

# ME 417

## Design of Alternative Energy Systems

### Fuel Cell Problems Solutions

1. Determine the ideal voltage, current, and required mass flow rate of hydrogen for an ideal air/hydrogen fuel cell operating at 101 kPa and 350 K that is producing 25 kW.

**Solution:**

Our general energy equation is

$$\bar{w}_{\text{elec}} = \sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j - T_{\text{FC}} \left\{ \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \right\}$$

For a hydrogen/air fuel cell the balanced chemical reaction equation is given by



which allows use to write our energy equation as

$$\begin{aligned} \bar{w}_{\text{elec}} = & \bar{h}_{\text{H}_2} + (0.5)\bar{h}_{\text{O}_2} + (1.88)\bar{h}_{\text{N}_{2,r}} - \bar{h}_{\text{H}_2\text{O}} - (1.88)\bar{h}_{\text{N}_{2,p}} \\ & - T_{\text{FC}} \left\{ \bar{s}_{\text{H}_2} + (0.5)\bar{s}_{\text{O}_2} + (1.88)\bar{s}_{\text{N}_{2,r}} - \bar{s}_{\text{H}_2\text{O}} - (1.88)\bar{s}_{\text{N}_{2,p}} \right\} \end{aligned}$$

We assume that we have an isothermal fuel cell at 350 K, so that

$$T_{\text{FC}} = T_p = T_r = 350 \text{ K}$$

So we can now evaluate our enthalpies

$$\bar{h}_{\text{H}_2} = \bar{h}_{f,\text{H}_2} + \Delta\bar{h}_{\text{H}_2} = 0 + (9971 - 8468) = 1503 \text{ kJ/kmole}$$

$$\bar{h}_{\text{O}_2} = \bar{h}_{f,\text{O}_2} + \Delta\bar{h}_{\text{O}_2} = 0 + (10,213 - 8682) = 1531 \text{ kJ/kmole}$$

$$\bar{h}_{\text{H}_2\text{O}} = \bar{h}_{f,\text{H}_2\text{O}} + \Delta\bar{h}_{\text{H}_2\text{O}} = -241,820 + (11,652 - 9904) = -240,070 \text{ kJ/kmole}$$

We have not included the enthalpies for the N<sub>2</sub>, since we know that they will cancel out. To evaluate the entropies we need to have our partial pressures.

Calculating them we have

$$y_{\text{H}_2} = \frac{1}{1+0.5+1.88} = 0.30$$

$$y_{\text{O}_2} = \frac{0.5}{1+0.5+1.88} = 0.15$$

$$y_{\text{N}_{2,r}} = \frac{1.88}{1+0.5+1.88} = 0.55$$

$$y_{\text{H}_2\text{O}} = \frac{1}{1+1.88} = 0.35$$

$$y_{N_2,p} = \frac{1.88}{1+1.88} = 0.65$$

Going to the ideal gas tables at 350 K we find

$$\bar{s}_{H_2}^0 = 135.21 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{O_2}^0 = 209.765 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{N_2,r}^0 = 196.173 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{H_2O}^0 = 194.125 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{N_2,p}^0 = 199.173 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

Now substituting

$$\bar{w}_{elec} = 1503 + (0.5)(1531) - (-240,070) - (298) \left\{ \begin{array}{l} (135.21 - (8.314)\ln\{0.3\}) + (0.5)(209.768 - (8.314)\ln\{0.15\}) \\ + (1.88)(196.173 - (8.314)\ln\{0.55\}) - (194.125 - (8.314)\ln\{0.35\}) \\ - (1.88)(196.173 - (8.314)\ln\{0.65\}) \end{array} \right\}$$

$$\bar{w}_{elec} = 242,340 - (350)(34.22) = 230,360 \text{ kJ/kmole}$$

The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{FC} \left\{ \begin{array}{l} \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \end{array} \right\}}{\sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j} = 1 - \frac{(350)(34.22)}{(242,340)} = 0.95$$

or 95%. Next we find

$$\dot{m}_{fuel} = \frac{MW_{fuel} \dot{W}_{elec}}{\bar{w}_{elec}} = \frac{(2)(25)}{230,360} = 2.17 \times 10^{-4} \text{ kg/s}$$

Our ideal voltage is then given by

$$V = \frac{\bar{w}_{elec}}{(96,487)N_e} = \frac{(230,360)}{(96,487)(2)} = 1.19 \text{ V}$$

The current is

$$I = \frac{\dot{W}_{elec}}{V} = \frac{25,000}{1.19} = 20,943 \text{ amps}$$

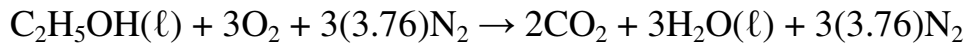
2. Determine the ideal voltage, current, and required mass flow rate of ethanol ( $C_2H_6O$ ) for an ideal air/ethanol–water fuel cell operating at 101 kPa and 280 K that is producing 500 kW. You may take the  $c_p$  of ethanol to be 2.77 kJ/(kg·K).

**Solution:**

Our general energy equation is

$$\bar{w}_{\text{elec}} = \sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j - T_{\text{FC}} \left\{ \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \right\}$$

For a ethanol-water/air fuel cell the balanced chemical reaction equation is given by



which allows use to write our energy equation as

$$\begin{aligned} \bar{w}_{\text{elec}} = & \bar{h}_{C_2H_6O(\ell)} + 3\bar{h}_{O_2} + (11.28)\bar{h}_{N_{2,r}} - 2\bar{h}_{CO_2} - 3\bar{h}_{H_2O(\ell)} - (11.28)\bar{h}_{N_{2,p}} \\ & - T_{\text{FC}} \left\{ \bar{s}_{C_2H_6O(\ell)} + 3\bar{s}_{O_2} + (11.28)\bar{s}_{N_{2,r}} - 2\bar{s}_{CO_2} - 3\bar{s}_{H_2O(\ell)} - (11.28)\bar{s}_{N_{2,p}} \right\} \end{aligned}$$

We assume that we have an isothermal fuel cell at 280 K, so that

$$T_{\text{FC}} = T_p = T_r = 280 \text{ K}$$

So we can now evaluate our enthalpies

$$\bar{h}_{C_2H_6O(\ell)} = \bar{h}_{f,C_2H_6O(\ell)} + \Delta\bar{h}_{C_2H_6O(\ell)} = -277,690 + (2.44)(46)(280 - 298)$$

$$= -279,110 \text{ kJ/kmole}$$

$$\bar{h}_{O_2} = \bar{h}_{f,O_2} + \Delta\bar{h}_{O_2} = 0 + (8150 - 8682) = -532 \text{ kJ/kmole}$$

$$\bar{h}_{H_2O(\ell)} = \bar{h}_{f,H_2O(\ell)} + \Delta\bar{h}_{H_2O(\ell)} = -285,830 + (4.2)(18)(280 - 298)$$

$$= -286,230 \text{ kJ/kmole}$$

$$\bar{h}_{CO_2} = \bar{h}_{f,CO_2} + \Delta\bar{h}_{CO_2} = -393,520 + (8697 - 9364) = -394,190 \text{ kJ/kmole}$$

To evaluate the entropies we need to have our partial pressures. Calculating them we have

$$y_{O_2} = \frac{3}{3 + 11.28} = 0.21$$

$$y_{N_{2,r}} = \frac{11.28}{3 + 11.28} = 0.79$$

$$y_{CO_2} = \frac{2}{2 + 11.28} = 0.15$$

$$y_{N_{2,p}} = \frac{11.28}{2 + 11.28} = 0.85$$

Determining the entropies at 280 K we find

$$\bar{s}_{C_2H_6O(\ell)}^0 = 160.70 + (2.44)(46) \ln(280/298) = 153.71 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{\text{O}_2}^{\circ} = 203.191 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{\text{N}_2, \text{r}}^{\circ} = 189.673 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{\text{H}_2\text{O}}^{\circ} = 69.29 + (4.2)(18) \ln(280/298) = 64.58 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{\text{CO}_2}^{\circ} = 213.685 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{\text{N}_2, \text{p}}^{\circ} = 189.673 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

Now substituting

$$\begin{aligned} \bar{w}_{\text{elec}} = & -279,110 + (3)(-532) - (2)(-394,190) - (3)(-286,230) \\ & - (280) \left\{ \begin{aligned} & (153.71) + (3)(203.191 - (8.314) \ln\{0.21\}) \\ & + (11.28)(189.673 - (8.314) \ln\{0.79\}) - (2)(213.685 - (8.314) \ln\{0.15\}) \\ & - (3)(64.58) - (11.28)(189.673 - (8.314) \ln\{0.85\}) \end{aligned} \right\} \end{aligned}$$

$$\bar{w}_{\text{elec}} = 1,366,400 - (280)(156.42) = 1,322,600 \text{ kJ/kmole}$$

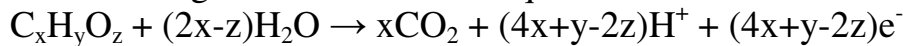
The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{\text{FC}} \left\{ \begin{aligned} & \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \end{aligned} \right\}}{\sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j} = 1 - \frac{(280)(156.42)}{(1,366,400)} = 0.97$$

or 97%. Next we find

$$\dot{m}_{\text{fuel}} = \frac{\text{MW}_{\text{fuel}} \dot{W}_{\text{elec}}}{\bar{w}_{\text{elec}}} = \frac{(46)(500)}{1,322,600} = 1.74 \times 10^{-2} \text{ kg/s}$$

To obtain our ideal voltage we need to determine the number of electrons released at the anode. Our general ion reaction equation at the anode is



For the isopropanol this becomes



So that

$$N_e = 12 \text{ kmole of electrons/kmole of C}_2\text{H}_6\text{O}$$

Our ideal voltage is then given by

$$V = \frac{\bar{w}_{\text{elec}}}{(96,487)N_e} = \frac{(1,322,600)}{(96,487)(12)} = 1.14 \text{ V}$$

The current is

$$I = \frac{\dot{W}_{\text{elec}}}{V} = \frac{500,000}{1.14} = 438 \text{ kA}$$

3. Determine the ideal voltage, current, number of stacks required, and required mass flow rate of isopropanol ( $C_3H_7OH$ ) for an ideal air/isopropanol –water fuel cell operating at 101 kPa and 298 K that is producing 50 kW with 4 volts. You may take the following properties for liquid isopropanol.

$$c_p = 2.35 \text{ kJ/(kgK)}$$

$$\bar{h}_f = -316,000 \text{ kJ/kmole}$$

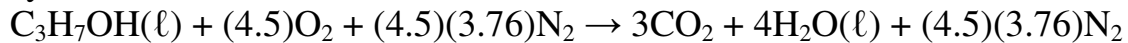
$$\bar{s}^0 (\text{at } 298\text{K}) = 194.6 \text{ kJ/(kmole} \cdot \text{K)}$$

**Solution:**

Our general energy equation is

$$\bar{w}_{\text{elec}} = \sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j - T_{\text{FC}} \left\{ \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \right\}$$

For a isopropanol-water/air fuel cell the balanced chemical reaction equation is given by



which allows use to write our energy equation as

$$\begin{aligned} \bar{w}_{\text{elec}} = & \bar{h}_{C_2H_6O(\ell)} + (4.5)\bar{h}_{O_2} + (16.92)\bar{h}_{N_{2,r}} - 3\bar{h}_{CO_2} - 4\bar{h}_{H_2O(\ell)} - (16.92)\bar{h}_{N_{2,p}} \\ & - T_{\text{FC}} \left\{ \bar{s}_{C_2H_6O(\ell)} + (4.5)\bar{s}_{O_2} + (16.92)\bar{s}_{N_{2,r}} - 3\bar{s}_{CO_2} - 4\bar{s}_{H_2O(\ell)} - (16.93)\bar{s}_{N_{2,p}} \right\} \end{aligned}$$

We assume that we have an isothermal fuel cell at 298 K, so that

$$T_{\text{FC}} = T_p = T_r = 298 \text{ K}$$

So we can now evaluate our enthalpies

$$\bar{h}_{C_3H_8O(\ell)} = \bar{h}_{f,C_3H_8O(\ell)} + \Delta\bar{h}_{C_3H_8O(\ell)} = -316,000 + 0 = -316,000 \text{ kJ/kmole}$$

$$\bar{h}_{O_2} = \bar{h}_{f,O_2} + \Delta\bar{h}_{O_2} = 0 + 0 = 0 \text{ kJ/kmole}$$

$$\bar{h}_{N_{2,r}} = \bar{h}_{f,N_2} + \Delta\bar{h}_{N_{2,r}} = 0 + 0 = 0 \text{ kJ/kmole}$$

$$\bar{h}_{CO_2} = \bar{h}_{f,CO_2} + \Delta\bar{h}_{CO_2} = -393,520 + 0 = 393,520 \text{ kJ/kmole}$$

$$\bar{h}_{H_2O(\ell)} = \bar{h}_{f,H_2O(\ell)} + \Delta\bar{h}_{H_2O(\ell)} = -285,830 + 0 = -285,830 \text{ kJ/kmole}$$

$$\bar{h}_{N_{2,p}} = \bar{h}_{f,N_2} + \Delta\bar{h}_{N_{2,p}} = 0 + 0 = 0 \text{ kJ/kmole}$$

To evaluate the entropies we need to have our partial pressures. Calculating them we have by only including our gaseous components

$$y_{O_2} = \frac{4.5}{4.5 + 16.92} = 0.21$$

$$y_{N_{2,r}} = \frac{16.92}{4.5 + 16.92} = 0.79$$

$$y_{CO_2} = \frac{3}{3 + 16.92} = 0.15$$

$$y_{N_2,p} = \frac{16.92}{3 + 16.92} = 0.85$$

For the entropies at 298 K we find

$$\bar{s}_{C_3H_8O}^0 = 194.6 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{O_2}^0 = 205.033 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{N_2,r}^0 = 191.502 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{CO_2}^0 = 213.685 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{H_2O(\ell)}^0 = 69.92 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

$$\bar{s}_{N_2,p}^0 = 191.502 \text{ kJ}/(\text{kmole} \cdot \text{K})$$

Now substituting

$$\begin{aligned} \bar{w}_{\text{elec}} = & -316,000 + (4.5)(0) + (16.92)(0) - (3)(-393,520) - (4)(-285,830) - (16.92)(0) \\ & - (298) \left\{ \begin{aligned} & (194.6) + (4.5)(205.033 - (8.314)\ln\{0.21\}) \\ & + (16.92)(191.502 - (8.314)\ln\{0.79\}) - (3)(213.685 - (8.314)\ln\{0.15\}) \\ & - (4)(69.92) - (16.92)(191.502 - (8.314)\ln\{0.85\}) \end{aligned} \right\} \end{aligned}$$

$$\bar{w}_{\text{elec}} = 2,007,900 - (298)(217.88) = 1,943,000 \text{ kJ/kmole}$$

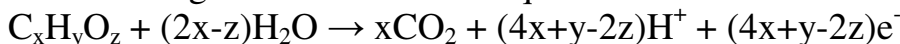
The ideal efficiency is then given by

$$\eta_i = 1 - \frac{T_{\text{FC}} \left\{ \sum_{\text{reactants}} v_i \bar{s}_i - \sum_{\text{products}} v_j \bar{s}_j \right\}}{\sum_{\text{reactants}} v_i \bar{h}_i - \sum_{\text{products}} v_j \bar{h}_j} = 1 - \frac{(298)(217.88)}{(2,007,900)} = 0.97$$

Next we find

$$\dot{m}_{\text{fuel}} = \frac{MW_{\text{fuel}} \dot{W}_{\text{elec}}}{\bar{w}_{\text{elec}}} = \frac{(60)(50)}{1,943,000} = 1.54 \times 10^{-3} \text{ kg/s}$$

To obtain our ideal voltage we need to determine the number of electrons released at the anode. Our general ion reaction equation at the anode is



For the isopropanol this becomes



So that

$$N_e = 18 \text{ kmole of electrons/kmole of } C_3H_8O$$

Then our ideal voltage is then given by

$$V = \frac{\bar{W}_{\text{elec}}}{(96,487)N_e} = \frac{(1,943,000)}{(96,487)(18)} = 1.12 \text{ V}$$

The number of stacks required is

$$\text{number of stacks} = V_{\text{required}}/V_{\text{cell}} = (4)/(1.12) = 4 \text{ stacks}$$

The current is

$$I = \frac{\dot{W}_{\text{elec}}}{V} = \frac{50,000}{4} = 12,500 \text{ amps} = 12.5 \text{ kA}$$