{elec}}{\bar{w}_{elec}} = \frac{(46)(500)}{1{,}322{,}600} = 1.74 \times 10^{-2}\ \text{kg/s}$$

To obtain our ideal voltage we need to determine the number of electrons released at the anode. Our general ion reaction equation at the anode is

$$C_xH_yO_z + (2x\text{-}z)H_2O \rightarrow xCO_2 + (4x\text{+}y\text{-}2z)H^+ + (4x\text{+}y\text{-}2z)e^-$$

For the isopropanol this becomes

$$C_2H_6O + (3)H_2O \rightarrow 2CO_2 + (12)H^+ + (12)e^-$$

So that

$N_e$ = 12 kmole of electrons/kmole of $C_2H_6O$

Our ideal voltage is then given by

$$V = \frac{\bar{w}_{elec}}{(96{,}487)N_e} = \frac{(1{,}322{,}600)}{(96{,}487)(12)} = 1.14\,\text{V}$$

The current is

$$I = \frac{\dot{W}_{elec}}{V} = \frac{500{,}000}{1.14} = 438\,\text{kA}$$

3. Determine the ideal voltage, current, number of stacks required, and required mass flow rate of isopropanol ($C_3H_7OH$) for an ideal air/isopropanol –water fuel cell operating at 101 kPa and 298 K that is producing 50 kW with 4 volts. You may take the following properties for liquid isopropanol.

$c_P = 2.35$ kJ/(kgK)

$$\bar{h}_f = -316{,}000 \text{kJ/kmole}$$

$$\bar{s}^o(\text{at}298\text{K}) = 194.6 \text{kJ/(kmole} \cdot \text{K)}$$

**Solution:**

Our general energy equation is

$$\bar{w}_{elec} = \sum_{reactants} \nu_i \bar{h}_i - \sum_{products} \nu_j \bar{h}_j - T_{FC}\left\{\sum_{reactants} \nu_i \bar{s}_i - \sum_{products} \nu_j \bar{s}_j\right\}$$

For a isopropanol-water/air fuel cell the balanced chemical reaction equation is given by

$$C_3H_7OH(\ell) + (4.5)O_2 + (4.5)(3.76)N_2 \rightarrow 3CO_2 + 4H_2O(\ell) + (4.5)(3.76)N_2$$

which allows use to write our energy equation as

$$\bar{w}_{elec} = \bar{h}_{C_2H_6O(\ell)} + (4.5)\bar{h}_{O_2} + (16.92)\bar{h}_{N_2,r} - 3\bar{h}_{CO_2} - 4\bar{h}_{H_2O(\ell)} - (16.92)\bar{h}_{N_2,p}$$
$$- T_{FC}\left\{\bar{s}_{C_2H_6O(\ell)} + (4.5)\bar{s}_{O_2} + (16.92)\bar{s}_{N_2,r} - 3\bar{s}_{CO_2} - 4\bar{s}_{H_2O(\ell)} - (16.93)\bar{s}_{N_2,p}\right\}$$

We assume that we have an isothermal fuel cell at 298 K, so that

$$T_{FC} = T_p = T_r = 298 \text{ K}$$

So we can now evaluate our enthalpies

$$\bar{h}_{C_3H_8O(\ell)} = \bar{h}_{f,C_3H_8O(\ell)} + \Delta\bar{h}_{C_3H_8O(\ell)} = -316{,}000 + 0 = -316{,}000 \text{ kJ/kmole}$$
$$\bar{h}_{O_2} = \bar{h}_{f,O_2} + \Delta\bar{h}_{O_2} = 0 + 0 = 0 \text{ kJ/kmole}$$
$$\bar{h}_{N_2,r} = \bar{h}_{f,N_2} + \Delta\bar{h}_{N_2,r} = 0 + 0 = 0 \text{ kJ/kmole}$$
$$\bar{h}_{CO_2} = \bar{h}_{f,CO_2} + \Delta\bar{h}_{CO_2} = -393{,}520 + 0 = 393{,}520 \text{ kJ/kmole}$$
$$\bar{h}_{H_2O(\ell)} = \bar{h}_{f,H_2O(\ell)} + \Delta\bar{h}_{H_2O(\ell)} = -285{,}830 + 0 = -285{,}830 \text{ kJ/kmole}$$
$$\bar{h}_{N_2,p} = \bar{h}_{f,N_2} + \Delta\bar{h}_{N_2,p} = 0 + 0 = 0 \text{ kJ/kmole}$$

To evaluate the entropies we need to have our partial pressures. Calculating them we have by only including our gaseous components

$$y_{O_2} = \frac{4.5}{4.5 + 16.92} = 0.21$$
$$y_{N_2,r} = \frac{16.92}{4.5 + 16.92} = 0.79$$
$$y_{CO_2} = \frac{3}{3 + 16.92} = 0.15$$

$$y_{N_2,p} = \frac{16.92}{3+16.92} = 0.85$$

For the entropies at 298 K we find

$$\bar{s}^o_{C_3H_8O} = 194.6\ \text{kJ/(kmole}\cdot\text{K)}$$

$$\bar{s}^o_{O_2} = 205.033\ \text{kJ/(kmole}\cdot\text{K)}$$

$$\bar{s}^o_{N_2,r} = 191.502\ \text{kJ/(kmole}\cdot\text{K)}$$

$$\bar{s}^o_{CO_2} = 213.685\ \text{kJ/(kmole}\cdot\text{K)}$$

$$\bar{s}^o_{H_2O(\ell)} = 69.92\ \text{kJ/(kmole}\cdot\text{K)}$$

$$\bar{s}^o_{N_2,p} = 191.502\ \text{kJ/(kmole}\cdot\text{K)}$$

Now substituting

$$\begin{aligned}\bar{w}_{elec} &= -316{,}000 + (4.5)(0) + (16.92)(0) - (3)(-393{,}520) - (4)(-285{,}830) - (16.92)(0)\\ &\quad - (298)\left\{\begin{aligned}&(194.6) + (4.5)(205.033 - (8.314)\ln\{0.21\})\\ &+ (16.92)(191.502 - (8.314)\ln\{0.79\}) - (3)(213.685 - (8.314)\ln\{0.15\})\\ &- (4)(69.92) - (16.92)(191.502 - (8.314)\ln\{0.85\})\end{aligned}\right\}\end{aligned}$$

$$\bar{w}_{elec} = 2{,}007{,}900 - (298)(217.88) = 1{,}943{,}000\ \text{kJ/kmole}$$

The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{FC}\left\{\sum\limits_{reactants}\nu_i\bar{s}_i - \sum\limits_{products}\nu_j\bar{s}_j\right\}}{\sum\limits_{reactants}\nu_i\bar{h}_i - \sum\limits_{products}\nu_j\bar{h}_j} = 1 - \frac{(298)(217.88)}{(2{,}007{,}900)} = 0.97$$

Next we find

$$\dot{m}_{fuel} = \frac{MW_{fuel}\dot{W}_{elec}}{\bar{w}_{elec}} = \frac{(60)(50)}{1{,}943{,}000} = 1.54 \times 10^{-3}\ \text{kg/s}$$

To obtain our ideal voltage we need to determine the number of electrons released at the anode. Our general ion reaction equation at the anode is

$$C_xH_yO_z + (2x-z)H_2O \rightarrow xCO_2 + (4x+y-2z)H^+ + (4x+y-2z)e^-$$

For the isopropanol this becomes

$$C_3H_8O + (5)H_2O \rightarrow 3CO_2 + (18)H^+ + (18)e^-$$

So that

$$N_e = 18 \text{ kmole of electrons/kmole of } C_3H_8O$$

Then our ideal voltage is then given by

$$V = \frac{\overline{w}_{elec}}{(96{,}487)N_e} = \frac{(1{,}943{,}000)}{(96{,}487)(18)} = 1.12\text{ V}$$

The number of stacks required is

number of stacks = $V_{required}/V_{cell}$ = (4)/(1.12) = 4 stacks

The current is

$$I = \frac{\dot{W}_{elec}}{V} = \frac{50{,}000}{4} = 12{,}500\text{ amps} = 12.5\text{ kA}$$