Business Mathematics & Statistics

NCWEB Hansraj Centre

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- **Topics discussed:**
- **Methods of Constructing Numbers**
  1. **Unweighted Index Numbers**
  2. **Weighted Index Numbers**
- **Tests for Index Numbers**
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- **Consumer Price Index Number (CPI)**

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Section A

METHODS OF CONSTRUCTING INDEX NUMBERS

They may be categorized into two broad groups as given below:

I) **Unweighted Indices; and**

II) Weighted Indices

Let us first acquaint ourselves with the symbols used in construction of index numbers. They are as follows:

P_0 denotes price per unit of a commodity in the base period.

P_1 denotes price per unit of the same commodity in the current period (current period is one in which the index number is calculated with reference to the base period).

Similar measurements are assigned to Q_0, Q_1 and V_0, V_1.

Capital letters P, Q, and V are used for denoting price index, quantity index, and value index numbers, respectively.

It may be noted that indices are expressed in per cent.

Unweighted Index Numbers:

This type of indices are also referred to as simple index numbers. In this method of constructing indices, weights are not expressly assigned. These are further classified under two categories:

1) Simple Aggregative Index
2) Simple Average of Relatives Index

Let us study the construction of indices under these two methods:

1) **Simple Aggregative Index :** This is the simplest and least satisfactory method of constructing indices. In the case of price indices, through this method, the total of unit cost of each commodity in the current year is divided by the total of unit cost of the same commodity in the base year and the quotient is multiplied by 100. Symbolically,

$$P_{01} = \left(\frac{\Sigma P_1}{\Sigma P_0}\right) \times 100$$

Similarly, the quantity index may be expressed as:

$$Q_{01} = \left(\frac{\Sigma q_1}{\Sigma q_0}\right) \times 100$$

Illustration 1: By considering the hypothetical data for the year 1990 and 2000 the following computation was done for construction of price index and quantity index.

Computation of Index by Simple Aggregative Method

Item	Year 1990		Year 2000	
	Price (Rs.)	Quantity	Price (Rs.)	Quantity
Wheat	700	4 qts	950	3.5 qts
Clothing	200	30 mts	300	35 mts
Gas	150	4 cylinder	220	6 cylinders
Electricity	0.80	800 units	1.10	1,000units
House Rent	400	1 dwelling	800	1 dwelling
	1450.80 ΣP_0	839 Σq_0	2271.1 Σp_1	1045.5 Σq_1

The price index for the year 2000 with reference to base year 1990 the simple aggregative method is

$$P_{01} = \left(\frac{\Sigma P_1}{\Sigma P_0}\right) \times 100 = \frac{2271.1}{1450.8} \times 100 = 156.54$$

Thus, the prices in respect of commodities considered in the index have shown an increase of 56.54 per cent in 2000 as compared to 1990.

2) **Simple Average of Relatives Index**

In this method of constructing price index, first of all price relatives have to be computed for the different items included in the index then the average of these is calculated symbolically,

$$P_{01} = \frac{\sum\left(\frac{P_1}{P_0} \times 100\right)}{N} \text{ or } \frac{\text{Sum of the Price Relatives}}{\text{No.of Items}}$$

Using the same data by considering only prices given in the illustration- 1, the computation of price index as simple average of price relatives is as follows:

Item	Units	Year 1990 Price (Rs.)	Year 2000 Price (Rs.)	Price relatives $\frac{P_1}{P_0} \times 100$
Wheat	Qts	700	950	(950 / 700) × 100 = 135.7
Clothing	Mts	200	300	(300 / 200) × 100 = 150.0
Gas	Cylinder	150	220	(220 / 150) × 100 = 140.7
Electricity	Units	0.80	1.10	(1.10 / 0.80) × 100 = 137.5
House Rent	dwelling	400	800	(800 / 400) × 100 = 200
	N = 5			$\sum\left(\frac{P_1}{P_0} \times 100\right) = 763.9$

$$P_{01} = \frac{\sum\left(\frac{P_1}{P_0} \times 100\right)}{N} = \frac{763.9}{5} = 152.78$$

Thus, the index of simple average of price relatives shows 52.78 per cent increase in price.

For construction of Quantity Index, quantity relatives should be obtained and averaged. The formula for quantity index in this method is:

$$Q_{01} = \frac{\sum\left(\frac{q_1}{q_0} \times 100\right)}{N}$$

This method also has its limitations.

First, each price/quantity relative is given equal importance, which is not realistic.

Secondly, the arithematic mean is not the right type of average for ratios, and percentages.

Weighted Index Numbers

In the earlier two methods each item received equal weight/importance in the construction of an index, whereas in the weighted index methods, weights are expressly assigned to each item which is included in an index construction.

This is further divided into two methods.

1) Weighted Aggregative Index, and

2) Weighted Average of Relatives Index.

Let us discuss these two methods one after another.

1) **Weighted Aggregative Index** : In this group, we shall study three specific methods commonly used in business research.

They are: (a) Laspeyre'sindex,

(b) Paasche's index, and

(c) Fisher's ideal index.

a) Laspeyre's Index: In this method, weights assigned to each commodity are the quantities consumed in the base year for price indices. For quantity index weights used are the prices of commodities in the base year. Thus, according to Laspeyre:

$$\text{Price Index } (P_{01}{}^{La}) = \left(\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0}\right) \times 100, \text{ and}$$

$$\text{Quantity Index} = (Q_{01}{}^{La}) = \left(\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0}\right) \times 100,$$

b) Paasche's Index : In this method, quantities consumed in the current year are used as weights in construction of price indices, where as in construction of quantity index, weights used are the prices of items in the current year. Thus according to Paasche:

$$\text{Price Index } (P_{01}{}^{Pa}) = \left(\frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right) \times 100, \text{ and}$$

$$\text{Quantity Index} = (Q_{01}{}^{Pa}) = \left(\frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}\right) \times 100,$$

c) Fisher's Ideal Index : Irving Fisher used geometric mean of the Laspeyre's and Paache's indices to overcome the shortcomings of the both.
Thus,

$$\text{Price Index } (P_{01}{}^{F}) = \sqrt{\left(\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\right)\left(\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}\right)} \times 100$$

Analogously, Fisher's quantity index is

$$\text{Quantity Index} = (Q^{F}{}_{01}) = \sqrt{\left(\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0}\right)\left(\frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}\right)} \times 100$$

Thus fisher's ideal index of price/quantity =

$$\sqrt{\text{Laspeyre's Index} \times \text{Paasche's Index}}$$

Fisher's index is superior because it uses geometric mean (which is best applicable for average of ratios and percentages) of Laspeyre's and Paache's indices. Also, because it is comparatively free from bias of over estimation and under estimation.

Fisher's index satisfies the requirement of time reversal test and factor reversal test. This index is, therefore, called ideal index.

TESTS FOR INDEX NUMBERS

A perfect index number, which measures the change in the level of a phenomenon from a specific period to another period, should satisfy certain tests. They are: (i) Time reversal test, and (2) factor reversal test .

The Time Reversal Test

If we observe the construction of index numbers, we found that there are two aspects. They are period and/ or quantity. Therefore, if we reverse the time subscripts, such as base period (0) and current period (1), of a price or/and quantity index, the result should be the reciprocal of the original index Algebraically, it is expressed as $P_{0.1} \times P_{1.0} = 1$

Where, $P_{0.1}$ = Index number for current period (P_1) with the base period (P_0)

$P_{1.0}$ = Index number for base period (P_0) with the current period (P_1)

Fisher's Ideal Index $P_{0.1} = \sqrt{\left(\frac{\sum P_1 q_0}{\sum P_0 q_0}\right)\left(\frac{\sum P_1 q_1}{\sum P_0 q_1}\right)}$

and if time subscripts are reversed i.e.,

$$P_{1.0} = \sqrt{\left(\frac{\sum P_0 q_1}{\sum P_1 q_1}\right)\left(\frac{\sum P_0 q_0}{\sum P_1 q_0}\right)}$$

With the above, now, we verify the result of time reversal test i.e.

$$P_{0.1} \times P_{1.0} = 1$$

Hence,

$$P_{0.1} \times P_{1.0} = \sqrt{\frac{\sum P_1 q_0}{\sum P_0 q_0} \times \frac{\sum P_1 q_1}{\sum P_0 q_1} \times \frac{\sum P_0 q_1}{\sum P_1 q_1} \times \frac{\sum P_0 q_0}{\sum P_1 q_0}}$$

The Factor Reversal Test

Irving Fisher suggested one more test i.e. Factor Reversal Test to be applied to weighted index numbers to verify the validity. According to him “Just as our formula should permit the interchange of the two times without giving inconsistent results so it ought to permit interchanging the prices (P) and quantities (q) without giving inconsistent results, i.e., the two results multiplied together should give the true ratio”.

Thus, with the usual notations a ‘value index’ (P0.1.× q1.0) formula is given by:

$$P_{0.1} \times q_{0.1} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$$

Where, $P_{0.1}$ = The price change for the current period over the base period.

$q_{0.1}$ = Quantity change for the current period over the base period.

$\sum P_1 q_1$ = The total value in the current period.

$\sum P_0 q_0$ = The total value in the base period.

The Fisher's ideal index only satisfied this test, as shown below:

$$P_{0.1}^{F} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}$$

and, if factors (part q) are reversed i.e.

$$q_{0.1} = \sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

Hence,

$$P_{0.1} \times q_{0.1} = \sqrt{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times \frac{\Sigma q_1 P_0}{\Sigma q_0 P_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}$$

$$= \sqrt{\frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} \times \frac{\Sigma q_1 P_1}{\Sigma q_0 P_0}} = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_0} = P_{0.1} \times q_{0.1}$$

CONSUMER PRICE INDEX NUMBER(CPI)

This method is also known as Cost of Living Index Number (CPI). This index is to serve as a measure of change in the prices of goods and services commonly consumed by a homogeneous section of people, such as the classes – lower middle, middle, upper middle, industrial workers, urban and rural areas etc. These indices are helpful in deciding dearness allowances, wages/ salaries, negotiations, framing price policy, taxation policy, other economic and welfare policies.

The common method for selecting from the consumption basket is to conduct a family living style survey among the population group (section) for which the consumer price index is to be constructed. Prices of selected commonly consumed items are also collected from various retail markets used by such consumers and also the quantity of consumption [normally expressed in terms of weights (w)]. When the price of one commodity varies, a simple average is applied. The overall index (CPI) is computed as an weighted average of group indices and the weights being again the proportional expenditure on different groups (e.g. 30 per cent on food).

$$\text{CPI: } I = \frac{\Sigma W\left(\frac{P_1}{P_0} \times 100\right)}{\Sigma W}$$

Where, $w = \frac{P_0 q_0}{\Sigma P_0 q_0}$, is the weight of a group index.

Let us observe how to construct the Consumer Price Index for food with the help of the following data pertains to current price, base price and weights of seven items:

Construction of an Index for food

Items	Price		$P\left(\frac{P_1}{P_0} \times 100\right)$	Weights (w)	Pw
	P_1	P_0			
Wheat	50	40	125.0	30	3750.0
Pulses	45	30	150.0	20	3000.0
Rice	60	40	150.0	10	1500.0
Sugar	40	50	200.0	5	1000.0
Oil	75	60	125.0	15	1875.0
Potato	60	50	120.0	15	1800.0
Meat	200	150	133.3	5	666.5
Total				100	13591.5

$$\text{CPI (Food)} = \frac{\Sigma W\left(\frac{P_1}{P_0} \times 100\right)}{\Sigma W} = \frac{13591.5}{100} = 135.92$$

Thank You