Application of Game Theory to Manpower Policy Development

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Abstract—**The choice of manpower policy is very strategic. Manpower policy affects the wage bill, the total product of labour, a firm's utility, as well as the historical probabilities of losses, gains and promotions. Hence, optimum policy that maximizes utility for both the workforce and the firm should be the equilibrium policy according to game theoretic predictions. Hence, evaluation tools, such as game theory, that take into account the manpower policy economics and dynamics, should evaluate alternative manpower policies very effectively. In this work a combination of Markov manpower model and game theoretic modelling is used to obtain optimal/equilibrium manpower policy. An oil company operating in the oil rich Niger Delta Region of Nigeria was used as case study. The results of the study show that game theoretic analysis would be a very useful tool for manpower policy planning and evaluation. In addition the model gives a more refined equilibrium wage policy than prescribed by Nash's wage bargaining model.**

Index Terms—**Manpower planning; Game theory; Markov processes; Stochastic processes**

I. INTRODUCTION

Quite a substantial number of business organizations in the world experience one form of manpower problem or another. This especially true in the oil-rich Niger Delta Region of Nigeria where a large number of firms, mostly indigenous companies, in the oil industry experience large labour turnover. Quite often many business organizations fail to motivate their workforce. Motivating the work force is very critical to the survival of any business organization. Hence, optimal manpower policies must always be sought by serious minded business enterprises. In a similar vein, a lot of industrial organizations have been run underground by unmotivated workers. Nigerian firms are not exempt from this. Practices by these workers that contributed to the collapse of these enterprises include but not limited to:

   a. Fraudulent sale of company properties.
   b. Soldering which culminate in low productivity to punish management.
   c. Vandalization of company properties.
   d. Fraudulent turn around maintenance (TAM) contracts.
   e. Inflated procurement contract prizes, including over invoicing.

   f. Lack of competitive bidding in supply contract award.
   g. Procurement of low quality spares, etc.

The choice of manpower policy is very strategic especially in this age of globalisation and rapid technological advancement. Motivated workforce can make any organization realize its goals effectively. This assertion is built around the motivation principle that what gets rewarded gets done. There are many approaches to manpower policy analysis and planning. The literature has reported four methods and they include: Markov chain models, computer simulation models, optimization models and supply chain management through System Dynamics [1].

Markov chain analysis has been at the heart of manpower policy analysis and assessment. Markovian models have been used to estimate distribution of future manpower as well as to maintain new recruitment and promotion or firing policies (see Kennington et al. [2], Abodunde and McClean [3], Zanakis and Maret [4], Price et al. [5], Bartholomew [6], Edwards [7], and Raghavendra [8]). A combination of Markov models and linear programming is used to obtain optimal policies taking into account not only the manpower requirements but also costs and conflicting objectives (see Young and Abodunde [9], and Zanakis and Maret [10]). Young and Abodunde related manpower policy to manpower capacity planning, an aspect of production planning. In their paper, they assumed that different manpower policies could cause either lead to undercapacity or overcapacity which leads to overproduction or underproduction to which they assigned costs. Hence, they used the principle of mathematical programming using Markov manpower modelling as the decision variable and minimization of the cost of overproduction or underproduction as their objective function in order to arrive at the optimal manpower policy. Zanakis and Maret used goal programming approach in their analysis of optimal manpower requirements.

In the setting of fierce competition among competing firms, choosing the optimal policy is very strategic. Game theory is the best decision making tool when active opponents are involved. With stiff competition existing among many firms in the modern world, it is obvious that game theoretic approach to manpower planning would be invaluable. The earliest attempt to use game theory in manpower planning was due to John Nash [11], [12]. Nash developed his wage bargaining model in order to obtain the equilibrium wage

policy for firms intending to higher workers. Nash's model is deterministic. The model used calculus and assumed zero wastage in order to determine the equilibrium wage. Secondly, Nash's model assumed that wage alone determines the manpower policy. But in practice, these assumptions are rarely true. For example, a firm may offer lower wage but other conditions of employment may be very favourable such that job seekers might prefer them to others who offer higher wages but employment conditions are generally unfavourable. Secondly, employment conditions are not generally clear to recruits until after acclimatization within the organization. Furthermore, manpower flow is stochastic, hence in order to obtain a refined equilibrium manpower/wage policy statistical analysis is necessary.

In this work an attempt is made to combine Markov models and game theory in order to obtain equilibrium manpower policy among competing policies according to game theoretic prescriptions. This approach measures not only manpower flow but takes into account economic considerations in order to obtain the optimal/equilibrium policy. The presentation in this work is kept simple enough to be understood by management science/operations research (OR) professional and students. An industrial case study was used for illustration.

II. THEORETICAL BACKGROUND

A. Games Theory

Game theory is a branch of applied mathematics that is applied in many fields including: economics, managment, engineering, psychology, computer science, sociology, biology and political science. Game theory mathematically capture behaviour in strategic situations where an individual's chances of making success based on his choices depends on the choice of others. Game theory studies the ways in which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players, none of which might have been intended by any of them. Since the mathematical theory of games was formalized by John von Neumann and Oskar Morgenstern [13], a lot of work has been done to expound and advance the theory of games. There are many types of games but the emphasis in this work is on Markov queue game [14]. For better understanding of the concepts in game theory, there is a rich literature and these include: Binmore [15], von Stengel and Turocy [16], von Stengel [17], Osborne [18], Fudenberg & Tirole [19], Ritzberger [20], Nwobi-Okoye [21], Nwobi-Okoye [22], Nwobi-Okoye [23], Nwobi-Okoye [24], Nwobi-Okoye [25]etc.

B. Markov Queue Game

DEFINITION: A general Markov queue game is defined as a game with a finite set of players i Є J, a grand/secondary payoff matrix, GG, a set of strategies, V = $(V_1, V_2, V_3, V_4 … V_n)$ for each player with each strategy tied to a payoff matrix, G_n, an associated probability

matrix/vector, E (v), and an effective payoff matrix, G_{ev}, where $G_{ev} = E (v) \cdot G_n$.

C. Markov Chain Model for Manpower Planning

Consider a Markov process which has state space S such that:
S = {1, 2, 3, …k}
The states are:
M = {1, 2, 3 … k}
These represent various manpower states, such as: active staff, recruitment, wastage, retirement etc. Wastages refer to any of the situations when recruited staff leaves an organization in any manner other than retirement. They include: termination, sack, dismissal, interdiction, resignation, redundancy, diseased, blank, voluntary withdrawal

Denoting PT_{ij} as the transition probability of the system from state i to state j, the transition matrix is defined as

$$PT = \begin{vmatrix} P_{11} & P_{12} & ... & P_{1k} \\ P_{21} & P_{22} & ... & P_{2k} \\ ... & ... & ... & ... \\ P_{k1} & P_{k2} & ... & P_{kk} \end{vmatrix}$$

Workforce systems could be described by the terminology: stocks and flows (Bartholomew et al, 1991). The stock $n_i(t)$ is the expected number of people in class i at time t. The flow $n_{ij}(t) = n_i(t)PT_{ij}$ denotes the expected number of members moving from class i to class j in an interval of unit length of time from t to $t + 1$ with PT_{ij} being the transition probability that an individual in class i at the start of the time interval sitting in class j at the end [6].

The transition probabilities PT_{ij} could be estimated from the historical data of stocks and flows using the method of maximum likelihood [6], [26], [27].

$$\hat{P}T_{ij} = \sum_t n_{ij}(t) \bigg/ \sum_t n_i(t)$$

D. The Markov Game Theory Model

The game theoretic model used to approach manpower policy choice game is Markov queue game model developed by Nwobi-Okoye [14] which has been defined above. The following assumptions are made for the purpose of the modelling:

Game Characteristics and Assumptions

The game model used in the analysis in this work is based on the following characteristics and assumptions:

1. The game corresponds to model/variant 2 of the game developed by Nwobi-Okoye [14].
2. An infinite population source [28], [29] which represents the population from which the possible number of potential recruits could be drawn.
3. There is no possibility of moving from an absorbing state to any other state.

Mathematical Analysis

Here it is assumed that competitors try to use strategies (manpower policies) that would attract and retain employees in their respective organizations. The potential employees could queue behind any organization they wish to be employed in. The entire mathematical analysis is based on the model developed by Nwobi-Okoye [14].

Since the game corresponds to variant 2 as has earlier been mentioned, the applicable equations for modelling the game theoretic aspect of manpower policies are:

$$EG = P \cdot G \dots\dots\dots\dots\dots\dots\dots (1)$$

The dot operator in equation 1 carries out the operation of multiplying each payoff by its associated probability of occurrence.

$$GT = G - R \dots\dots\dots\dots\dots\dots\dots (2)$$

Here R is the payoff reduction.

R represents the extra cost incurred due to the specific manpower policy implementation.

$$GT_{IXJX} = G_{IXJX} - R_X \dots\dots\dots\dots\dots (3)$$
$$GT_{IYJY} = G_{IYJY} - R_Y \dots\dots\dots\dots\dots (4)$$

$$EG = PT \cdot GT \dots\dots\dots\dots\dots\dots\dots (5)$$

The grand payoff matrix, GG, is shown in Figure 1.

$$
\begin{array}{c|cccc}
 & 1 & 2 & \dots & N \\
\hline
1 & GG_{1X,1X}, GG_{1Y,1Y} & GG_{1X,1X}, GG_{1Y,1Y} & \dots GG_{1X,NX}, GG_{1Y,NY} \\
2 & GG_{2X,1X}, GG_{2Y,1Y} & GG_{2X,2X}, GG_{2Y,2Y} & \dots GG_{2X,NX}, GG_{2Y,NY} \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
N & GG_{NX,1X}, GG_{NY,1Y} & GG_{NX,2X}, GG_{NY,2Y} & \dots GG_{NX,NX}, GG_{NY,NY}
\end{array}
$$

Figure 1: Grand payoff matrix, GG

Here

$GG_{iX,jX}$ = the cumulative payoff for player X in the effective payoff matrix, EG, when X uses strategy i-1. $GG_{iY,jY}$ = the cumulative payoff for player Y in the effective payoff matrix, EG, when Y uses strategy j-1. The strategies are denoted by numbers 1 to N.

$$GG_{IYJY} = \sum_{I=1}^{N} \sum_{J=1}^{N} EG_{IYJY} \dots\dots\dots\dots (6)$$

$$GG_{IXJX} = \sum_{I=1}^{N} \sum_{J=1}^{N} EG_{IXJX} \dots\dots\dots\dots (7)$$

N= maximum value of strategies i.e. possible number of manpower policies/strategies.

Replacing E (v) with the manpower transition probabilities matrix PT_{ij}, a Markov queue game applicable to manpower planning is redefined as a game with a finite set of players i Є J, a grand/secondary payoff matrix, GG, a set of strategies, V = (V_1, V_2, V_3, V_4 … V_n) for each player with each strategy tied to a payoff matrix, G_n, an associated probability matrix/vector, PT_{ij}, and an effective payoff matrix, G_{ev}, where $G_{ev} = PT_{ij} \cdot G_n$.

In order to obtain the payoffs from the policies we consider the firms utility function which is given by (Carmichael, 2005):

$$U_f(w) = TP_L - Lw \qquad (8)$$

where TP_L is the total product of labour, the total contribution of labour to output, L is the number of workers employed by the firm, w is the wage per worker and Lw is the wage bill. With this utility function, the firm like the union, is risk neutral and the firm's utility is equal to its profits if labour is the firm's only input and product price equals 1. In this case if the wage per worker w is equal to the average product of labour, $\frac{TP_L}{L}$, the firm profits are zero.

Hence, $U_f(w)$ constitute elements of the payoff matrix G_n.

Thus the equilibrium of the game GG [30], i.e. the equilibrium/optimal manpower policy, is given by:

$$\max (TP_{Li} - Lw_i) P_{Ri} \qquad (9)$$

Or

$$\max U_{fi}(w) P_{Ri} \qquad (10)$$

Where P_{Ri} is the proportion of manpower of player/policy i that stays till retirement. Since the payoff from wastages is assumed to be zero (0).

III. METHODOLOGY

The appropriate class of virtual games to be used for the modelling as have has been mentioned earlier is the Markov Queue Game. In this game the matrix P is a Markov transition matrix whose nature is shown in Figure 2. In matrix P, $P_{iX,jX}$ represents elements for competitor x, while

$P_{iY,jY}$ represents elements for competitor y assuming a 2-persons game. The strategies are denoted by numbers 1 to N.

$$
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & \dots & N \end{array} \\
\begin{array}{c} 1 \\ 2 \\ \vdots \\ N \end{array}
\left|
\begin{array}{cccc}
P_{1X,1X}P_{1Y,1Y} & P_{1X,2X}P_{1Y,2Y} & \dots & P_{1X,NX}P_{1Y,NY} \\
P_{2X,1X}P_{2Y,1Y} & P_{2X,2X}P_{2Y,2Y} & \dots & P_{2X,NX}P_{2Y,NY} \\
\vdots & \vdots & \vdots & \vdots \\
P_{NX,1X}P_{NY,1Y} & P_{NX,2X}P_{NY,2Y} & \dots & P_{NX,NX}P_{NY,NY}
\end{array}
\right|
\end{array}
$$

Figure 2: Transition Matrix, P

Proposition 1

Let $GG_{IX_n JX_n}$ represent elements of competitor X of the matrix GG at period n and $GG_{IX_{n+1}JX_{n+1}}$ at period n+1.

It follows that $GG_{IX_n JX_n} < GG_{IX_{n+1}JX_{n+1}}$ provided the equilibrium of the transition matrix P is not reached at period n.

Corollary

Similarly, $GG_{IX_n JX_n} = GG_{IX_{n+1}JX_{n+1}}$ provided the equilibrium of the transition matrix P is reached at period n.

Proof

It can easily be shown that for the transition matrix P, that

$$\sum_{n=1}^{\infty} P_{ii}^{(n)} \neq \infty \text{ , Provided } i \neq N$$

Therefore, each state is transient except N. Since state N is the absorbing state and transition matrix P is reducible; hence, at equilibrium the probability of staying at state N is one, while the probability of staying in any other state is zero. But from equations 6 and 7

$$GG_{IX_n JX_n} = \sum_{I=1}^{N}\sum_{J=1}^{N} EG_{IX_n JX_n} \quad \text{and}$$

$$GG_{IX_{n+1}JX_{n+1}} = \sum_{I=1}^{N}\sum_{J=1}^{N} EG_{IX_{n+1}JX_{n+1}}.$$

It follows that provided the equilibrium of the transition Matrix P is not reached,

$$GG_{IX_n JX_n} < GG_{IX_{n+1}JX_{n+1}}.$$

Proposition 2

For a given payoff reduction factor z, if strategy n is optimal i.e. an equilibrium at period n, it may not necessarily be optimal at period n+1 provided the equilibrium state of the transition matrix P is not reached at period n and the game is not biased in favour of any player.

Corollary

For a given payoff reduction factor z, if strategy n is optimal i.e. an equilibrium at period n, it is also optimal at period n+1 provided the equilibrium state of the transition matrix P is reached at period n and the game is not biased in favour of any player.

Proof

Assuming some values of payoff reduction factor z denoted by z_1 and z_2. Let $GG_{IX_{nz1}JX_{nz1}}$ represent the gain for competitor X while playing the optimal strategy n at period n when the payoff reduction factor is z1 and $GG_{IX_{(n+1)z2}JX_{(n+1)z2}}$ represent the gain for competitor X while playing the optimal strategy n at period n+1 when the payoff reduction factor is z2. Since z1=z2 and from proposition 1 it is shown that at equilibrium $GG_{IX_n JX_n} = GG_{IX_{n+1}JX_{n+1}}$, it follows that for a given payoff reduction factor z, if strategy n is optimal i.e. an equilibrium at period n, it is also optimal at period n+1 provided the equilibrium state of the transition matrix P is reached at period n and the game is not biased in favour of any player.

IV. APPLICATION CASE STUDY

Consider a Markov process which has state space S such that:
S = {1, 2, 3, 4}
The states are:
M = {1, 2, 3, 4}
The states represent various manpower states. State 1 assumes that manpower retirement occurred; state 2 assumes that wastage occurred; state 3 assumes the staff is in active employment of the firm while state 4 assumes that the staff is newly recruited.

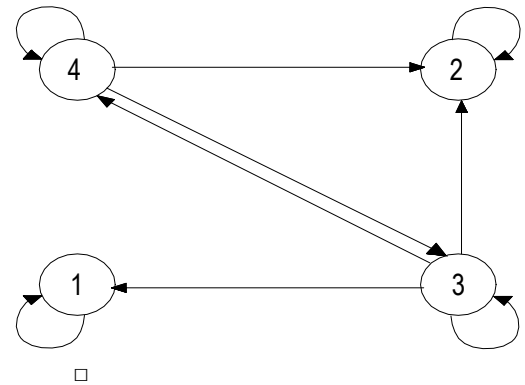The transition diagram for the system is shown in Figure 3 below:

Figure 3: Transition Diagram for the manpower transitions

An oil company at Port Harcourt, in the oil rich Niger Delta region of Nigeria is used as our case study. Two manpower policies were analysed in order to obtain the optimal/equilibrium policy.

A. Policy 1

In this case, 5 year manpower data of the oil service company in Port Harcourt, Rivers State, Nigeria was obtained as shown in Table 1. The manpower policy that resulted to Table 1 involves irregular recruitment, strict sack policy and pull-system of promotion.

Table 1: Company's Manpower Statistics

S/N	ITEM DESCRIPTION	NUMBER
1	Retirement	692
2	Wastages	1,903
3	Active Staff	35, 758
4	Recruitment	1,840

Markov chain analyses was used to determine the proportion of recruited staff that go on retirement and the proportion that are wasted as shown in the following analysis.

<u>Solution</u>

The states are: 1(Retirement), 2 (Wastages), 3 (Active Staff), 4 (Recruitment)

Consider Table 1 above, since states 1 and 2 are absorbing states, the proportion 'P' of active staff (state 3) that either move to states 1, 2, 3 or 4 is determined thus:

Let T1 = Wastages + Retirement + Active Staff
T1 = 38,353
Wastages:

$$\text{P (W)} = \frac{Wastages}{T1} = \frac{1903}{38,353} = 0.05$$

Retirement:

$$\text{P (R)} = \frac{Retirement}{T1} = \frac{692}{38,353} = 0.02$$

Active Staff:

$$\text{P (S)} = \frac{Staff}{T1} = \frac{35,758}{38,353} = 0.93$$

The proportion 'P' of recruited staff (state 4) that either move to states 1, 2, 3 or 4 is determined thus:

Let T2 = Wastages + Active Staff + Recruitment
T2 = 39,501

Wastages:

$$\text{P (W)} = \frac{Wastages}{T2} = \frac{1903}{39,501} = 0.05$$

Active Staff:

$$\text{P (S)} = \frac{Staff}{T2} = \frac{35,758}{39,501} = 0.90$$

Recruitment:

$$\text{P (W)} = \frac{Recruitment}{T2} = \frac{1840}{39,501} = 0.05$$

The description of the various transition probabilities are as follows:

$$P_{11}=1 \qquad P_{12}=0 \qquad P_{13}=0 \qquad P_{14}=0$$

$$P_{21}=0 \qquad P_{22}=1 \qquad P_{23}=0 \qquad P_{24}=0$$

$$P_{31}=0.02 \quad P_{32}=0.05 \quad P_{33}=0.93$$
$$P_{34}=0$$

$$P_{41}=0 \quad P_{42}=0.05 \quad P_{43}=0.90$$
$$P_{44}=0.05$$

Table 2: The Transition Matrix

	Retirement	Wastages	Active Staff	Recruitment
Retirement	1	0	0	0
Wastages	0	1	0	0
Active Staff	0.02	0.05	0.93	0
Recruitment	0	0.05	0.90	0.05

The following transition matrix PT is developed from Table 2.

$$\text{PT} = \begin{vmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.02 & 0.05 & 0.93 & 0 \\ 0 & 0.05 & 0.90 & 0.05 \end{vmatrix}$$

$$PT = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.02 & 0.05 & 0.93 & 0 \\ 0 & 0.05 & 0.90 & 0.05 \end{vmatrix} = \begin{vmatrix} I & 0 \\ R & Q \end{vmatrix}$$

Where

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \qquad 0 = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$R = \begin{vmatrix} 0.02 & 0.05 \\ 0 & 0.05 \end{vmatrix}$$

$$Q = \begin{vmatrix} 0.93 & 0.00 \\ 0.90 & 0.05 \end{vmatrix}$$

Multiplying the transition matrix PT by itself to infinity we obtain the steady state probabilities of the transition matrix, hence:

$$PT^\infty = \begin{vmatrix} I & 0 \\ R & Q \end{vmatrix}^\infty = \begin{vmatrix} I & 0 \\ R(I + Q + Q^2 +, .. + Q^\infty) & Q^\infty \end{vmatrix} \quad (11)$$

Remembering that the sum to infinity of a geometrical progression is given by:

$$S^\infty = \frac{a}{1-r} \qquad\qquad (12)$$

Where a = first term and r = common ratio.

$$PT^\infty = \begin{vmatrix} I & 0 \\ R(I-Q)^{-1} & 0 \end{vmatrix}$$

If $(I-Q) = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$

then

$$(I-Q)^{-1} = \frac{1}{ad-bc}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = N$$

$$(I-Q) = \begin{bmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 0.93 & 0.00 \\ 0.90 & 0.05 \end{vmatrix} \end{bmatrix} = \begin{vmatrix} 0.07 & 0 \\ -0.90 & 0.95 \end{vmatrix}$$

$$|I-Q| = (0.07 \times 0.95) - 0 = 0.0665 \cong 0.07$$

$$(I-Q)^{-1} = N = \begin{vmatrix} 0.07 & 0 \\ -0.90 & 0.95 \end{vmatrix}^{-1}$$

Where N = the fundamental matrix.

$$N = \frac{\begin{vmatrix} 0.95 & 0 \\ 0.90 & 0.07 \end{vmatrix}}{0.07}$$

$$N = \begin{vmatrix} 13.57 & 0 \\ 12.86 & 1 \end{vmatrix}$$

By multiplying N by R we obtain the new probabilities that each of the states will end up with.

$$NR = \begin{vmatrix} 13.57 & 0 \\ 12.86 & 1 \end{vmatrix}\begin{vmatrix} 0.02 & 0.05 \\ 0 & 0.05 \end{vmatrix}$$

For the transition matrix PT above, at equilibrium the proportion of manpower in states 1 and 2 is shown in matrix NR below:

$$NR = \begin{vmatrix} 0.3 & 0.7 \\ 0.3 & 0.7 \end{vmatrix}$$

From the matrix above it is apparent that approximately 30% of recruited staff retire normally, while 70% are wasted. This suggests that the conditions of service in the organization are relatively poor. Hence, the manpower policy requires drastic improvement in order to reduce wastages.

In order to obtain the payoffs from the policies equation (8) was used. Hence:

$$U_f(w) = TP_L - Lw$$

For the first organization studied, $U_f(w)$ is given by:

$$U_f(w) = 500,000,000 - 200,000,000$$

$$U_f(w) = 300,000,000$$

For wastages it is assumed that $U_f(w) = 0$.

Assuming that the payoff matrix **GT** is as shown below and the elements represent the payoff or possible gain per state.

$$GT = \begin{matrix} 1 & 2 \\ |0 & 300,000,000| \end{matrix}$$

The effective payoff matrix for the game, EG_1, is given by:

$EG_1 = GT \cdot NR$

Here $NR \equiv P_0$ in equation 1.

$$EG_1 = |0 \quad 90,000,000|$$

Possible total payoff for state 1 = 0
Possible total payoff for state 2 = 90,000,000

The transition and payoff matrices P_0 and GT above represent the state of the game using strategy 1 or manpower policy 1. The matrix NR represents the long term labour/manpower reaction to the utility state $U_f(w)$.

B. Policy 2
The manpower policy of the company was changed. The new manpower policy involves constant yearly recruitment, relaxed sack policy and push-system of promotion. In this case, 5 year manpower data the oil service company is as shown in Table 3.

Table 3: Company's Manpower Statistics

S/N	ITEM DESCRIPTION	NUMBER
1	Retirement	1,903
2	Wastages	692
3	Active Staff	35, 758
4	Recruitment	1,840

Markov chain analyses was used to determine the proportion of recruited staff that go on retirement and the proportion that are wasted as shown in the following analysis.

The states are: 1(Retirement), 2 (Wastages), 3 (Active Staff), 4 (Recruitment)

Consider Table 2 above, since states 1 and 2 are absorbing states, the proportion 'P' of active staff (state 3) that either move to states 1, 2, 3 or 4 is determined thus:

Let T1 = Wastages + Retirement + Active Staff
T1 = 38,353
Wastages:

$$P(R) = \frac{Retirement}{T1} = \frac{1903}{38,353} = 0.05$$

Retirement:

$$P(W) = \frac{Wastages}{T1} = \frac{692}{38,353} = 0.02$$

Active Staff:

$$P(S) = \frac{Staff}{T1} = \frac{35,758}{38,353} = 0.93$$

The proportion 'P' of recruited staff (state 4) that either move to states 1, 2, 3 or 4 is determined thus:

Let T2 = Wastages + Active Staff + Recruitment
T2 = 38290

Wastages:

$$P(W) = \frac{Wastages}{T2} = \frac{692}{38,290} = 0.018$$

Active Staff:

$$P(S) = \frac{Staff}{T2} = \frac{35,758}{38,290} = 0.934$$

Recruitment:

$$P(W) = \frac{Recruitment}{T2} = \frac{1840}{38,290} = 0.048$$

The description of the various transition probabilities are as follows:

$$P_{11} = 1 \qquad P_{12} = 0 \qquad P_{13} = 0 \qquad P_{14} = 0$$

$$P_{21} = 0 \qquad P_{22} = 1 \qquad P_{23} = 0 \qquad P_{24} = 0$$

$$P_{31} = 0.05 \quad P_{32} = 0.02 \quad P_{33} = 0.93$$
$$P_{34} = 0$$

$$P_{41} = 0 \quad P_{42} = 0.02 \quad P_{43} = 0.93$$
$$P_{44} = 0.05$$

Table 4: The Probability Matrix

	Retirement	Wastages	Active Staff	Recruitment
Retirement	1	0	0	0
Wastages	0	1	0	0
Active Staff	0.05	0.02	0.93	0
Recruitment	0	0.02	0.93	0.05

The following transition matrix PT is developed from table 4.

$$PT = \begin{vmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.05 & 0.02 & 0.93 & 0 \\ 0 & 0.02 & 0.93 & 0.05 \end{vmatrix}$$

$$PT = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.05 & 0.02 & 0.93 & 0 \\ 0 & 0.02 & 0.93 & 0.05 \end{vmatrix} = \begin{vmatrix} I & 0 \\ R & Q \end{vmatrix}$$

Where

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \qquad 0 = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$R = \begin{vmatrix} 0.05 & 0.02 \\ 0 & 0.02 \end{vmatrix}$$

$$Q = \begin{vmatrix} 0.93 & 0.00 \\ 0.93 & 0.05 \end{vmatrix}$$

$$(I-Q) = \left[\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 0.93 & 0.00 \\ 0.93 & 0.05 \end{vmatrix} \right] = \begin{vmatrix} 0.07 & 0 \\ -0.93 & 0.95 \end{vmatrix}$$

$$|I-Q| = (0.07 \times 0.95) - 0 = 0.0665 \cong 0.07$$

$$(I-Q)^{-1} = N = \begin{vmatrix} 0.07 & 0 \\ -0.93 & 0.95 \end{vmatrix}^{-1}$$

Where N = the fundamental matrix.

$$N = \frac{\begin{vmatrix} 0.95 & 0 \\ 0.93 & 0.07 \end{vmatrix}}{0.07}$$

$$N = \begin{vmatrix} 13.57 & 0 \\ 13.29 & 1 \end{vmatrix}$$

By multiplying N by R we obtain the new probabilities that each of the states will end up with.

$$NR = \begin{vmatrix} 13.57 & 0 \\ 13.29 & 1 \end{vmatrix} \begin{vmatrix} 0.05 & 0.02 \\ 0 & 0.02 \end{vmatrix}$$

For the transition matrix PT above, at equilibrium the proportion of manpower in states 1 and 2 is shown in matrix NR below:

$$NR = \begin{vmatrix} 0.7 & 0.3 \\ 0.7 & 0.3 \end{vmatrix}$$

From the matrix above it is apparent that approximately 70% of recruited staff retire normally, while 30% are wasted. This suggests that the conditions of service in the organization have improved.

For the second organization studied, $U_f(w)$ is given by:

$$U_f(w) = 500,000,000 - 250,000,000$$

$$U_f(w) = 250,000,000$$

Assuming that the payoff matrix **GT** is as shown below and the elements represent the payoff or possible gain per state.

$$GT = \begin{matrix} 1 & 2 \\ |0 & 250,000,000| \end{matrix}$$

The effective payoff matrix for the game, EG$_2$, is given by:

$$EG_2 = GT \cdot NR$$

Here NR ≡ P in equation 1.

$$EG_2 = |0 \quad 175,000,000|$$

Possible total payoff for state 1 = 0
Possible total payoff for state 2 = 175,000,000

The transition and payoff matrices P and GT above represent the state of the game using strategy 2 or manpower policy 2.

C. Summary

Considering firms 1 and 2 as players 1 and 2, let us consider the two-person coordination game represented by GG.

$$GG = \begin{array}{cc} 1 & 2 \\ \left| \begin{array}{c|c} 90000000,90000000 & 90000000,175000000 \\ \hline 175000000,90000000 & 175000000,175000000 \end{array} \right| \end{array}$$

A look at the payoff matrix of the coordination game GG above shows that the optimum strategy for players 1 and 2 corresponds to strategy 2, hence, Nash equilibrium point corresponds to strategy 2.

As has been previously noted, in general, the equilibrium/optimal manpower policy is given by:

$$\max \ (TP_{Li} - Lw_i)P_{Ri}$$

Or

$$\max \ U_{fi}(w)P_{Ri}$$

V. RESULTS AND DISCUSSION

From the analysis above, it is obvious that the choice of manpower policies has different effects on the business enterprises studied. The second policy studied has optimal manpower policy even though the wage bill required for its implementation was higher than that of the first policy studied. This is because the proportion of recruited staff that stay till requirement is much higher in policy 2 than policy 1. The optimal policy did not correspond to the highest utility for the firm; this is in conformity with game theoretic results from Nash wage bargaining model where the optimal wage does not correspond to the highest utility for the firm [11]. Nash's wage bargaining model is built on the assumption that at equilibrium the possibility of wastage is zero, as the equilibrium wage must always be higher than any alternative wage outside the firm. But this assumption is not always true; hence, game theoretic approach to manpower policy planning could result to more efficient equilibrium wage. This is because it makes use of historical data and stochastic methodology to arrive at equilibrium/optimal manpower/wage policy. The assumption of zero wastage in Nash's wage bargaining model, though idealistic, is similar to game theoretic prescriptions of optimal manpower policy, where the optimal policy aims to reduce as much as feasible the possibility of wastage.

The consequence of proposition 1 is that a player that starts the game earlier than his opponent is more likely to win. Hence any firm that adopts a particular manpower policy first is most likely to make more profits provided all conditions remain the same for the competing firms.

Favourable manpower policies lead to higher proportion of recruited workers working in the organization till retirement. Also, favourable manpower policies lead to higher profits for enterprises because employees give in their best for the overall good of the firm. The implication of proposition 2 is that the full effect of manpower policy takes time to materialise. Nevertheless, huge/bloated wage bills could wipe off the benefits of a highly effective manpower policy.

VI. CONCLUSIONS

Manpower planning is concerned with the allocation of the right number of personnel to different tasks in order to achieve short and long term goals of an organization without violating organizational policies [2]. Sustainability of enterprises depends a lot on manpower performance. Hence, optimal manpower policy must always be sought by practitioners. The benefits of active and well motivated workforce are numerous. Hence, the well being of its workforce should be the priority of any good organization.

In this work we have been able to prescribe a guide to manpower policy planners using game theoretic modelling. The analysis done here will be very useful to game theorists, management scientists, systems scientists and operations researchers who help in manpower policy development and planning. This will result to optimal manpower policies beneficial to the firm and its workforce.

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