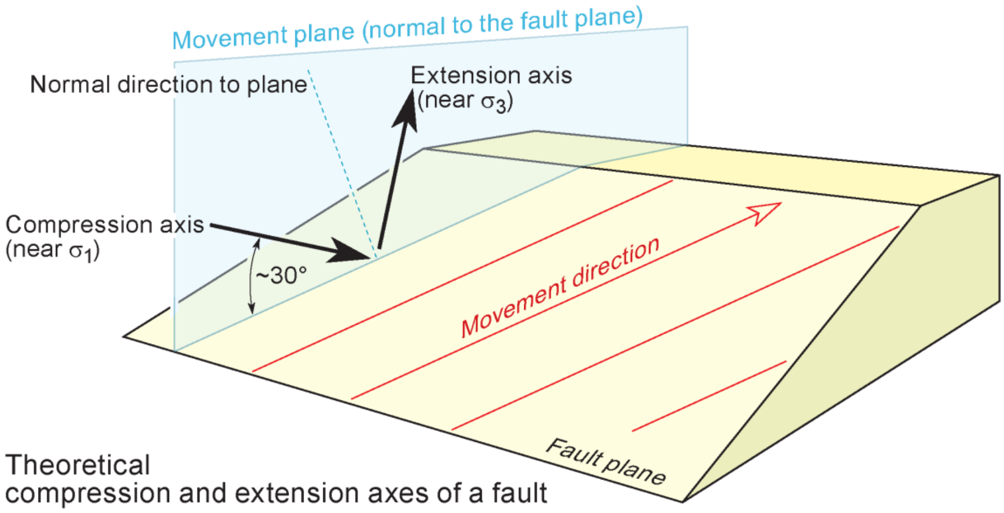
PALEO-“STRESS” ANALYSIS FROM FAULT DATA

Slicken-lines on a fault plane represent the direction of some relative displacement between the two blocks separated by the fault. Fault data include both the fault plane and slicken-line orientations, the latter including the relative sense of movement along the line.



The goal is to use these measurements to calculate a so-called **paleostress tensor**. Paleostress tensors provide a dynamic interpretation (in terms of stress orientation) to the kinematic (movement) analysis of brittle features. The calculation does not yield a true paleo- stress tensor since it is a statistic calculation on fractures that integrate a geologically significant amount of time. In that sense, the term stress is misused, because the result does not provide instantaneous forces applied at a point. The calculated axes would more appropriately be called kinematic axes. However, the information is an interpretation of stress conditions that are responsible for the brittle deformation events under consideration. Several applications in different tectonic contexts have yielded regionally consistent results, which makes paleostress reconstruction a useful structural analysis to recreate to some extent the stress environments and tectonic regimes operating in the past.
Historically, the analysis has grown from three steps:

- Hypothesis about the orientation of the stress field (one principal stress is vertical).
- The failure criterion that gives some relationship between stress and fault plane + line for new fractures.
- Reactivation criteria for pre-existing fractures in which case the relationships defined above are more complex because many patterns become possible (thus calculation with inverse methods).

Fault measurements and data set

All methods are statistic. It is therefore important to collect in the field as many measurements as possible. Field observation is important in separating sets that homogeneously represent the same deformation event because they provide independent information that may support or help to correct calculation inconsistencies. Several criteria exist to distinguish the different fault rock types in the field, and relate them to successive tectonic regimes, which in turn allow a relative timing and reconstruction of successive stress fields:

- Geometrical relationships that indicate whether the fault planes are contemporaneous or result from superimposed events with different directions.
- The qualitative aspect and morphology of fault planes, reflecting different geological environments (for example higher temperature, mineral-coated planes versus likely younger sharp and clean fractures).
- The influence of lithology.
- The size of the fault plane and, eventually, of the relative movement.

Practically, orientation data should be collected from small sites (also called **stations**) that are structurally homogeneous domains.

Principal stress orientations from field data

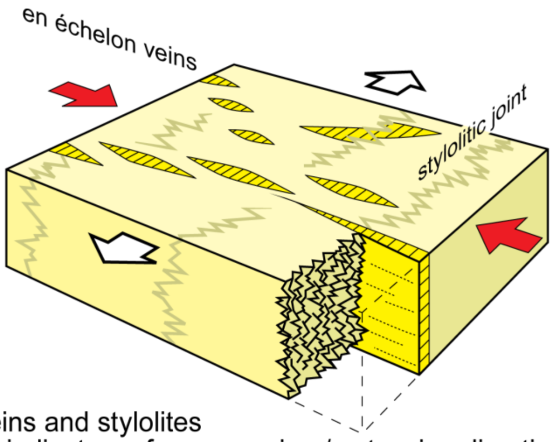
The systematic relationship that exists between brittle structures and principal stress directions provides a basis for interpreting paleostress directions. In particular, it is important to separate, in the field, the different compression directions that may correspond to separate paleostress tensors responsible for successive deformation event.

Convention

We distinguish the principal stresses directions, i.e. the eigenvectors of the stress tensor, represented by the three unit vectors $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ from the principal stress magnitudes, i.e. the corresponding eigenvalues $\sigma_1 \geq \sigma_2 \geq \sigma_3$, taken positive in compression.

Application

In low strain regions deformed under brittle conditions, some structures other than faults may guide the stress analysis, in particular in terms of orientation of successive events when superposed fracture fabrics and other indicators can be identified.



Veins and stylolites
as indicators of compression / extension directions

Joints are the most dangerous structures to use for any stress calculation. However, they may be the only structures available for structural analysis in tabular regions. The principal criteria for classifying joints into extension or shear fractures are the bulk organisation, symmetry, surface morphology and dihedral angles between conjugate sets.

Stylolites are irregular dissolution surfaces characterised by mm to cm-long peaks. The peaks (or columns) are considered to point in the direction of maximum dissolution, i.e. parallel to the maximum shortening direction (equated here with compression $\vec{s}_1$).

Tension gashes are fractures opening orthogonal to the smallest principal stress $\vec{s}_3$.

If the directions of $\vec{s}_1$ (stylolites) and $\vec{s}_3$ (tension gashes) are known, then the direction of $\vec{s}_2$ can be reconstructed perpendicular to both.

Dynamic classification of faults

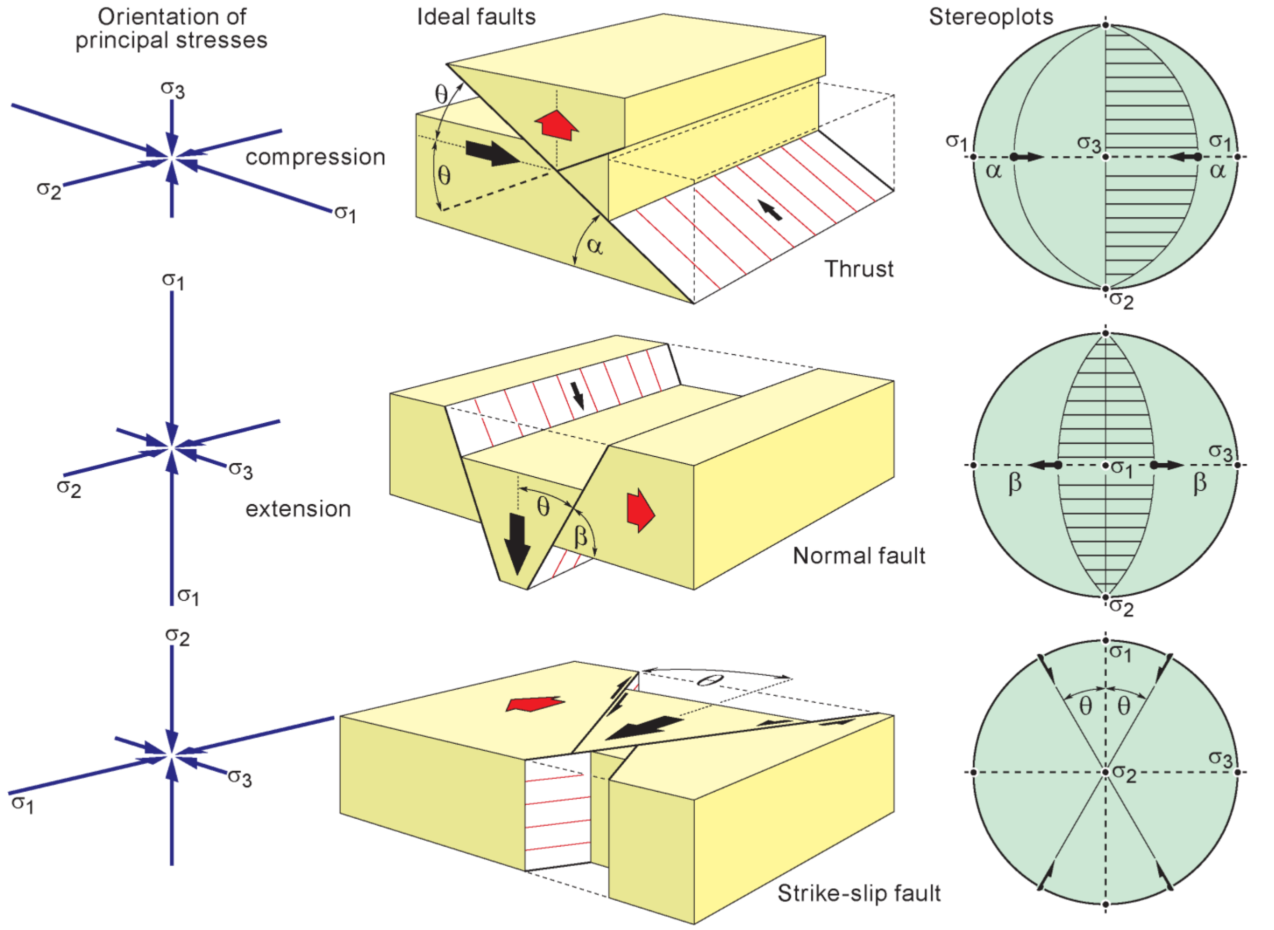
Classification

The co-existence of stress and equal counter-stress on opposite sides of a surface element is a general property on any internal surface of a body in equilibrium. An important implication of this condition concerns its application to the surface of the Earth. Since the air is incapable of sustaining a shearing stress, there is no shearing stress across the surface of the Earth because it is an air-rock interface.

Hence, the Earth's surface is a principle plane of stress. Therefore, where the Earth's surface is reasonably horizontal, one of the principal directions of stress is approximately vertical at, and for a shallow depth beneath the surface (this assumption was proposed by Anderson in 1905 and is sometimes referred to as the "Anderson" condition). The other two principal directions of stress must be horizontal.

Assumptions on the stress field

The basic premise of stress fields oriented with axes perpendicular to the overall crustal surfaces and of uniform intensity and orientation over considerable masses of the crust is a very broad approximation. Stress fields in general vary considerably in both intensity and orientation. Thus, the dip of faults will vary as the orientation of the maximum principal stress axis (e.g. listric faults).
Equilibrium also requires that no distributed torque acts in the medium, which implies that the stress tensor is symmetrical.



Dynamic interpretation of faults: Anderson's "standard" relationship between stresses and ideal faults

Failure within isotropic rocks (new faults)

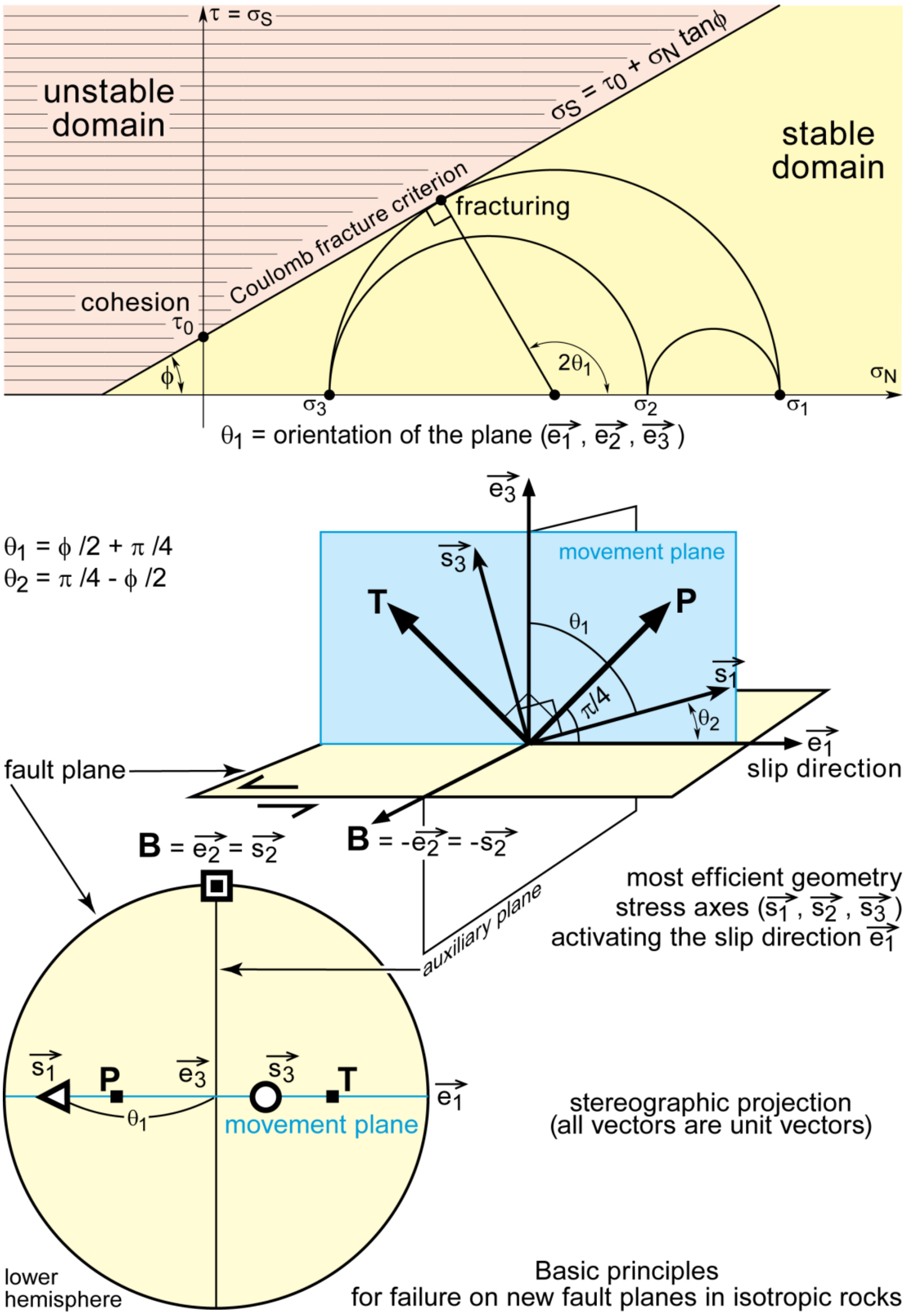
Basic principles

The dynamic principle of faulting is very simple and relates on the stress geometry when the failure envelope is reached.

- The analysed body of rock is homogeneous and isotropic.
- A state of stress will tend to cause shear fracture in accordance with the Mohr-Coulomb criterion: Faulting occurs on the plane on which the shear to normal stress ratio reaches the failure envelope. Therefore, the local stress field determines the attitude of a newly created fault.
- A cataclastic lineation is movement direction and has the same direction and sense as the resolved shear stress on the fault plane. Striation orientations on a given fault thus define the intersection of

the fault surface with the $(\vec{s}_1, \vec{s}_3)$ plane. This statement implies that fault displacements are small with respect to fault length and there is no ductile deformation of the material and thus rotation of the fault plane.

- The intersection of conjugate faults in an isotropic rock defines the intermediate principal stress direction $\vec{s}_2$.
- The acute angle between the conjugate faults is bisected by the greatest principal stress $\vec{s}_1$. If the yield envelope is a line of slope $\tan\phi$ as in the Coulomb criteria, then the angle between $\vec{s}_1$ and each fault plane is $45° - (\phi/2)$. Typically $\phi \approx 30°$, and this angle is about 30°.
- Neighbouring faults are independent from each other.



Basic principles
for failure on new fault planes in isotropic rocks

Classification

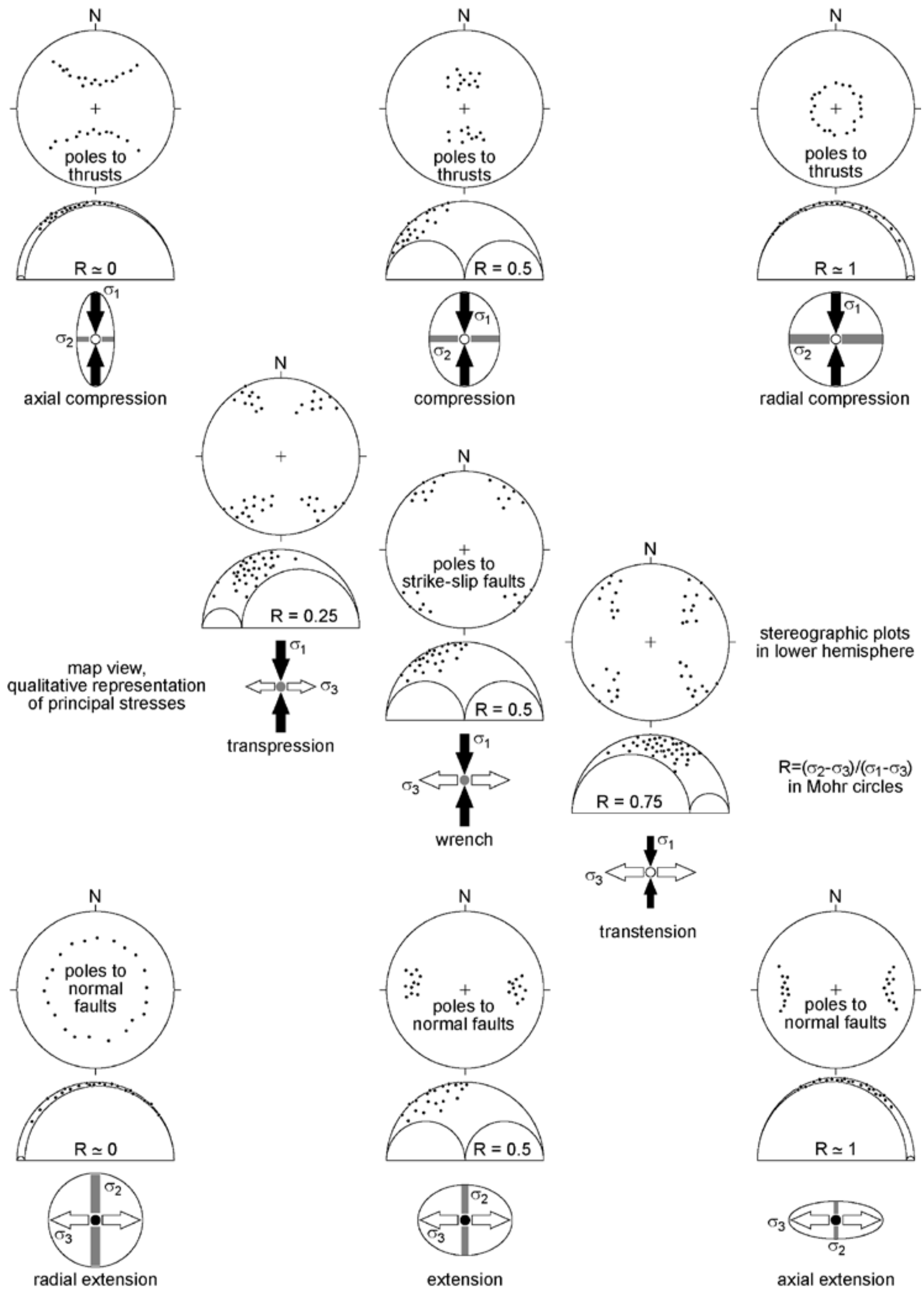
The assumption ensuring that $\vec{s}_2$ is either vertical or horizontal leads to a dynamic classification of faults:

- For $\vec{s}_2$ horizontal, the resulting faults are either normal ($\vec{s}_1$ vertical) or reverse ($\vec{s}_1$ horizontal).
- For vertical $\vec{s}_2$, the resulting faults are vertical strike slip faults.

Inversely, determination of the orientations of $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ prior to faulting requires that one knows the strike and dip of a fault and both the direction and the sense of initial displacement.

Tectonic regime

Since $\vec{s}_1$ bisects, or at least lies within the acute angle (about 60°) between conjugate faults, we can anticipate that normal faults related to $\vec{s}_1$ vertical should be steep planes dipping about 60°; conversely, thrusts related to $\vec{s}_1$ horizontal should be shallow dipping planes (about 30°).



Tectonic regimes defined by paleostress calculations

Accordingly, 3 main tectonic regimes are differentiated with reference to the vertical principal stress:
$\vec{s}_1$ vertical: extensional tectonic regime;
$\vec{s}_2$ vertical: wrench (strike slip) regime;
$\vec{s}_3$ vertical: compression regime.

The relative magnitudes of principal stresses vary continuously between these three end-member tectonic regimes and the transition from one regime to another is fundamentally due to the permutation of one of the horizontal principal stress axes with the vertical one. Each tectonic regime may vary continuously from radial to axial, depending on the relative magnitudes of the intermediate principal stresses, σ_2, with respect to the extreme principal stresses, σ_1 and σ_3. This information is embedded in the R ratio:

$$R=\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3} \qquad \text{(note that } 0 \le R \le 1 \text{)}$$

Transpression is a deformation style with the simultaneous occurrence of strike-slip faults, thrusts and oblique-slip thrust faults. Tectonic regimes near the transition between compression (vertical $\vec{s}_3$, small R) and wrench (vertical $\vec{s}_2$, small R) are favourable to the development of such a deformation style.

Transtension reports the simultaneous occurrence of strike-slip faults, normal faults and oblique-slip normal faults. Tectonic regimes near the transition between extension (vertical $\vec{s}_1$, high R) and wrench (vertical $\vec{s}_2$, high R) are favourable to this deformation style.

Paleostress directions – geometrical and graphical approximation

Several techniques allow measuring or estimating some or all the components of the local stress tensor. Present-day stresses can be directly measured (overcoring, hydraulic fracturing, see lecture 1 on mechanical aspects of deformation). In the following, more emphasis is given on geological / structural information indirectly derived from theoretical stress-strain relationships.

Single fault: P-T method

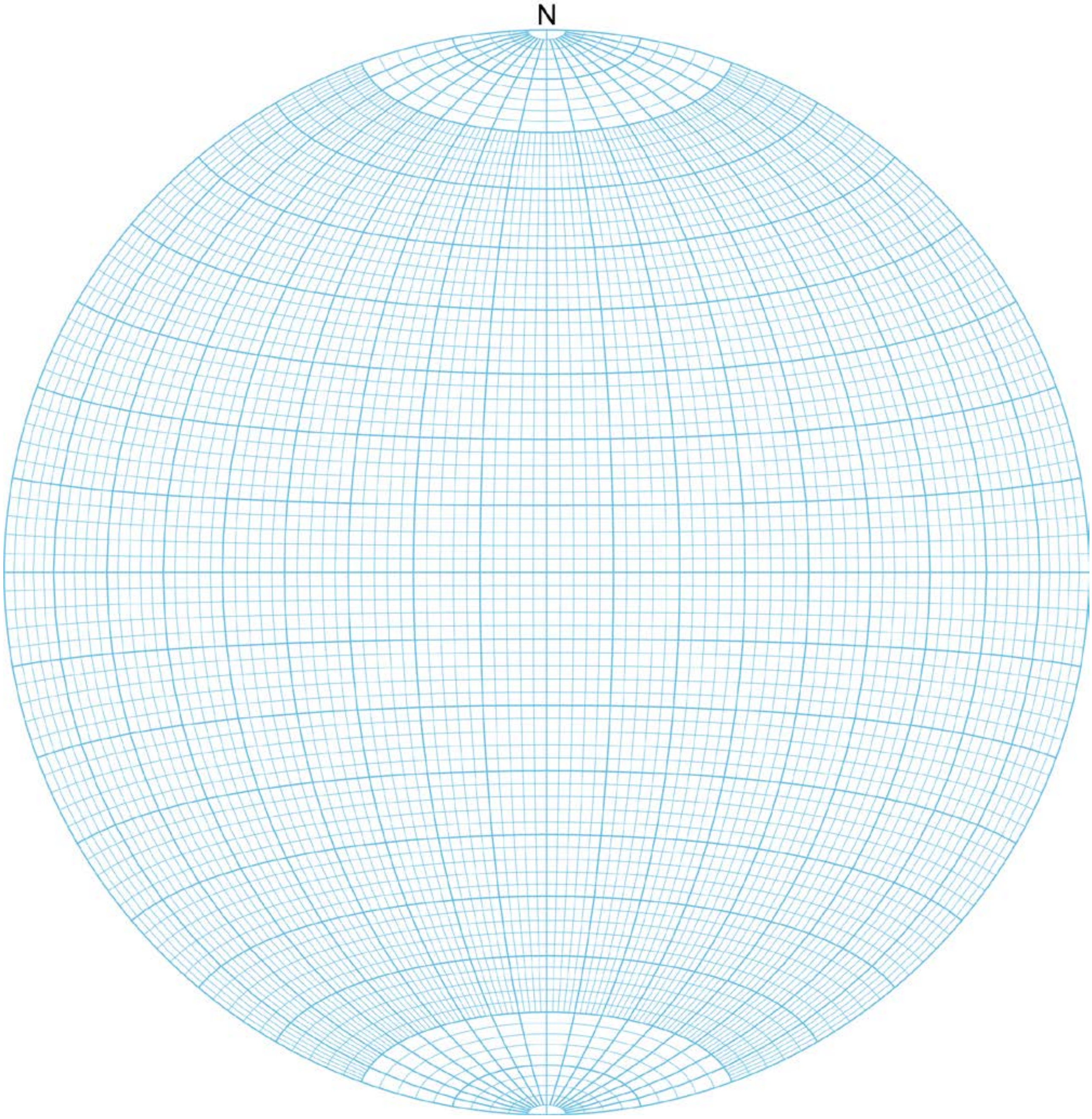
The common but not easily interpreted situation is that of a single fault in isotropic rock with a determined sense and direction of displacement. The **movement plane** is defined as the plane that is perpendicular to the fault plane and includes the slip direction. In this method it is assumed that the movement plane contains the greatest ($\vec{s}_1$) and least ($\vec{s}_3$) principal stresses directions (more appropriately the infinitesimal shortest and longest strain axes, respectively). The line normal to the slickenline and lying in the fault plane is tentatively taken as the direction of $\vec{s}_2$. If the angle between the fault plane and $\vec{s}_1$ is now taken as about 30° (Mohr-Coulomb criterion), for instance, with the sense of slip in the direction of $\vec{s}_1$, the directions of $\vec{s}_3$ may be determined.

The graphical technique works as follows: For each fault measured:
- Plot both the fault plane and its pole on an equal area projection;
- Plot the slicken-line on the great circle that represents the fault;
- Add an arrow on the slickenside point to indicate the relative movement of the hanging-wall;
- Rotate the plot to find the great circle that contains the slip direction and the pole to the fault; this circle defines the movement plane direction.
- Along the great circle of the movement plane, the compression direction (P) is plotted at 60° to the pole to the fault, and 30° to the slip direction, in a sense consistent with the sense of movement.
- Similarly, the extension direction (T) is plotted at 30° to the pole of the fault and 60° to the slip direction (the arrow points away from the extension direction).

Obviously, the bigger the number of striated planes orientation has been measured, the best it is to find approximate positions of the stress axes.

Exercise

Plot a fault stricking 020 and dipping 60°W with a slicken-line whose pitch is 30°N. Define the compression and stretching directions when the fault is normal, thrust fault. Do the same when the line has a pitch of 30°S.



Conjugate faults

The easiest approximation is provided by the observation, in the field, of conjugate faults. Per definition, conjugate faults intersect parallel to $\vec{s}_2$, which is also the pole to the $(\vec{s}_1;\vec{s}_3)$ great circle. In that case $\vec{s}_1$ bisects the acute (about 60°) angle between the two fault orientations. However one should check that the wedge containing $\vec{s}_1$ moves inward.

Exercise

Plot a fault striking 060 and dipping 60°SE with a slicken line whose pitch is 30°N. This fault is supposed to be conjugate to the single fault treated above. Define the compression and stretching directions when both faults are normal, thrust faults. Try the same exercise when one is normal and the other is a thrust fault. What are conclusions?

Dihedral angle

This method displays results in a form resembling earthquake focal mechanisms. Its precision is based on a large number of measurements of fault plane-striation pairs. The justification refers to the fact that pre-existing faults are geologically common and may dominate measurements. In that case, the largest principal stress can be at any angle between 0 and 90 ° (anticlockwise from the movement direction) to the slip plane and line.

The graphical technique seeks for kinematic axes of shortening and extension on each fault to deduce the dynamic (stress) axes for a fault population; the procedure is the following: For each data pair measured:

- Plot both the fault plane and its pole on an equal area projection;
- Plot the slickenside striae on the great circle that represents the fault;
- Plot the great circle that represents a plane orthogonal to both the movement plane and the movement direction (the slicken-line is the pole to the plane, known as **auxiliary plane**). This plane divides, with the fault plane, the sphere in four quadrants, called the **dihedral right angles**. Opposed angles make two pairs, one in compression contains $\vec{s}_1$ the other, in tension, contains $\vec{s}_3$.
- The plot is repeated for all other fault/movement plane pairs.
- Superimposing all dihedra of all the considered fault planes progressively narrows down the areas of compression and extension. Only that small area for which all quadrants containing either compression or extension are superposed is accepted as $\vec{s}_1$ and $\vec{s}_3$ paleostress directions, respectively, representing the entire fault system.

Exercise

Plot in a stereonet a thrust fault oriented 60/190 with a line 20/110. Define its compressive and extensional quadrants; Repeat the exercise on the same plot for a fault 30/130 with a line 20/080.

Define the probability surface for the compression and extension directions

Inherited fractures

In real rock there are many closed and uniformly distributed planes of discontinuity (e.g. grain boundaries). Inherited planes may produce a critical structural anisotropy, especially when considered on a scale large enough to include joints or contacts between rocks of different lithology. As a consequence two failure criteria coexist:

(1) The failure criterion for new fault plane orientations in the intact rock;

(2) A second criterion for the shearing resistance to friction sliding on inherited, weak plane orientations.

At depth and for positive normal stresses, the linear Coulomb criterion is usually consistent with both cases. However, in the second case the cohesion may vary down to zero (from equation 5 in the lecture on Faulting). In addition, the simple relation between the orientation of Mohr-Coulomb shear fractures and stress directions no longer holds:

- With inherited fractures, a fault can be inclined at any angle (other than 0 or 90°) to $\vec{s}_1$. Actually $\vec{s}_1$ may lie anywhere in the dilatational quadrant of the corresponding fault plane solution.
- On pre-existing planes of weakness that are not parallel to any of the principal stress directions, the direction of fault displacement depends on the applied shear stress directions. This direction, in turn, depends on the relative magnitudes of the intermediate principal stress (embedded in the R ratio), and the plane orientation with respect to the principal stress axes.
- The fault plane needs not include principal stress directions.

These are fundamental factors that determine which direction in the potential fault plane feels the greatest shear stress.

Graphical representation

One may illustrate these characteristics on a Mohr diagram representing a triaxial state of stress and considering a small stress difference, which increases with tectonic loading.

Principle

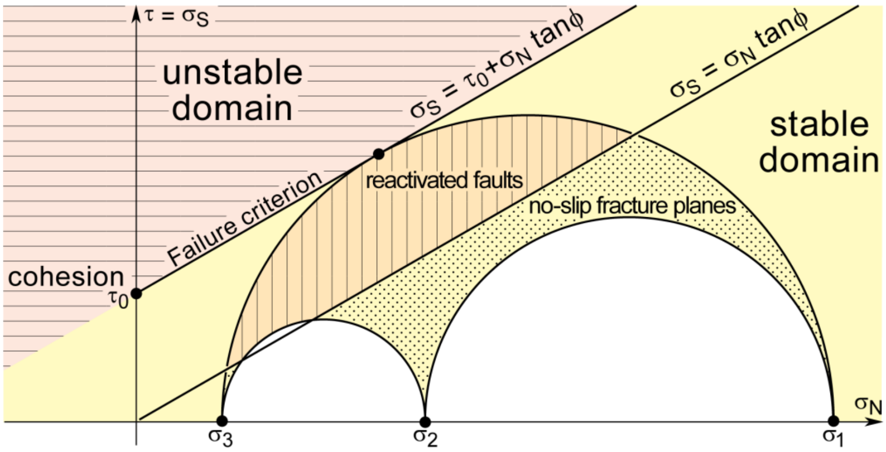
We know that:

- Any fracture plane is a point with coordinates (σ_N, σ_S) that plots on (if they are created) and within (if there is no slip) the (σ_1, σ_3)circle.

- A new fault is created when the outer (σ_1, σ_3)circle reaches the Mohr-Coulomb failure criterion. When this happens, this circle cannot grow anymore and the stress difference $(\sigma_1 - \sigma_3)$ is limited by the movement on new faults. The Mohr-Coulomb envelope is therefore a maximum friction law.

- The minor circles (σ_3, σ_2) and (σ_2, σ_1) represent planes that contain only one of the principal stresses defining the differential stress; hence, they contain no relevant fault solution because slip requires (σ_N, σ_S)conditions on and out of these minor circles.

All pre-existing rock discontinuities (e.g. fractures, cleavage, bedding plane…) may slip under the considered state of stress if they correspond to (σ_N, σ_S) points within the curvilinear triangle between the (σ_1, σ_3), (σ_3, σ_2) and (σ_2, σ_1) Mohr circles; these points must actually correspond to the orientation of a fracture plane and lay on a friction line defining the failure condition (Byerlee law).



Domain of reactivated, inherited fractures
in a Mohr diagram with the three principal stresses

The zero-cohesion line going through the origin represents a pre-existing discontinuity. It cuts the curvilinear triangle into two areas.

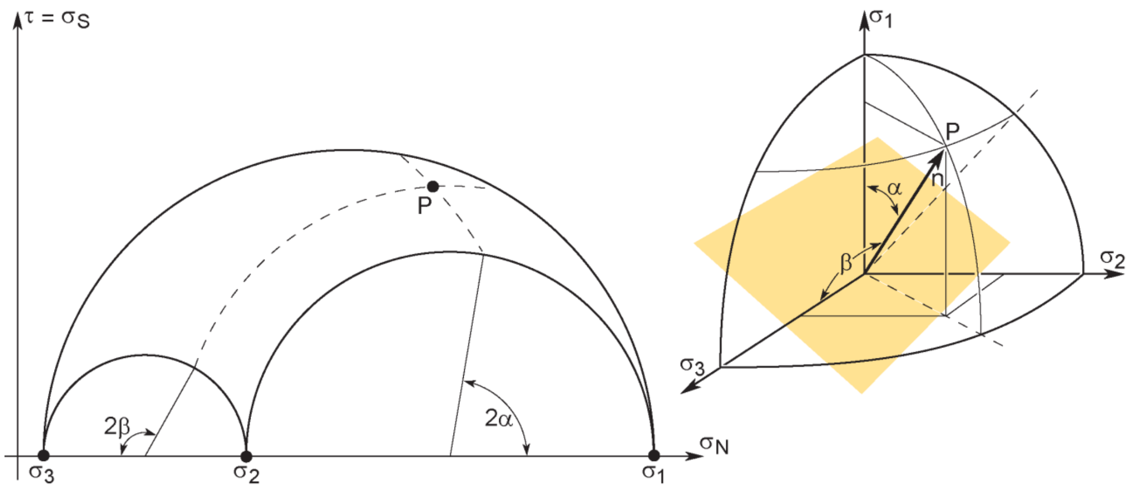
- Fault planes plotting below this line should not slip, otherwise cohesion would be negative. The zero-cohesion line is thus a minimum friction law.
- Under a uniform state of stress, slip is theoretically possible for the fault planes that plot along the zero-cohesion line.
- Allowing stress variations in space and time (hence not a single state of stress) considers possible slip between the smallest and the largest stress tensors, the latter producing slip on inherited surfaces just before creating new faults. These two conditions are represented by the minimum friction line and the inner side of the (σ_1, σ_3)Mohr circle.

On the Mohr construction, all points contained in the area between the minimum-friction line and the (σ_1, σ_3)-Mohr circle are potentially reactivated discontinuities.

Plotting a plane in Mohr construction

Any plane is defined by its normal unit vector. The position of a plane P in the three-dimensional Mohr circle uses the two Euler angles α and β between the normal to the plane and the $\vec{s}_1$ and $\vec{s}_3$ axes, respectively.

- As all angles in the Mohr construction, one utilizes the angles 2α and 2β to place the orientation points on the (σ_2, σ_1) and (σ_3, σ_2) small circles, respectively.
- Then one construct arcs passing through these points, drawn from the center of the other small circle
- P is determined at the intersection of the two arcs.



Representation of a fault plane P in a three-dimensional Mohr diagram

Practical consequences

There is a major difference between faults created under some stress conditions and slip along a reactivated pre-existing fracture. The attitude of new faults is controlled by the stress state that produces them, whereas inherited fault planes may have a wide variety of orientation with respect to the principal stress axes. This statement is consequential for field geologists who do not always find observation to separate new from inherited fault planes. What is done is to identify certain orientations of the stress ellipsoid that could not give rise to the observed displacements, for example orientations giving the wrong sense of displacement. This should be done from place to place in order to depict the causal stress field.

Computer calculation

Numerical techniques have gained preeminence since computer-aided calculations have become convenient. The stress tensor $\mathbf{T}$ with components σ_{ij} relates $\vec{t}$, the traction vector with t_i

components on a plane, to the normal vector $\vec{n}$ with components n_j that defines the orientation of the plane (see lecture 1). This mathematical relation is the Cauchy's law:

$$t_i = \sigma_{ij} n_j$$

This equation can be used to calculate the stress on any plane once one knows the value of the stress tensor in a chosen coordinate system.

Assumptions

The principal directions of a regional stress field can be calculated from the analysis of slip motions induced by this stress field on faults with various dip and strike. The direction of slip motions can be derived either from striated fault planes (structural analysis) or from the focal mechanisms of seismic events associated with these motions.
There are several methods of paleostress calculation but all rely on the same basic assumptions:
- (1) The rock is isotropic
- (2) All movements are caused by the same stress tensor (i.e. are all contemporaneous in a geological time frame).
- (3) On each fault plane, the slip is parallel to the maximum resolved shear stress from the regional stress tensor (this is the so-called Wallace-Bot assumption).
- (4) The strain is non-rotational

In practice, these assumptions require that:
- The considered area is large compared to the fault scale.
- The state of stress is homogeneous and constant in space and time during faulting or the fault and slip data can be separated in subsets where this is expected to hold reasonably true.
- Movements on each fault are sufficiently independent from each other (i.e. movement on a fault is not influenced by a stress perturbation due to movement on another fault).

If these assumptions are satisfied, then the deviatoric tensor of a tectonic event can be obtained from several independent data related to this event.

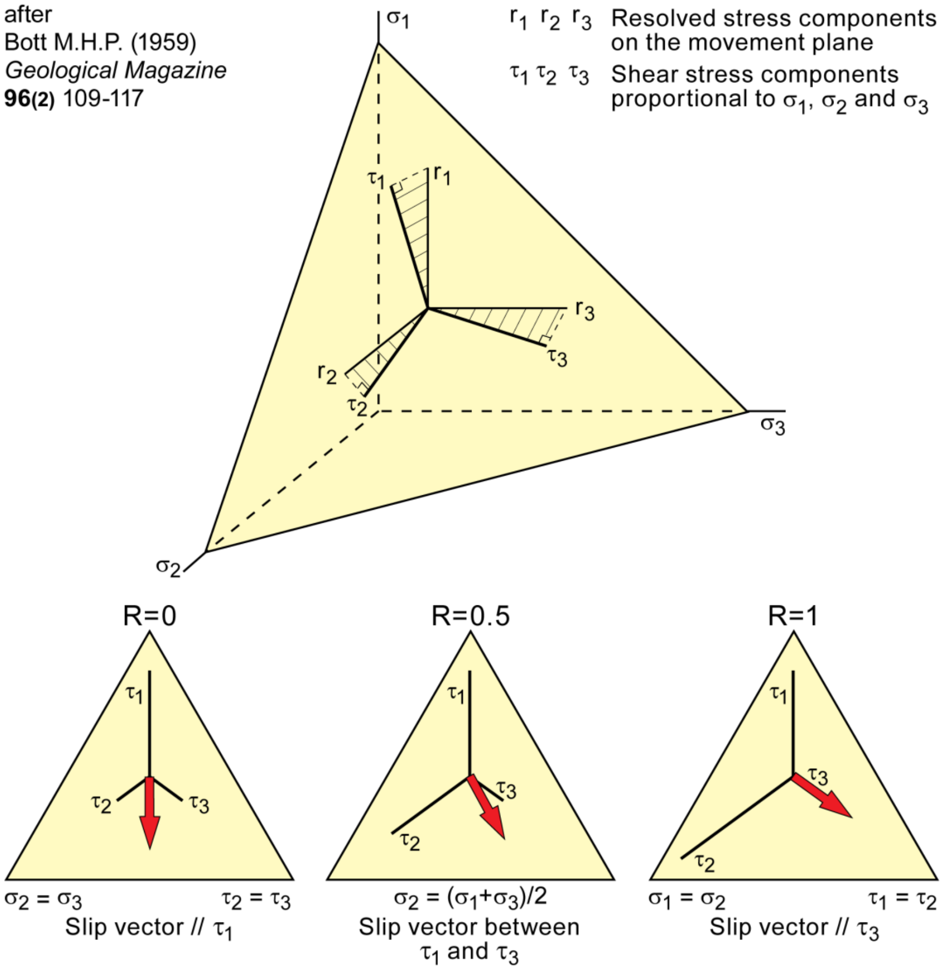
Relationship between slickenline and stress

The relationship between a slickenline and the stress that produced it has been established by Bott (1959). He showed that the **rake** (or **pitch**, i.e. the angle between the slicken-line with the horizontal strike of the fault plane) depends on four parameters:
- 3 angles (generally Euler angles) that define the orientation of the fault plane with respect to the orientation of the principal stresses i.e. the eigenvectors $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ (with the corresponding eigenvalues $\sigma_1 \geq \sigma_2 \geq \sigma_3$, in geology taken positive in compression)
- the aspect ratio of the stress tensor that locally describes the relative magnitudes of the principal stresses and is defined as:

$$R = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$$

This equation implies that $0 \leq R \leq 1$. A low R-value indicates a prolate stress ellipsoid where σ_2 is closer in magnitude to σ_3 than σ_1. R increases as σ_2 moves closer in magnitude to σ_1, producing an oblate stress ellipsoid.
The stress ratio controls, for any given plane, the direction of shear stress and determines the geometry of the slip on fault planes. This consequence of Bott's demonstration is easier to understand when considering a particular case.
Consider the particular plane P that makes the same angle (about 34°) with the three principal stresses $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$. The three direction cosines of the plane are equal and, therefore, the three components of resolved shear stress on the plane are only proportional to the principal stresses. One can project

on the plane the three components of resolved shear stress τ_1, τ_2 and τ_3 with no angular consideration.



Bott principle in the particular case where the shear plane is equally inclined to the three principal stresses σ_1, σ_2 and σ_3

Now consider three different values of R:

- R = 0 $\sigma_2 = \sigma_3$; uniaxial compression
- R = 0.5 $\sigma_2 = \left(\sigma_1 + \sigma_3\right)/2$
- R = 1 $\sigma_1 = \sigma_2$; uniaxial extension

One obtains three different striations (two end members and the intermediate case) parallel to the resolved shear stress, from assumptions. Therefore, for the same fault plane the orientation of the striation line varies according to the relative magnitudes of principal stresses. Inversely, a single striation may correspond to an infinity of state of stress and does not constrain the direction of the stress-tensor principal axes. Therefore, a statistic treatment dealing with a large number of data is necessary to approach the state of stress. As a reward, compared to the geometrical approaches, one can calculate the relative magnitudes of the three principal stresses.

Inverse methods

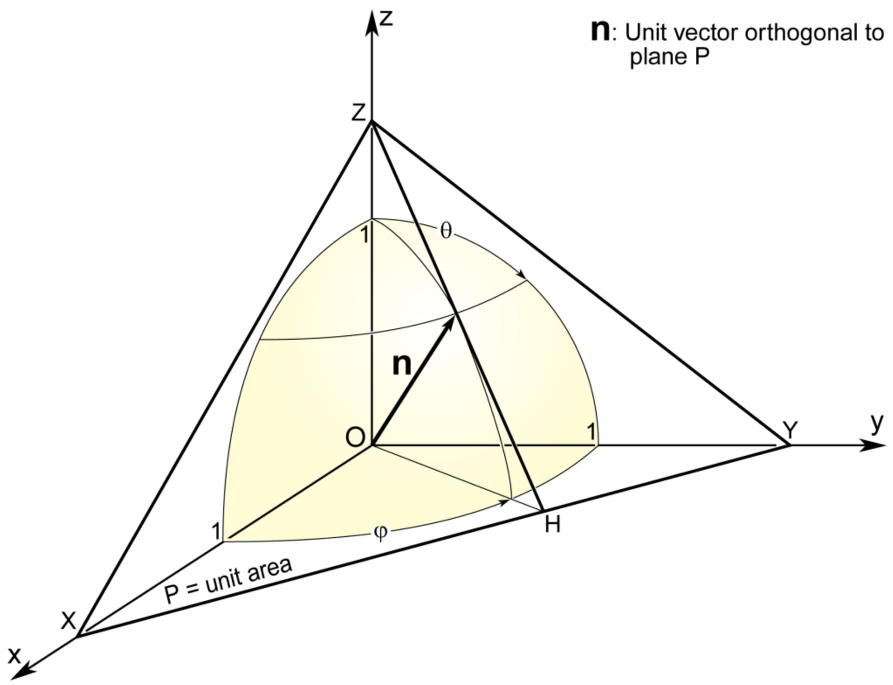
Stress inversion uses fault data measured at outcrop to infer the remote, paleostress tensor that produced or reactivated the striations and planes observed. The challenge is deducing dynamic information (stress) from kinematic data (strain). Parallelism between the principal stress axes and the principal strain axes is an important assumption extrapolated from elasticity. Calculation provides both the principal directions and the stress ratio.

Principle

The inverse problem is to calculate four (out of six) components of a symmetric stress tensor that minimizes the difference between measured and computed slip directions on the fault planes, with the requirement that for each fault plane the striation is parallel to the resolved shear stress.
The high number of unknown factors and the non-linearity of the problem require the use of computer software for the calculations. A commonly employed method consists in selecting numerically, out of a large number of randomly chosen tensors, the tensor for which the sum of angular misfit between measured striations and calculated slip-lines is the smallest.
It is then usual to verify whether the solution tensor yields resolved shear stress higher or equal to the resistance to slip on each plane. This is done on a Mohr diagram where the rupture envelope fixes the highest limit of the stress values.

Analytical considerations

In geology, the orthogonal coordinate system with axes x = 1, y = 2 and z = 3 sensibly refers to the geographical NS and EW horizontal directions and the vertical, respectively.
A fault plane F is defined by the unit vector $\vec{n}$ normal to the plane, with geographical coordinates (n_1, n_2, n_3).



Definition of a unit area with an infinitesimally small tetrahedron

The slicken-line on F is defined by the unit vector $\vec{L}$ with geographical coordinates (L_1, L_2, L_3).

The traction applied to F, $\vec{t}$, can be computed from the stress tensor **T** represented by the symmetric matrix (symmetric because the rock undergoes no torsion):

$$\mathbf{T} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

so that in this coordinate system (Cauchy law):

$$\vec{t} = \mathbf{T}\vec{n}$$

Thus $\vec{t}$ is given by:

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} . \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

Since $\sigma_{ij} = \sigma_{ji}$, that can be written as:

$$\begin{aligned} t_1 &= \sigma_{11}.n_1 + \sigma_{12}.n_2 + \sigma_{13}.n_3 \\ t_2 &= \sigma_{12}.n_1 + \sigma_{22}.n_2 + \sigma_{23}.n_3 \\ t_3 &= \sigma_{13}.n_1 + \sigma_{23}.n_2 + \sigma_{33}.n_3 \end{aligned}$$

In principal stress frame, **T** is represented by the diagonal matrix:

$$\mathbf{T} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

with $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

As previously seen (lecture on stresses and mechanical aspects of deformation), six quantities are necessary to define **T**: the 3 Euler angles (θ, ϕ, ψ) giving the orientation of the principal axes with respect to the coordinate axes along with the 3 principal stress magnitudes.

T can be separated into an isotropic pressure part **P** and a deviatoric one **D**. Only the deviatoric part **D** contributes to the shear stress since the isotropic pressure-part **P** results in a force normal to any plane.

Take $\vec{\tau}$ the shear component on F. The orientation and direction of this shear component, reduced to a unit vector, remains unchanged when the hydrostatic pressure **P** is taken out of **T** and **T** is multiplied by an arbitrary positive constant (homothetic ratio). Therefore the shear component depends only on 4 parameters: the three principal stress orientations (θ, ϕ, ψ) and the relative ratio of principal stresses, which refers to stress magnitudes. These four parameters define a **reduced stress tensor**. They do not define a unique tensor but a set of stress tensors that would cause on any fault a displacement with the same unit vector.

Attention: The amount of displacement is not considered in this calculation. Consequently, the tensor calculated is NOT a strain tensor.

Assuming parallelism between the maximum resolved shear stress on the fault plane and the slip line for inversion, the basic equation to be solved is:

$$\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0$$

where b_i are components of the unit vector $\vec{b}$ perpendicular to the slip line L on the fault plane (vectors $\vec{n}$ and $\vec{L}$ are orthogonal to each other).

The set of vectors obtained for one fault is ambiguous, unless additional assumptions (such as values of the rock cohesion or the internal friction angle, the lithostatic pressure, etc) can be made.

Consider N fault planes with unit normals $\vec{n}_j$ (j=1,...N) on which striations with unit vectors $\vec{L}_j$ correspond to the same tectonic event (stress tensor). Inversion consists in determining out of a set of M tensors **T** the tensor $\mathbf{T}_i$ that explains the data, i.e. such that for each i and j the unit vector of the tangential force $\vec{\tau}_{ij}$ due to $\mathbf{T}_i$ corresponds to the unit vector $\vec{L}_j$ of the measured striation on the fault $\vec{n}_j$ (best fit agreement). The angle error a_{ij} between $\vec{\tau}_{ij}$ and $\vec{L}_j$ ranges from 0 to 180°. The calculation consists in seeking tensors that minimise the vector function $F_i = \sum_{j=1}^{N} |a_{ij}|$.

This function depends on the four parameters (θ, ϕ, ψ) and R, which are the results to be found.

Method

The analytical method proceeds in two steps.

Best fit solution

A computer program first applies thousands of arbitrarily generated stress tensors (variations of orientation and relative magnitudes of the principal stresses) to the fault data set. A trial and error calculation, consisting of randomly chosen parameters with a uniform probability density over their variation range, provides a first approximation (a Monte Carlo inversion). This approximation of the tensor position and shape is further optimised through a non-linear success and failure test tuning separately the three Euler angles and the ratio R until any change of these four parameters no longer improves the minimisation. Linearisation, which is now sufficient because one is very close to the final answer, refines this second result to the “best fit solution”, which has the smallest angle difference between observed and calculated slip vectors in the fault planes, for a maximum of faults. This calculation requires at the very least four, well distributed in space fault plane – striation pairs to provide a practical answer. The error on the four parameters (θ, ϕ, ψ) and R indicates the confidence interval (usually 95%).

Compatibility with a friction law

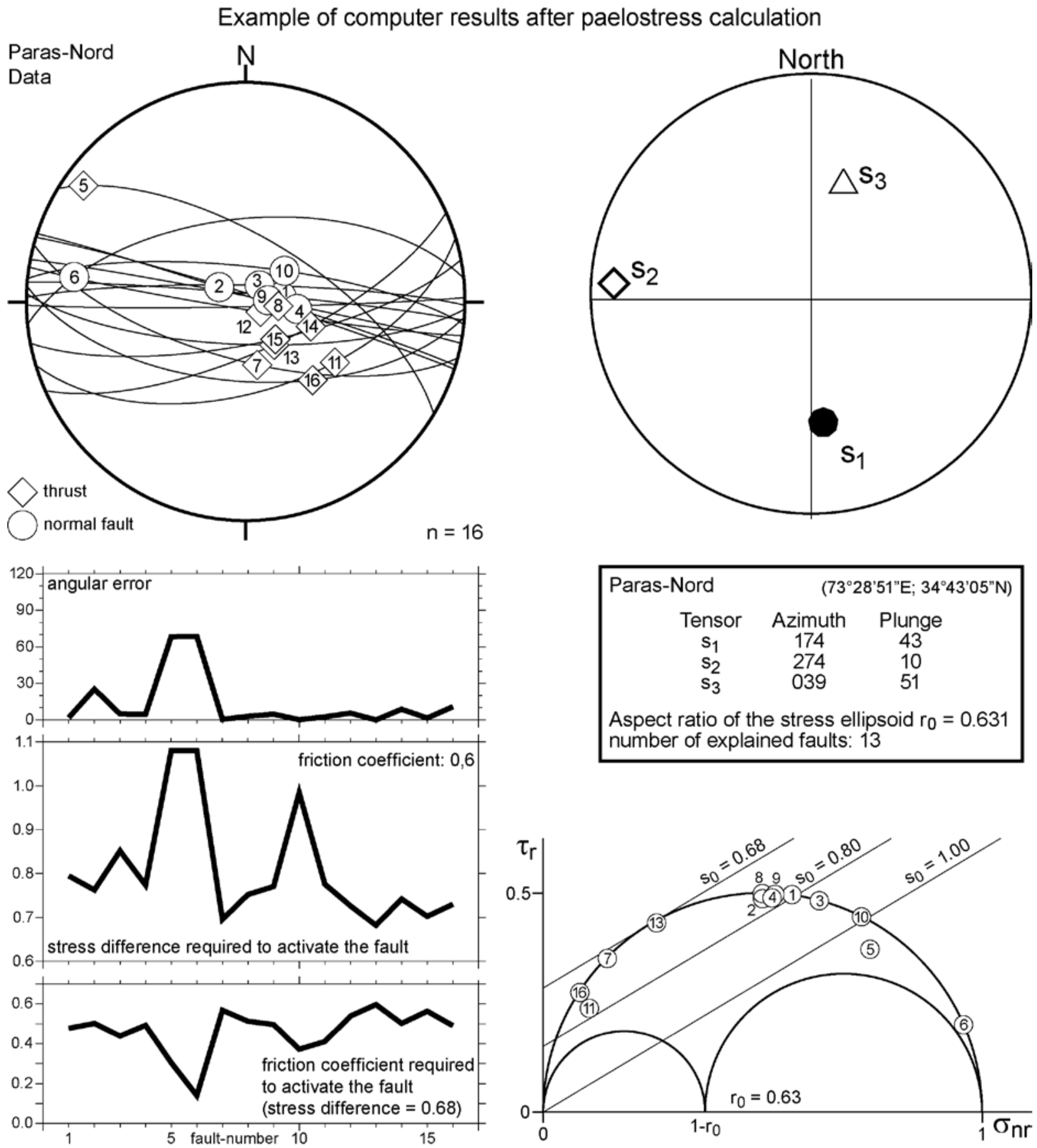
Second, the compatibility of the reduced stress tensor solution with a true stress tensor that satisfies a friction law on the activated faults planes is estimated through graphical representations (for example Mohr diagrams).

Representation of results

An important assumption is that at a particular place a given tectonic event is characterised by one homogeneous stress tensor.

The graphic representation of the results comprises a histogram of calculation errors and a Mohr diagram illustrating the tectonic regime. They provide complementary information in evaluating the quality of the solution.

- The histogram displays the angular errors between the measured striations and the shear stress from the calculated tensor on each individual fault of the data set. This representation allows appreciating at a glance the quality of the result. The peak that contains most faults should stand next to the 0°-error axis. The result is considered to be correct if the fault data stand within less than 20° from the calculated axis. Striations at a larger angle are not explained in a satisfactory way. These faults may pertain to another tectonic event or signal that the stress system is not homogeneous within a single event.
- The Mohr diagram illustrating the magnitudes of calculated principal stresses further allows showing whether the calculation yields a sensible result. Fault planes whose slickenline is compatible with the determined stress tensor have an acceptable ratio of shear versus normal stress components and, therefore, are plotted near the point where the failure envelope is tangential to the Mohr circle. Fault planes that plot in the lower part of this diagram are not acceptable because the normal stress component is too large with respect to the shear component. These latter faults may not be related to the calculated tensor. However, there are often inherited fractures that are easy to activate, and on which the magnitude of the shear component with respect to the normal component is not a determining criterion. Finally, the Mohr diagram represents the spatial distribution of fault planes.



One can now identify and exclude unsatisfactory fault measurements from the data set in order to refine the solution with the remaining, acceptable measurements and run a new calculation to know whether the rejected faults from a first calculation do not belong to a separate tectonic event. However, determination in the field of superimposed tectonic events is more reliable than blind confidence in calculations.

Superimposed faulting phases

Fault reactivation and interaction is a well-established natural phenomenon. Evidence that faults characteristically undergo repeated movements comes from seismological studies, from multidirectional sets of slip lineations on exposed fault surfaces, and from varying stratigraphic separation.

In the case of several tectonic phases, the most difficult problem consists in defining the relevant stress tensors and selecting corresponding striated fault planes. Separation of homogeneous populations from heterogeneous data sets is a prerequisite to obtain results of geological significance.

Unfortunately, outcrops often do not display intersection criteria or any structural relationship that would define a relative chronology.

Assumption

The basic mechanical assumption is that an applied stress field can reactivate only pre-existing faults that have favourable orientations, where shear stresses are sufficiently high to overcome the frictional resistance.

Separation of fault sets

A first separation boundary is that fault planes should yield directions of principal stresses contained within an angle smaller than 20 to 40°.
A man-assisted mathematical technique proceeds in several steps:

- Run a first calculation.
- Separate dubious faults from both the histogram and Mohr circle
- Use the remaining acceptable faults to run a second calculation and refine the tensor. The procedure must go until a stable population yields a stable tensor.
- Run calculations and separations on the extracted “wrong” faults in the same way.

A purely mathematical method consists in two simultaneous calculations with different goals:

- Determine the best tensor that explains only a limited percentage of faults (e.g. n% out of N data)
- Determine for each tensor the percentage of fault data to accept.

The first calculation can be summarised as a random calculation with thousands of tensors, but for each tried tensor the acceptable n fault population does not necessarily include the same data. The minimisation will yield the best way to take into account n% fault among the N data so that the angular function is the smallest. The main problem lies in defining n. If this is too large, the calculation will include data unrelated to the searched event. If n is too small, the result is partial. There is no direct way to define n. Therefore the procedure is a succession of try-and-fail using two criteria:

- convergence of results from initially different tensors
- analysis of the histogram and Mohr circles.

In practice, fully automatic procedures designed for separating homogeneous data sets (i.e. resulting from a single faulting phase) from heterogeneous (multiphase) ones are unsuccessful. Different tensor populations may reflect temporal as well as spatial heterogeneity of brittle strain. It is difficult to ascertain timely successive tectonic stress events rather than partitioned fault movements or short term fluctuations of the stress field. The geologist has to use structural data such as fault plane characteristics, stylolites and tension gashes to guide the correct choice of stress tensors from fault-slip data.

Hydraulic fracturing

The three-dimensional stress state and the driving pressure associated with the opening of veins and dykes can be estimated from vein (dyke) orientations. The driving fluid pressure ratio D_R is obtained from considering a fracture whose normal is inclined to the principal stress axes $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ at angles θ_1, θ_2 and θ_3, respectively. The directional cosines of these angles are written $\mathbf{n_1} = \cos\theta_1$, $\mathbf{n}_2 = \cos\theta_2$ and $\mathbf{n}_3 = \cos\theta_3$ and satisfy the relationship (equation 9 in lecture on stresses):

$$\mathbf{n_1}^2 + \mathbf{n}_2^2 + \mathbf{n}_3^2 = 1$$

The far-field normal stress is the sum of the components:

$$\sigma_N = \sigma_1\mathbf{n}_1^2 + \sigma_2\mathbf{n}_2^2 + \sigma_3\mathbf{n}_3^2$$

Using these equations, the condition to open a fracture is:

$$p-\left\{\sigma_1\mathbf{n}_1^{\,2}+\sigma_2\mathbf{n}_2^{\,2}+\sigma_3\left(1-\mathbf{n}_1^{\,2}-\mathbf{n}_2^{\,2}\right)\right\}\geq 0$$

By rearranging one gets:

$$p-\left\{(\sigma_1-\sigma_3)\mathbf{n}_1^{\,2}+(\sigma_2-\sigma_3)\mathbf{n}_2^{\,2}+\sigma_3\right\}\geq 0$$

$$p-(\sigma_1-\sigma_3)\mathbf{n}_1^{\,2}-(\sigma_2-\sigma_3)\mathbf{n}_2^{\,2}-\sigma_3\geq 0$$

Dividing each term by $(\sigma_1 - \sigma_3)$:

$$\frac{p-\sigma_3}{(\sigma_1-\sigma_3)}-\mathbf{n}_1^{\,2}-\frac{(\sigma_2-\sigma_3)}{(\sigma_1-\sigma_3)}\mathbf{n}_2^{\,2}\geq 0 \tag{1}$$

This equation identifies two ratios:
The previously defined aspect ratio of the stress tensor $R=(\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$.
The driving pressure ratio $D_R=(p-\sigma_3)/(\sigma_1-\sigma_3)$, which also varies between 0 and 1 and describes the balance between the value of the fluid pressure and the minimal and maximal principal stress components.
Dividing equation (1) by R yields:

$$1-\frac{\mathbf{n_1}^{2}}{R}-\frac{D_R}{R}\mathbf{n_2}^{2}\geq 0 \tag{2}$$

This equation is a curve that, for a given pair of R and D_R, can be transferred from a 3-D Mohr circle to a stereographic projection with $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ as its vertical and horizontal axes. The curve is symmetric with respect to the principal stress planes and separates two regions: One includes poles to fractures with a positive driving pressure, i.e the fractures able to dilate in this particular stress state, and the other includes poles to fractures with a negative driving pressure, which remain closed. This principle can be inverted, using the distribution of vein or dyke data on a stereogram to determine the relative values of stress and fluid pressure on a Mohr circle, thereby providing information on the stresses controlling fracture opening. For example, take the case of a low driving pressure, which produces few opening fractures nearly orthogonal to $\vec{s}_3$. $\vec{s}_3$ is the center of the region with poles. One may define the size of the cluster through its length and width measured as half-angles, the length in the $\vec{s}_2$ direction and the width in the $\vec{s}_1$ direction. In a 3D space, these size parameters are angles in the (s_2, s_3) and (s_1, s_3) planes, respectively.

Consider the intersection of the cluster envelope with the (s_1, s_3) plane. This point is defined by three directional cosines, with angles θ_{1w}, θ_{2w} and θ_{3w} (w for width) the inclinations to the principal stress axes $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$, respectively.

All points in the (s_1, s_3) plane have $\theta_{2w}=90°=\pi/2$, hence $\cos^2\theta_{2w}=0$.

$\theta_{3w}=(\pi/2)-\theta_{1w}$ which means that $\sin^2\theta_{3w}=\cos^2\theta_{1w}$ (trigonometric rule).
In this plane, equation (2) is reduced to:

$$1\geq\frac{\cos^2\theta_{1w}}{R}$$

which one can write:

$$\sin^2\theta_{3w}\leq R. \tag{3}$$

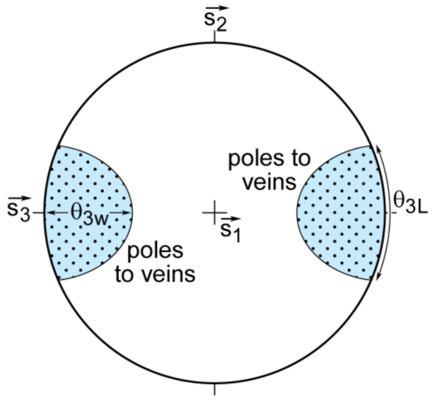
Consider the intersection of the cluster envelope with the (s_2, s_3) plane. All points in this length (L) plane have $\theta_{1L} = 90° = \pi/2$, hence $\cos^2\theta_{1L} = 0$, and $\theta_{3L} = (\pi/2) - \theta_{2L}$ which means that $\sin^2\theta_{3L} = \cos^2\theta_{2L}$. In this plane, equation (2) is reduced to:

$$1 \geq \frac{D_R}{R}\sin^2\theta_{3L}$$

Replacing R as defined in equation (3) and rearranging, this expression is:

$$\frac{\sin^2\theta_{3w}}{\sin^2\theta_{3L}} \geq D_R \tag{4}$$

Equations (3) and (4) indicate that two angles provide a measure of the stress ellipsoid shape and the fluid driving pressure. These angles can directly be measured on a stereographic projection:
First one plots the poles to opened fractures (veins) on a stereographic projection. Typically, these poles are clustered in a limited region of the projection.



Two stereographic angles providing an estimate of the shape of the stress ellipsoid and the driving pore pressure
after Baer *et al.* 1994 *J. Geophys. Res.* **99B12**, 24039-24050

From the center of the pole cluster (presumably $\vec{s}_3$) and the envelope of the cluster one can measure θ_{3w} along the (s_1, s_3) diagram axis, and θ_{3L} is half the angle containing the cluster in the (s_1, s_3) direction. Then equations (3) and (4) can be solved.

Stress or strain directions?

The similarity of relation of fault orientations to stress directions on the one hand, and strain directions on the other hand opens to some confusing interpretations. Paleostress reconstructions suffer from several points of dissatisfaction:

- Fault patterns change with time mostly because of displacements and rigid rotations accumulated by early faults.
- It is often necessary to include measurements from several exposures in order to have enough fault-data to calculate the paleostress axes. Obviously, the local stress situation may have been different than the averaged, calculated stress orientation. It is difficult to estimate this statistical error.
- Fault interaction strongly influences slip directions because movement on one fault plane may deviate the regional stresses near that fault, and this change in orientation affects the movement on other faults nearby.
- Large faults generate small splay faults, which in turn generate smaller splay faults so that the stress orientations inferred from fault-slip data is scale dependent. Furthermore, local geometries may produce local and transient paleostress regimes (e.g. near a ramp).

- Many major faults move under low resolved shear stress (“weak” fault) at high angles to $\vec{s}_1$ rather than at high shear stress predicted by Byerlee’s law and with small angle to the $\vec{s}_1$, according to Mohr-Coulomb criterion.

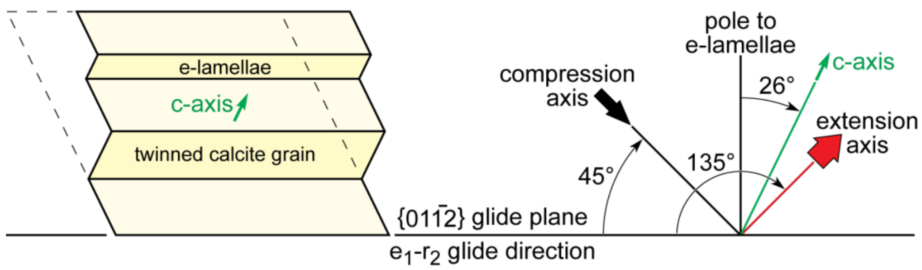
Therefore, fault data are often complex and do not simply reflect local stress or strain tensors. In many cases they form a heterogeneous pattern, which is strongly influenced by rotations between blocks. The calculated, homogeneous tensor axes deduced from the analysis of fault-slip data appear in some cases to be consistent with finite displacements at block boundaries, thus are pertinent to finite deformation rather than stress directions. The point encourages limitation of the paleostress calculations to very little deformed areas in which strain and stress orientations can be assumed to be close to each other, as during elasticity. On another hand, quantitative estimates of strain from fault data require knowing the amounts of slip on all faults, which, in practice, is not available. Thus, the most that can be inferred are the principal directions. The analysis of fault slip data leads to a mean stress tensor which is averaged on time (over several thousands or millions of years) and space (the volume of rock that contains the analysed faults).

Mechanical twinning in minerals

Twinning depends on the magnitude of the shear stress which has been applied on the twinned mineral. Twinning planes correspond to specific planes of the crystal lattice, and twinning takes place in a specific direction of this crystal lattice. The lattice orientation can be measured through standard microscopic analysis with a universal stage and / or a goniometer. This property is used for evaluating the stresses that a rock has supported during its history.

Owing to the widespread occurrence of *e*-twins, calcite in particular has been the object of several methods for paleostress analysis. These twins are equated with simple shear zones in a particular sense and direction along crystallographically defined *e* planes {01-12} of the calcite grains. As such, they are is geometrically comparable to slip along a slickenside lineation within a fault plane and an inversion method similar to that for fault slip data can be employed.

Relationship between principal stress directions and calcite twin lamellae
from Spang J.H. (1972) *Geol. Soc. Am. Bull.* **83(2)**, 467-472



These methods are best applied to very small strains in aggregates deformed at low pressure and temperature and share the fundamental assumption that the measured twins were formed in a homogeneous state of stress at the grain scale and were not passively rotated after their formation. As for faults, the inversion of gliding data along twin planes leads only to four parameters of the complete stress tensor: the orientations of the three principal stress axes and the stress ellipsoid shape ratio R.

Conclusion

Fault-slip data reflect passed tectonic activity under some stress environment. Within the limits of a few assumptions (material isotropy, strain-stress coaxiality) and under conditions of brittle deformation resulting in strain sufficiently small for involving no rotation and fault interaction, one system of principal stress axes can valuably represent the paleo-stress-environment. An ellipsoid quantitatively embodies these “paleostress” conditions. The geological approximation neglects variations in stress intensity and orientation related to material heterogeneities. In addition to the orientation of the principal paleostresses, the relative paleostress magnitudes indicate to some extent under which tectonic regime the studied faults have formed, and how they evolved through time. Yet, calculations do not yield absolute magnitudes of the stress tensor. Calculations depend on consistency of fault sets and subsets at the scale of sampling site, which must be verified to infer that stresses were in equilibrium. The consistency of results in regional studies of little deformed areas indicates that paleostress calculations are a valid geological application. Thanks to this success, paleostress inversion procedures are becoming a routine analytical technique in structural geology. They result in a **kinematic model**, which includes maps and sections illustrating the relation between fault orientations and regional stress field or large-scale crustal displacements, and their evolution through geological time. As such, the knowledge of recent and active stress fields is important to seismic hazard assessment and risk mitigation; knowledge of the stress field evolution in time is fundamental to the understanding of the earth dynamic system.

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Recommended free software

http://www2.arnes.si/~jzaloh/t-tecto_homepage.htm