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by

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I. Introduction

Previous attempts to stabilize the prices of commodities by purchases and sales from buffer stocks have ended in failure. In the past buffer stocks of gold, silver, and tin were each attacked by speculators anticipating price increases.[1/] And yet similar stabilization schemes continue to be proposed and, in some cases, adopted. International agreements have recently been negotiated to stabilize the prices of tin, wheat, sugar, coffee, cocoa, and natural rubber by operating separate stockpiles of each commodity, and consideration is being given to similar agreements for twelve other commodities. Proponents of such schemes either ignore the historical record[2/] or feel confident the mistakes of the past can be avoided simply by creating larger buffer stocks in the future.

Such confidence rests on shaky foundations. Although the recent agreements have been evaluated favorably, the methodology underlying the analyses renders their favorable conclusions worthless. The consequences of a particular commodity agreement cannot be forecasted without taking proper account of how it will affect the price expectations of speculators and hence their behavior. And yet, previous analyses have ignored completely the impact of such schemes on speculative demand. This is no small omission since it has been speculators who have thwarted all previous attempts at price stabilization.

This paper develops a methodology in which the consequences of buffer-stock proposals can be studied. The approach is to examine the competitive equilibrium under rational expectations when the government adheres to a specified rule. For analytic simplicity, it is assumed that the government attempts to peg the price. This policy is shown under plausible assumptions to induce speculative attacks. The methodology developed and the insights gained about speculative attacks and one-sided bets apply equally to policy regimes where ceiling and floor prices differ. [4/]

In Section II, a pegging policy is introduced in the Hotelling model of pure depletion under certainty. The resulting equilibrium is shown to contain a speculative attack -- a situation where previously inactive speculators suddenly purchase the remaining government stock. The attack occurs when total stocks fall below a predictable threshold -- as they eventually must because of the continual depletion. An appendix adapts the analysis of the section to the attack on gold, which occurred in 1968. In addition, it has been applied by Koromzay (1979) and Krugman (1979) to balance-of-payments crises.

In Section III, the model is generalized to include the possibility of random additions to the existing stock. These additions are interpreted as new harvests of an agricultural good. It has been shown by Townsend (1977) under related circumstances that the government cannot successfuly peg the price forever by operating a buffer stock, but he did not go on to investigate the stochastic equilibrium which results when such a futile policy is attempted. Such an analysis is conducted Section III, where a pegging policy is introduced into the standard agricultural carry-over model of Samuelson [1973] and Kohn [1978]. As in Section II, a speculative attack is shown to occur in equilibrium whenever total stock falls below a predictable threshold. But now the possibility exists that random additions will keep the total stock above the threshold. The probability of this event depends on the type of policy pursued and its underlying parameters. If the policy is attempted pegging and the official price is set so low that it induces more consumption than the mean harvest, then a speculative attack occurs with probability one. A more revealing case is when the consumption induced by the official price just matches the expected harvest. Then, on average, no depletion of the government stock will occur and it seems natural to think that the attack threshold will never be crossed -- at least if the initial government stockpile is sufficiently large. This turns out to be true only if harvests do not flucturate at all. Otherwise, an attack is virtually inevitable. Indeed, such attacks almost surely recur infinitely often -- regardless of the initial size of the government stockpile.

Section IV discusses the economic function of speculative attacks and useful extensions of the analysis.

II. Pure Depletion Under Certainty

In this section, we introduce a government pegging policy into the Hotelling model of an exhaustible resource. Since this model contains neither randomness nor additions to stocks, it permits attention to be focused without distraction on the causes of speculative attacks. Once they are understood, we will be able to discuss the occurrence of attacks in a model where random harvests periodically augment existing stocks.

To begin simply, suppose the government attempts to peg the real price of a non-renewable resource (call it "oil") by standing ready to purchase whatever oil is offered or to sell, if necessary, <u>all</u> of its limited stock to maintain the official price relative to that of some other storable good. It is assumed that this other good is produced each period, and that the government initially has the capacity to purchase with it whatever oil is offered at the official price. Assume for simplicity that both extraction and stockpiling are costless, that both activities are competitive, and that non-speculative demand for oil is a stationary function of the contemporaneous relative price.

If the official price is set below the price which would completely choke off demand (the "choke price"), it will induce positive demand. But since the reserves of the government are finite, they cannot satisfy this demand forever at the official price -- even it the government buffer stock initially contains every drop of oil on the planet.

A. Equilibrium with Sepculators Excluded

The market incentives which trigger an attack can best be understood by considering initially the equilibrium price path which the government policy would induce in the absence of speculators (by definition, private agents who purchase for subsequent resale). Since (by assumption) the government follows its policy rule, it would prevent the price from falling below the official level ($\bar{P}$) by purchasing at that price all the oil which extractors offered, and would allow the price to rise above that level only if it ran out of reserves to sell. Since (by assumption) the extractors maximize discounted profits, they would sell their stock (I) at the highest discounted price (if the highest discounted price occurred at different times, there would be indifference as to the allocation of the stock among such periods). In equilibrium, the extractors must voluntarily sell oil to the government at the first instant and, in the absence of speculators, directly to consumers after the buffer stock is exhausted. Thus, in the absence of speculators, the equilibrium price path must consist of two discontinuous pieces and can be constructed from the curves in Figure I:

[Figure I goes here]

The first piece of the path is horizontal at the official price. The second, higher piece rises at the rate of interest from a level which has a present value equal to $\bar{P}$ until the choke price (P^c) is reached, and then is horizontal at the choke price. Given such a price path, extractors would be willing to sell either at the first instant or between the time the price jumps up and the time it reaches the choke price. To construct the equilibrium path from Figure I, pick the date of the upward jump in price arbitrarily and compute cumulative demand along each piece of the path. If cumulative demand exceeds (falls short of) the initial stock (I), the horizontal piece before the jump should be shortened (lengthened) and the rising piece after the jump should be lengthened (shortened). Such a procedure reduces cumulative demand along the new path since it replaces one (flat) segment of the old path with a uniformly higher segment of the same length. In matching cumulative demand to the initial stock, it will never be necessary to eliminate the horizontal piece of the price path entirely, however, as long as the official price set by the government exceeds the free market price (a necessity if the government is to acquire any reserves). The maintenance of the official price for more than an instant insures that in equilibrium the price will jump upward when the buffer stock is exhausted.

Other characteristics of the equilibrium without speculators deserve mention. The magnitude of the reserves acquired by the government in establishing the official price is endogenous. Suppose, for example, that the equilibrium price path has an upward jump at $\hat{t}$ (see Figure I). Then, extractors initially sell part of their stocks ($I-\hat{S}$) to the government and retain the rest ($\hat{S}$) for sale (at prices with the same discounted value) to consumers after $\hat{t}$, the time when the buffer stock runs out. The reserves initially acquired by the government are exactly enough ($\hat{t}D(\overline{P})$) to satisfy the cumulative needs of consumers during the phase when extraction is suspended.

B. Equilibrium with Speculators Permitted to Enter

It has been shown that the equilibrium price path in the absence of speculators always has an upward jump. We now consider the equilibrium with speculators. If speculators are introduced, such a price path would give them an incentive to buy on the instant before the jump and to sell sometime afterwards. Given any amount to be acquired, the optimal acquisition strategy at constant prices is to purchase at the last instant before the jump -- rather than more gradually -- since gradual acquisition at the official price would cause the speculator a needless loss of interest income.

Such sudden acquisitions by speculators would in aggregate cause the buffer stock to be depleted sooner, and the corresponding subsequent sales would cause the second piece of the price path to begin at a lower level. Indeed, if speculators acquired a large enough stock, the upward jump in the price path would be eliminated altogether and no incentive for a larger acquisition would remain. The absence of a price jump would induce extractors to sell their entire stock to the government in the first instant. It will be shown that such a situation constitutes the equilibrium when speculators are permitted to enter.

If speculators acquired stock S in the attack, the price would (in accordance with the Hotelling principle (1931)) immediately adjust to a level (P_A) permitting absorption along a path rising over time at the rate of interest. The larger the stock to be absorbed, the lower the initial price on the second piece of the price path:

$$P = P_A(S),\ P_A'(S) < 0.$$

In the simple case under consideration, this function can be written implicitly as a single equation:

$$\int_{x=0}^{\infty} D\{P_A e^{rx}\}dx = S.$$

The size of the government buffer stock at the instant of the attack (S*) can be determined by solving the following equation, which insures that speculators make zero profits in equilibrium: $P_A(S^*) = \bar{P}$. S* is determined graphically in Figure II.

[Figure II goes here]

Given the stock in private hands following the attack (S*), the date of the attack may be computed ($t_A = \frac{I-S^*}{D(\overline{P})}$). Prior to t_A, the official price is maintained by the government. To do so, the government initially must purchase the offerings of extractors who sell their entire stock[6/] at the highest discounted price (the initial price). The government must then sell to the market at the rate $D(\overline{P})$. When t_A is reached, speculators suddenly attack the remaining reserves (S*); they then sell over time at prices rising without a jump from the official price by the rate of interest. This scenario is an equilibrium since (1) the actions of all agents (extractors, speculators, consumers, and the buffer-stock manager) are compatible (markets clear); (2) the government is following its policy rule; and (3) no speculator or extractor can increase his discounted profits at the given prices by alternative behavior.

A comparison of the equilibrium with and without speculation may be useful.[7/] In both cases, the government ultimately loses all its reserves and is forced to terminate its stabilization program. But the stock held by the public after the program terminates is larger when termination results from an attack ($S^* > \hat{S}$) because the rapid depletion at the rate $D(\overline{P})$ terminates sooner ($t_A < \hat{t}$) in that case. Moreover, consumer-plus-producer surplus is higher in the equilibrium containing the attack since (1) the two equilibria are identical until only S* remains; and (2) the allocation of the remaining S* in the attack equilibrium dominates all other allocations of S* since it obeys the Hotelling principle.

In conclusion, two points should be noted. First, this simple model predicts speculators will attack swiftly, with no prior activity, and before consumers have depleted the government buffer stock. Such behavior, which is often regarded as bizarre, has been shown to be the entirely rational response to the market incentives generated by the government stabilization program. 8/ Second, although the buffer-stock program reduces the combined surplus of extractors and consumers relative to the Pareto-optimal, laissez-faire allocation, none of this welfare loss is borne by the extractors. Indeed, introduction of the program raises the initial price and therefore increases the wealth of the extractors relative to laissez-faire. In such a circumstance, it is not surprising that the program has its political adherents.

III. A. Generalization to Incorporate Stochastic Replenishment

In the previous section, we introduced a government pegging policy into the simplest Hotelling model of an exhaustible resource. Since there are no additions to aggregate stocks in that model, the reserves of the government decline steadily--providing an ideal environment for studying the incentives for speculative attacks which a falling government stock creates.

With this preliminary understanding of the determinants of speculative attacks, we are ready to consider whether they occur in more complex environments. In particular, we examine the effects of a stabilization policy in the standard agricultural carry-over model analyzed by Samuelson (1973), Kohn (1978), and many others. In that model private carry-overs are augmented each period by a random exogenous harvest drawn independently from a stationary distribution. As in the Hotelling model, there are two kinds of private agents: consumers--whose demand depends only on contemporaneous price--and producer-speculators--whose sequential decisions maximize their expected wealth. All costs are assumed to be zero. Hence, if the random harvest is degenerate at zero the model of this section would collapse to the discrete-time version of the Hotelling model. The policy introduced is once again pegging by means of a buffer stock. But the approach is designed to facilitate analysis of other government policies, as will become evident.

The model of the previous section can be summarized by the following system of equations:

$P(t) = P^*(S(t))$	Price Equation
$\dot{S}(t) = -D(P(t))$	Transition Equation
$S(0) = \bar{S}$	Initial Condition

where $P^*(S) = \max(\bar{P}, P_A(S))$ and S is the sum of private and official stocks. The carry-over model of this section will be put in a similar form. In the next subsection, it is shown that a unique equilibrium price function exists which indicates the market price induced by attempted pegging as a function of the combined stocks of the government and private sector (the social stock). The price function is derived by backward induction. Subsection B is devoted to displaying other aspects of the point-in-time equilibrium associated with any particular social stock, while subsection C adjoins to the price function the initial condition and transition equation for the state variable (social stock) and analyzes the stochastic dynamics of the system.

A. Existence and Uniqueness of the Infinite-Horizon Equilibrium Price Function.

We begin the analysis by showing inductively how a function can be constructed indicating the market price in the infinite-horizon, carry-over model with attempted pegging as a function of the social stocks brought into any period.

If the price in any period is a known function of the state vector at the beginning of the period, and -- in the previous period --

(1) the government obeys its policy rule;

(2) risk-neutral private speculators act optimally, with rational expectations; and

(3) the market clears,

then it will be shown that the price in the previous period is uniquely determined for any state at the beginning of the previous period. It is therefore possible to deduce the equilibrium sequence of price functions for a horizon of length N. Since the horizon is finite, it is assumed that stocks held beyond its terminus have no value. One therefore begins with a price function of zero and works backwards, recursively constructing the N previous functions. These functions, in conjunction with the difference equations governing stochastic transitions of the state variables from their specified initial values, completely describe the equilibrium. If the horizon is infinite, it must be shown that this sequence of price functions converges to a limiting price function. This limiting function can then be used to study the stochastic equilibrium in the infinite-time model.

Our first objective must therefore be to show that a unique limiting price function exists and to understand its properties. We begin by showing how the price function in period n - 1, denoted $P_{n-1}(\cdot)$, can be constructed from a given price function in period n ($P_n(\cdot)$). If it is assumed that $P_n(\cdot)$ is a continuous, decreasing, bounded function of the combined government-plus-private stock at the beginning of period n, then the function constructed for the previous period will be shown to inherit these characteristics. Since the zero function in fact has these characteristics, so must all the previous price functions.

To show that this sequence converges to a limiting price function, we verify that the rule transforming one price function into its predecessor satisfies Blackwell's [1965] sufficiency conditions for a contraction mapping. Since the space of continuous, decreasing, bounded functions is a complete metric space, the Banach fixed-point theorem then implies that every sequence of functions generated by this transformation (including the one whose first term is the zero function) converges uniformly to a unique limiting price function. Once the existence of this price function is established, properties of the equilibrium of particular economic interest are discussed.

The following notation is used:

$\tilde{H}_n$ denotes the random harvest at the beginning of period n;

K_n denotes the private stock at the beginning of the n^{th} period (including $\tilde{H}_n$);

G_n denotes the government stock at the beginning of the n^{th} period;

θ_n denotes total stock $(K_n + G_n)$ at the beginning of the n^{th} period -- the counterpart to S in the continuous-time model of Section II;

X_n denotes the decumulation (sales) of private stocks during the n^{th} period (hence, $K_{n+1} = K_n - X_n + H_{n+1}$);

R_n denotes the government stock at the end of the n^{th} period (hence $R_n = G_{n+1}$);

C denotes the maximum "capacity" of the government stockpile (its initial stock of grain plus the additional amount of grain which can be acquired at the official rate in exchange for stocks of the "other good". C can be made arbitrarily large without affecting the principal results);

$\bar{P}$ denotes the price the government attempts to defend;

Z denotes consumption desired at $\bar{P}$ ($D(\bar{P}) = Z$).

β denotes the real discount factor ($0 \leq B = 1/1+r < 1$).

The market-clearing price in period n - 1 depends on the combined amount which private agents and the government supply to consumers. If the government begins period n - 1 with a stock of G_{n-1} and ends the period with a stock of $R_{n-1}(=G_n)$, then it must have sold $G_{n-1} - R_{n-1}$ to the makket. If private speculators sell an additional X_{n-1} units, the market-clearing price will be

$$P_{n-1} = D^{-1}(X_{n-1} + G_{n-1} - R_{n-1}).$$

It is assumed that $D^{-1}(0) = p^c$ (the vertical intercept of the inverse demand curve), $D^{-1}(X) > 0$, $\lim_{X \to \infty} D^{-1}(X) = 0$, and $D'^{-1}(\cdot) < 0$. These assumption insure that $0 \geq P_{n-1}(\cdot) \leq P^c$.

So far, it has been shown that P_{n-1} can be determined from X_{n-1} and R_{n-1}. But what determines these variables? We will adopt the following approach. First, we will deduce -- for any specified government stock (R_{n-1}) at the end of the period -- the speculative sales (X_{n-1}) and hence the market-clearing price (P_{n-1}) for the period. We will then determine which R_{n-1} and hence P_{n-1} is consistent with the particular policy rule under consideration. In this way, we determine the unique market price associated with each level of entering social stocks.

We begin by determining speculative sales during the period for a given R_{n-1}. Private sales (X_{n-1}) depend on speculators' attempts to make money. Speculators are assumed to know the current price and to anticipate correctly the mean discounted price next period. If a capital loss is expected, no private stock will be carried. Conversely, if stocks are carried no loss must be expected. More formally, $K_{n-1} - X_{n-1} \geq 0$ and $P_{n-1} - \beta EP_n \geq 0$, with complementary slackness. 9/

If the subjective expectations of the speculators are "rational," their behavior is determined by the objective expected price. It is assumed that we are given the price for period n as a function of the social stocks at the beginning of the period: $P_n(K_n + G_n)$. Since $G_n = R_{n-1}$ and $K_n = K_{n-1} - X_{n-1} + H_n$, the expected discounted price in period n depends on the sum of the stocks carried into the n^{th} period by the government and private sector:

$$\beta EP_n = \beta EP_n(K_{n-1} - X_{n-1} + H_n + R_{n-1}).$$

We now have the ingredients to determine X_{n-1} and P_{n-1} as functions of K_{n-1}, G_{n-1} and R_{n-1}. It is assumed that the given function $P_n(\cdot)$ is bounded, continuous, and decreasing. Then the discounted expected price function derived from $P_n(\cdot)$ will also possess these properties. Given K_{n-1}, G_{n-1} and R_{n-1}, X_{n-1} is the unique solution to the following conditions: $K_{n-1} - X_{n-1} \geq 0$ and

$$D^{-1}(X_{n-1} + G_{n-1} - R_{n-1}) - \beta EP_n(K_{n-1} - X_{n-1} + H_n + R_{n-1}) \geq 0,$$

with complementary slackness.

To determine X_{n-1}, one first solves the condition that expected per unit losses are zero and then checks to see if the sales required for that solution exceed private inventories. If they do, the candidate solution is infeasible and X_{n-1} is set equal to K_{n-1}. The determination of $X_{n-1}(K_{n-1}, G_{n-1}, R_{n-1})$ is illustrated in Figure III:

[Figure III goes here]

In the particular case portrayed, the sales required for expected per unit losses to be zero are feasible and hence X_{n-1} is the horizontal intercept.

Changes in K_{n-1}, G_{n-1}, and R_{n-1} shift the curve in Figure III and, therefore, alter X_{n-1}. The sensitivity of X_{n-1} to changes in these variables may be obtained by totally differentiating ( ).

	for $X_{n-1} < K_{n-1}$	for $X_{n-1} = K_{n-1}$
$\left.\frac{\partial X_{n-1}}{\partial G_{n-1}}\right\vert_{K_{n-1}, R_{n-1}} =$	$\frac{-D'^{-1}(X_{n-1} + G_{n-1} - R_{n-1})}{D'^{-1}(\cdot) + \beta EP'_n(K_{n-1} - X_{n-1} + H_n + R_{n-1})} < 0$	0
$\left.\frac{\partial X_{n-1}}{\partial K_{n-1}}\right\vert_{G_{n-1}, R_{n-1}} =$	$\frac{\beta EP'_n(K_{n-1} - X_{n-1} + H_n + R_{n-1})}{D'^{-1}(X_{n-1} + G_{n-1} - R_{n-1}) + \beta EP'(\cdot)} > 0$	1
$\left.\frac{\partial X_{n-1}}{\partial R_{n-1}}\right\vert_{K_{n-1}, G_{n-1}} =$	1	0

To determine P_{n-1} as a function of K_{n-1}, G_{n-1}, and R_{n-1} we substitute the speculative sales function into the inverse demand curve:

$$P_{n-1} = D^{-1}(X_{n-1}\{K_{n-1}, G_{n-1}, R_{n-1}\} + G_{n-1} - R_{n-1}).$$

The properties of this price function follow from the properties of the speculative sales function:

$$\left.\frac{\partial P_{n-1}}{\partial G_{n-1}}\right|_{K_{n-1}, R_{n-1}} = \left.\frac{\partial P_{n-1}}{\partial K_{n-1}}\right|_{G_{n-1}, R_{n-1}} < 0 \qquad \text{for } X_{n-1} \leq K_{n-1}$$

and

$$\left.\frac{\partial P_{n-1}}{\partial R_{n-1}}\right|_{K_{n-1}, G_{n-1}} = \begin{cases} 0 & \text{for } X_{n-1} < K_{n-1} \\ D'^{-1}(K_{n-1} + G_{n-1} - R_{n-1}) > 0 & \text{for } X_{n-1} = K_{n-1}. \end{cases}$$

The first property implies that P_{n-1} can be expressed as a strictly decreasing function of the sum of private and government stocks. The second implies that increases in R_{n-1} do not alter P_{n-1} as long as $X_{n-1} < K_{n-1}$ since they induce offsetting increases in X_{n-1}. Once X_{n-1} can increase no further ($X_{n-1} = K_{n-1}$), however, increases in R_{n-1} raise the price.

Denote $K_{n-1} + G_{n-1} = \theta_{n-1}$. Given $P_n(\cdot)$, we plot the field of functions $P_{n-1}(\theta_{n-1}, R_{n-1})$ against R_{n-1} (treating θ_{n-1} as a parameter) in Figure IV. Higher θ's underly lower price functions.

[Figure IV goes here]

Each price function in Figure IV is defined for $0 \leq R_{n-1} \leq \theta_{n-1}$. If R_{n-1} is increased from zero, the price associated with a given θ_{n-1} remains constant for as long as $X_{n-1} < K_{n-1}$. Once $X_{n-1} = K_{n-1}$, further increases in R_{n-1} raise the price. When R_{n-1} reaches θ_{n-1}, $X_{n-1} + G_{n-1} - R_{n-1} = 0$ so $P_{n-1} = p^c$ -- the vertical intercept of the inverse demand curve. Thus, each price function has a flat region where speculators carry inventories and a sloped region where they carry none. Moreover, the θ - parameter underlying a particular curve is determined graphically as the horizontal component of the point where the price function crosses the horizontal line of height p^c.

So far, we have determined the equilibrium price function for any given level of government stocks at the end of period n - 1. This analysis applies <u>regardless</u> of the policy pursued by the government. But to complete the determination of $P_{n-1}(\cdot)$, the government behavior rule must be specified. In general, the rule might depend on price, private stocks, and government stocks at the beginning of period n - 1: $R(P_{n-1}, K_{n-1}, G_{n-1})$.10/ However, to make our points simply we will restrict ourselves to the case of government pegging:

$$R(P) = \begin{cases} 0 & \text{if } P > \overline{P} \\ [0\ ,\ C] & \text{if } P = \overline{P} \\ C & \text{if } P < \overline{P} \end{cases}$$

That is, the government will allow the market price to rise above the official price only when there are no more grain reserves to sell and will let the market price fall below the official level only when it has nothing left to exchange for additional grain reserves.

In Figure V(a) the government behavior rule and the field of contingent price functions are plotted together on the same diagram.

[Figure V(a and b) goes here]

Since each price function is continuous and positive for $0 \leq R_{n-1} \leq \theta_{n-1}$, it must intersect the policy rule at least once. Furthermore, since the government rule decreases while the price function associated with any given θ_{n-1} increases, the intersection point must have a unique vertical component. This component tells us the unique price -- for any given θ_{n-1} -- simultaneously consistent with optimal private behavior, adherence by the government to its policy rule, and market clearing. It should be noted that $P_{n-1}(\theta_{n-1})$ -- like its predecessor -- is a bounded, continuous, decreasing function of one variable. The price function derived from Figure V(a) is sketched in Figure V(b). The procedure outlined above can be applied repeatedly to construct P_{n-2}, P_{n-3},.... We postpone further discussion of properties of the constructed functions until we have shown that backward recursion from the zero function converges to a unique limiting function.

We have shown how a price function in one period can be used to determine the price function in the previous period. Denote this transformation by $T(\cdot)$. $T(\cdot)$ takes any element in the space of continuous, bounded, decreasing functions to a particular element in the same space. Since the space is a complete metric space, every sequence generated by repeated application of $T(\cdot)$ converges uniformly to a unique limit (the fixed point of $T(\cdot)$) provided that $T(\cdot)$ is a contraction mapping.

To demonstrate that $T(\cdot)$ is such a mapping we verify that it satisfies Blackwell's sufficiency conditions:

(1) If $P(\theta)$, $p(\theta)$ are two price functions with $P(\theta) \geq p(\theta)$, then $T(P(\theta)) \geq T(p(\theta))$ for all θ.

(2) If $\delta > 0$, $T(P(\theta) + \delta) \leq T(P(\theta)) + \beta\delta$, where $0 \leq \beta < 1$.

The verification proceeds as follows. If we are given a larger price function in period n ($P_n(\theta) \geq p_n(\theta)$), the expected discounted price in period n will be larger for any K_{n-1}, G_{n-1}, R_{n-1} and X_{n-1}. Consequently X_{n-1} will be smaller for any K_{n-1}, G_{n-1}, R_{n-1} and accordingly, P_{n-1} will be larger for any $K_{n-1} + G_{n-1}$, R_{n-1}. As a result P_{n-1} will be larger for any $K_{n-1} + G_{n-1}$, establishing that $T(\cdot)$ possesses the first property. As for the second, suppose the price function in period n were higher by a constant (δ). Then, the expected discounted price in period n for any given K_{n-1}, G_{n-1}, R_{n-1}, and X_{n-1} would be higher by $\beta\delta$. If, for the particular θ_{n-1}, $X_{n-1} < K_{n-1}$ previously, then the increase in the expected price will result in a reduction in X_{n-1} of sufficient magnitude to raise P_{n-1} by $\beta\delta$. If, for the particular θ_{n-1}, $X_{n-1} = K_{n-1}$ previously, then $P_{n-1} > \beta EP_n$ and the increase in the expected discounted price may either (1) induce no contraction in X_{n-1} and hence no change in P_{n-1} or (2) induce a contraction in X_{n-1} below K_{n-1} and hence an increase in P_{n-1} of less than $\beta\delta$. In either case, $T(\cdot)$ satisfies Blackwell's second condition. It follows that every sequence generated by repeated application of $T(\cdot)$ converges uniformly. Hence the sequence whose first term is the zero function $(0, P_N(\cdot), P_{N-1}(\cdot), \ldots)$ has a unique limit.[11/] Moreover, the limit is continuous, bounded, and decreasing. We denote it by $P(\theta)$.[12/] [13/]

B. Properties of the Equilibrium for a Given θ

Given P(θ), we can construct the P(θ, R) curves for the previous period. Since P(θ) is continuous, bounded, and decreasing, the field of curves (P(θ, R)) derived from it will have the same appearence as those in Figure Va. For convenience, we shall assume the curves in that diagram were derived from the limiting function, P(θ).

Some price functions in Figure Va emanate from the vertical axis with a horizontal slope and then are kinked, while others emanate with a positive slope and have no kinks. It can be shown in general that the locus of kinks rises to the left until it intersects the vertical axis, and that the price functions with higher vertical intercepts have no kinks.[14/]

In Figure Va, the locus of kinks intersects the vertical axis above the official price ($\overline{P}$). In principle, this need not be the case and a weak assumption must be introduced to guarantee it. Without this restriction, the intersection may occur below or above $\overline{P}$, as is illustrated respectively in cases (a) and (b) of Figure VI.

[Figure VI goes here]

In each case, NK is the locus of kinks. The smallest social stock (θ) at which the official price can be maintained is denoted "A". In each case, the price function associated with θ = A (P(A, R)) is sketched.

Whether case (a) or case (b) occurs depends on the relation of two numbers, A and Z. Since a social stock of A induces consumption of Z, $A \geq Z$. Case (a) of Figure VI occurs when $A = Z$, while case (b) occurs when $A > Z$ as the following considerations indicate. If (1) $A = Z$, (2) social stocks are A, and (3) the government ends up with no stocks ($R = 0$), then the entire social stock is consumed and speculators carry no inventories ($K - X = 0$) -- the case illustrated in (a). If, instead, $A > Z$ under those circumstances, part of the social stock is consumed ($Z < \theta$) and the remainder is carried out of the period by speculators ($K - X > 0$) -- the case illustrated in (b).

The size of the government stockpile at the end of the period (R) can be expressed as a function of the social stock (θ) at the beginning of the period. As Figure VII illustrates, the shape of $R(P(\theta))$ depends on the relation of A to Z.

[Figure VII goes here]

If $A = Z$, government stocks approach zero smoothly as social stocks decline below A. If $A > Z$, however, a slight reduction in social stocks can cause a precipitous disappearance of the government stockpile. Since consumption is a continuous function of θ ($D(P(\theta))$), the sudden reduction in government stocks has as its counterpart a sudden acquisition by speculators. For this reason, A is referred to in case (b) as the "attack threshold." The shape of the functions in Figure VII can be derived graphically from Figure VI. The indeterminacy in R arises in case (b) because the horizontal segment of P(A, R) coincides with the horizontal segment of the government policy rule.[15/]

To eliminate the possibility of case (a) and therefore to insure that a speculative attack occurs whenever social stocks fall below A, it is sufficient to introduce a weak restriction. In terms of exogenous functions and parameters, it is assumed that:

$$\bar{P} < \beta\{\int_0^Z D^{-1}(H)dF(H) + \bar{P}\int_Z^{Z+C} dF(H) + \int_{Z+C}^{\infty} D^{-1}(H - C)dF(H)\}$$

or equivalently,[16] $\bar{P} < \beta EP_N(H)$.

Since the sequence of price functions whose first term is the zero function is increasing, the condition can be shown to imply that $\bar{P} < \beta EP(H)$[17] and consequently $A > Z$[18]

The restriction that $\bar{P} < \beta EP_N(H)$ can be readily understood. It states that the mean discounted price in the final period of a finite-time model would exceed the official price if no stocks were carried into the final period by either speculators or the government. This prospect of expected profits insures that speculators would attack if the government stocks were sufficiently small on the next-to-last period.

Other characteristics of the equilibrium for a given θ are apparent from Figure VII b. Since for social stocks slightly above the attack threshold ($\theta > A$), speculators carry no inventories ($K - X = 0$) and the official price is maintained, larger social stocks at the beginning of the period mean larger government stocks at the end of the period. For some θ, maintenance of the official price requires the government to hold a stockpile of C at the end of the period. This occurs when $\theta = Z + C$.[19]

The price function associated with this social stock (P(Z + C, R)) is plotted in Figure VIII.

Larger social stocks depress the market price below the official price ($\bar{P}$), but induce no speculative carry-overs (K - X = 0).[20/] Only when θ exceeds some level (F in Figure VIII) are speculators motivated to carry stocks (K - X > 0). Figure VIII plots P(A, R), P(Z + C, R), and P(F, R) against R, along with the government policy rule. The intersections determine the equilibrium P and R associated with each size social stock.

[Figure VIII goes here]

The behavior of speculators when F > Θ > Z + C is easily understood. For Θ in this region, the market price is only slightly below the official ceiling price. Thus, the government policy truncates capital gains but not capital losses. Under this circumstance, it is not profitable for speculators to carry stocks. If social stocks exceed F, however, the market price is depressed enough below the ceiling that the potential capital gains attract speculation. Similar properties are to be expected in models with price bands. Speculators will hold no stocks when the market price is slightly below a ceiling but will hold inventories when the market price is slightly above a floor. Such situations are often labelled "one-sided bets."

Given P(θ), we can compute the discounted expected price on the following period as a function of θ: $\beta EP(\theta + H - D(P(\theta)))$. Current and discounted expected price are plotted together in Figure IX.

[Figure IX goes here]

For any θ, the discounted expected price either equals or is less than the current price. Whenever a capital loss is expected (for example, when $A < \theta < F$), speculators carry zero inventories ($K - X = 0$).

Figure IX can be used to clarify an important question. The mystery of speculative attacks is why they are so precipitous. Why is there never a range of social stocks in which the government maintains the official price while speculators hold stocks? Why is there only one size social stock which induces this behavior?

If -- for some range of θ -- speculators held stocks while the government maintained the official price, then the discounted expected price would have to equal the official price over that range of θ. But as Figure IX illustrates,[21/] the expected discounted price function is strictly decreasing when its height is $\overline{P}$ and is non-increasing throughout. Hence only one size social stock is consistent with private holding at the official price. A similar result holds for any government policy with a ceiling.

C. Stochastic Evolution of θ

It has been shown that a unique limiting price function exists and that it exceeds or falls short of the official price depending on the magnitude of θ. Three regions may be distinguished:

Region 1	$\theta < A$,	$(P(\theta) > \bar{P})$
Region 2	$Z + C \geq \theta \geq A$	$(P(\theta) = \bar{P})$
Region 3	$\theta > Z + C$	$(P(\theta) < \bar{P})$.

Transitions from one region to its neighbor have the following economic interpretation. If θ crosses from region 2 to region 1, the government loses its entire stockpile in a sudden attack and can no longer prevent the market price from exceeding the official level. As long as θ remains in region 1, all stocks carried between periods are held by private speculators. When θ crosses back into region 2, speculators expect inadequate gains and sell their stocks to the government at the official price. As long as θ remains in region 2, all stocks carried between periods are held by the government. When θ crosses from region 2 to region 3, the government is no longer able to purchase the additional grain necessary to support the market price and it falls below the official level. As long as θ remains in region 3, the government continues to carry its full capacity. If additional stocks are carried they must be held by private speculators. When θ crosses back into region 2, all stocks are once again carried by the government.

To complete the analysis, the stochastic evolution of θ must be studied. θ evolves according to the following first-order Markov process:

$P_t = P(\theta_t)$ Price Function

$\theta_{t+1} = \theta_t + \{\tilde{H}_{t+1} - D(P_t)\}, \ t = 0, 1, \ldots$ Transition Equation

$\theta_0 = \bar{\theta},$ Initial Condition

Various events of economic interest depend on θ and it is sometimes possible (even without Monte Carlo simulation) to compute the probabilities that events of particular interest occur. To illustrate, we will investigate the probability that θ eventually declines below A regardless of the initial size of the private plus government stocks and also the probability that this event will recur frequently. We will assume that the official price is set to induce consumption equal to the expected harvest ($D(\bar{P}) = E(H)$). Consider the stochastic difference equation above. If it were true that $P(\theta) = \bar{P}$ for all θ, θ would execute an ordinary, additive random walk with zero drift. In fact, although $P(\theta) = \bar{P}$ for $\theta \epsilon [A, Z + C]$, the function is strictly decreasing elsewhere. When stocks exceed Z + C, the price falls -- stimulating additional consumption; when stocks fall below A, the price rises -- rationing consumption. Hence the stochastic process under consideration is a variant of the much-studied, classical random walk[22/] and can be analyzed by using well-known results.

The motion of θ_t is represented graphically in Figure X. The question of whether an attack must occur is equivalent to the question of whether θ

must eventually drop below the attack threshold. The question of its recurrence is equivalent to whether with "virtual certainty" θ drops below the attack threshold more than any finite number of times.

[Figure X goes here]

The claim that a speculative attack is virtually inevitable rests on the standard theorem in random-walk theory popularized in the proposition that playing a fair game against an opponent with infinite wealth "assures" ruin. That is, in n rounds of play, various distinct ordered sequences of events (wins and losses) might occur. Each sequence of length n has a probability attached to it. The sum of the probabilities of events (sequences) which imply ruin (zero wealth) approaches one for large n.

Little need be changed to apply this theorem to our modified random walk. Assume for the moment that the disturbance term $(H_t - D(P(\theta_t)))$ conformed exactly to the conventional random-walk assumption (zero mean, i.i.d.). Then θ would hit zero with probability one -- and would, <u>a fortiori</u>, also hit the attack threshold. More precisely, for any n, some sequences of n harvests would cause the stock to drop below the attack threshold and some would not; but the probability of generating sequences in the former class can be made arbitrarily close to one by suitable choice of n. Consider the set of sequences of harvests which generate attacks. All such sequences would still generate attacks if proper account is now taken of the fact that -- when combined stocks exceed Z + C -- the additive disturbance term should in fact be smaller than was supposed when the additional induced demand was neglected. Indeed, some sequences of harvests which did not result in an attack when the additional component of demand was ignored, would result in an attack when proper account of it is taken. Hence, for any n, the probability of an attack is at least as great in the case of our modified random walk as in the conventional case. This proves that if the official price induces demand equal to (or exceeding) the expected harvest, the buffer-stock manager will almost surely experience a speculative attack. As for the possibility of repeated attacks, the appendix contributed by R. Wenocur extends the recurrence property of the classical random walk with zero drift to the case of our modified random walk. Provided the demand induced by the official price exactly matches the expected harvest, the buffer-stock manager can look forward with virtual certainty to an infinite number of subsequent attacks.

IV. Conclusion

In this paper, we have considered situations where the government is unable to use a buffer stock to stabilize a price forever. Its futile attempts to do so result either in upward price jumps if speculators are absent or in speculative attacks if speculators are present. The economic function of such attacks deserves emphasis. An attack prevents the occurrence of dislocations which result from sudden cutbacks in consumption. Such cutbacks accompany upward jumps in price.

One general consequence of competitive speculation is the elimination of all upward price jumps which can be foreseen.[23/] When the government stocks approach a level beyond which a future price jump would be expected, the prospective jump is avoided by the swift "transfer" of stocks of the commodity which has become scarce from the hands of irresponsible bureaucrats who would continue to sell the remaining reserves at the low official price to speculators who will sell them at prices which change gradually to reflect economic scarcity. The attack is instantaneous because slower acquisition is unnecessarily costly to speculators.

Whether such attacks actually occur under the circumstances specified in this paper is an empirical matter which no amount of theorizing can resolve. The predictions of the paper follow logically from the assumptions. Readers who doubt the validity of the assumptions concerning optimal behavior or rational expectations may question the validity of the theoretical predictions. Nonetheless, it should be emphasized that although these assumptions have been shown to be sufficient to generate the conclusions, no one has contended that they are necessary. On the contrary, similar conclusions may well follow from more "realistic" assumptions.

Like any other, this theory should be judged by the accuracy of its predictions. Fortunately, experiments have recently been designed and executed [Miller et. al., 1977] to test the predictions of the theory of intertemporal competitive speculation in the absence of government intervention. To extend these laboratory experiments to situations involving attempted pegging seems straightforward. Such an extension will permit a replicable test of the theory. If the theory predicts well, a general computerized version of the model can be used to forecast the consequences of any buffer stock policy which is contemplated.

Appendix to Section II

Attack on Gold

Between 1934 and 1968, the U.S. pegged the nominal price of gold at $35 per ounce. Hence, the real price varied inversely with the price level. When speculators attacked in 1968, the government salvaged some of its buffer stock by closing the gold window. It has recently begun to auction the portion retained.

For simplicity, we will assume that inflation over the period proceeded at a constant rate (π) so that the real official price declined exponentially. Furthermore, we will assume that speculators anticipated that the government would be unwilling to sell its entire stock to defend the official price, but would instead initially retain $\bar{G}$ for subsequent disposition. We will suppose speculators anticipated that the portion retained would be auctioned at some unknown interval (t^+) after the rest of the government buffer stock was exhausted and that t^+ was taken to be exponentially distributed with mean $1/\alpha$.

The government might not have behaved as the speculators anticipated. But simply because we must make some assumption about the relation of the prior expectations of the speculators to the subsequent conduct of the government -- and no alternative assumptions seem superior -- we will assume that speculators were correct both about the amount of gold retained by the government and about the manner and timing of its subsequent disposition.

We begin once again by considering the equilibrium in the absence of speculators. Inflation would have progressively eroded the real official price -- stimulating increasing demand -- until the government stocks eventually dwindled to $\bar{G}$. At that point, the government would have suspended its pegging operation, and the real price would have soared high enough either to eliminate demand entirely or -- alternatively -- to provide enough incentive for foresighted extractors initially to have withheld some of their stocks from the government when the stockpile was first created in order to sell them after its exhaustion.

In the presence of speculators, this price path ceases to be an equilibrium since it gives speculators an incentive to purchase infinite stocks immediately before the upward jump in price. Such behavior would "tend" to shorten the duration of the pegging operation and reduce the market price which would prevail following its demise. In the new equilibrium, speculators would attack, the government would close its window on $\bar{G}$ units, and there would be no upward jump in price.

The size of the government stockpile (S^{+}) just prior to the attack may be determined as before by expressing the price immediately before and after the attack as functions of S and then finding that stock which equalizes the two prices.

Since the nominal price (f_n) is pegged in this case, the real official price (f) prior to the attack is no longer constant as was its counterpart ($\bar{P}$) in Figure II. As time elapses, the stock held by the government falls from I and the real official price falls from f_n. Hence the plot of f against S (varying time parametrically) in Figure A1 is an upward sloping curve passing through the point (I, f_n).[24/]

As before, the market price immediately following the attack (P_B) is a decreasing function of the stocks held privately ($S-\bar{G}$). But the market price would also be affected by two exogenous parameters -- the amount retained by the government ($\bar{G}$) and the probability per period that it would be auctioned (α). Hence we denote P_B as follows:

$$P_B = P_B\ (S-\bar{G};\ \bar{G};\ \alpha\ ).$$

In a previous paper, Salant and Henderson (1978, p. 634) showed how the initial price on an equilibrium path can be determined when there is a probability α that the government might auction $\bar{G}$ units of gold and speculators own a stock of any given size. Hence, the P_B function above is a reduced form of the Salant-Henderson gold model, just as the P_A function is a reduced form of the Hotelling oil model. The P_B function is plotted in Figure A1.

[Figure A1 goes here]

The two curves in Figure A1 intersect at the point (S^+, f^+). The coordinates indicate that the speculative attack occurs when the real price reaches f^+ and the buffer stock declines to S^+ ($> \bar{G}$). At that point, speculators purchase $S^+-\bar{G}$, the government salvages $\bar{G}$ for subsequent auction at an unknown date, and the price rises without a jump from f^+ along the trajectory described in detail in Salant-Henderson (1978, p. 635-6).

Changes in government policy parameters alter the date of the speculative attack. A decrease in the rate of inflation causes the upward-sloping curve in Figure A1 to flatten, pivoting around (I, f_n) so that a smaller stock is associated with each real official price.[25/] The point of intersection shifts to the left, implying that the smaller the rate of inflation, the smaller the buffer stock at the time of the attack. Since it takes longer at higher real prices for the stock to be reduced to any given level, lowering the rate of inflation must also delay the speculative attack.

In Salant-Henderson (1978, p. 637) increases in the probability of an expected auction (α) are shown to depress the price. An increase in α would therefore shift the downward-sloping curve in Figure A1 to the left so that the two curves intersected at a lower real price. Hence, the shorter the expected interval between the date of the attack and the subsequent auction, the longer speculators would postpone the attack. Finally, it can be shown that if a smaller portion of a given stock is in fact retained by the government for subsequent auction, the price will initially be lower $\left(\frac{\partial P_B}{\partial \bar{G}}\Big|_{\alpha, s} = P_{B2} - P_{B1} > 0\right)$. Thus, if speculators anticipate that less will be withheld, the downward-sloping curve in Figure A1 shifts to the left and the intersection occurs at a lower real price -- implying once again that the attack occurs later. Hence, if the government reduces the rate of inflation to zero and fostered the belief that the entire stockpile would be committed to the defence of the official price, the attack would be delayed the longest. But this is the case of the Hotelling model studied in Section II.

Appendix to Section III

A Recurrence Property of a Modified Random Walk

by R.D.S. Wenocur, Drexel University

We examine a basic recurrence property of a specific discrete parameter Markov process, $\{X_n, n \in N\}$, which we choose to call a modified general one-dimensional random walk, although the term "random walk" is usually reserved for more restricted cases. Before proceeding, let us establish our notation. N denotes the set of natural (counting) numbers, including zero; that is,

$$N = \{0, 1, 2, \ldots\};$$

R represents the set of real numbers. For the sake of brevity, we refer to the robability triple (Ω, A, P) with basic set Ω, Borel field (σ - algebra) A of subsets (events) of Ω, and probability measure P, without any discussion of Kolmogorov's extension theorem or of the distinction between an abstract probability space and one of the function-space type, involving realizations of the process. (See, for example, Chung (1974), Chapters 3 and 8.) Generally, ω denotes a sample point of Ω. $\epsilon_1, \epsilon_2, \ldots$ are independent, identically distributed (i.i.d.), non-degenerate real-valued random variables with finite mean μ and common distribution function F; $g: R \rightarrow R$ is a monotone, nondecreasing function satisfying.

$$g(t) \begin{cases} \leq \mu & t < a \\ = \mu & a \leq t \leq b \\ \geq \mu & t \geq b \end{cases}$$

for some fixed a, $b \in R$. The process X_n is defined as:

$$X_0 = a_0 \qquad\qquad a \leq a_0 \leq b \;\; (a_0 \text{ constant})$$

$$X_n = X_{n-1} + \epsilon_n - g(X_{n-1}),$$

and the process $S_n^{(m)}$ is defined as

$$S_0^{(m)} = X_m$$

$$S_n^{(m)} = S_{n-1}^{(m)} + (\epsilon_{m+n} - \mu).$$

For convenience, $S_n = S_n^{(0)}$. We note that $\{S_n, n \in N\}$ is a classical random walk, and that the process $\{X_n, n \in N\}$ is identical to $\{S_n, n \in N\}$ when $g(t) \equiv \mu$.

Now, let us present our main result. <u>Theorem</u>: If ϵ_1, ϵ_2,.... are i.i.d. real-valued random variables with common distribution function F and finite mean μ, in the sense that

$$\int_{-\infty}^{\infty} |x| \, dF < \infty,$$

then

$$P\{X_n < a \text{ infinitely often}\} = 1,$$

and

$$P\{X_n > b \text{ infinitely often}\} = 1.$$

Proof: Let

$$A_\infty = \{ -\infty = \underline{\lim}_{n\to\infty} S_n < \overline{\lim}_{n\to\infty} S_n = +\infty\}. \tag{1A}$$

In other words, A_∞ consists of all $\omega \in \Omega$ such that $\{S_n(\omega);\ n \in N\}$ is unbounded from both above and below. By a slight modification of a theorem of Chung and Fuchs (1951),

$$P(A_\infty) = 1. \tag{2A}$$

Consider $\omega \in A_\infty$. By (1A), for $\omega \in A_\infty$, there exists a stopping time $N_1 < +\infty$ such that

$$S_{N_1} < a$$

and

$$S_i \geq a \qquad \text{for all } i < N_1.$$

Certainly,

$$X_{N_1} \leq S_{N_1} < a.$$

Suppose

$$X_n \leq b \qquad \text{for all } n \geq N_1.$$

Then

$$S_{n-N_1}^{(N_1)} \leq X_n \leq b \quad \text{for all } n \geq N_1.$$

But this contradicts $\omega \in A_\infty$, since a translation of the S-process at the N_1-th step does not affect its recurrence. Therefore, there exists $N_2 > N_1$ such that

$$X_{N_2} > b$$

and

$$X_n \le b \text{ for all } N_1 \le n < N_2.$$

Continuing in this fashion, employing an induction argument, we obtain

$$A_\infty \subset \{X_n < a \text{ infinitely often}\}$$

and

$$A_\infty \subset \{X_n > b \text{ infinitely often}\}.$$

An appeal to (2A) establishes our result. ■

The stopping times N_i employed in the proof are, of course <u>entrance times</u> into certain Borel sets.

The result stated and proved above is sufficient for our present purposes. For further details on the processes $\{X_n, n \in N\}$ and $\{S_n, n \in N\}$ as defined here, refer to Chung and Fuchs (1951), Chung and Ornstein (1962), and Wenocur and Salant (1979).

Returning to the notation employed in the body of the paper, let

$$\begin{aligned} X_t &= \theta_t \\ \epsilon_t &= \tilde{H}_t \\ g(x) &= D(P(x)) \\ a &= A \\ b &= Z + C. \end{aligned}$$

In addition, we stipulate that

$$\mu = D(\overline{P}) = E(\tilde{H}_t). \tag{3A}$$

Under assumption (3A), remarks contained in the last part of Section II regarding losses of funds and speculative attacks as recurrent events are mathematically justified.

Footnotes

*/ The views expressed in this paper are solely the author's and are not intended to reflect the opinions of the Federal Trade Commission. The author wishes to acknowledge useful discussions on attacks with Katherine Blair, John Bryant, Dale Henderson, Val Koromzay, Pauline Ippolito, Steven Salop, T.N. Srinivasan, Sheldon Switzer and Peter Zadrozny.

1/ The government's desperate attempts to stabilize silver prices are summarized in Burke and Levy (1969). The futile attempts by the private sector to keep a lid on tin prices are summarized in Fox (1974).

2/ See, for example, Bergsten (June 8, 1977; p. 9) or Bergsten (June 8, 1976; p. 9).

3/ In Goreux's evaluation of bands [1978], he assumes away private speculators altogether although he stresses the desirability of including them and "their reaction to the bufferstock intervention rule" in future work. Two econometric studies include private speculators but fail to capture their reaction to the policy regime facing them. In one study prepared jointly for the Department of State, Treasury, and Commerce by the Commodity Research Unit (1977), an econometric model of the copper industry was estimated under the conventional (but erroneous) assumption that agents react only to current and lagged prices. Then -- under the (false) assumption that behavior would not be affected by the operation of the government buffer stock -- a price forecast was generated. In many of the simulations reported, the stockpile was exhausted during the 15-year simulation period and an upward jump in price resulted. Such price paths are disequilibria and, as such, have no legitimacy as forecasts. No properly specified model would generate them. In another study, Schink and Smith [1976] use the Wharton EFA model to evaluate how the tin market would have functioned with and without a commodity agreement -- implicitly assuming speculative behavior would not be affected by the change in regime. Both the CRU study and the article by Schink and Smith are typical of a class the methodological basis of which has recently been questioned. As Lucas (1976) has pointed out:

> "given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in structure will systematically alter the structure of econometric models... for issues involving policy evaluation ... (this conclusion) is fundamental, for it implies that comparisons of the effects of alternative policy rules using current macro-economic models are invalid regardless of the performance of these models over the sample period or in ex-ante short term forecasting." (my emphasis)

4/ A general computerized version of the model could be used first to analyze specific proposals (be they bands, pegs, ceilings, or floors) and later to forecast prices once a proposal is implemented. An excellent start on such a model has been developed independently by Gardner [1979]. The merit of the present analytical treatment is its insights into the properties of such a model.

5/ If the commodity is a pure depletable, any buffer stock used to enforce a ceiling on its price (a band, for example) will induce an attack. If the commodity can be replenished stochastically, the probability of an attack depends on the parameters of the stabilization scheme. For example, if the band width is infinite (laissez-faire) no attack will occur, if its width is zero (pegging) -- and if $D(\overline{P}) \geq E(H)$ -- an attack will occur with probability one. For any specific proposal a computerized version of the model would permit calculation of all probabilities of interest.

6/ That extractors sell their entire reserves initially is a consequence of the assumption that marginal extraction costs are constant. Readers finding the implication unrealistic, should consider the case where marginal extraction costs are an increasing function of the rate of extraction. This case also generates a speculative attack, but is not considered in the text because it is slightly more complicated.

7/ The models of Section II can be generalized as follows. Given any real pegging policy of the government ($\phi(x)$) from the restricted set where $\phi(x) < P_c$ and $\phi'(x) < r\phi(x)$, equilibrium without speculators requires that

$$(1)\ P(s) = \min\{\phi(0)e^{rt}, P_c\}$$

and

$$(2)\ I - \int_{x=0}^{t} D(\phi(x))dx = S.$$

These two equations determine t and S, where t is the length of time before the buffer stock is depleted and S is the total stock when pegging is terminated.

In the presence of speculators, the new equilibrium satisfies equation (2) above, but (1) is replaced by the zero-profit condition $\hat{P}(S) = \phi(t)$. In the case considered in the text, $\phi(x) = \overline{P}$ and $\hat{P}(S) = P_A(S)$, the reduced for of the Hotelling model. In the case considered in the appendix, $\phi(x) = f_n e^{-\pi x}$ and $\hat{P}(S) = P_B(S - \overline{G}; \overline{G}; \alpha)$, the reduced form of the Salant-Henderson model.

8/ Speculative attacks are commonly thought to result from mistaken beliefs about future prices. By assuming rational expectations, however, we have emonstrated that biases in forecasting prices are not needed to explain peculative attacks.

9/ We are relying here on an intuitively plausible characteristic of the optimal policy for a competitive speculator proved rigorously by Kohn (1978, p. 1065): "the speculator's behavior depends on his price expectations for the next period only" (his emphasis). That is, a speculator expecting a capital loss in the next period, but expecting a capital gain thereafter will never find it optimal to hold stocks until the market rebounds. Whenever an immediate capital loss is expected stocks should be sold. If a capital gain is expected in the period after next, next period is the time to acquire stocks.

10/ If the policy were laissez-faire, the government sales rule would coincide with the vertical axis ($R(P) = 0$, for $P \geq 0$). If the government attempted to defend a real price band, R(P, K, G) would be defined as follows:

$$\begin{array}{ll} R = 0 & \text{if} \quad P > P_u \\ R \varepsilon [0, G] & P = P_u \\ R = G & P_\ell < P < P_u \\ R \varepsilon [G, C] & P = P_\ell \\ R = C & P < P_\ell, \end{array} \qquad (2')$$

where P_u and P_ℓ denote the upper and lower limits of the band ($P_u \geq P_\ell$ -- if the limits are equal, the policy is identical to attempted pegging). In the case of this policy, each price function in the sequence whose first term is the zero function depends on two variables (K and G). But the principles developed in the text can still be applied.

11/ Alternatively, we know this sequence must converge pointwise to a unique limit since it is an increasing sequence of functions bounded from above. Furthermore, the limit function must be decreasing and bounded since each element in the sequence has these properties. Continuity of the limit function, however, does not follow from continuity of the elements of the sequence. To establish it, we must first show that each element is piecewise differentiable with a first derivative which is bounded for all n and θ-- a consequence of assuming the derivative of the underlying inverse demand curve is uniformly bounded. By implication, the sequence generates an equicontinuous set of functions. The Arzela-Ascoli theorem may therefore be applied to establish continuity of the limit function.

12/ Since P(θ) is the fixed-point of the contraction operator T(•) it is the only continuous, bounded, decreasing function to satisfy the following functional conditions:

$$K - [D(P(\theta)) + R(P(\theta)) - G] \geq 0 \quad \text{and}$$

$$P(\theta) \geq \beta E P\{K - [D(P(\theta)) + R(P(\theta)) - G] + \tilde{H} + R(P(\theta))\}$$

with complementary slackness where

$$R(P) = \begin{cases} 0 & \text{if} \quad P > \bar{P} \\ \epsilon[0, C] & \text{if} \quad P = \bar{P} \\ = C & \text{if} \quad P < \bar{P}. \end{cases}$$

13/ At the end of Section II, we remarked that the imposition of pegging in the case of certainty raises the market price above its laissez-faire level. We now prove that the same proposition holds for the model of Section III. The method of proof should indicate the power and versatility of the methodology just presented for analyzing the consequences of government policies.

Since laissez-faire can be regarded as another government policy, the equilibrium price function for laissez-faire ($P_L(\theta)$) can be deduced from the procedure in subsection A. That is, given any continuous, bounded decreasing price function in period n, the price function for the previous period under laissez-faire can be determined by first constructing the field ($P(\theta,R)$) of price functions and then examining the locus of its intersections with the new policy rule, $R_L(P)=0$.

The price function generated will also be continuous, bounded and decreasing. Denote by $T_L(\cdot)$ the rule which generates the previous price function from any given price function under laissez-faire. By repeating the argument on p.18, it can be verified that $T_L(\cdot)$ satisfies Blackwell's sufficiency conditions and hence has a unique fixed point.

To prove that $P(\theta)\geq P_L(\theta)$, apply $T_L(\cdot)$ repeatedly to $P(\theta)$ to construct $T^n(P(\theta))$. This can be done since $P(\theta)$ is an element in the space on which $T_L(\cdot)$ is defined. We first observe that $P(\theta)\geq T_L(P(\theta))$, as the following argument indicates. If we applied $T(\cdot)$ to $P(\theta)$ by constructing the field of price functions in figure Va in conjunction with the intervention rule $R(P)$, we would generate $P(\theta)$-- since it is the fixed point of $T(\cdot)$. But if we instead examine the locus of intersections of that same field of price functions with the laissez-faire rule $R_L(P)$, the equilibrium price will be lower whenever the intervention policy would have required the government to hold stocks at the end of the period (whenever $R>0$). Hence, $P(\theta)=T(P(\theta))\geq T_L(P(\theta))$. Since $T(\cdot)$ satisfies Blackwell's first condition (monotonicity), $P(\theta)\geq T_L^n(P(\theta))$ for any n. In the limit, $P(\theta)\geq P_L(\theta)$--

14/ Each price function is kinked where

$$X = K \text{ and } D^{-1}(X + G - R) = \beta EP(K - X + H + R)$$

$$\text{or} \quad D^{-1}(\theta - R) = \beta EP(H + R).$$

Since $\frac{dR}{d\theta} > 0$, the locus of kinks in Figure VIa is downward sloping. To show that price functions above the point where the locus of kinks cuts the vertical axis emanate with a positive slope, note that if $R = 0$ and G is reduced (for a fixed K), X must remain at K(since $\frac{\partial X}{\partial G}\Big|_{K,R} = 0$, for $X = K$).

15/ In each case, $R(P(\theta))$ is increasing and bounded above by C. Furthermore $R(P(\theta)) = 0$ for $\theta < A$ while $R(P(\theta)) = \theta - Z$ when $P(\theta) = \overline{P}$ and $X = K$. In (b) however, when $\theta = A$, X and R are indeterminate -- restricted only by the conditions that: $X \leq K$, $R \geq 0$, and $X + G - R = Z$. Hence $0 \leq R \leq A - Z$, depending on X. When $X = K$, $R = A - Z$.

16/ In the final period, $X_N(K_N, G_N, R_N) = K_N$. Hence $P_N(\theta_N, R_N) = D^{-1}(\theta_N - R_N)$ or

$$P_N(\theta_N) = \begin{cases} D^{-1}(\theta_N) & \text{for } 0 \leq \theta_N < Z \\ \overline{P} & Z + C \geq \theta_N \geq Z \\ D^{-1}(\theta_N - C) & \theta_N > Z + C. \end{cases}$$

Hence $\beta E P_N(H) = \beta\{\int_0^Z D^{-1}(H)dF(H) + \overline{P}\int_Z^{Z+C} dF(H) + \int_{Z+C}^{\infty} D^{-1}(H - C)dF(H)\}$.

17/ To prove that $\overline{P} < \beta EP_N(H)$ implies that $\overline{P} < \beta EP(H)$, we show that $P(\theta) \geq P_N(\theta)$, where $P_N(\theta) = T0(\theta)$. Since $P_N(\theta) > 0$ and $T(\cdot)$ satisfies the first Blackwell condition, $T^2 0(\theta) \geq T0(\theta)$ or $T^{n+1}0(\theta) \geq T^n 0(\theta) \geq T0(\theta)$. Hence, $\lim_{n\to\infty} T^n 0(\theta) \equiv P(\theta) \geq T0(\theta) \equiv P_N(\theta)$.

18/ If $\beta EP(H) > \overline{P}$, we show $P(Z) > \overline{P}$ and hence $A > Z$. To show that $P(Z) > \overline{P}$, set $K + G = Z$ and examine the condition:

$$D^{-1}(X + G - R) \geq \beta EP(K - X + H + R)$$

$$\text{or } D^{-1}(X + G - R) \geq \beta EP(Z - G - X + H + R)$$

$$\text{or } D^{-1}(Y) \geq \beta EP(Z + H - Y), \text{ where } Y = X + G - R.$$

If $\beta EP(H) > \overline{P}$ and both functions are decreasing, $Y \geq Z$ will not satisfy the condition. Hence, $Y^* < Z$. This implies that $P(Z) = D^{-1}(Y^*) > \overline{P}$. Since $P(A) = \overline{P}$ and $P(\cdot)$ is decreasing, $A > Z$.

19/ Since $X = K$, $R = C$, and $X + G - R = Z$, it follows that $\theta = Z + C$.

20/ There is an exceptional case (not considered in the text) where the locus of NK in Figure VI exceeds $\bar{P}$ for all R<C so that the official price can be defended only when θ=A. In this circumstance, the flat section of the price function in Figure V(b) shrinks to a point. It will be true even then that attacks are recurrent events. However, in this limiting case speculators will be willing to hold stocks even when the market price is only slightly below the ceiling (that is, for all θ>A) since the ceiling can never be defended. The claim in the text is valid whenever there exists some range of stocks for which the ceiling can be defended.

21/ The argument is made for H a discrete random variable but can be extended for cases where H is continuous. $\beta EP(\theta + H - D(P(\theta)))$ can be regarded as the discounted, weighted sum of functions of θ. Each function is similar in shape to Figure VIb but is horizontally displaced by $H - D(P(\theta))$. Consequently, each function strictly decreases when its value differs from $\bar{P}$ and is constant (or non-differentiable) when its value equals $\bar{P}$. By assumption, the mean discounted price next period equals $\bar{P}$. Accordingly, at least one function is the discounted weighted average must exceed $\bar{P}$ and the discounted mean price must therefore be strictly decreasing in θ when its value equals $\bar{P}$. Reversing the argument, the discounted expected price function can have a zero derivative with respect to θ only when the function has height $\beta\bar{P}$.

22/ By a classical random walk is meant any process of the form $X_{t+1} = X_t + \tilde{\mu}$, where $\tilde{\mu}$ is i.i.d. $\tilde{\mu}$ can be either a discrete or a continuous random variable. The extension of the usual theorems for discrete states to the case of continuous states is due to Chung and Fuchs [1951].

23/ In this sense at least, speculators do stabilize prices.

24/ Since $f(t) = f_n e^{-\pi t}$ and

$$S(t) = I - \int_{x=0}^{t} D(f_n e^{-\pi x})dx,$$

the official price and the government reserves can be computed for each t. As time elapses, both f and S decline monotonically from (I, f_n). Hence $\frac{df}{dS} > 0$.

25/ Formally, the claim is that $\left.\frac{dS}{d\pi}\right|_f > 0$. From the previous footnote

when df = 0, $dS = -D(f_n e^{-\pi t})\, dt + [\int_0^t D'(f_n e^{-\pi x})Xf_n e^{-\pi x}]d\pi$

or $$\left.\frac{dS}{d\pi}\right|_f = -D(f_n e^{-\pi t}) \left.\frac{dt}{d\pi}\right|_f + [\int_0^t D'(f_n e^{-\pi x})\, Xf_n e^{-\pi x} dx].$$

Since $df = -f_n e^{-\pi t}(\pi dt + t d\pi)$,

$$\left.\frac{dt}{d\pi}\right|_f = \frac{-t}{\pi}.$$

Hence $$\frac{dS}{d\pi} = \frac{t}{\pi} D(f_n e^{-\pi t}) + [\int_0^t D'(f_n e^{-\pi x})\, Xf_n e^{-\pi x} dx].$$

When integrated by parts, the second term reduces to

$$\left.\frac{XD(f_n e^{-\pi x})}{-\pi}\right|_0^t + \int_0^t \frac{D(f_n e^{-\pi x})dx}{\pi}.$$

Thus $$\left.\frac{dS}{d\pi}\right|_f = \int_0^t \frac{D(f_n e^{-\pi x})dx}{\pi} > 0,$$ as was claimed.

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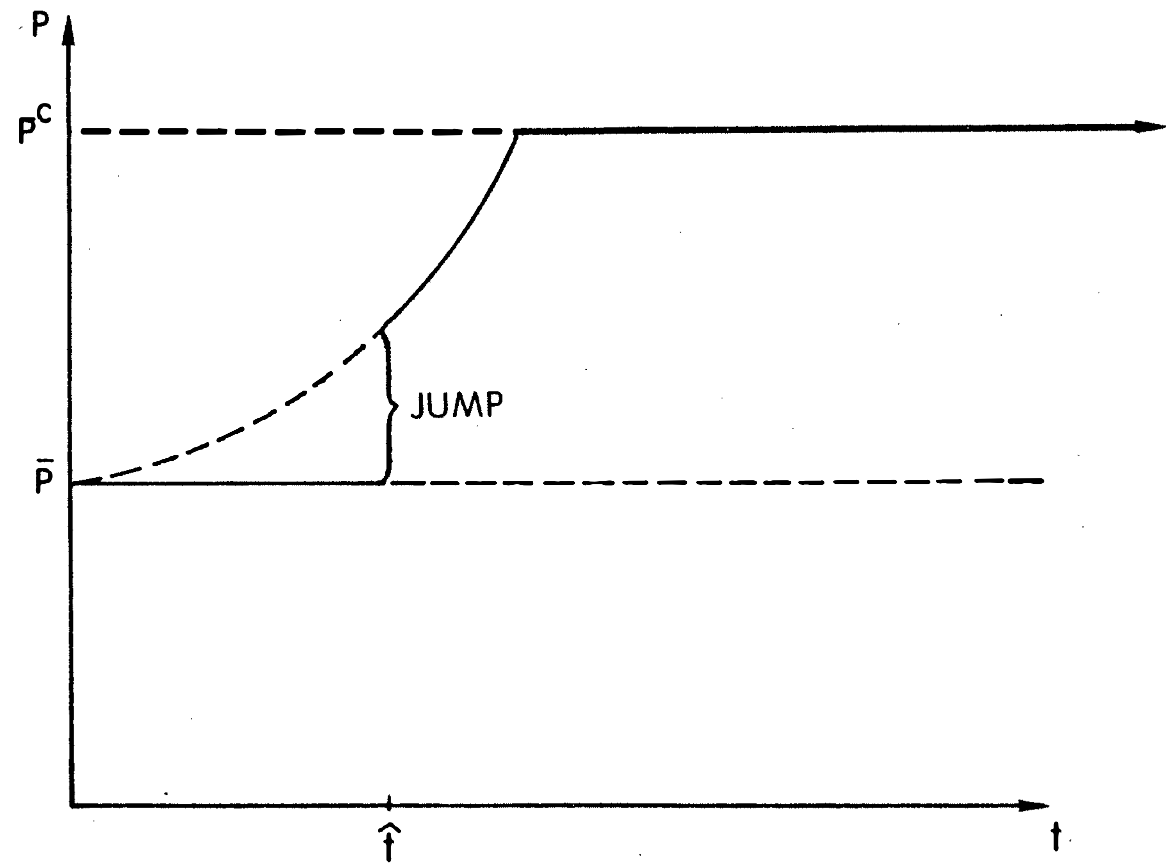


FIGURE I

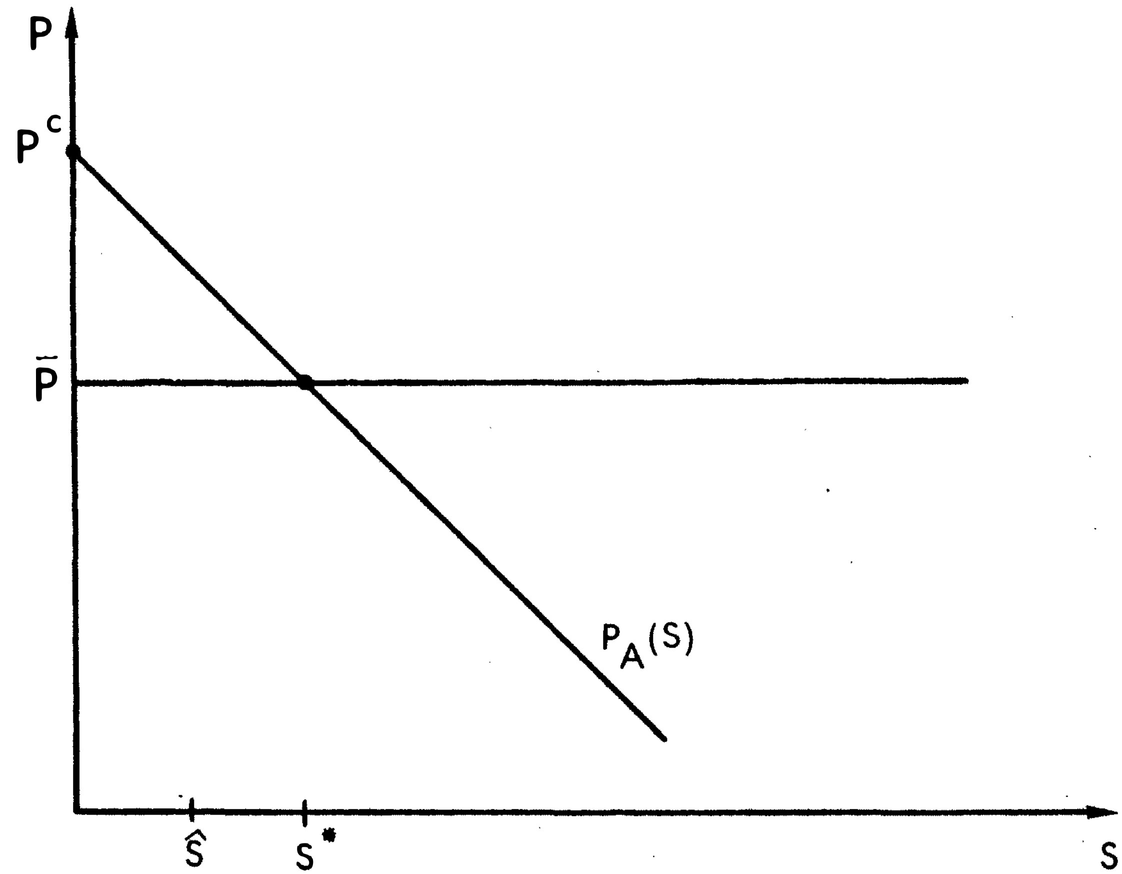


FIGURE II

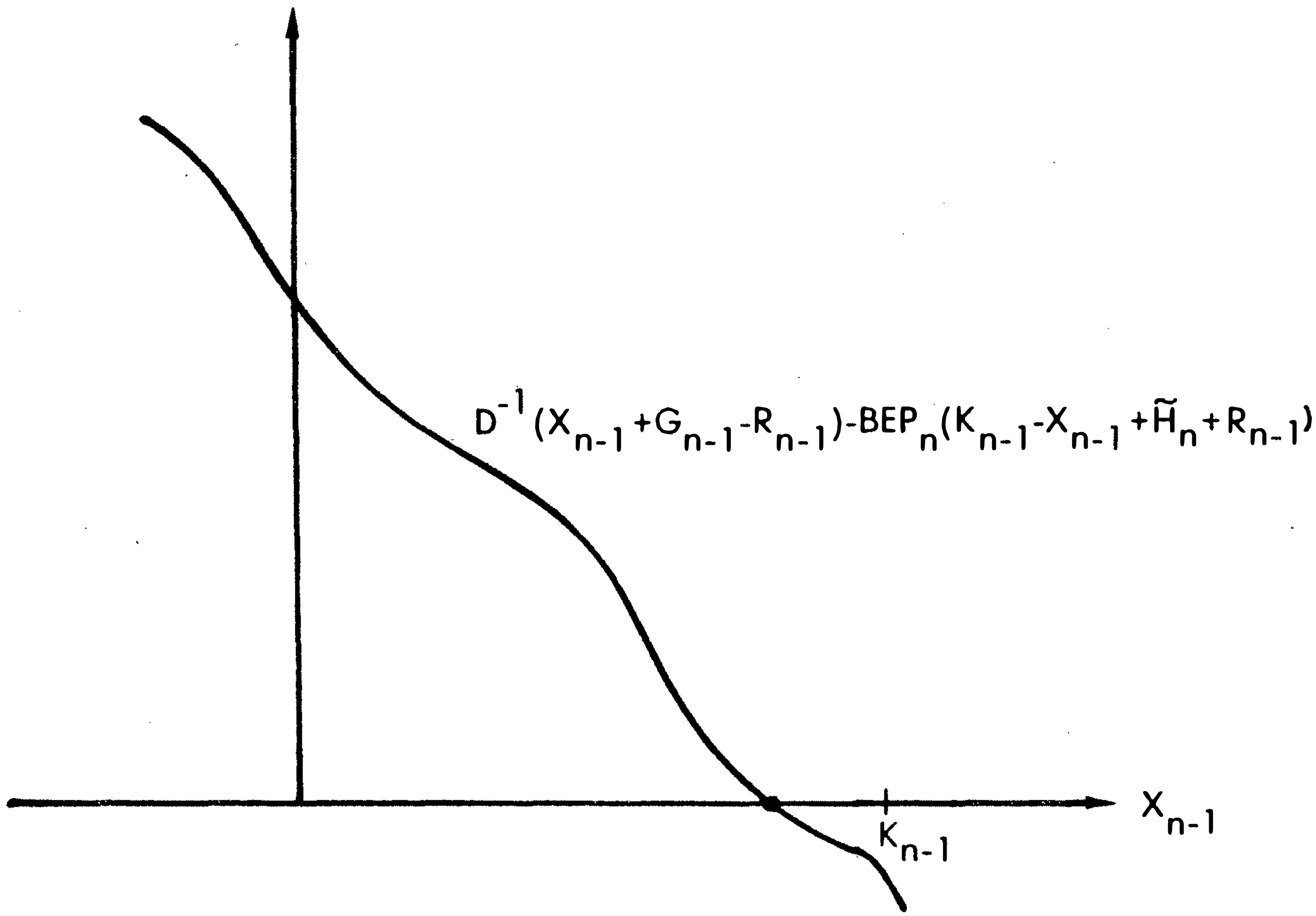


FIGURE III

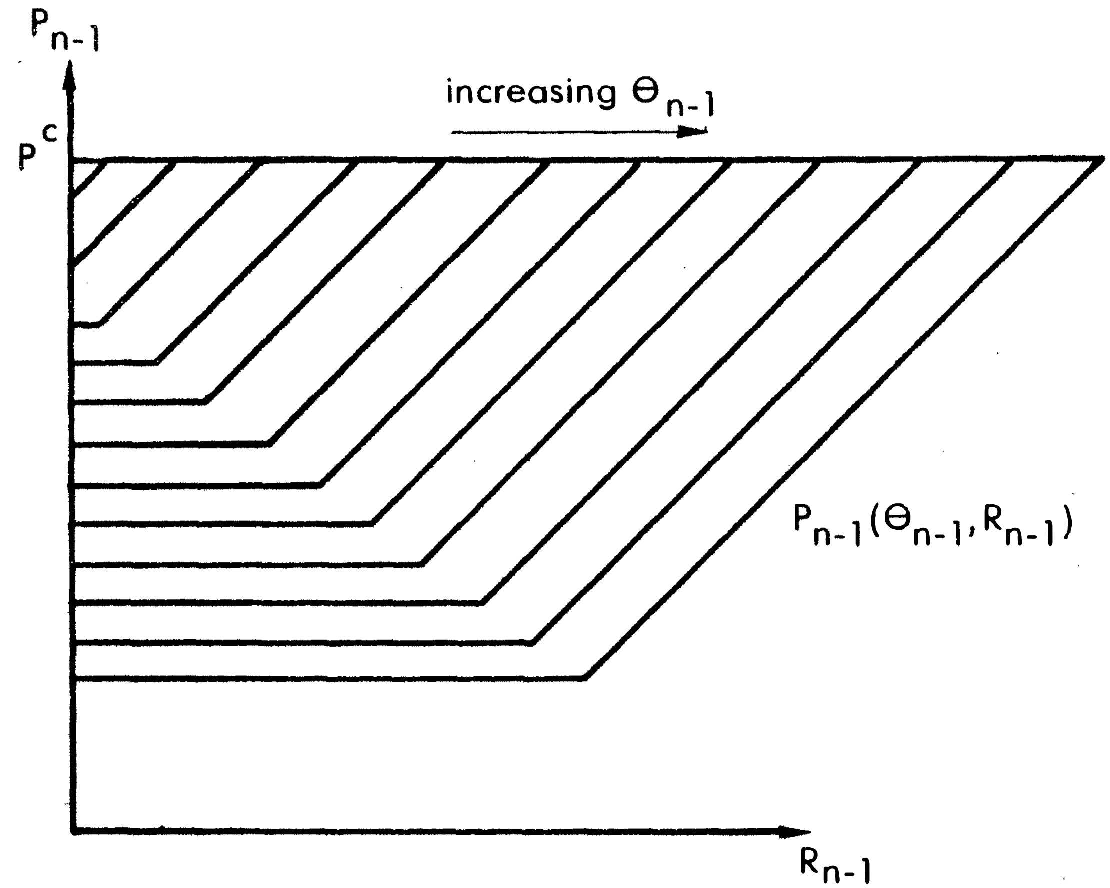


FIGURE IV

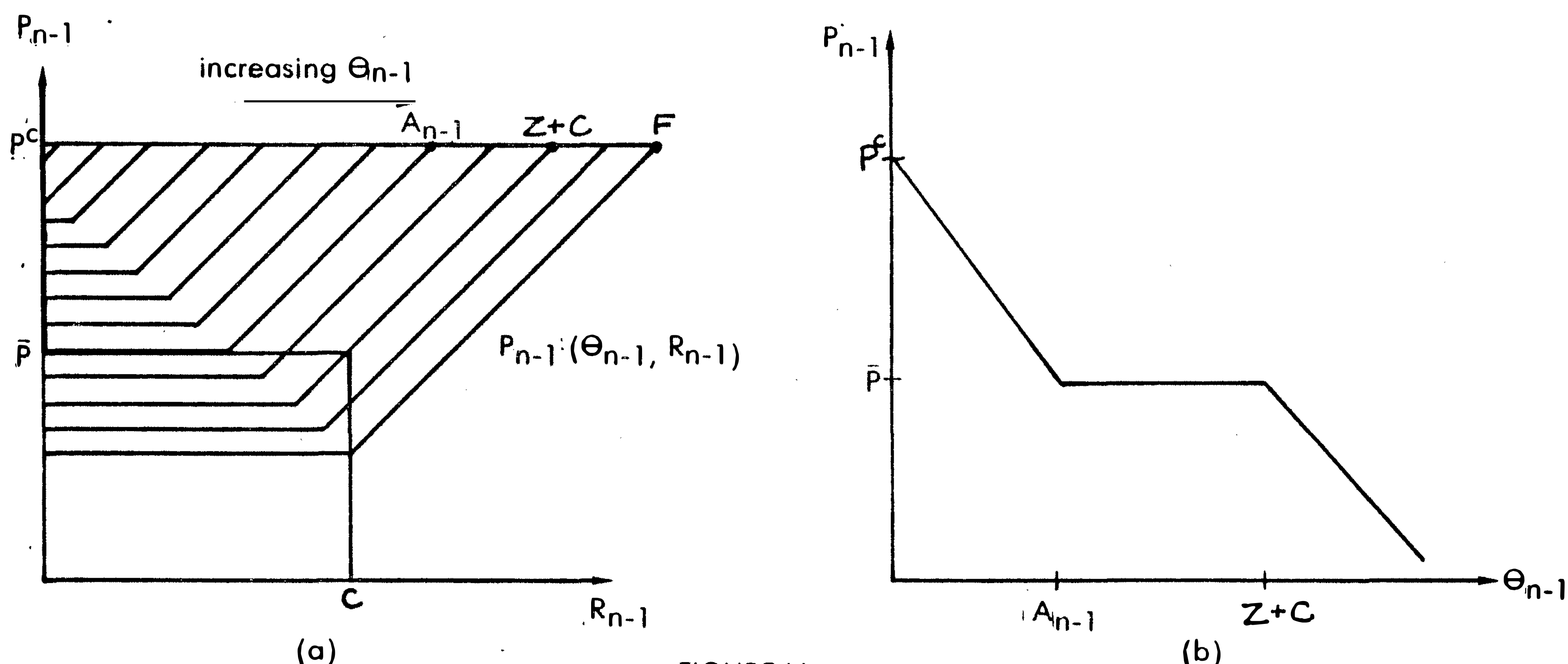

P_{n-1}
increasing θ_{n-1}
P^c
$\bar{A}_{n-1}$
$Z+C$
F
$\bar{P}$
$P_{n-1}(\theta_{n-1}, R_{n-1})$
C
R_{n-1}
(a)
P_{n-1}
P^c
$\bar{P}$
θ_{n-1}
A_{n-1}
$Z+C$
(b)

FIGURE V

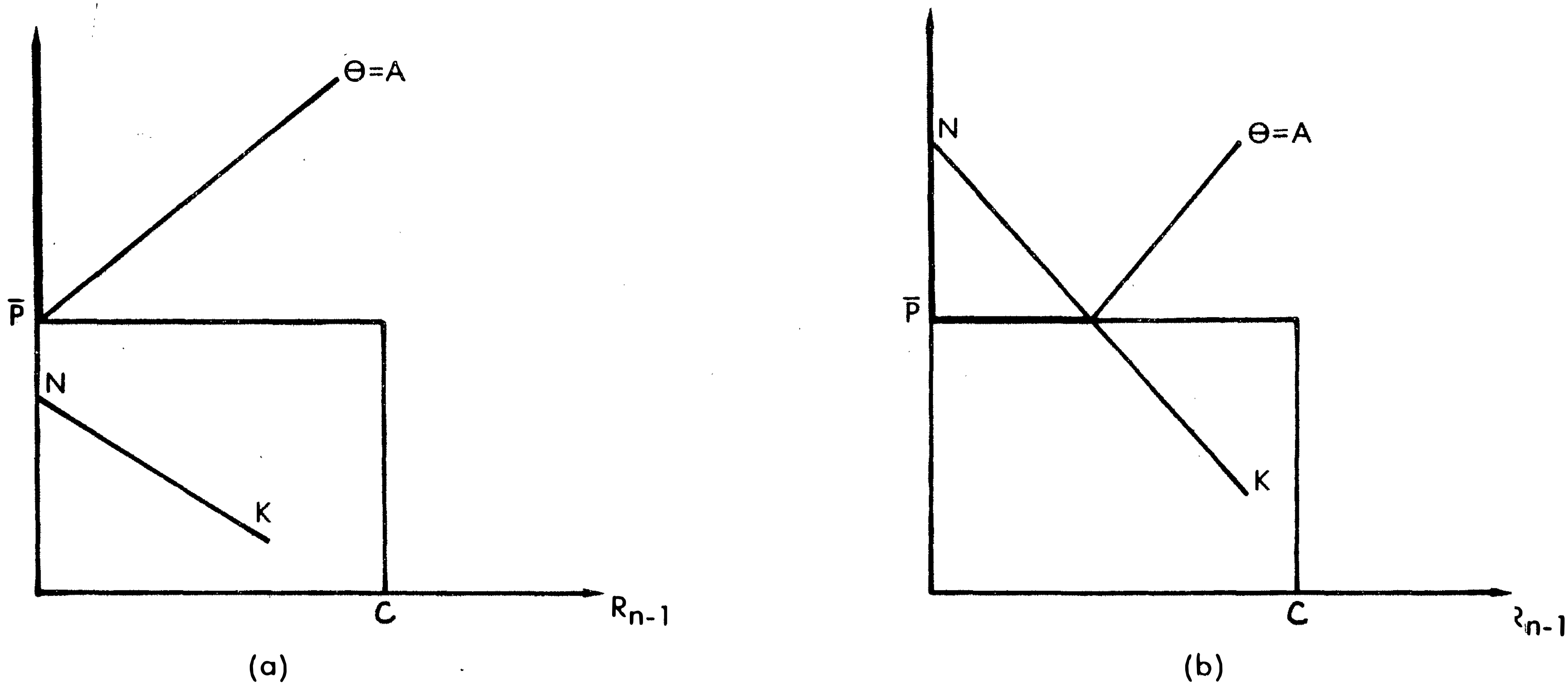
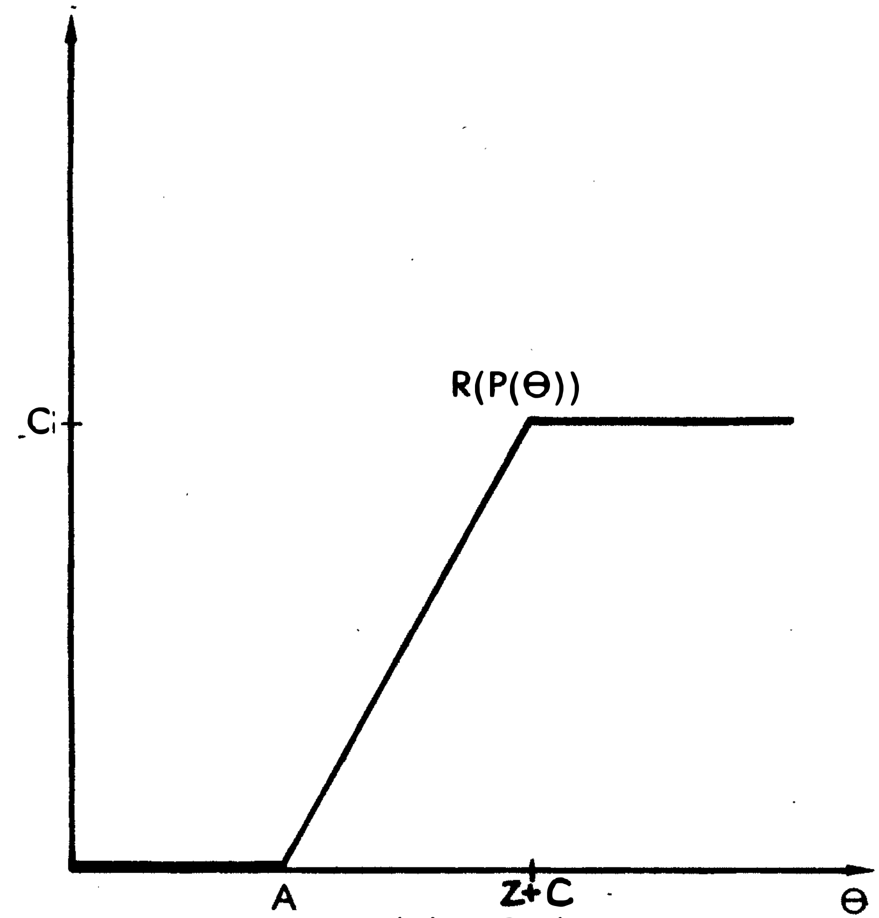
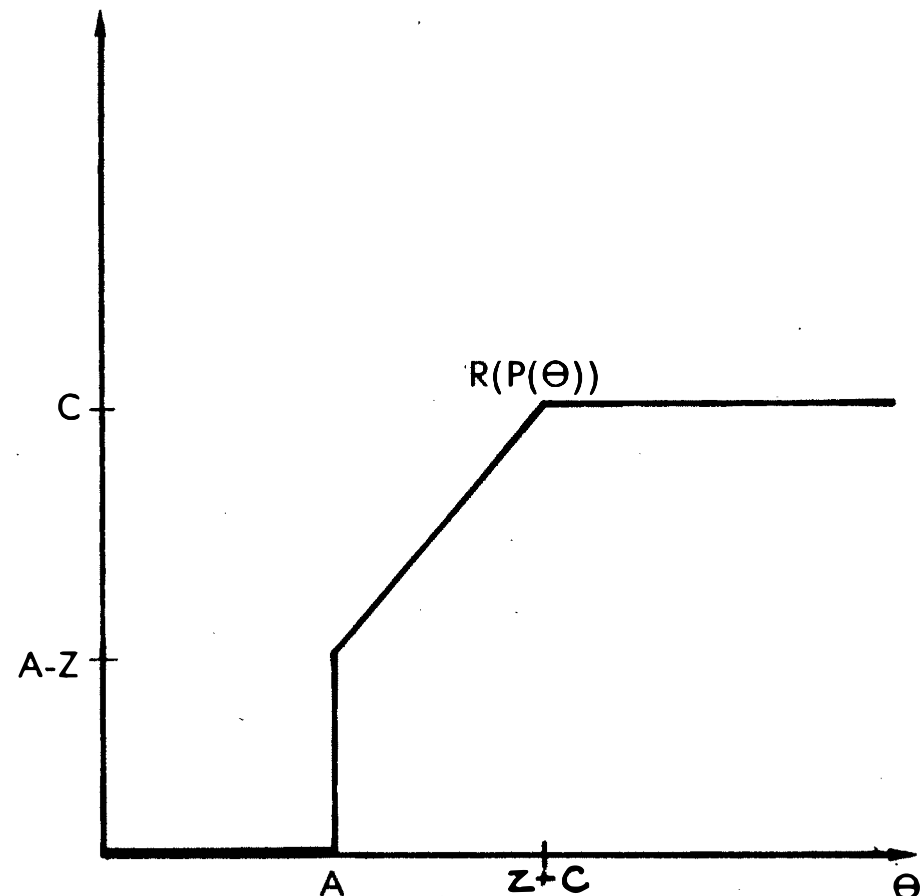


FIGURE VI



(a) A=Z: the run out



(b) A>Z: the attack

FIGURE VII

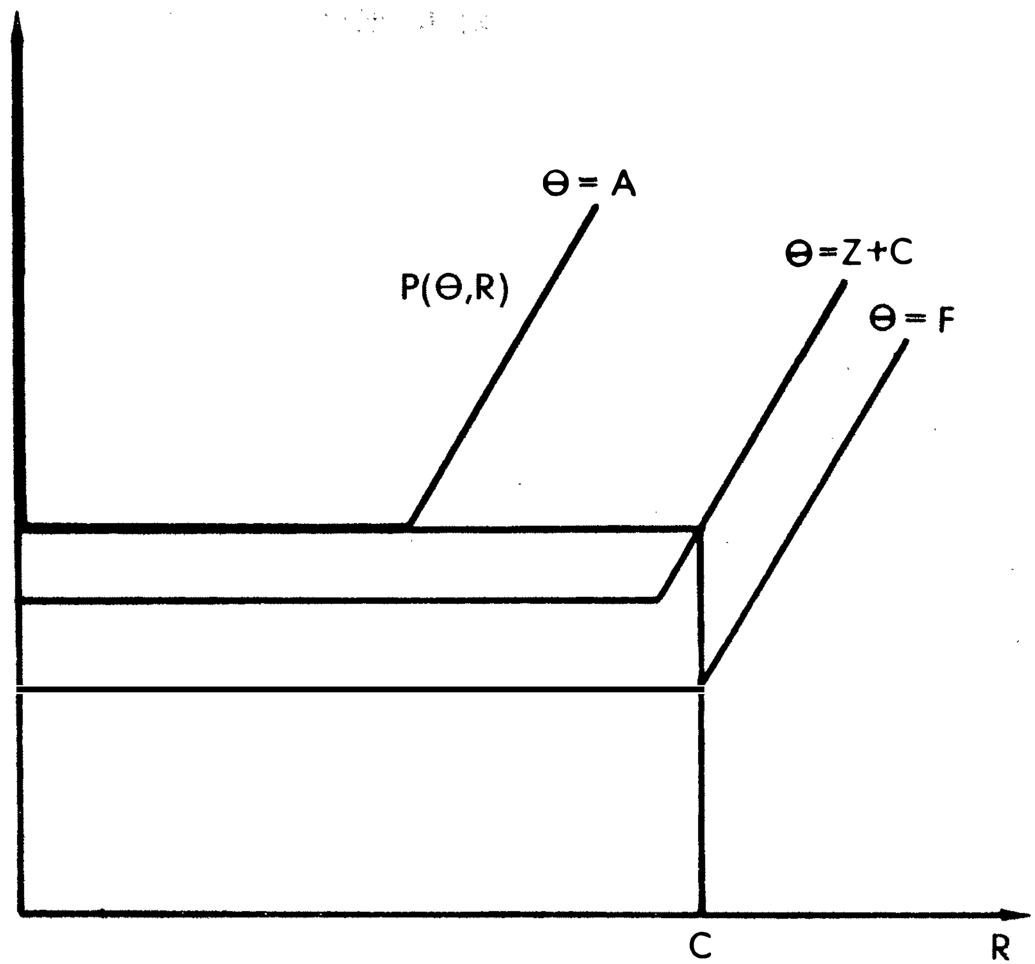


FIGURE VIII

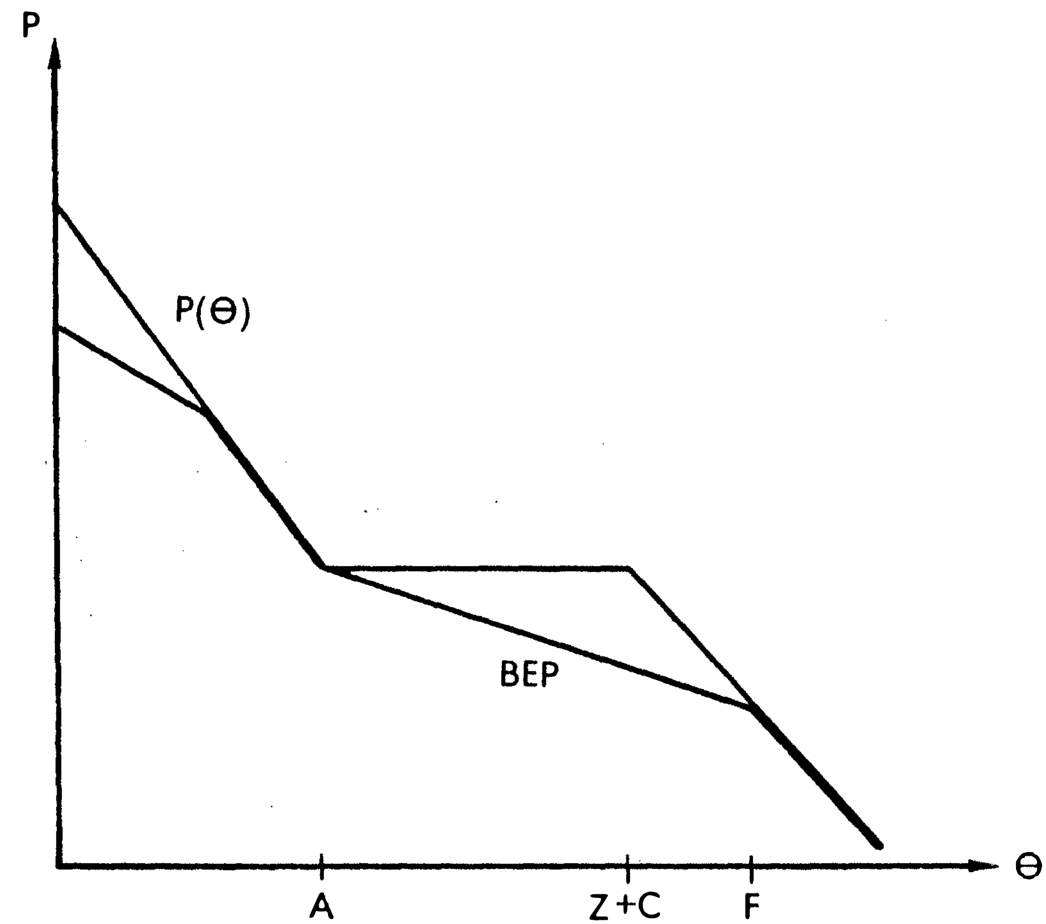
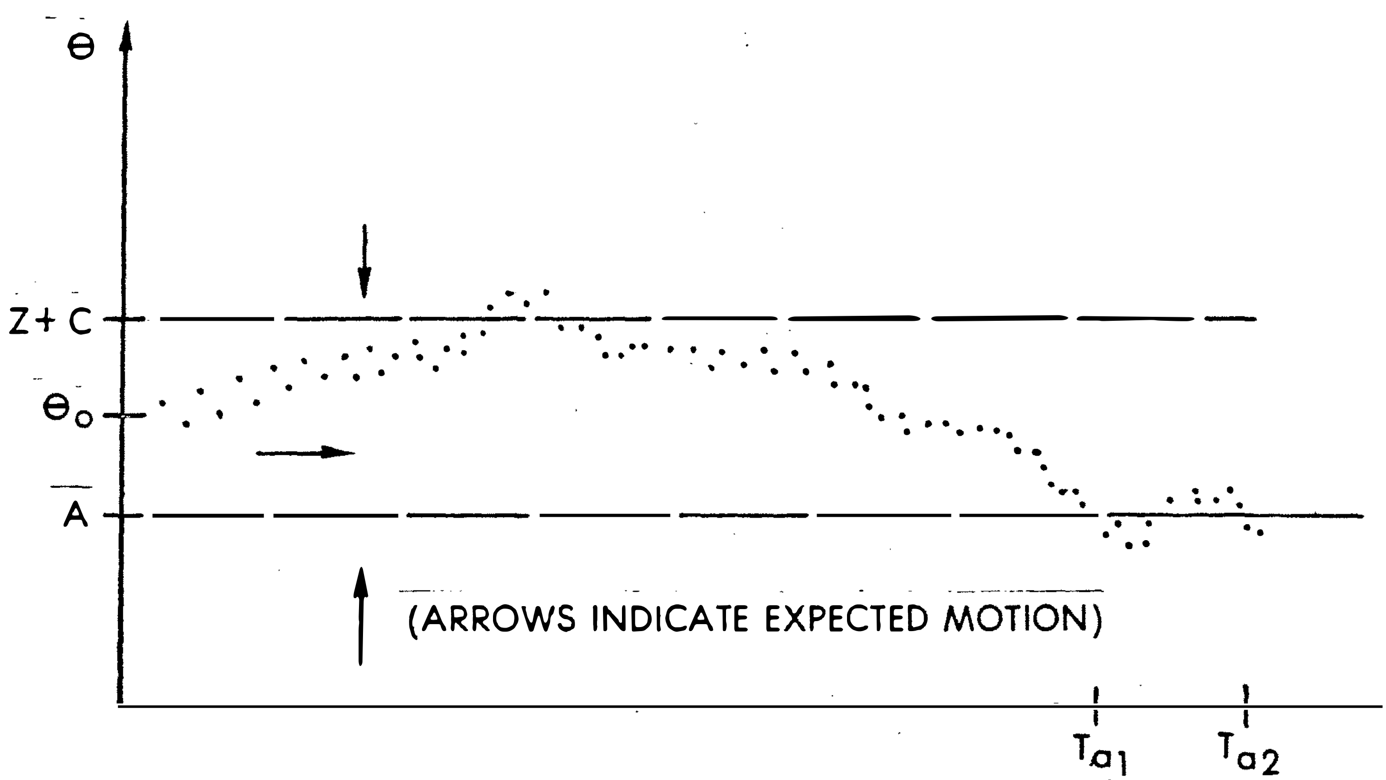


FIGURE IX



$\bar{\Theta}$
$\bar{Z}+\bar{C}$
$\bar{\Theta}_o$
$\bar{A}$
(ARROWS INDICATE EXPECTED MOTION)
T_{a1}
T_{a2}

FIGURE X

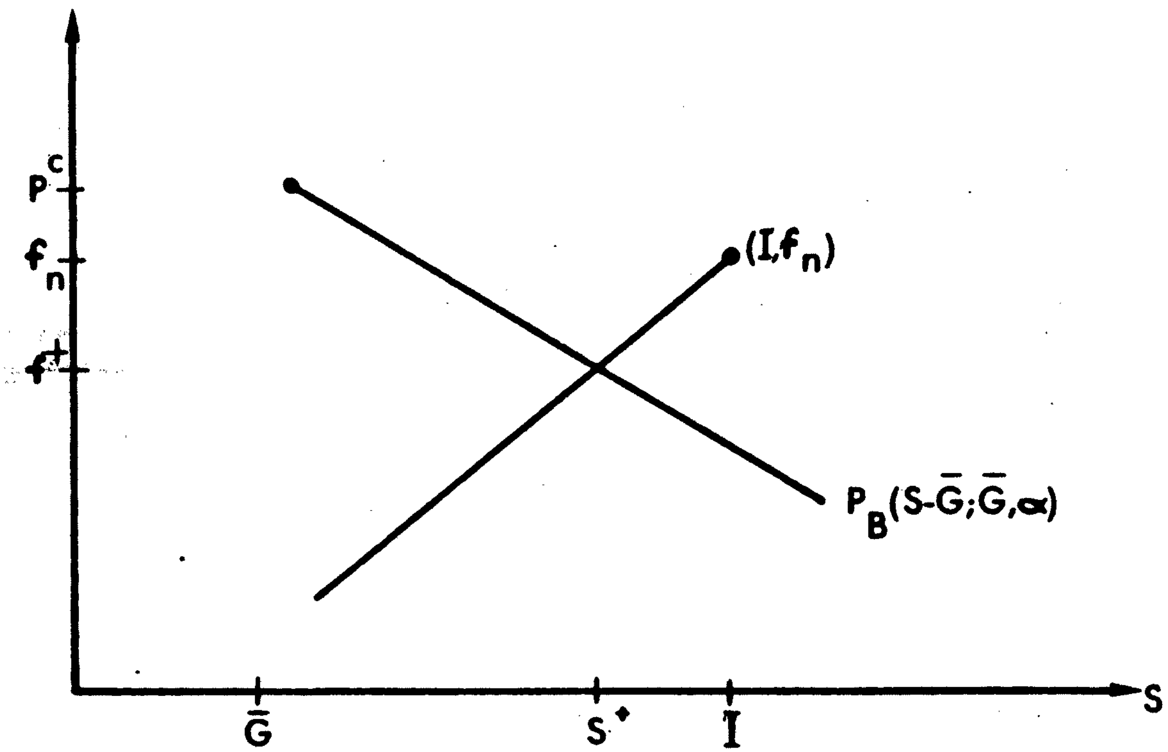


FIGURE AI