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MOŽNOST DOLOČITVE VISOKO NATANČNEGA GEOIDA ZA OZEMLJE EGIPTA

POSSIBILITY TO DETERMINE HIGHLY PRECISE GEOID FOR EGYPT TERRITORY

Moamen Awad Habib Gad, Oleg Odalović, Sofija Naod

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IZVLEČEK

V članku predstavljamo rezultate študije, s katero smo preverjali, ali je mogoče na temelju dosegljivih globalnih in lokalnih podatkov o težnostnem polju določiti geoid z nekajcentimetrsko natančnostjo za ozemlje Egipta. Pri tem smo si zastavili dva cilja. Najprej podajamo pregled objav na temo izračuna geoida, sledi predstavitev aktualnih globalnih geopoetencialnih modelov (GGM) za območje Egipta. Analiza obstoječih modelov geoida v Egiptu je pokazala, da so izračunani na temelju neustrezne porazdelitve podatkov z nekonsistentnim vzorčenjem. Tudi danes za območje Egipta ni dovolj podatkov za izračun geoida. Za rešitev težave smo izbrali pristop podatkovnega okna ter uporabili tehniko kolokacije po metodi najmanjših kvadratov (angl. least square collocation – LSC). Rezultati so zanimivi in sprejemljivi, saj znaša standardna deviacija za odstopanja izračunanega geoida od vrednosti na kontrolnih točkah 11 centimetrov.

ABSTRACT

This paper presents an attempt to consider whether it is possible to determine a geoid at the centimetre level in the territory of Egypt based on recently available global and local gravity field data. The paper has two main objectives. Firstly, the paper overviews previously published geoid solutions, while the second objective investigates the performance of the recent global geopotential models (GGM) in Egypt. The existing geoid solutions have illustrated that there is an insufficient distribution of data which is sampled inconsistently. At this time, data deficiency still exists, and to overcome it, we have selected a "data window" and applied the Least Square Collocation (LSC) technique. The outcome from LSC was interesting and acceptable, and we obtained a "sample" geoid that has a standard deviation of 11 cm for the external control points.

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1 INTRODUCTION

Over the last few decades, the demand for high-resolution geoid models has grown, especially after the widespread use of the Global Navigation Satellite System (GNSS). With the broad use of the GNSS in surveying and mapping, vertical geodetic datum transformations are still a critical issue in Egypt. Many countries have already developed geoid models that play an essential role in the derivation of orthometric heights from GNSS observations. In Egypt, there have been several geoid evaluations until now, but the determination of the geoid is still an important and essential issue (Shaker, El Sagheer and Saad, 1997).

The determination of Earth's shape and its external gravity field by using data collected on Earth's surface represents a solution to the geodetic boundary value problem (GBVP). Depending on the boundary data given on the boundary surface and the type and number of unknown functions to be solved, different formulations of the GBVP exist. Stokes' Boundary Value Problem deals with the determination of a potential field, harmonic outside the masses, associated with Laplace's differential equation. Observables, in this case, are gravity anomalies given everywhere on the geoid. Thus, this approach is based on a reduction of observational data for gravitational effects induced by the topographical masses. In a mathematical sense, Stokes' BVP belongs to the class of the "free" boundary value problem (boundary surface is of unknown geometry) with oblique derivative associated with the Laplace operator. Many reference materials are available for this subject (Heiskanen and Moritz, 1967; Heck, 2003; Sansò and Rummel, 1997; Grafarend and Niemeier, 1971).

Practically, the modified Stokes' formula that combines local terrestrial data with a global geopotential model (GGM) is often used nowadays. The optimum modification of Stokes' formula, introduced by (Sjöberg, 2003), is employed so that the expected mean square error of the combined geoid height is minimised. According to this stochastic method, the approximate geoid height is first computed from the modified Stokes' formula using surface gravity data and a GGM. The precise geoid height is then obtained by downward continuation and by adding topographic and ellipsoidal corrections to the approximate geoid height.

The commonly adopted and applied approach to regional gravimetric geoid determination is the so-called Remove-Compute-Restore (RCR) technique (Schwarz, Sideris and Forsberg, 1990). The complete expansion of GGMs is used in conjunction with regional terrestrial gravity data via the spherical Stokes' integral. In principle, in the RCR technique, any a priori known signal features are first removed from the observed data, before the prediction process, and then appropriately added back (restored) to the predicted features. After the removal of those effects, the resulting residual could be treated practically as centred data with minimum signal mean and standard deviation. The removal usually involves low-frequency global features, implied by an appropriate high-resolution harmonic model of global geopotential. Moreover, a suitable high-resolution Digital Terrain Model (DTM) is usually used to account for local topographic effects (Figure 1).

The following factors that affect the quality and accuracy of the necessitated precise geoid model for Egypt are:

– The amount, quality, and distribution of the available terrestrial gravity data,

– Existing GGMs quality and spatial resolution, and

– Quality and accuracy of GPS/levelling (GPS/H) derived undulations

The LSC algorithm is a flexible tool for combining all possible heterogeneous data in one unified solution to solve any desired type of anomalous signals. The LSC estimated signals, which have minimum error variance among all other solutions, could also be evaluated at any optional point (Moritz, 1978; Moritz, 1980). This study's main objective is whether it is possible to determine the geoid at the centimetre level in the territory of Egypt by using LSC.
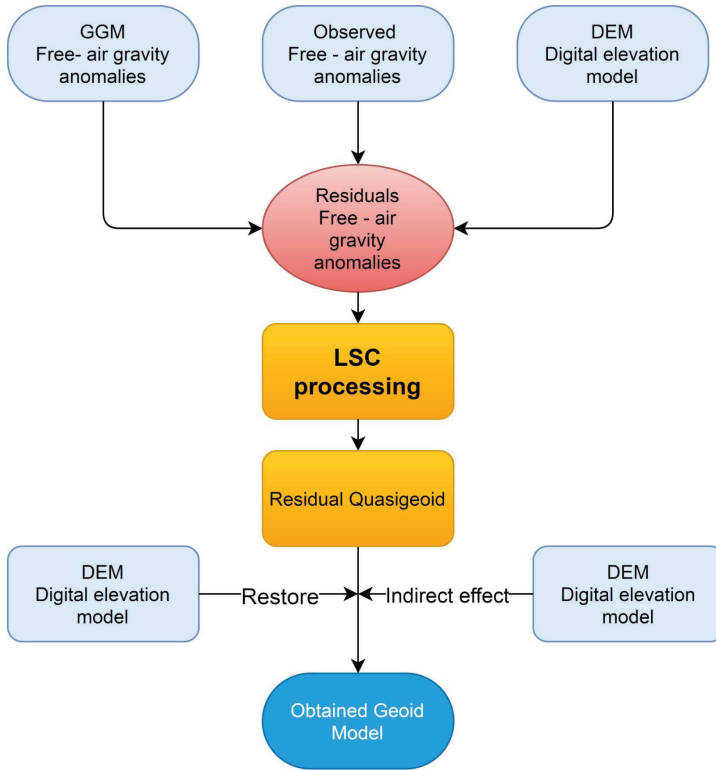
Figure 1:   Overview of data processing steps for remove-compute-restore procedure.

The gravity anomaly Δg is the difference between the observed gravity value g, after the atmospheric correction δ_A and a normal or theoretical gravity value $\gamma_{\circ}$ at the mean Earth's ellipsoid (Moritz, 1980):

$$\Delta g = g + \delta_A - \gamma_{\circ} \tag{1}$$

Especially, if the homogeneous input data are free-air anomalies, Δg, then the GGM low-frequency part, Δg_{GGM}, and topographic effect, Δg_{TOPO}, could be removed from the local data to obtain the residual anomalies, Δg_R (Forsberg and Tscherning, 1981):

$$\Delta g_R = \Delta g - \Delta g_{GGM} - \Delta g_{TOPO} \tag{2}$$

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Hence, the obtained Δg_R values are then used as input for LSC to predict the corresponding values of undulation, while the values of height anomalies are determined as shown in eq. (3):

$$\zeta = \zeta_{REF} - \zeta_{GGM} - \zeta_{TOPO}, \tag{3}$$

where ζ_{REF} is computed from residual gravity anomalies employing, for example, the Stokes' integral or LSC, ζ_{GGM} is the long-wavelength part and ζ_{TOPO} is the short-wavelength part determined by using DTM from Shuttle Radar Topography Mission (SRTM).

Based on the determined height anomalies, it was necessary to use the following formula to determine geoid undulation from height anomalies and Bouguer anomaly as (Heiskanen and Moritz, 1967):

$$N = \zeta + \frac{\Delta g_B}{\bar{\gamma}} H^\circ, \tag{4}$$

where Δg_B are Bouguer anomalies, H° is the orthometric height, and $\bar{\gamma}$ is the mean normal gravity along the normal plumb line between ellipsoid and telluroid.

A schematic diagram of the general computation procedure, which is considered as one of the possible solutions is depicted in Figure 2. The processing procedures consist of:

(1) Computation of gravity anomalies and geoid undulation values from observed data.
(2) Calculating the corresponding gravity anomalies values and height anomalies from GGM to obtain the differences between them (i.e., the residual values).
(3) Using the remove-restore strategy to:
  - remove the effect of a GGM
  - remove the effect of the topography.
(4) Estimating the empirical covariance function for the residual data after the computation of terrain correction.
(5) Determining the parameters of analytic representation of the empirical covariance function.
(6) Using LSC to determine residual quasigeoid undulations from gravity data only (gravimetric quasigeoid), verifying error estimates of data, and checking for gross errors, which could be removed if the errors are large. Otherwise, new data could be added, and the previous step could be repeated.
(7) Restoring the effect of the residual topography and adding the contributions from the GGMs.
(8) Applying the ellipsoidal correction.
(9) Transform height anomaly into geoid undulation.
(10) Checking the developed model by comparing its results against the known GPS/H control points.

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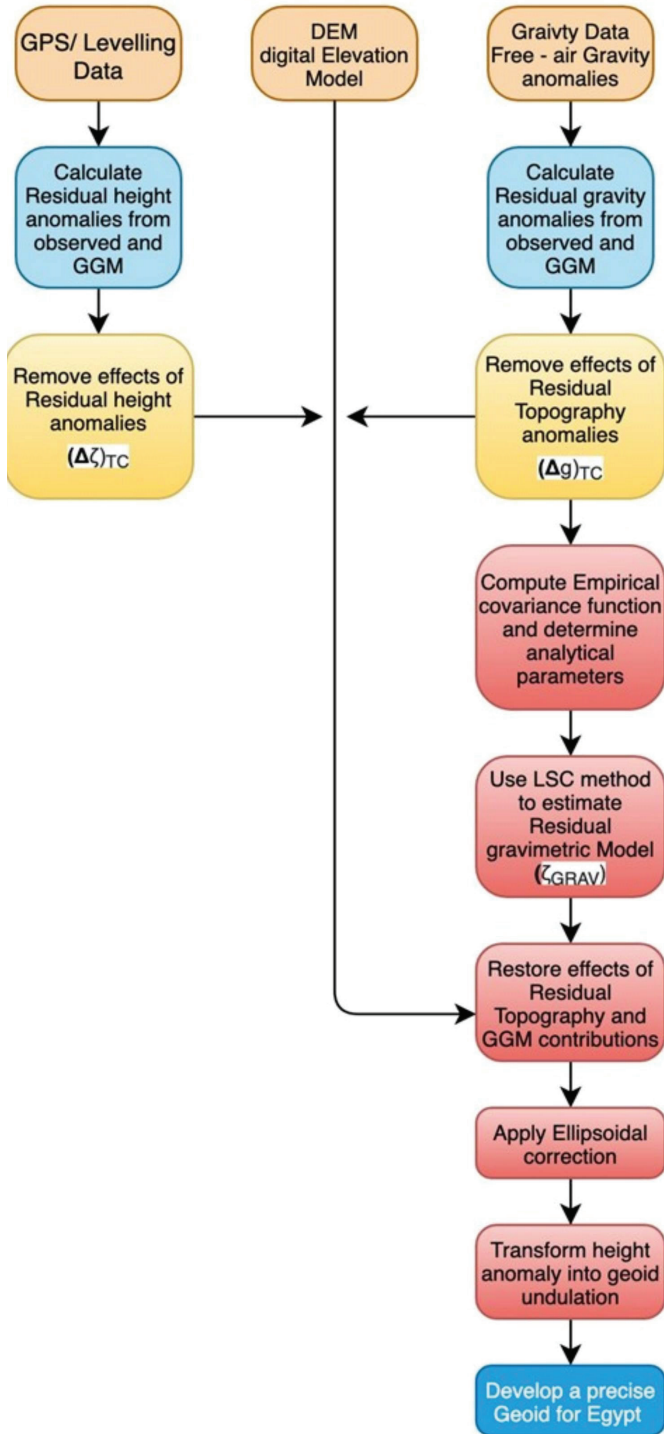
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Figure 2:   Flowchart of the procedure of developing a precise geoid model for Egypt.

2 AVAILABLE DATA AT THIS TIME

The local gravity data used in this study has been grouped into four sets.

The first set mostly consists of previously available free-air gravity anomalies gathered at 1849 points. The sources of this data are documented in many previous works (Amin, El-Fatairy and Hassouna, 2002; 2003a; El Tokhy, 1993), with -4.37 mGal as mean (average) and 0.73 mGal as standard deviation. Besides, free-air gravity anomaly values at 267 points were obtained from the Bureau Gravimetric International (BGI), where their mean value is -6.09 mGal, and the standard deviation is 0.24 mGal. The total number of the first set is 2116 data points. Figure 3 shows an irregular distribution of the available gravity anomaly data with large gaps, especially over the land. The data coverage is relatively homogeneously distributed over the seas.
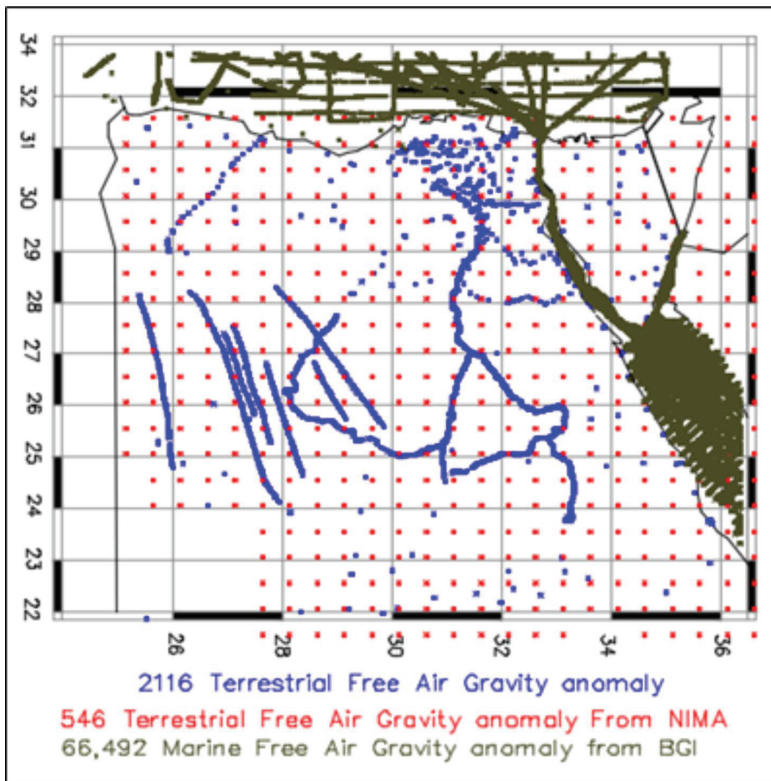

Figure 3:   Spatial distribution of the available terrestrial and marine gravity anomalies data points.

The second set consists of marine free-air gravity anomalies at 66,492 points, which have been gathered by BGI with an average -26.15 mGal and 7 mGal standard deviations. Most of the measurements were executed and supervised by the General Petroleum Company under the Egyptian Academy of Sciences and Technology (Kamel and Nakhla, 1987). The National Gravity Standard Base Net of Egypt (NGSBN77) consists of 71 stations with standard deviations ranging from 0.01 to 0.18 mGal (Figure 4). It includes existing stations of the International Gravity Standardization Net 1971 (IGSN71) in Egypt (11 stations). The standard error of the adjusted IGSN71 gravity values was less than ± 0.1 mGal (Morelli et al., 1972).

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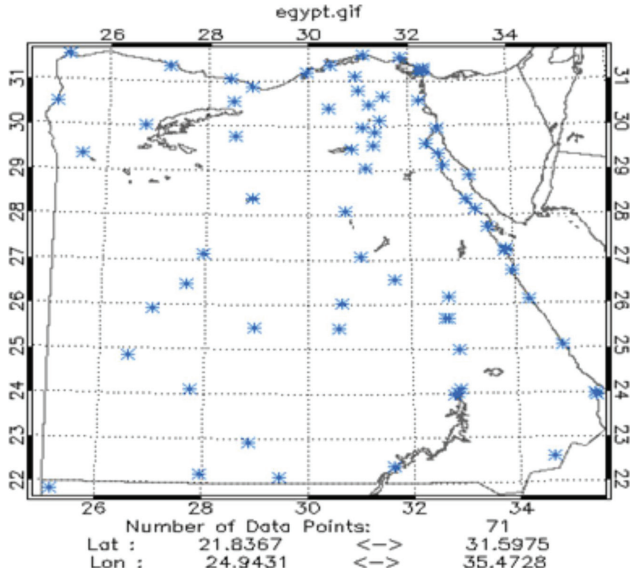
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Figure 4: The National Gravity Standard Base Net (NGSBN77).

The third set covers 546 gravimetric data points from the National Imagery and Mapping Agency (NIMA) terrestrial data (Figure 3) and 1240 local Ground Control Points gathered from different construction projects.
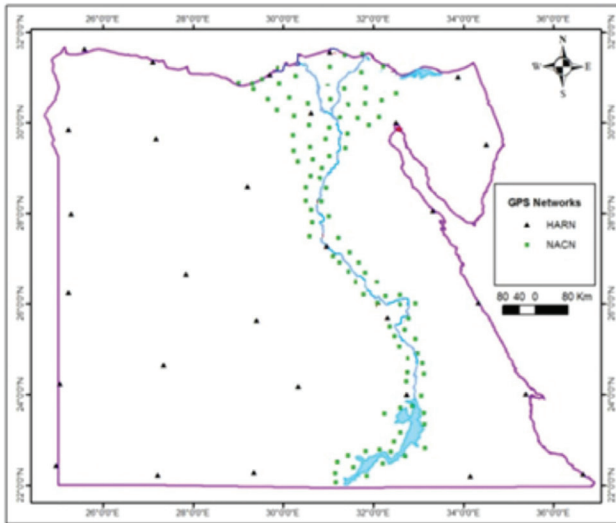

Figure 5: First and second-order geodetic networks HARN and NACN (Dawod and Ismail, 2005).

The fourth set includes the GPS/H undulation data points obtained from the first order geodetic network (High Accuracy Reference Network – HARN) that covers the entire Egyptian territory and consists of 30 stations with an approximate separation of 200 km. The second network is the National Agricultural Cadastral Network (NACN) that mainly covers the Nile valley and the Delta. NACN consists of 112 stations, with an approximate station separation of 50 km. Both networks are depicted in Figure 5 (Da-

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wod and Ismail, 2005). Also, additional 670 data points had been collected from several sources, such as construction or petroleum projects.

3 CHRONOLOGY OF ATTEMPTS FOR GEOID DETERMINATION IN EGYPT

Several attempts have been performed for geoid determination in Egypt. A typical result for all these attempts is the low accuracy of the estimated geoid undulations, especially over areas lacking enough data.

Egyptian Geodetic Datum (EGD) was defined in 1907 with Helmert 1906 ellipsoid as a reference ellipsoid. The geoid undulation was set to zero at the initial point (station F1 located at Al-Mouqatam Mountain). Mean Sea Level computed at Alexandria tide gauge, based on low/high water daily readings through the period 1898-1906, was adopted as Egypt's geoid (Cole, 1944). Alnaggar developed the first geoid in 1986 using the LSC technique and heterogeneous data. That geoid referred to the WGS72 global geodetic datum with 16.47 m as a mean value, and 3.3 m in terms of root mean square (RMS) (Dawod, 2009).

In 1993, El-Tokhey developed a geoid relative to the GRS80 datum using gravity, astronomic, Doppler, and GPS data. In the same year, (Saad, 1993) investigated the accuracy improvements and redefinition of the Egyptian vertical control networks (Dawod, 2009). At the same time, Nassar, Hanafy and El-Tokhey (1993) developed the Ain Shams University (ASU93) national geoid, which was based on applying the LSC.

In 1995 El-Sagheer applied Fast Fourier Transformation (FFT) techniques to generate a DTM called DTM-95, which was used to predict the geoid with 32.19 m mean value and 3.71 m RMS. In the same year, El-Shazly (1995) investigated the redefinition of Egypt's vertical datum based on analysing Sea Surface Topography and levelling by GPS. Two national GPS geodetic control networks were established by the Egyptian Survey Authority in 1995. The first was the HARN network that covers the entire Egyptian territory, and the second network was the NACN network (Dawod, 2009).

In 1997, an accurate gravity framework for Egypt was established through the Egyptian National Gravity Standardization Network 1997 (ENGSN97). The ENGSN97 150 gravity points (plus another 100 stations) have been utilised by (Dawod, 2001).

The data used during the previously mentioned period was the gravimetric data, mostly. However, the data suffer from significant gaps, particularly in the eastern and western deserts, which is eventually directly reflected in the computed geoid's accuracy (Dawod and Alnaggar, 2000).

A significant accomplishment in the field of geodesy was initiated in the last two decades. It started with the development of Ain Shams University's (ASU2000) national geoid, with estimated internal and external accuracy of 0.6 m and 1 m on average. Tscherning et al. (2001) developed a geoid model for the Lake Nasser area with 10.08 m as a mean value and 0.26 m in terms of RMS. A generation of geoid models for Egypt on a five-minute grid was developed by (Saad and Dawod, 2002).

From 2007 to 2015, many attempts have been made, including Abd-Elmotaal (Abd-Elmotaal, 2006), who developed and applied a high-degree tailored reference model by merging the available gravity anomalies with EGM96. (Dawod, 2008) utilised local data sets from terrestrial gravity and GPS/H stations in the context of developing a precise geoid model for Egypt. Its average accuracy is estimated to be 0.36 m (Dawod, 2009).

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Recently, Al-Krargy, Doma and Dawod (2014) developed a national geoid for Egypt using recent surveying data with an estimated accuracy of ±18.4 cm. Additionally, El-Ashquer, Elsaka and El-Fiky (2017) have improved the hybrid local geoid model for Egypt, with hybrid residual height anomalies ranging from −1.5 m to +0.9 m. The mean of 0.22 m and the standard deviation of 0.17 m, were obtained. In the last two decades, new observations of gravimetric data have been added to the previously collected data (Figure 6). However, insufficient and irregular distribution of data still exists over Egypt territory.
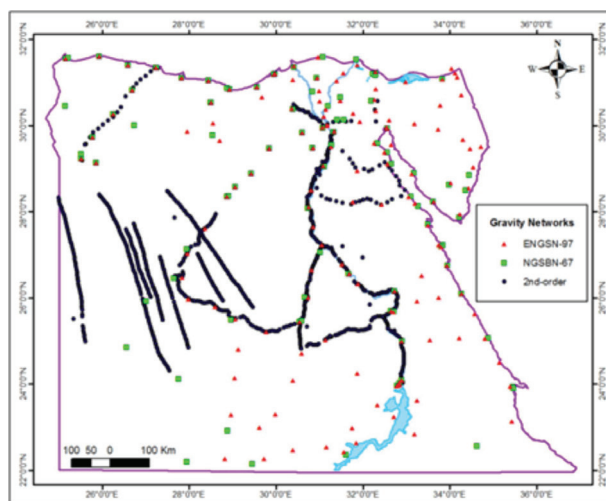

Figure 6:   Spatial distribution of the available gravimetric data utilised up to the year 2017 (Dawod and Hosny, 2017).

4 PERFORMANCE EVALUATION OF USED GGMs OVER EGYPT TERRITORY

One of the essential processes to determine precise geoid is checking the performance of GGM over Egypt. The aim is to choose the best fitting GGM to the local gravity field. The evaluation process is applied for the next six GGMs:

– Earth Gravitational Model EGM96 (Lemoine et al., 1998);
– High degree combined Earth Gravitational Model EGM2008 (Pavlis et al., 2008);
– Combined gravity field model GOCO05C (Mayer-Gürr et al., 2015);
– Combined gravity field model GECO (Gilardoni, Reguzzoni and Sampietro, 2016);
– Invariant of the Gravitational Gradient Tensor IGGT_R1 based on the second invariant of the GOCE gravitational gradient tensor (Lu et al., 2017);
– Gravity field products from GOCE satellite using the space-wise approach release 5 (GO-CONS -GCF -2SPW-R5) (Gatti et al., 2016).

All mentioned models are compared with 2116 terrestrial gravity data points and 812 GPS/H data points.

We can notice from Table 1, the range of mean residual values among GGMs is 7 mGal in terms of standard deviation. That means the resolution of GGMs does not have a significant effect on the GGMs accuracy over Egypt due to the small variations of gravity values. Most of the regions of Egypt are flatlands, and that is the reason for slight variations of gravity values. Therefore, for precise gravity field modelling in Egypt, any of the mentioned GGMs may be used as a reference model with the same quality level.

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Examined GGMs indicated the superior performance of GGM GO-CONS-GCF-2SPW-R5 concerning free-air gravity anomalies in terms of standard deviation.

Table 1: The results of the comparison among the terrestrial gravity anomaly data at scattering points and those computed from the different harmonic models at the same points.

Models	Degree/ Order	Mean mGal	Minimum mGal	Maximum mGal	St. dev. mGal
Terrestrial Data		-21.689	-143.300	115.550	41.757
IGG.R1	240	-9.067	-158.746	128.905	43.401
EGM2008	2190	-1.682	-161.322	232.128	49.010
EGM96	360	-8.166	-157.758	200.832	44.790
GECO	2190	-18.167	-161.230	205.003	47.153
GOCO05c	720	-5.717	-161.595	199.002	44.577
GO-CONS-GCF-2SPW-R5	330	-12.789	-159.613	121.923	41.835

Table 2: The results of the comparison among GPS/levelling undulation residuals at scattering points and those computed from the different harmonic models at the same points.

Models	Degree/ Order	Mean m	Minimum m	Maximum m	St. dev. m
Terrestrial Data		13.917	9.204	19.331	1.976
IGG.R1	240	-0.748	-4.545	0.742	0.588
EGM2008	2190	-0.797	-5.092	0.278	0.519
EGM96	360	-0.951	-1.904	0.053	0.431
GECO	2190	-0.708	-4.508	0.579	0.578
GOCO05c	720	-0.696	-4.187	0.366	0.508
GO-CONS-GCF-2SPW-R5	330	-0.762	-4.176	0.392	0.514

According to Table 2, differences of geoid undulation among GGMs, in terms of standard deviation, reached 16 cm between the minimum value of 43 cm for EGM96 and a maximum residual of 59 cm for the IGG.RI model. In other terms, terrestrial GPS/H data have a contribution of EGM96, while no terrestrial data had been included in the other GGMs.

From the above explanations, we can choose EGM96 as a reference model for precise gravity field modelling in Egypt because it has a good performance concerning gravity anomaly residuals.

To determine a few centimetres accurate geoid, by applying the LSC, well coverage and distribution of data are needed. That condition cannot be achieved regarding available data. Therefore, an area with dense data has been chosen as a window for calculations as shown in Figure 7. Hence, 260 data points of free-air gravity anomalies were used to determine geoid solutions as a sample of the whole Egypt territory, and the solution is checked by using 59 GPS/H values.

This data set window refers to the WGS84 and IGSN71 and covers the window $30° < \phi < 32°$ and $29° < \lambda < 31°$. The statistic calculations for those values are shown in Table 3.

SRTM 5'×5' arc-minute was utilised to remove high-frequency features of the gravity field in Egypt, which also plays an important role in the smoothing strategy of the data. The SRTM with the resolution of SRTM 5'×5' arc-minute was utilised because the selected area is, in general, a flat area, and this resolution is less demanding in computation time needed.
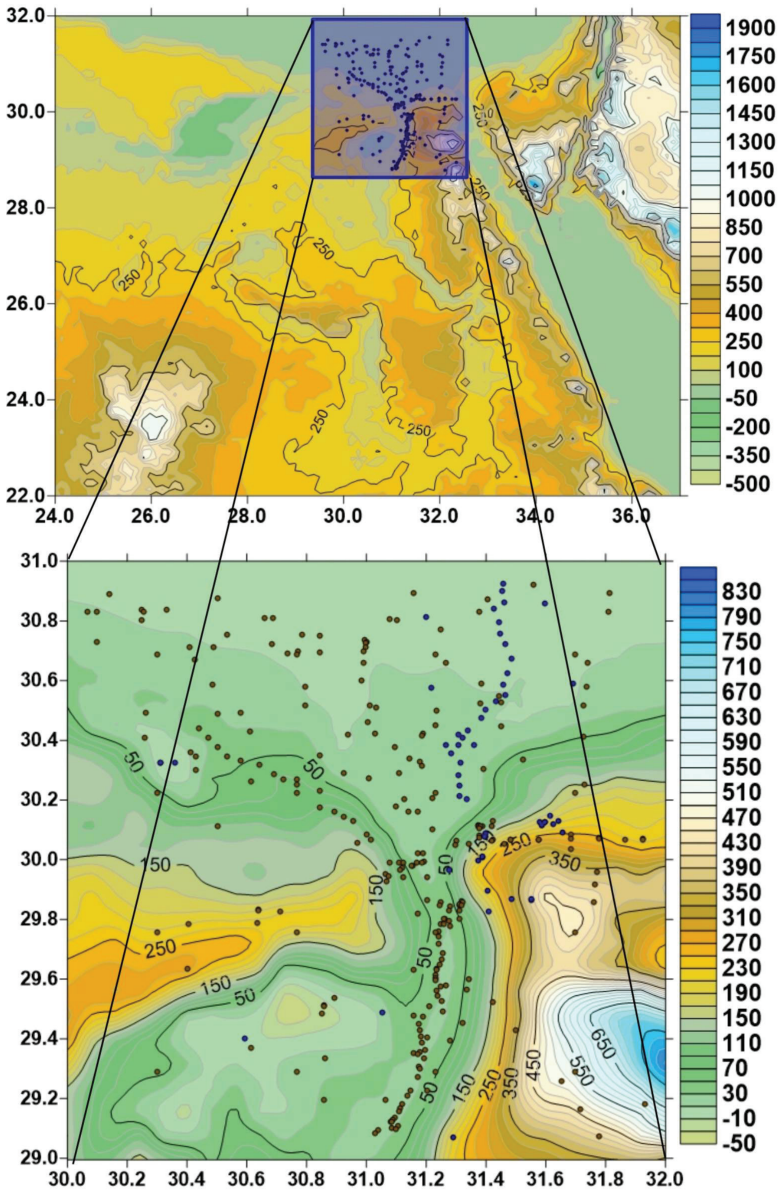
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Figure 7: DEM for whole Egypt territory and data used (blue circle – GPS/H data, grey circle gravity data).

Table 3: Statistics of original and residual gravity anomaly data

Item	Mean mGal	Minimum mGal	Maximum mGal	St. dev. mGal
Free air gravity anomaly	-4.552	-78.234	57.787	20.581
RTM reduced gravity anomaly	-7.915	-37.643	26.981	9.363
Final (RTM+EGM96) residual gravity anomaly	-13.812	-32.430	57.519	17.623

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Topographic correction for gravity anomalies, and geoidal heights were computed according to Eq. (5) and (6). This step is performed by using TC software from GRAVSOFT package (Amin, El-Fatairy and Hassouna, 2003b).

$$\Delta g_{DEM} = 2\pi G \rho \left(h - h_{ref} \right) - T_c = 2\pi G \rho \left(h - h_{ref} \right) - \left(\frac{G\rho R^2}{2} \right) (\Delta\phi.\Delta\lambda) \sum \left(\left(h' - h \right)^2 / l^3 \right) \qquad (5)$$

$$N_{DEM} = -\left(\frac{2\pi G \rho}{\gamma}\right) \left(h - h_{ref} \right)^2 - \left(\frac{G\rho R^2}{6\gamma} \right) (\Delta\phi.\Delta\lambda) \sum \left(\left(h'^3 - h^3 \right) / l^3 \right) \qquad (6)$$

where T_C is the classical terrain correction concerning the Bouguer plate and the first term in the equation is the Bouguer plate effect on the anomaly, h is the orthometric height of the computation point, h' is the orthometric height of the running point, G is the gravitational constant, ρ is the mean crustal density, h_{ref} is the relevant elevation of the average surface, and l is the spatial distance between the computation point and the running point.

The LSC algorithm is a powerful tool in combining all possible heterogeneous data in one unified solution. Therefore, this technique will be used for geoid modelling in the current work. An empirical covariance function should be determined and used in the computation to obtain the best LSC approximation to the real potential field in a particular area. However, LSC's practical solution presupposes that the gravity field, and hence the relevant covariance functions, are homogeneous and isotropic, i.e., location and azimuth independent, which implies that the data should be as smooth as possible so that they behave purely randomly. That is why the removal steps mentioned above have been conducted.
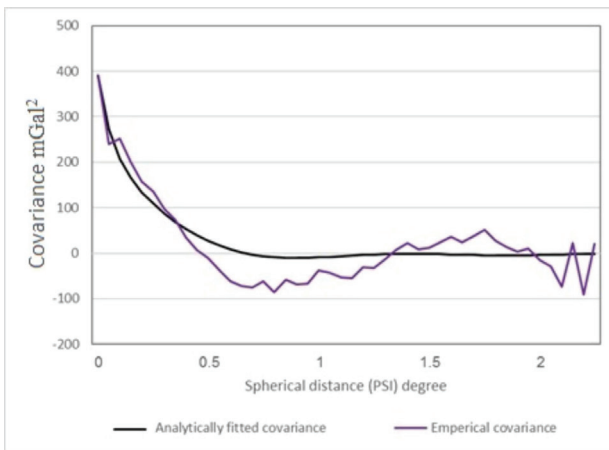

Figure 8:   Residual gravity disturbance empirical and analytically fitted covariance functions

One element of those smooth signals (residuals), usually the residual gravity anomaly, is used to obtain an isotropic empirical covariance function that represents the statistical characteristics of the local field. As this function merely depends on the separation between the data points, it describes the spatial variability of the local residual field under consideration.

The modelled (analytical) covariance function is fitted to the residual anomaly empirical covariance function via a nonlinear 3-parameter iterative least-squares adjustment until the convergence arrives,

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resulting in the final three parameters: the unit less scale factor (c), the constant A in mgal², and $(R_B - R)$ in meters. In the current study, the fitting has been done through the FORTRAN program COVFIT, written by P. Knudsen, as shown in Figure 8.

After covariance-function determination, the residual heights anomalies at all points of the regional gravimetric survey were predicted. Next, the effect of the reference field ζ_{REF}, topographic mass effect ζ_{TOPO}, and the total values of height anomalies were obtained in accordance with eq. (3).

Table 4: Statistics calculations of the residuals and full predicted geoid for the chosen area

Item	Mean m	Minimum m	Maximum m	St. dev. m
Residual N	0.085	-0.630	0.420	0.207
Final N	15.814	15.079	17.481	0.475

To obtain the respective full spectrum geoid values and their error estimates, the contribution of harmonic model and digital elevation model (DEM), in terms of the topographic indirect effect, were then added back (restored) to the residual geoid values predicted at discrete points. Table 4 shows the statistics of the predicted geoid for the considered study area, while Figure 9 represents the obtainable geoid as a contour map with an interval of 10 cm.

To estimate the external accuracy of the geoid, discrete geoid values were predicted at the available 59 control points. At those control points, geoidal heights are known in terms of GPS and levelling observations.
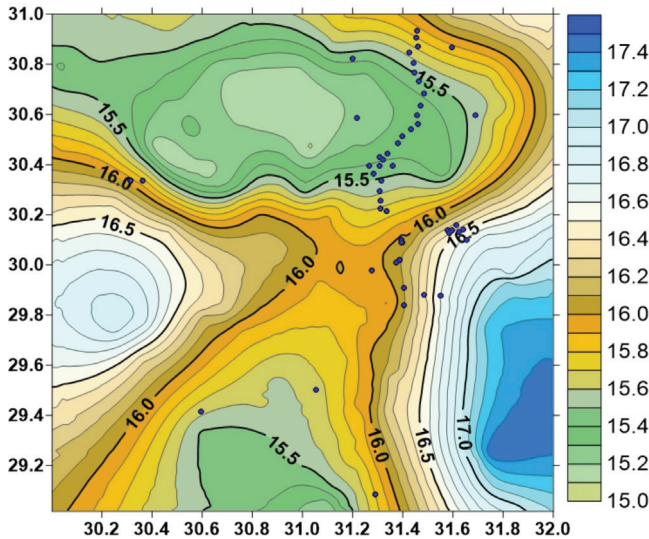

Figure 9: Represents the obtained geoid contour map for the chosen area (interval: 0.10 m)

Table 5 shows the resulting external accuracy as a function of the statistics of the differences between the observed and predicted geoid values at control points. Those control points have not been used as data points in the geoid solution. Finally, Figure 10 shows the general representation of residuals surface.

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Table 5: Statistics of the differences among the observed and predicted geoid heights at GPS/H control points

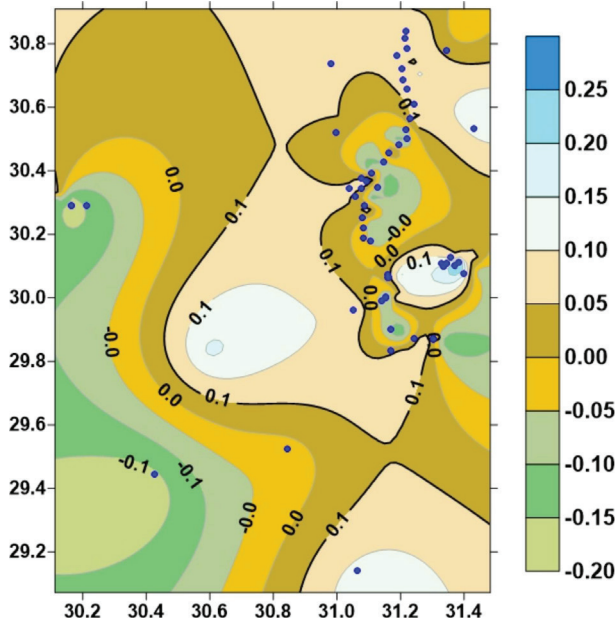| Item	Mean m	Minimum m	Maximum m	St. dev. m
N(observed) − N(predicted)	0.024	-0.380	0.360	0.111

Figure 10: General representation of residuals surface

5 CONCLUSION AND RECOMMENDATIONS

Based on the studied area results, as outlined by Table 5 and shown by Figure 10, the obtained accuracy by the LSC technique in terms of the standard deviation of the differences is considered rather satisfactory. The obtained accuracy in terms of the standard deviation of the differences at control points, as outlined by Table 5, amount 0.11 m. Still, to generalise this solution to Egypt's whole area, we need to apply and achieve the below recommendations.

Unfortunately, some of the data is out of date, and other data has inadequate and insufficient distribution all over Egypt's territory, especially in the eastern and northern deserts. Hence, to obtain a geoid with centimetre accuracy, it is recommended to have new gravimetric measurements in Egypt's territory and exchange the data with neighbouring countries.

For this reason, these are some of the ways to fill in these gaps:

– Airborne gravity surveys may significantly improve the gravity coverage over Egypt,
– Using available gravity data observations from three gravity field missions (CHAMP, GRACE, and GOCE) for better gravity field modelling in Egypt,
– Perform terrestrial gravity measurements where is suitable and possible.

Adding new observations from levelling and GPS data points to cover most of the Egyptian territory by suitable GPS/H data points.

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It is advisable to use the satellite-only models of the GOCO series and in combination with the available local geodetic data for better modelling of the Egyptian gravity field. It is also necessary to have active cooperation between the Egyptian surveying authority representatives and the responsible organisations dealing with GGM computations.

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